Engineering Applications of Artificial Intelligence 129 (2024) 107578

Contents lists available at ScienceDirect

Engineering Applications of Artificial Intelligence

journal homepage: www.elsevier.com/locate/engappai

Optimal cloud service provider selection: An MADM framework on correlation-based TOPSIS with interval-valued q-rung orthopair fuzzy soft set

Rana Muhammad Zulqarnain [a,b], Harish Garg [c,*], Wen-Xiu Ma [a,d,e,f], Imran Siddique [g]

[a] School of Mathematical Sciences, Zhejiang Normal University, Jinhua, Zhejiang, 321004, China
[b] Department of Mathematics, Graphic Era Deemed to be University, Dehradun, 248002, Uttarakhand, India
[c] School of Mathematics, Thapar Institute of Engineering & Technology (Deemed University), Patiala, 147004, Punjab, India
[d] Department of Mathematics, King Abdulaziz University, Jeddah, 21589, Saudi Arabia
[e] Department of Mathematics and Statistics, University of South Florida, Tampa, FL, 33620-5700, USA
[f] School of Mathematical and Statistical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho, 2735, South Africa
[g] Department of Mathematics, University of Sargodha, Sargodha, 40100, Pakistan

ARTICLE INFO

Keywords:
Interval-valued q-rung orthopair fuzzy soft set
Correlation coefficient
Weighted correlation coefficient
TOPSIS method
MADM
Cloud service provider

ABSTRACT

The interaction between two factors significantly measures the results' exactness. But, acquiring information for statistical computation is frequent, and the data obtained could be challenging to interpret. To predict how a particular factor will fluctuate regarding another as well, the correlation coefficient (CC) is usually employed. But this approach is rarely utilized for interval-valued q-rung orthopair fuzzy soft set (IVq-ROFSS). The situation in which IVq-ROFSS grows as it is modern, along with a broad depiction of the q-rung orthopair fuzzy soft set (q-ROFSS), engaging for a more reflective and precise assessment. The current study explores the CC and weighted correlation coefficient (WCC) for IVq-ROFSS and their fundamental characteristics. This research is designed to improve the prioritization technique for order preference by similarity to the ideal solution (TOPSIS) with expanded measures. Also, to check the linearity of the intended approach, we integrated mathematical formulations of correlation constrictions. This study demonstrates that the suggested methodology is a robust multi-attribute decision-making (MADM) tool for intricate information set interpretation and prioritizing. We presented a numerical illustration demonstrating the actual application of our recommended decision-making strategy for choosing Cloud Service Providers (CSPs) in cloud service management. The approach developed in this research is superior to conventional models in maintaining the precise structure of the determined studies. Thus, the algorithm produces more reliable and consistent decisions. The influence of our studies grows within the scope of this research, as the originated algorithm can enhance the ability to analyze realities and make informed decisions in light of the information provided. Therefore, this study can significantly impact data analysis and decision-making by revealing the importance of the proposed TOPSIS approach and the vitality of perpetual development in methods for making decisions to get more reliable and precise outcomes.

1. Introduction

Cloud computing is a service delivery model in which users can access aggregated computing resources such as processing power, storage, and software applications via the Internet. The word "cloud" refers to a collection of resources available to users on demand. It changed our expectations of obtaining computing power with high versatility, availability, and minimal management effort (Varghese and Buyya, 2018). As a result, organizations can concentrate on their main areas of expertise as CSPs maintain their technology infrastructure. CSPs are vendors who contract multiple services to their clients (e.g., IaaS, PaaS, SaaS) that are frequently delivered depending on client demand on a per-user basis. Customers and CSPs have a Service Level Agreement (SLA) that regulates their connection (Agheeb and Mazinani, 2023; Almishal and Youssef, 2014). Because of CC's numerous advantages to organizations, including economies of scale, investments in this

* Corresponding author.
E-mail address: harishg58iitr@gmail.com (H. Garg).

https://doi.org/10.1016/j.engappai.2023.107578
Received 9 June 2023; Received in revised form 31 October 2023; Accepted 17 November 2023

technology are skyrocketing. As a result, the number of cloud services and CSPs providing these services has increased (Upadhyay, 2017). Large IT corporations like Google, Microsoft, and Amazon are increasingly fighting to provide their clients with reliable services that meet their expectations. This competitive environment stimulates the expansion of CC technology and inspires many IT companies to enhance their Quality of Service (QoS). All CSPs serve similar services at various costs, quality levels, and diverse features. Whereas a single supplier may be cheaper for storage spaces. It may be costly for the computation. Considering a wide range of cloud computing choices, clients face an important obstacle in determining which CSP best meets their needs. This is critical to securing future performance and adhering to laws, policies, and standards (Garg et al., 2013; Butler, 2022). On the other hand, choosing the wrong CSP may result in an inability to deliver future services, compromised data confidentiality or integrity, and noncompliance with using the cloud as data storage. Cloud computing has various potential business advantages, including reduced expenses, more mobility and collaboration, improved disaster resilience, and simpler upgrades and maintenance. However, cloud computing has some possible negatives, such as worries about security and privacy and the danger of vendor lock-in. Cloud computing is a dynamic and adaptable approach to delivering IT assistance that can benefit organizations. However, like with any new invention, it is important to assess risks and benefits before employing a cloud computing solution.

Corporations depend on cloud storage solutions to manage and save important data in the current information technology landscape. Determining the proper cloud storage provider is an important decision affecting an organization's management information strategy, operational efficacy, and overall effectiveness. Multi-attribute decision-making (MADM), an area focused on determining alternatives using multiple variables, includes this decision-making method. Selecting a cloud storage provider is a significant MADM challenge because it could have profound implications. Organizations must sort among an extensive spectrum of cloud service providers, which fluctuate in price, reliability, safety, scalability, and other important aspects. A poor decision might result in data breaches, interruptions, failures, or extra expenses. So, determining the most suitable cloud storage provider that matches an organization's particular requirements and preferences involves an intelligent approach to decision-making. This study emphasizes the importance of the MADM barrier and the difficulties and complexities involved in making this conclusion. We intend to provide decision-makers with an adequate basis for evaluating cloud storage alternatives through an innovative technique, utilizing modern statistical methods, and developing technological advances. Enhancing the precision and relevance of the decision-making process is required to make better decisions in the challenging context of cloud data management. In cloud computing, where decision-makers must weigh numerous options according to various factors, including availability, reliability, performance, security, cost, and others, MADM techniques have a lot of potential. Choosing a vendor that provides cloud services is a significant decision in cloud computing, requiring careful consideration of various providers' advantages and disadvantages concerning several factors. Decision-makers in cloud computing may gain from MADM methods by evaluating different cloud service providers using distinct based on needs considerations. Also, MADM techniques may assist in determining the best cloud service provider, deployment strategy, or pricing model. They provide an organized and systematic methodology to compare and compare different companies that offer cloud services. By integrating risk management strategies into the decision-making process, MADM methodologies may be useful in detecting possible threats or challenges with various cloud providers. The MADM considers multiple variables to tackle the risks, benefits, and consequences. MADM techniques are beneficial for selecting and evaluating cloud service providers based on specific requirements. Decision-makers can reach qualified choices that align with their objectives and needs for cloud services by applying MADM methodologies. It has been shown that the MADM appears to be a viable strategy to assess the best alternative when it comes to an absence of information or confusing information. Specific objectives and obstacles might be considered during a comprehensive evaluation. However, in cases of selecting options with undetermined intent and boundaries, fuzzy mathematical frameworks, especially fuzzy sets (FS) (Zadeh, 1965) and interval-valued FS (IVFS) (Turksen, 1986), can contend with imprecise and insecure data. Ansari et al. (2020) developed a robust precautionary engineering technique for the construction of dependable software for healthcare. Ashtiani et al. (2009) used the TOPSIS approach in the IVFS scenario to address complications with multi-criteria decision-making. To solve the shortcomings of the preceding FS and IVFS, Atanassov designed the intuitionistic fuzzy sets (IFS) (Atanassov, 1986) and interval-valued IFS (IVIFS) (Atanassov, 1999). Despite these modifications, previous IFS cannot deal with inconsistent and erroneous data, irrespective of whether a team of specialists with membership degree (MD) and non-membership degree (NMD) levels that exceed 1 could deal with it. The commonly used FS and IVFS methodologies fail to adequately tackle the complex aspect of MD and NMD in DM assessment. Rouyendegh et al. (2020) used the TOPSIS strategy with IFS to address MCDM issues in managing sustainable supply chains. Hung and Wu (2002) presented a centroid algorithm for determining the CC of IFS. Afterwards, this strategy expanded to comprise interval-valued IFS (IVIFS). The CC for IVIFS has been established by Bustince and Burillo (1995), who also offered proposals for its decomposition. For CC in IFS and IVIFS, Mitchell (2004) and Hong (1998) gave decomposition theorems. Zhang and Yu (2012) offered a TOPSIS approach for IVIFS. Jana et al. (2021) stated hybrid Dombi aggregation operators (AOs) for IFS and employed these operators to build a MADM approach.

