General Instructions: This is a close-book and close-notebook examination. You may use any calculator that does not have the feature of symbolic differentiation and integration. You must show your work to receive points. Read the specific instructions of each problem carefully.

1. (30%) Find the derivatives of the following functions. Simplify your answer when possible
   (a) $f(x) = \frac{\sin x}{x}$.
   
   (b) $f(x) = \frac{\sqrt{x^2+3}}{x}$.

   (c) $g(x) = \tan^3(x) + \tan(x^3)$.
2. (10%) Let \( \cos(x + y^2) = xy \). Use implicit differentiation to find \( \frac{dy}{dx} \).

3. (12%) Sand is falling into a conical pile so that the radius of the base is always equal to one-half of its altitude. The sand is falling at a rate of 12 cubic feet per minute. How fast is the altitude of the pile increasing when the pile is 6 feet high?
4. (20%) Let $f(x) = (2x + 3)(x - 1)^3$.

(a) Find all critical numbers of $f$ and draw a sign chart for $f'$. (In order to answer the questions in (b) and (c), you should also include suitable information about $f$ in your chart.)

(b) Determine the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing.

(c) Find all local extreme (maximum and minimum) values of $f$. Also find all absolute extreme values of $f$. Indicate where these extreme values are attained.
5. (20%) City A is on a highway which runs east-west. Village B is located in the desert 80 miles east of and 30 miles north of City A. (See the figure below.) A motorist tries to drive from City A to Village B in the shortest time. His car travels 70 miles per hour on the highway and 30 miles per hour in the desert. His plan is to stay on the highway for a while and then drive through the desert to B. How far should he stay on the highway so that he can reach B in the shortest time?

![Diagram of City A, Village B, and the highway and desert paths]

6. (8%) Let \( f(x) = x^4 - 24x^2 \). Determine the intervals on which the graph of \( f \) is concave up and the intervals on which the graph of \( f \) is concave down. Find coordinates (both \( x \) and \( y \)-coordinates) of the points of inflection on the graph of \( f \).