General Instructions: This is a close-book and close-notebook examination. You may use any calculator that does not have the feature of symbolic differentiation and integration. You must show your work to receive points unless specified otherwise. Read the specific instructions of each problem carefully.

1. (9%) Let

\[ f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0}{b_k x^k + b_{k-1} x^{k-1} + \cdots + b_0} \]

be a rational function, where \( a_0, \ldots, a_n, b_0, \ldots, b_k \) are constants and \( a_n \neq 0 \) and \( b_k \neq 0 \). Answer the following questions. (No work is required for this problem.)

(a) If \( n > k \), the horizontal asymptote of the graph of \( f \) is ________________.

(b) If \( n = k \), the horizontal asymptote of the graph of \( f \) is ________________.

(c) If \( n < k \), the horizontal asymptote of the graph of \( f \) is ________________.

2. (11%) Find the equations of the horizontal and vertical asymptotes of the function

\[ f(x) = \frac{3x}{\sqrt{x^2 + 1}}. \]

To find the horizontal asymptote(s), you must calculate \( \lim_{x \to \pm\infty} f(x) \). If \( x = c \) is a vertical asymptote, calculate \( \lim_{x \to c^-} f(x) \) and \( \lim_{x \to c^+} f(x) \).
3. (30%) Let $f(x) = \frac{x+1}{2x^2}, x \in (-\infty, 0) \cup (0, +\infty)$.

(a) Find the vertical and horizontal asymptotes, if any, of the graph of $f$.

(b) Compute $f'(x)$ and $f''(x)$. Determine the $x$-intercepts, the critical numbers and the candidates for points of inflection of $f$. Then complete the following sign chart for $f'$ and $f''$. In last row of the chart, you should use appropriate arrows to indicate whether $f$ is increasing or decreasing, concave up or concave down, and you should provide the values of $f$ at the important points (the $x$-intercepts, the critical numbers and the candidates for points of inflection).

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<th>$x$</th>
<th>$f'(x)$</th>
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3. (continued)
(c) Use the information in (a) and (b) to sketch the graph of \( f \) in the coordinate system below. Make sure that you label both \( x \) and \( y \) coordinates of the important points (the \( x \)-intercepts, critical numbers and candidates for points of inflection).

4. (10%) Let \( f(x) = 1 + \sin x \) and let \( P = \{0, \frac{\pi}{3}, \pi, \frac{3\pi}{2}, 2\pi\} \) be a partition of \([0, 2\pi]\). Find the upper sum \( U_f(P) \) and the lower sum \( L_f(P) \).
5. (20%) Find the derivatives of the following functions. Simplify your answer whenever possible.

(a) \( F(x) = \int_{20}^{x} \frac{\cos(\sin t)}{\sin(\cos t)} \, dt. \)

(b) \( F(x) = \int_{0}^{x^{4+1}} \sqrt{t-1} \, dt. \)

6. (20%) Evaluate the following integrals using the Fundamental Theorem of Calculus.

(a) \( \int_{-1}^{3} (x + 1)(x + 2) \, dx \)

(b) \( \int_{0}^{\frac{\pi}{4}} (\sin x + \sec^2 x) \, dx \)