Yager (2013) designed the Pythagorean fuzzy set (PFS) to illustrate the drawbacks of current FS methods in conducting inconsistent and obscure data. These disparities impacted the core state $\mathscr{T}+\mathscr{J}\leq 1$ was revised to $\mathscr{T}^2+\mathscr{J}^2\leq 1$. Rahman et al. (2017) advocated Einstein-weighted geometric AOs for multi-attribute group decision-making (MAGDM), whereas Wei and Lu (2018) presented power AOs for MADM in PFS. Wang and Li (2020) examined the relationships between Pythagorean fuzzy numbers and power Bonferroni mean operators. Hajiaghaei-Keshteli (Hajiaghaei-Keshteli et al., 2023) offered a TOPSIS approach for green supplier selection in the food industry. Peng and Yang (2016) enhanced the IVPFS with the required features and a DM structure. Rahman et al. (Wang, 2018) devised a DM methodology for IVPFS that uses weighted geometric AOs. Yager (2016) introduced q-rung orthopair sets with fuzzy values, modifying $\mathscr{T}^2+\mathscr{J}^2\leq 1$ to $\mathscr{T}^q+\mathscr{J}^q\leq 1$, where $q>2$. Joshi et al. (2018) constructed the interval-valued q-rung orthopair fuzzy sets (IVq-ROFS) by modifying the specifications from $(\mathscr{T}^u)^2+(\mathscr{J}^u)^2\leq 1$ to $(\mathscr{T}^u)^q+(\mathscr{J}^u)^q\leq 1$, where $q>2$. Yu et al. (Ju et al., 2019) invented the MADM strategy for solving unidentified issues using weighted average AOs. Weighted geometric AOs for IVq-ROFS were generated by Li et al. (2020a) and employed to develop the MCDM technique. However, the above methods have drawbacks in addressing ambiguity and indeterminacy in parametric chemistry. The structures mentioned above are insufficient when dealing with the parametric values of alternatives. Molodtsov (1999) developed soft sets (SS), a broad mathematical tool, to deal with and conquer that issue. Maji et al. (2001a) combined fuzzy sets (FS) and soft sets (SS) to develop fuzzy soft sets (FSS), which are eventually modified into intuitionistic fuzzy soft sets (IFSS) (Maji et al., 2001b) with important operations and features. Based on their designed measures, Das et al. (2022) presented the similarity measures for generalized IFSS. Jiang et al. (2010) enlarged the IFSS to interval-valued IFSS (IVIFSS) and confirmed its fundamental abilities. Considering the choice and score values, Ma et al. (2020) proposed a new DM methodology for IVIFSS. Khan et al. (2020) stated a MADM technique for generalized IVIFSS, incorporating basic problem-solving processes. To address MADM complications, Zulqarnain et al. (2021a) implemented the TOPSIS approach in conjunction with IVIFSS AOs. Ghosh et al. (2022) stated similariy measure based approach using IFSS.

Recently, there has been a lot of focus on soft sets' overall design and progress. Peng et al. (2015) proposed the Pythagorean fuzzy soft set (PFSS), which combines two popular models, PFS and SS, and has outstanding features to address this issue. Krisci and Simsek (Kirişci and Şimşek, 2022) established a decision-making algorithm by using PFSS features, while Wang and Khalil (2023) presented a generalized PFSS and their application to decision-making process which have been more adaptable than IFSS or FSS. The TOPSIS and VIKOR methods for analyzing linguistic PFSS data in the stock market investing forum have been expanded by Naeem et al. (2019). Riaz et al. (2020) proposed a similarity measure and built the TOPSIS methodology for m-polarity PFSS. Han et al. (2019) improved the TOPSIS approach to solving MAGDM issues with PFSS information. Zulqarnain et al., 2021b, 2022a modified Einstein-ordered operational rules for PFSS and presented Einstein-ordered weighted AOs to deal with complicated real-life situations. Zulqarnain et al. (2021c) developed the TOPSIS method for PFSS and employed it in green supply chain management. Zulqarnain et al. (2022b) established the AOs for IVPFSS and a MAGDM technique to resolve DM difficulties. Hussain et al. (2020) developed weighted average AOs under q-ROFSS, a most generalized variant of PFSS. Chinram et al. (2021) developed geometric AOs in MCDM barriers using the q-ROFSS scheme. Zulqarnain et al., 2022c, 2022d, 2022e stated the interactive and Einstein AOs for q-ROFSS. Hamid et al. (2020) proposed an MCGDM framework by modifying the TOPSIS method with a q-rung orthopair fuzzy soft topology. Yang et al. (2022) enhanced the q-ROFSS to IVq-ROFSS with fundamental operations and built AOs and interaction AOs based on their established algebraic operational principles. They added an MCDM technique relying on their developed approach and used it to assess automation enterprises. Hayat et al. (2023) introduced the generalized interval-valued q-rung orthopair fuzzy soft set. In another investigation, Zulqarnain et al., 2021a, 2022b used the CC to sort out the TOPSIS technique for IVIFSS and AOs for IVPFSS and then designed MADM and MCDM strategies to cope with DM shortcomings. To address circumstances, a decision-maker cannot deliver a precise value for the MD and NMD. In such cases, the person making the decision can only provide an interval of possible MD and NMD values. As a result, the IVq-ROFSS displays variability and imperfection in DM activities accurately. Consequently, IVq-ROFSS provides a more precise depiction of variation and inaccuracy in DM processes. Moreover, IVq-ROFSS can be used for DM issues involving interval-valued data, common in real-world fields such as finance, economics, and engineering.

