General Instructions: This is a close-book and close-notebook examination. You may use any calculator that does not have the feature of symbolic differentiation and integration. You must show your work to receive points. Read the specific instructions of each problem carefully.

1. (20%) For each of the following functions, determine the intervals on which the graph of the function is concave up and the intervals on which the function is concave down. Also determine the points of inflections. (You should find both $x$ and $y$ coordinates of the points of inflection.)

(a) $f(x) = 3x^4 - 4x^3 + 1$.

(b) $g(x) = \sin 2x, \ x \in (0, \pi)$. 


2. (10%) Find the equations of the horizontal and vertical asymptotes of the function \( f(x) = \frac{\sqrt{x^2 + 1}}{2x} \). To find the horizontal asymptote(s), you must calculate \( \lim_{x \to +\infty} f(x) \) and \( \lim_{x \to -\infty} f(x) \). If \( x = c \) is a vertical asymptote, calculate \( \lim_{x \to c^-} f(x) \) and \( \lim_{x \to c^+} f(x) \).

3. (10%) Determine if the graph of \( f(x) = x(2x + 1)^{\frac{1}{3}} \) has a vertical tangent or a vertical cusp at the point \( (-\frac{1}{2}, 0) \).

4. (30%) Let \( f(x) = \frac{x - 1}{x^2} \), \( x \in (-\infty, 0) \cup (0, +\infty) \).

(a) Find the vertical and horizontal asymptotes, if any, of the graph of \( f \).
4. (continued)
(b) Compute $f'(x)$ and $f''(x)$. Determine the $x$-intercepts, the critical numbers and the candidates for points of inflection of $f$. Then complete the following sign chart for $f'$ and $f''$. In last row of the chart, you should use appropriate arrows to indicate whether $f$ is increasing or decreasing, concave up or concave down, and you should provide the values of $f$ at the important points (the $x$-intercepts, the critical numbers and the candidates for points of inflection).

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f'(x)$</th>
<th>$f''(x)$</th>
<th>$f(x)$</th>
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(c) Use the information in (a) and (b) to sketch the graph of $f$ in the coordinate system below. Make sure that you label both $x$ and $y$ coordinates of the important points (the $x$-intercepts, critical numbers and candidates for points of inflection).
5. (10%) Let \( f(x) = \cos x \) and let \( P = \{0, \frac{\pi}{6}, \frac{\pi}{3}, \pi, \frac{3\pi}{2}, 2\pi\} \) be a partition of \([0, 2\pi]\). Find the upper sum \( U_f(P) \) and the lower sum \( L_f(P) \).

6. (20%) Find the derivatives of the following functions.

(a) \( F(x) = \int_{3}^{x} \sqrt{1 + \sqrt{1 + td}} \, dt \).

(b) \( F(x) = \int_{0}^{3x^2 + 1} (3t^2 + 1) \, dt \).