1.1. Motivation

Interval-valued q-rung orthopair fuzzy soft sets become more significant in DM, especially for managing partial data and unpredictability. The advantages of both SS and IVq-ROFSS are merged in IVq-ROFSS, creating an effective method for tackling uncertainty, discrepancies, and insufficient information. Assume an ordinary e-commerce company contemplating moving its systems to the cloud for storage. The business must choose a reliable CSP to guarantee top-notch performance, security, and scalability while effectively managing costs. The absence of complete awareness of how various CSP properties will communicate causes anxiety for the firm. Its performance may impact the security capabilities of a CSP, and expenses may be impacted by extensibility. The business can employ correlation analysis methods, such as the CC, to measure correlations among attributes employing the defined standards. For example, the CC can show that enhanced safety precautions have a beneficial relationship with improved productivity, assisting the business in balancing these considerations during the DM phase. The online retailer collects inquiries from various CSPs, each offering unique security, reliability, and cost criteria. Because reporting requirements differ, these measures may not be identical. In this example, the predetermined standards are used by offering a systematic way to normalize and standardize these various data. This eliminates differences in communication structures and enables the organization to evaluate the CSPs unbiasedly on an equal footing. The company discovered through the review that particular CSPs offer scant details regarding their unique and future-ready competencies. In this case, the established criteria provide an organized strategy to use current facts and future possibilities. The organization evaluates each CSP's dedication to keeping intellectually relevant and adjusting to changing patterns in response to attributes like "Innovation and Future-Readiness." It allows the organization to compensate for data that is not full by incorporating an optimistic viewpoint into the process of DM. Significant progress has been achieved by using IVq-ROFSS to overcome these difficulties in recent years. Even though the TOPSIS approach is an important tool for DM issue-solving, the CC has not been utilized in previous studies on amalgamating SS and IVq-ROFS. The study of CC and its applications to real-world issues was motivated by developments in theory and the need to understand these concepts. Compared to interval-valued fuzzy sets, IVq-ROFSS contains both the degree of MD and NMD intervals. The IVIFSS (Jiang et al., 2010) and IVPFSS (Zulqarnain et al., 2022b) also address the intervals of MD and NMD. However, both strategies have specific limitations and regulations on selecting these intervals. However, the IVq-ROFSS approach does not include these kinds of confines. For example, it is not conceivable to classify $\mathscr{T}^{\mho} = 0.5$ and $\mathscr{J}^{\mho} = 0.6$ as $\mathscr{T}^{\mho} + \mathscr{J}^{\mho} > 1$ within the context of the IVIFSS. In the framework of IVPFSS, it is not feasible to consider the intervals $\mathscr{T}^{\mho} = 0.7$ and $\mathscr{J}^{\mho} = 0.8$ in the context being examined because $(\mathscr{T}^{\mho})^2 + (\mathscr{J}^{\mho})^2 > 1$. This drawback is caused by the prerequisite that the sum of the squares of these intervals must not exceed 1. However, the IVq-ROFSS enables an extensive choice of numbers to be allocated as MD and NMD intervals. This implies that all possible values can be specified within these ranges. The IVq-ROFSS structure has an enhanced framework compared with various existing structures, encompassing and extending its predecessors' capabilities. Therefore, the frequently employed TOPSIS strategy for IVIFSS (Zulqarnain et al., 2021a) and AOs for IVPFSS (Zulqarnain et al., 2022b) are unable to deal with situations in which the higher $(MD)^2 + (NMD)^2 > 1$. Also, the TOPSIS approach using correlation for IFSS (Das et al., 2022), PFSS (Zulqarnain et al., 2021c), and q-ROFSS (Hamid et al., 2020) cannot calculate the aggregate IVq-ROFSN or intentionally correlate with MD and NMD. Moreover, the model's outcome is confined, and the depiction bias of alternatives is not determined. The abovementioned limitations provide significant motivation to develop a more competent methodology capable of resolving various specialty choices in interval form. We suggest an amendment to the boundaries of current DM methods when addressing IVq-ROFSS. This method includes CC and WCC metrics developed for IVq-ROFSS, allowing us to rank preferences based on their resemblance to the optimal solution. The CC and WCC are helpful statistical tools that can help us better understand the interactions between factors. In the case of IVq-ROFSS, these measurements can be very useful when analyzing large amounts of data. By analyzing the correlation between dissimilar variables, we can raise perceptions of their association and make more conversant judgments based on that fact. The TOPSIS method is then applied to MADM situations using the derived correlation measures. Because it is customized to the unique challenges of IVq-ROFSS, our technique exceeds conventional TOPSIS algorithms. We exhibit our methodology's usefulness over a statistical case study and an empirical comparison to verify its authenticity and productivity. Our contribution is a novel DM methodology for IVq-ROFSS, which is more stable than frequently employed models. The following crucial research questions will be investigated to address the above intentions: How can we formulate CC and WCC measures that precisely reflect relationships in IVq-ROFSS? Can CC measures and TOPSIS be integrated to develop the most consistent and efficient MADM approach via IVq-ROFSS data? How does the proposed technique compare against existing techniques regarding exactness, sensitivity, and practicality in different DM domains? To what extent does executing the proposed approach optimize decision-makers' competence to precisely evaluate and rank alternatives, especially while confronted with unpredictability and ambiguity in IVq-ROFSS data?

1.2. Contribution

Decision-making (DM) structures seek to quantify confusing and scarce data in light of recent research in the field of DM. This is due to real-life issues, including indeterminate or vague data, causing choices to be troubling. IVq-ROFSS, which incorporates the benefits of SS and IVq-ROFS, is one notable measure to regulate these events. The above frequently utilized CC measure failed to accurately represent an adequate evaluation for alternates in a setting of DM strategies with IVq-ROFSS due to the fact it is insufficiently adjusted for validated features. The systematic strategy IVq-ROFSS is specifically helpful in addressing insecurity, conflicts, and inadequate details. This study attempts to put forward innovation CC and weighted CC (WCC) measures that integrate erroneous information provided in the setting of IVq-ROFSS to address it. The following are the main goals of this study.

- ❖ Growing a structure that makes it viable to assess the informational energies that occur in IVq-ROFSS scenarios constitutes a few of the benefits of this investigation. Examining these energies, which indicate the quantity of data an FS contains, is important to establishing effective CC and WCC measures.
- ❖ The research proposes new CC and WCC measures for IVq-ROFSS that use informational energies and correlation measures. These measures consider IVq-ROFSS's insufficient information and enable a more accurate assessment of the real value of alternatives in DM activities.
- ❖ This study intends to design an enhanced version of the TOPSIS strategy to address DM obstacles, including various variables, and enhance its effectiveness and durability by integrating settled CC and WCC. This approach delivers a more exact picture of the viable viability of alternatives by coping for contradictory data in IVq-ROFSS.
- ❖ Using a TOPSIS approach to explain MADM challenges, determining DM inattention and CSP decision-making, and making the most achievable for comparing gives a substantial amount of data concerning the expected layout of FS in DM.
- ❖ Analyze comparison to determine how effectively the suggested method relates to the business standards. This investigation will emphasize the TOPSIS strategy's benefits over other methods to proceed with the MADM framework issues and its most necessary advantages and durability.

The first section addresses the need to take the uncertain and inadequate information with it when making decisions. This section also addresses the faults of the CC measure used to address DM challenges. Section 2 outlines the most important concepts and beliefs that will regulate the growth of this research in the subsequent study. This part defines the conditions for the rest of the plan by establishing fundamentals for acknowledging the various aspects of DM obstacles and arguing for an improved, robust, specific strategy. Section 3 explains informational energy and explores how it influences CC measures for IVq-ROFSS. The most significant components of this technique are addressed in the same section, along with how it boosts the reliability and precision of the DM procedure despite unreliability and incomplete information. Section 4 promotes the accuracy and reliance of DM mechanisms within the context of IVq-ROFSS by explaining and analyzing the WCC's essential characteristics. Section 5 presents the correlation-based TOPSIS approach for dealing with MADM concerns. A numerical study takes place in section 6 to illustrate the advantages of the proposed strategy. The study determines the most suitable cloud provider for the task and demonstrates how the suggested approach can be employed to deal with practical DM issues. A comparison study takes place to validate how the offered model is feasible in section 7. This examination shows that the proposed model is superior in clarity and stability compared to current models. The outcomes are then carefully summarized, and the implications are explored in section 8. Moreover, this investigation provided a framework for additional research in the field by identifying areas for possible future studies in the same section.

2. Preliminaries

This section recalls compulsory notions such as IVFS, SS, PFSS, IVIFSS, IVPFSS, and q-ROFSS.

Definition 2.1. (Zadeh, 1965) A fuzzy set $\mathscr{A}$ in a universe of discourse U is defined as:

$$\mathscr{A} = \left\{ \left(\mathfrak{u}_i, \mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i)\right) \middle| \ \mathfrak{u}_i \in U \right\}$$

Where, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i)$ be the MD, and indicating the uncertainty or imprecision.

Definition 2.2. (Turksen, 1986) An interval-valued fuzzy set $\mathscr{A}$ in a universe of discourse U is defined as:

$$\mathscr{A} = \left\{ \left(\mathfrak{u}_i, \mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i)\right) \middle| \ \mathfrak{u}_i \in U \right\}$$

Where, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD interval, and $\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)$ is indicating the uncertainty or imprecision in the MD interval.

Definition 2.3. (Atanassov, 1999) An interval-valued intuitionistic fuzzy set $\mathscr{A}$ in a universe of discourse U is defined as:

$$\mathscr{A} = \left\{ \left(\mathfrak{u}_i, \left(\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)\right) \middle| \ \mathfrak{u}_i \in U \right\}$$

Where, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals. Also, $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \leq \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \leq 1$, such as $0 \leq \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) + \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \leq 1$.

Definition 2.4. (Peng and Yang, 2016) An interval-valued Pythagorean fuzzy set $\mathscr{A}$ in a universe of discourse U is defined as:

$$\mathscr{A} = \left\{ \left(\mathfrak{u}_i, \left(\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)\right) \middle| \ \mathfrak{u}_i \in U \right\}$$

Where, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals. Also, $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \leq \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \leq 1$, such as $0 \leq (\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^2 + (\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^2 \leq 1$.

Definition 2.5. (Joshi et al., 2018) An interval-valued q-rung orthopair fuzzy set $\mathscr{A}$ in a universe of discourse U is defined as:

$$\mathscr{A} = \left\{ \left(\mathfrak{u}_i, \left(\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)\right) \middle| \ \mathfrak{u}_i \in U \right\}$$

Where, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{J}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals. Also, $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \leq \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \leq 1$, such as $0 \leq (\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^q + (\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^q \leq 1$, where $q > 2$.

Definition 2.6. (Molodtsov, 1999) Let U and ς be the universe of discourse and set of attributes, $\mathscr{P}(U)$ be the power set of U and $\mathscr{A} \subseteq \varsigma$. Then, a pair $(\mathscr{F}, \mathscr{A})$ is called a soft set over U, where $\mathscr{F}$ is a mapping:

$$\mathscr{F} : \mathscr{A} \rightarrow \mathscr{P}(U)$$

Also, it can be defined as follows:

$(\mathscr{F},\mathscr{A}) = \{\mathscr{F}(\varsigma) \in \mathscr{P}(U) : \varsigma \in ʕ, \mathscr{F}(\varsigma) = \varnothing \ if \ \varsigma \notin \mathscr{A}\}$

Definition 2.7. (Jiang et al., 2010) Let U and $ʕ$ be the universe of discourse and set of attributes, $\mathscr{P}(U)$ be the power set of U and $\mathscr{A} \subseteq ʕ$. Then, a pair $(\mathscr{F},\mathscr{A})$ is called an IVIFSS over U.

$$(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, (\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)))|\ \mathfrak{u}_i \in U\}$$

Where $\mathscr{F} : \mathscr{A} \to \mathscr{P}(U)$ is a mapping between a set of attributes and a power set of U. Also, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals, such as $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \le \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \le 1$, and $0 \le \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) + \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \le 1$.

If $\mathscr{T}^{\mho}_{ij} + \mathscr{I}^{\mho}_{ij} > 1$, the IVIFSS (Jiang et al., 2010) cannot accommodate the case. An interval-valued Pythagorean fuzzy soft set should be presented to assist such instances, a combination of the IVPFS and SS and the most generalized extension of IVIFSS and PFSS. Furthermore, it is a more advanced variant of the interval-valued Pythagorean fuzzy set.

Definition 2.8. (Zulqarnain et al., 2022b) Let U and $ʕ$ be the universe of discourse and set of attributes, $\mathscr{P}(U)$ be the power set of U and $\mathscr{A} \subseteq ʕ$. Then, a pair $(\mathscr{F},\mathscr{A})$ is called an IVPFSS over U.

$$(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, (\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)))|\ \mathfrak{u}_i \in U\}$$

Where $\mathscr{F} : \mathscr{A} \to \mathscr{P}(U)$ is a mapping between a set of attributes and a power set of U. Also, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals, such as $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \le \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \le 1$, and $0 \le (\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^2 + (\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^2 \le 1$.

If $(\mathscr{T}^{\mho}_{ij})^q + (\mathscr{I}^{\mho}_{ij})^q > 1$, for $q > 2$, the IVIFSS (Jiang et al., 2010) and IVPFSS (Zulqarnain et al., 2022b) are incapable of accommodating the case. An interval-valued q-rung orthopair fuzzy soft set was proposed by (Yang et al., 2022) to assist such instances, a combination of the IVq-ROFS (Joshi et al., 2018) and SS (Molodtsov, 1999) and the most generalized extension of IVIFSS and the IVPFSS. Furthermore, it is a more advanced variant of the interval-valued q-rung orthopair fuzzy set. We may more nuancedly characterize hesitancy and fuzziness with IVq-ROFSS, facilitating a more precise and comprehensive evaluation of complicated data sets. This shows that IVq-ROFSS is a valuable tool for DM and statistical analysis, and implementing it into our technique can improve the accuracy and reliability of the outcomes.

Definition 2.9. (Yang et al., 2022) Let U and $ʕ$ be the universe of discourse and set of attributes, $\mathscr{P}(U)$ be the power set of U and $\mathscr{A} \subseteq ʕ$. Then, a pair $(\mathscr{F},\mathscr{A})$ is called an IVq-ROFSS over U.

$$(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, (\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)))|\ \mathfrak{u}_i \in U\}$$

Where $\mathscr{F} : \mathscr{A} \to \mathscr{P}(U)$ is a mapping between a set of attributes and a power set of U. Also, $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ and $\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$ be the MD and NMD intervals, such as $[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$ and $[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)] \subseteq [0,1]$, $0 \le \mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) \le 1$, and $0 \le (\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^q + (\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i))^q \le 1$, where $q \ge 3$.

Definition 2.10. (Yang et al., 2022) Let $\mathscr{F}_{\varsigma} = ([\mathscr{T}^{\ell}, \mathscr{T}^{\mho}], [\mathscr{I}^{\ell}, \mathscr{I}^{\mho}])$, $\mathscr{F}_{\varsigma_{11}} = ([\mathscr{T}^{\ell}_{\varsigma_{11}}, \mathscr{T}^{\mho}_{\varsigma_{11}}], [\mathscr{I}^{\ell}_{\varsigma_{11}}, \mathscr{I}^{\mho}_{\varsigma_{11}}])$, and $\mathscr{F}_{\varsigma_{12}} = ([\mathscr{T}^{\ell}_{\varsigma_{12}}, \mathscr{T}^{\mho}_{\varsigma_{12}}], [\mathscr{I}^{\ell}_{\varsigma_{12}}, \mathscr{I}^{\mho}_{\varsigma_{12}}])$ be the IVq-ROFSNs and $\beta > 0$. Then, the algebraic operational laws for IVq-ROFSNs are given:

1) $\mathscr{F}_{\varsigma_{11}} \oplus \mathscr{F}_{\varsigma_{12}} = \left(\begin{bmatrix} \sqrt[q]{(\mathscr{T}^{\ell}_{\varsigma_{11}})^q + (\mathscr{T}^{\ell}_{\varsigma_{12}})^q - (\mathscr{T}^{\ell}_{\varsigma_{11}})^q (\mathscr{T}^{\ell}_{\varsigma_{12}})^q}, \\ \sqrt[q]{(\mathscr{T}^{\mho}_{\varsigma_{11}})^q + (\mathscr{T}^{\mho}_{\varsigma_{12}})^q - (\mathscr{T}^{\mho}_{\varsigma_{11}})^q (\mathscr{T}^{\mho}_{\varsigma_{12}})^q} \end{bmatrix}, [\mathscr{I}^{\ell}_{\varsigma_{11}} \mathscr{I}^{\ell}_{\varsigma_{12}}, \mathscr{I}^{\mho}_{\varsigma_{11}} \mathscr{I}^{\mho}_{\varsigma_{12}}]\right).$

2) $\mathscr{F}_{\varsigma_{11}} \otimes \mathscr{F}_{\varsigma_{12}} = \left([\mathscr{T}^{\ell}_{\varsigma_{11}} \mathscr{T}^{\ell}_{\varsigma_{12}}, \mathscr{T}^{\mho}_{\varsigma_{11}} \mathscr{T}^{\mho}_{\varsigma_{12}}], \begin{bmatrix} \sqrt[q]{(\mathscr{I}^{\ell}_{\varsigma_{11}})^q + (\mathscr{I}^{\ell}_{\varsigma_{12}})^q - (\mathscr{I}^{\ell}_{\varsigma_{11}})^q (\mathscr{I}^{\ell}_{\varsigma_{12}})^q}, \\ \sqrt[q]{(\mathscr{I}^{\mho}_{\varsigma_{11}})^q + (\mathscr{I}^{\mho}_{\varsigma_{12}})^q - (\mathscr{I}^{\mho}_{\varsigma_{11}})^q (\mathscr{I}^{\mho}_{\varsigma_{12}})^q} \end{bmatrix}\right).$

3) $\beta \mathscr{F}_{\varsigma} = \left(\begin{bmatrix} \sqrt[q]{1 - (1 - (\mathscr{T}^{\ell})^q)^{\beta}}, \\ \sqrt[q]{1 - (1 - (\mathscr{T}^{\mho})^q)^{\beta}} \end{bmatrix}, [(\mathscr{I}^{\ell})^{\beta}, (\mathscr{I}^{\mho})^{\beta}]\right) = (\sqrt[q]{1 - (1 - [\mathscr{T}^{\ell}, \mathscr{T}^{\mho}]^q)^{\beta}}, [(\mathscr{I}^{\ell})^{\beta}, (\mathscr{I}^{\mho})^{\beta}]).$

4) $\mathscr{F}^{\beta}_{\varsigma} = \left([(\mathscr{T}^{\ell})^{\beta}, (\mathscr{T}^{\mho})^{\beta}], \begin{bmatrix} \sqrt[q]{1 - (1 - (\mathscr{I}^{\ell})^q)^{\beta}}, \\ \sqrt[q]{1 - (1 - (\mathscr{I}^{\mho})^q)^{\beta}} \end{bmatrix}\right) = ([(\mathscr{T}^{\ell})^{\beta}, (\mathscr{T}^{\mho})^{\beta}], \sqrt[q]{1 - (1 - [\mathscr{I}^{\ell}, \mathscr{I}^{\mho}]^q)^{\beta}}).$

Based on the stated algebraic operational laws, Yang et al. (2022) introduced the AOs for IVq-ROFSS, which are given as follows:

$$\text{IVq-ROFSWA}(\mathscr{F}_{\varsigma_{11}}, \mathscr{F}_{\varsigma_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\varsigma_{nm}}) = \left(\begin{array}{c} \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - [\mathscr{T}^{\ell}_{ij}, \mathscr{T}^{\mho}_{ij}]^q\right)^{\Omega_i}\right)^{\gamma_j}}, \\ \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left([\mathscr{I}^{\ell}_{ij}, \mathscr{I}^{\mho}_{ij}]\right)^{\Omega_i}\right)^{\gamma_j} \end{array}\right) \quad (1)$$

$$\text{IVq-ROFSWG}(\mathscr{F}_{\varsigma_{11}}, \mathscr{F}_{\varsigma_{12}}, \ldots\ldots\ldots, \mathscr{F}_{\varsigma_{nm}}) = \left(\begin{array}{c} \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left([\mathscr{T}^{\ell}_{ij}, \mathscr{T}^{\mho}_{ij}]\right)^{\Omega_i}\right)^{\gamma_j}, \\ \sqrt[q]{1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - [\mathscr{I}^{\ell}_{ij}, \mathscr{I}^{\mho}_{ij}]^q\right)^{\Omega_i}\right)^{\gamma_j}} \end{array}\right) \quad (2)$$

3. Correlation coefficient for interval valued q-rung orthopair fuzzy soft set

In this section, we will present the CC for IVq-ROFSS. We will explore the specific properties of this measure and demonstrate how they can be applied to IVq-ROFSS data.

3.1. Informational energies

To determine the CC, we include the informational energies for assessing the quantity of facts mutually debated between two data sets. The degree of association, cohesion, or correlation between these sets can be clarified by defining their informational energies, which facilitates the analysis of how they interact. The relevance of informational energy depends upon its capability to precisely characterize the rate of information connect or reliance within two sets. The classification of associations among data sets is imperative in multiple fields, mostly in decision-making and pattern recognition. Also, this research extends to the study's conceptual framework by accurately indicating and analyzing these energies and presenting a systematic and empirical strategy for interpreting data propagation between various sets. The preceding details provide a basis for potential studies into innovative data extraction, fusion, and modeling approaches. The suggested structure offers a robust mathematical foundation for quantifying informational energies inside sets, enhancing our ability to comprehend and leverage data interactions. This framework also presents opportunities for more accurate data mining, data modeling, and knowledge extraction techniques. It can be defined as follows.

Definition 3.1. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,(\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,(\mathscr{T}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}_{\mathscr{B}_j}(\mathfrak{u}_i)))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS over a set of attributes $\varsigma=\{\varsigma_1,\varsigma_2,\varsigma_3,\ldots,\varsigma_m\}$, where $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i)=[\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$, $\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)=[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$, $\mathscr{T}_{\mathscr{B}_j}(\mathfrak{u}_i)=[\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]$, $\mathscr{I}_{\mathscr{B}_j}(\mathfrak{u}_i)=[\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]$. Then the informational energies of $(\mathscr{F},\mathscr{A})$ and $(\mathscr{G},\mathscr{B})$ are defined as:

$$\mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A})=\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\right) \tag{3}$$

$$\mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B})=\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\right). \tag{4}$$

Definition 3.2. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS. Then, the correlation of $(\mathscr{F},\mathscr{A})$ and $(\mathscr{G},\mathscr{B})$ is defined as:

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\sum_{j=1}^{m}\times\sum_{i=1}^{n}\begin{pmatrix}\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\\\left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{pmatrix}. \tag{5}$$

Proposition 3.1. *Let* $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ *and* $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ *be two IVq-ROFSS. Then*

1) $\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{F},\mathscr{A}))=(\mathscr{F},\mathscr{A})$.
2) $\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\mathscr{C}_{IVq-ROFSS}((\mathscr{G},\mathscr{B}),(\mathscr{F},\mathscr{A}))$.

proof: The proof is simple and easy to follow.

Definition 3.3. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS. Then, CC is defined as:

$$\mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\frac{\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))}{\sqrt{\mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A})}\sqrt{\mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B})}}$$

$$=\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\\+\left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{pmatrix}.}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\\+\left(\left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{pmatrix}}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\begin{pmatrix}\left(\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\\+\left(\left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{pmatrix}}} \tag{6}$$

Theorem 3.1. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS, then the following properties are held:

1. $0\leq\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\leq 1$.
2. $\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{G},\mathscr{B}),(\mathscr{F},\mathscr{A}))$.
3. If $(\mathscr{F},\mathscr{A})=(\mathscr{G},\mathscr{B})$, i.e., $\forall$ i, j, $\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, and $\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, then $\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=1$.

Proof 1. $\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\geq 0$ is obvious. Now, we will demonstrate $\mathbb{C}_{\text{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\leq 1$. Using Eq. (5).

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=$$

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\left\{\begin{array}{c}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q\right)+\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q\right)+\\ \vdots\\ +\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q\right)\end{array}\right\}$$

$$+\left\{\begin{array}{c}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q\right)+\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q\right)+\\ \vdots\\ +\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q\right)\end{array}\right\}$$

$$+\ \vdots\ +$$

$$\left\{\begin{array}{c}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q\right)+\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q\right)+\\ \vdots\\ +\\ \left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q\right)\end{array}\right\}$$

$$\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\right)$$

$$=\sum_{j=1}^{m}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q\right)$$

$$+\sum_{j=1}^{m}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q\right)$$

$$+\ \vdots\ +$$

$$\sum_{j=1}^{m}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q\right)$$

$$= \sum_{j=1}^{m}\left(\begin{array}{c}\left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q\right) + \left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q\right) \\ + \cdots\cdots + \left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q\right)\end{array}\right)$$
$$+ \sum_{j=1}^{m}\left(\begin{array}{c}\left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q\right) + \left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q\right) \\ + \cdots\cdots + \left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q\right)\end{array}\right)$$

Using Cauchy-Schwarz inequality

proof 2. The proof is simple and easy to follow.

Proof 3. It is known that

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq \sum_{j=1}^{m}\left\{\begin{array}{c}\left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^{2q}\right) + \left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^{2q}\right) + \ldots + \left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^{2q}\right) + \\ \left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^{2q}\right) + \left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^{2q}\right) + \ldots + \left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^{2q}\right)\end{array}\right\}$$
$$\times \sum_{j=1}^{m}\left\{\begin{array}{c}\left(\left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^{2q}\right) + \left(\left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^{2q}\right) + \ldots + \left(\left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^{2q}\right) + \\ \left(\left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^{2q}\right) + \left(\left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^{2q}\right) + \ldots + \left(\left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^{2q}\right)\end{array}\right\}$$

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq \sum_{j=1}^{m}\sum_{i=1}^{n}\left\{\left(\left(\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\right) + \left(\left(\left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^4\right)^q\right)\right\} \times \sum_{j=1}^{m}$$
$$\times \sum_{i=1}^{n}\left\{\left(\left(\left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\right) + \left(\left(\left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^4\right)^q\right)\right\}$$

$$\mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q \\ + \left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{array}\right).}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right)}}$$

$$\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq \mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A}) \times \mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B}).$$

Using Definition 3.3, we get

$\mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \leq 1.$

So, it is verified that $0 \leq \mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \leq 1$.

As.

$\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}'_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}'_{\mathscr{B}_j}(\mathfrak{u}_i)$, and $\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$. So,

$$\mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right).}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}'_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right)}}$$

$\mathbb{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))= 1.$

Definition 3.4. Let $(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ and $(\mathscr{G},\mathscr{B}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ be two IVq-ROFSS. Then, CC is also defined as:

$$\mathbb{C}^1_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))}{max\{\mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A}), \mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B})\}}$$

$$\mathbb{C}^1_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q \\ + \left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q * \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{array}\right)}{max\left\{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right), \sum_{j=1}^{m}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q} \\ + \left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^{2q}\end{array}\right)\right\}} \tag{7}$$

Theorem 3.2. Let $(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ and $(\mathscr{G},\mathscr{B}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ be two IVq-ROFSS, then the following properties are held:

1) $0 \le \mathbb{C}^1_{\mathrm{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \le 1.$
2) $\mathbb{C}^1_{\mathrm{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \mathbb{C}^1_{\mathrm{IVq-ROFSS}}((\mathscr{G},\mathscr{B}),(\mathscr{F},\mathscr{A})).$
3) If $\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)$, and $\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$ ∀ i,j. Then $\mathbb{C}^1_{\mathrm{IVq-ROFSS}}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = 1.$

proof. The proof for case 2 is simple and can be easily demonstrated. The proof for case 3 follows a similar pattern as shown in Theorem 3.1 for case 3. Also, $\mathbb{C}^1_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \ge 0$ is trivial in case 1. Here, we only need to prove $\mathbb{C}^1_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \le 1$. Since, $\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \le \mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A}) \times \mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B})$. So, $\mathscr{C}_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \le max\{\mathscr{E}_{IVq-ROFSS}(\mathscr{F},\mathscr{A}), \mathscr{E}_{IVq-ROFSS}(\mathscr{G},\mathscr{B})\}$. Hence, $\mathbb{C}^1_{IVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \le 1$.

4. Weighted correlation coefficient for interval valued q-rung orthopair fuzzy soft set

In today's world, it is crucial to consider the significance of IVq-ROFSS in practical decision-making. The results may vary depending on policymakers' weights to different alternatives during the planning process. Hence, determining the weights of decision-makers and alternatives is crucial before drawing any conclusions. To address this, we introduce the WCC for IVq-ROFSS. Let $\Omega = \{\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_n\}^T$ and $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_m\}^T$ represent the weights for experts and parameters, where $\Omega_i > 0$, $\sum_{i=1}^{m}\Omega_i = 1$ and $\gamma_j > 0$, $\sum_{j=1}^{m}\gamma_j = 1$.

Definition 4.1. Let $(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, (\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i)))| \ \mathfrak{u}_i \in U\}$ and $(\mathscr{G},\mathscr{B}) = \{(\mathfrak{u}_i, (\mathscr{T}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{I}_{\mathscr{B}_j}(\mathfrak{u}_i)))| \ \mathfrak{u}_i \in U\}$ be two IVq-ROFSS over a set of attributes $\varsigma = \{\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_m\}$, where $\mathscr{T}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$, $\mathscr{I}_{\mathscr{A}_j}(\mathfrak{u}_i) = [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]$, $\mathscr{T}_{\mathscr{B}_j}(\mathfrak{u}_i) = [\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]$, $\mathscr{I}_{\mathscr{B}_j}(\mathfrak{u}_i) = [\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]$. Then the weighted informational energies of $(\mathscr{F},\mathscr{A})$ and $(\mathscr{G},\mathscr{B})$ are defined as:

$$\mathscr{E}_{WIVq-ROFSS}(\mathscr{F},\mathscr{A}) = \sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\left(\begin{array}{c}\left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q \\ + \left(\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{array}\right)\right) \tag{8}$$

$$\mathscr{E}_{WIVq-ROFSS}(\mathscr{G},\mathscr{B}) = \sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\left(\begin{array}{c}\left(\left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q + \left(\left(\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q \\ + \left(\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{array}\right)\right) \tag{9}$$

Definition 4.2. Let $(\mathscr{F},\mathscr{A}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ and $(\mathscr{G},\mathscr{B}) = \{(\mathfrak{u}_i, ([\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)], [\mathscr{I}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_i), \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))| \ \mathfrak{u}_i \in U\}$ be two IVq-ROFSS. Then, the WCC between $(\mathscr{F},\mathscr{A})$ and $(\mathscr{G},\mathscr{B})$ is defined as:

$$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\begin{pmatrix}\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\\ \left(\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{pmatrix}\right). \tag{10}$$

Proposition 4.1. *Let* $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ *and* $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ *be two IVq-ROFSS. Then*

1) $\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{F},\mathscr{A}))=(\mathscr{F},\mathscr{A})$.
2) $\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\mathscr{C}_{WIVq-ROFSS}((\mathscr{G},\mathscr{B}),(\mathscr{F},\mathscr{A}))$.

proof: The proof is simple and straightforward.

Definition 4.3. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS. Then, the WCC is defined as:

$$\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\frac{\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))}{\sqrt{\mathscr{E}_{WIVq-ROFSS}(\mathscr{F},\mathscr{A})}\sqrt{\mathscr{E}_{WIVq-ROFSS}(\mathscr{G},\mathscr{B})}}$$

$$=\frac{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\begin{pmatrix}\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\\ \left(\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{pmatrix}\right).}{\sqrt{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\begin{pmatrix}\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\\ +\left(\left(\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{pmatrix}\right)}\sqrt{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\begin{pmatrix}\left(\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\\ +\left(\left(\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q+\left(\left(\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^2\right)^q\end{pmatrix}\right)}} \tag{11}$$

$$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\begin{pmatrix}\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\\ \left(\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q+\left(\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^q*\left(\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)\right)^q\end{pmatrix}\right)$$

Where $\Omega=\{\Omega_1,\Omega_2,\Omega_3,\ldots,\Omega_m\}^T$ and $\gamma=\{\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_m\}^T$ be the weight vectors for experts and attributes, respectively, such as $\Omega_i>0$, $\sum_{i=1}^{m}\Omega_i=1$ and $\gamma_j>0$, $\sum_{j=1}^{m}\gamma_j=1$.

Theorem 4.1. Let $(\mathscr{F},\mathscr{A})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ and $(\mathscr{G},\mathscr{B})=\{(\mathfrak{u}_i,([\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)],[\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i),\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)]))|\ \mathfrak{u}_i\in U\}$ be two IVq-ROFSS. If $\Omega=\{\Omega_1,\Omega_2,\Omega_3,\ldots,\Omega_m\}^T$ and $\gamma=\{\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_m\}^T$ be the weight vectors for experts and attributes, respectively, such as $\Omega_i>0$, $\sum_{i=1}^{m}\Omega_i=1$ and $\gamma_j>0$, $\sum_{j=1}^{m}\gamma_j=1$. Then, WCC satisfied the following properties:

1. $0\le\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\le1$.
2. $\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=\mathbb{C}_{WIVq-ROFSS}((\mathscr{G},\mathscr{B}),(\mathscr{F},\mathscr{A}))$.
3. If $(\mathscr{F},\mathscr{A})=(\mathscr{G},\mathscr{B})$, i.e., $\forall$ i, j, $\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{J}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{J}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, and $\mathscr{J}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)=\mathscr{J}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, then $\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))=1$.

Proof 1 $\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\ge0$ is trivial. Now, we will prove $\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))\le1$.

$$= \sum_{j=1}^{m} \gamma_j \left(\Omega_1 \left(\begin{array}{c} \left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q \\ + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_1)\right)^q \end{array} \right) \right)$$

$$+ \sum_{j=1}^{m} \gamma_j \left(\Omega_2 \left(\begin{array}{c} \left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q \\ + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_2)\right)^q \end{array} \right) \right)$$

$+$

$+$

$\vdots$

$+$

$$\sum_{j=1}^{m} \gamma_j \left(\Omega_n \left(\begin{array}{c} \left(\mathscr{T}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q \\ + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_j}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_j}(\mathfrak{u}_n)\right)^q \end{array} \right) \right)$$

$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),$

$$(\mathscr{G},\mathscr{B})) = \left\{ \begin{array}{l} \gamma_1 \left(\Omega_1 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q \right) \right) + \\ \gamma_2 \left(\Omega_1 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q \right) \right) + \vdots + \gamma_m \left(\Omega_1 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q * \right.\right. \end{array} \right.$$

$$+ \left\{ \begin{array}{c} \gamma_1 \left(\Omega_2 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q \right) \right) + \\ \gamma_2 \left(\Omega_2 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q \right) \right) + \\ \vdots \\ + \\ \gamma_m \left(\Omega_2 \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q \right) \right) \end{array} \right\}$$

$$\left\{ \begin{array}{c} \gamma_1 \left(\Omega_n \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q \right) \right) + \\ \gamma_2 \left(\Omega_n \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q \right) \right) + \\ \vdots \\ + \\ \gamma_m \left(\Omega_n \left(\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \left(\mathscr{T}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \left(\mathscr{J}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \left(\mathscr{J}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q \right) \right) \end{array} \right\}$$

$$\vdots$$

$$+$$

$$= \left\{ \begin{array}{c} \gamma_1 \left(\begin{array}{c} \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \\ \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_1)\right)^q \end{array} \right) + \\ \gamma_2 \left(\begin{array}{c} \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \\ \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_1)\right)^q \end{array} \right) + \\ \vdots \\ + \\ \gamma_m \left(\begin{array}{c} \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q + \\ \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q + \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^q * \sqrt{\Omega_1}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_1)\right)^q \end{array} \right) \end{array} \right\}$$

$$+$$

$$\vdots$$

$$+$$

$$\left\{ \begin{array}{c} \gamma_1 \left(\begin{array}{c} \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \\ \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q \end{array} \right) + \\ \gamma_2 \left(\begin{array}{c} \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \\ \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q \end{array} \right) + \\ \vdots \\ + \\ \gamma_m \left(\begin{array}{c} \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{T}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \\ \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\ell}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q + \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q * \sqrt{\Omega_n}\left(\mathscr{I}^{\upsilon}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q \end{array} \right) \end{array} \right\}$$

$$+\left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\\ \sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_2)\right)^q\end{array}\right)+\\ \left(\begin{array}{c}\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\\ \sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_2)\right)^q\end{array}\right)+\\ \vdots\\ +\\ \left(\begin{array}{c}\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{T}'_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\\ \sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{I}'_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q+\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_2}\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_2)\right)^q\end{array}\right)\end{array}\right\}$$

$$+$$

$$\vdots$$

$$+$$

$$\left\{\begin{array}{c}\left(\begin{array}{c}\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\\ \sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_1}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\mathfrak{u}_n)\right)^q\end{array}\right)+\\ \left(\begin{array}{c}\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\\ \sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_2}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\mathfrak{u}_n)\right)^q\end{array}\right)+\\ \vdots\\ +\\ \left(\begin{array}{c}\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{T}'_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\\ \sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{I}'_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q+\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^q*\sqrt{\gamma_m}\sqrt{\Omega_n}\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\mathfrak{u}_n)\right)^q\end{array}\right)\end{array}\right\}$$

Using Cauchy-Schwarz inequality

$$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq$$

$$\left\{\begin{array}{c}
\left(\begin{array}{c}
\gamma_1\Omega_1\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_1)\right)^{2q}\right\}+\\
\gamma_2\Omega_1\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_1)\right)^{2q}\right\}+\\
\vdots\\
+\\
\gamma_m\Omega_1\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_1)\right)^{2q}\right\}
\end{array}\right)+\\
\left(\begin{array}{c}
\gamma_1\Omega_2\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_2)\right)^{2q}\right\}+\\
\gamma_2\Omega_2\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_2)\right)^{2q}\right\}+\\
\vdots\\
+\\
\gamma_m\Omega_2\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_2)\right)^{2q}\right\}
\end{array}\right)+\\
\vdots\\
+\\
\left(\begin{array}{c}
\gamma_1\Omega_n\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_1}(\mathfrak{u}_n)\right)^{2q}\right\}+\\
\gamma_2\Omega_n\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_2}(\mathfrak{u}_n)\right)^{2q}\right\}+\\
\vdots\\
+\\
\gamma_m\Omega_n\left\{\left(\mathscr{T}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{T}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\ell}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^{2q}+\left(\mathscr{I}^{\upsilon}_{\mathscr{A}_m}(\mathfrak{u}_n)\right)^{2q}\right\}
\end{array}\right)
\end{array}\right\}$$

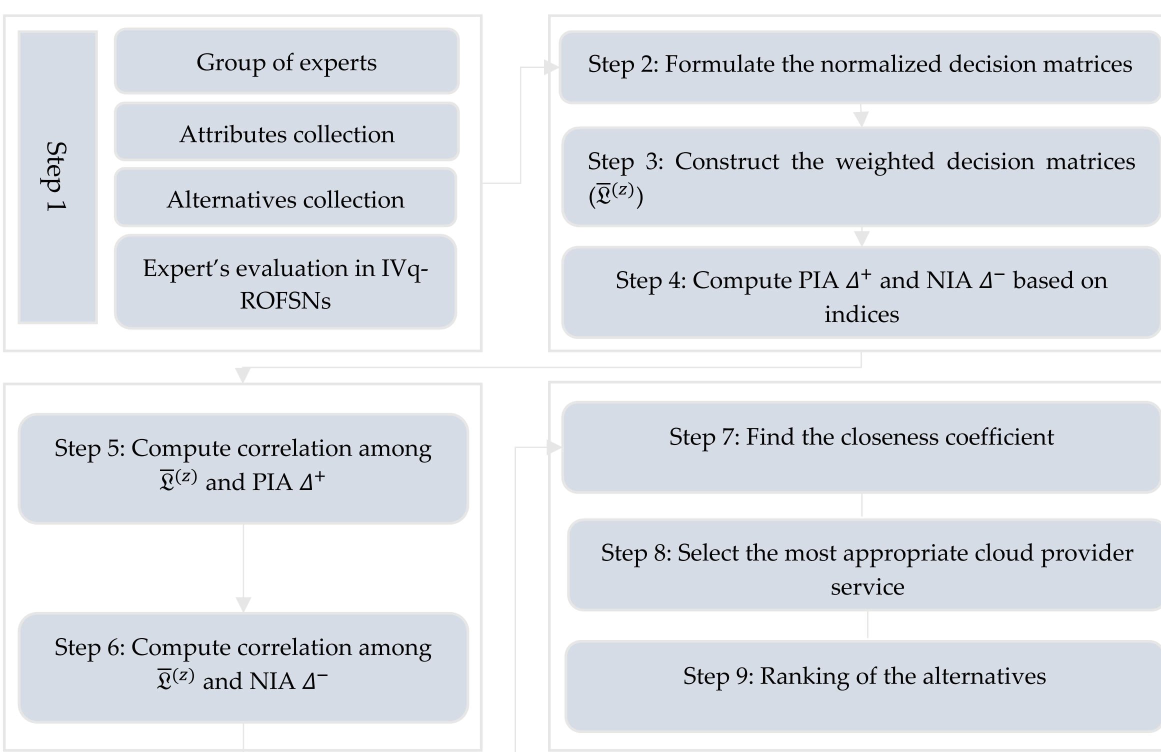

Fig. 1. Flow chart of the proposed TOPSIS model.

$$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq \mathscr{E}_{WIVq-ROFSS}(\mathscr{F},\mathscr{A}) \times \mathscr{E}_{WIVq-ROFSS}(\mathscr{G},\mathscr{B}).$$

Using Definition 4.3, we get

$$\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \leq 1.$$

$$\times \left\{ \begin{array}{c} \left(\begin{array}{c} \gamma_1\Omega_1\left\{\left(\mathscr{T}'_{\mathscr{B}_1}(\text{u}_1)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_1}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\text{u}_1)\right)^{2q}\right\}+ \\ \gamma_2\Omega_1\left\{\left(\mathscr{T}'_{\mathscr{B}_2}(\text{u}_1)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_2}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\text{u}_1)\right)^{2q}\right\}+ \\ \vdots \\ + \\ \gamma_m\Omega_1\left\{\left(\mathscr{T}'_{\mathscr{B}_m}(\text{u}_1)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_m}(\text{u}_1)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\text{u}_1)\right)^{2q}\right\} \end{array} \right)+ \\ \left(\begin{array}{c} \gamma_1\Omega_2\left\{\left(\mathscr{T}'_{\mathscr{B}_1}(\text{u}_2)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_1}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\text{u}_2)\right)^{2q}\right\}+ \\ \gamma_2\Omega_2\left\{\left(\mathscr{T}'_{\mathscr{B}_2}(\text{u}_2)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_2}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\text{u}_2)\right)^{2q}\right\}+ \\ \vdots \\ + \\ \gamma_m\Omega_2\left\{\left(\mathscr{T}'_{\mathscr{B}_m}(\text{u}_2)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_m}(\text{u}_2)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\text{u}_2)\right)^{2q}\right\} \end{array} \right)+ \\ \vdots \\ + \\ \left(\begin{array}{c} \gamma_1\Omega_n\left\{\left(\mathscr{T}'_{\mathscr{B}_1}(\text{u}_n)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_1}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_1}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_1}(\text{u}_n)\right)^{2q}\right\}+ \\ \gamma_2\Omega_n\left\{\left(\mathscr{T}'_{\mathscr{B}_2}(\text{u}_n)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_2}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_2}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_2}(\text{u}_n)\right)^{2q}\right\}+ \\ \vdots \\ + \\ \gamma_m\Omega_n\left\{\left(\mathscr{T}'_{\mathscr{B}_m}(\text{u}_n)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_m}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_m}(\text{u}_n)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_m}(\text{u}_n)\right)^{2q}\right\} \end{array} \right) \end{array} \right\}$$

$$\mathscr{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B}))^2 \leq \sum_{j=1}^{m}\gamma_m\left(\sum_{i=1}^{n}\Omega_i\left\{\left(\left(\mathscr{T}'_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}\right)+\left(\left(\mathscr{I}'_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}\right)\right\}\right)$$
$$\times \sum_{j=1}^{m}\gamma_m\left(\sum_{i=1}^{n}\Omega_i\left\{\left(\left(\mathscr{T}'_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}\right)+\left(\left(\mathscr{I}'_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}\right)\right\}\right)$$

So, it is verified that $0 \leq \mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) \leq 1$.

proof 2. The proof is simple and easy to follow.

Proof 3. It is known that

$$\mathbb{C}_{WIVq-ROFSS}((\mathscr{F},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\left(\begin{array}{c}\left(\mathscr{T}'_{\mathscr{A}_j}(\text{u}_i)\right)^{q}*\left(\mathscr{T}'_{\mathscr{B}_j}(\text{u}_i)\right)^{q}+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{q}*\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{q}+ \\ \left(\mathscr{I}'_{\mathscr{A}_j}(\text{u}_i)\right)^{q}*\left(\mathscr{I}'_{\mathscr{B}_j}(\text{u}_i)\right)^{q}+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{q}*\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{q}\end{array}\right)\right)}{\left(\begin{array}{c}\sqrt{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\left(\left(\mathscr{T}'_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\text{u}_i)\right)^{2q}\right)\right)} \\ \sqrt{\sum_{j=1}^{m}\gamma_j\left(\sum_{i=1}^{n}\Omega_i\left(\left(\mathscr{T}'_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{T}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}'_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}+\left(\mathscr{I}^{\mho}_{\mathscr{B}_j}(\text{u}_i)\right)^{2q}\right)\right)}\end{array}\right)}$$

As.
$\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{T}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$, $\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}^{\prime}_{\mathscr{B}_j}(\mathfrak{u}_i)$, and $\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i) = \mathscr{I}^{\mho}_{\mathscr{B}_j}(\mathfrak{u}_i)$. So,

$$\mathbb{C}_{WIVq-ROFSS}((\mathscr{T},\mathscr{A}),(\mathscr{G},\mathscr{B})) = \frac{\sum_{j=1}^{m}\gamma_m\left(\sum_{i=1}^{n}\Omega_i\left\{\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right) + \left(\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right)\right\}\right)}{\left(\begin{array}{c}\sqrt{\sum_{j=1}^{m}\gamma_m\left(\sum_{i=1}^{n}\Omega_i\left\{\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right) + \left(\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right)\right\}\right)} \\ \sqrt{\sum_{j=1}^{m}\gamma_m\left(\sum_{i=1}^{n}\Omega_i\left\{\left(\left(\mathscr{T}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{T}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right) + \left(\left(\mathscr{I}^{\prime}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q} + \left(\mathscr{I}^{\mho}_{\mathscr{A}_j}(\mathfrak{u}_i)\right)^{2q}\right)\right\}\right)}\end{array}\right)}$$

$\mathbb{C}_{WIVq-ROFSS}((\mathscr{T},\mathscr{A}),(\mathscr{G},\mathscr{B})) = 1.$

5. TOPSIS method on IVq-ROFSS for MADM problem based on the correlation coefficient

TOPSIS is a typical method for addressing MADM challenges. It is used to sort the priority order of feasible choices and use the most suitable option by considering complete information. The entire reliability of the outcomes may be improved in the decision-making procedure of the given rules by employing an integrated evaluation by a group of experts. The TOPSIS methodology describes the variability of real-life issues more effectively than previous IVIFSS and IVPFSS models in the interval-valued q-rung orthopair fuzzy soft environments. In this subsection, we will improve the TOPSIS approach under the principles of correlation coefficients under IVq-ROFSS Information to deliver a framework for navigating decision-making challenges. The TOPSIS method was established and employed by Hwang and Yoon (1981) to encourage assessing positive and negative ideal solutions to decision-making challenges. Using the TOPSIS strategy, we can identify the best options with the shortest and longest PIS and NIS distances. The TOPSIS approach demonstrates how correlation metrics distinguish between positive and negative ideals by selecting rankings. Researchers frequently use the TOPSIS approach to determine closeness coefficients using multiple distances, different types, and similarity measures. TOPSIS with the correlation coefficient is more effective in determining the closeness coefficient than distance and similarity measures. Since the correlation measure preserves the linear relationship throughout each factor studied, an algorithm based on the TOPSIS approach will be used to gain the most beneficial option employing the newly generated correlation measures.

5.1. Proposed TOPSIS approach

Consider a particular scenario in which we possess a set of alternatives signified by $\mathfrak{L} = \{\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3, \ldots, \mathfrak{L}_s\}$. Also, there is a team of specialists signified by $\mathfrak{H} = \{\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \ldots, \mathfrak{H}^n\}$, all of whom have unique weights vectors $\Omega = (\Omega_1, \Omega_1, \ldots, \Omega_n)^T$ and $\Omega_i > 0$, $\sum_{i=1}^{n}\Omega_i = 1$. We include a set of parameters $\varsigma = \{\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_m\}$, along with the weight of each parameter specified as $\gamma = (\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_m)^T$ such as $\gamma_j > 0$, $\sum_{j=1}^{m}\gamma_j = 1$.

In the present scenario, a team of specialists $\{\mathfrak{H}^i : i = 1, 2, \ldots, n\}$ express their viewpoints on all possibilities $\{\mathfrak{L}_z : z = 1, 2, 3, \cdots, s\}$ predicated on the stated features $\varsigma = \{\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_m\}$. The expert's recommendation for every alternate, described as IVq-ROFSNs, can be defined as $\Delta_{ij}^{(z)} = (\mathscr{T}_{ij}^{(z)}, \mathscr{I}_{ij}^{(z)})$, where $\mathscr{T}_{ij}^{(z)} = [\mathscr{T}^{\prime}_{ij}, \mathscr{T}^{\mho}_{ij}]$, $\mathscr{I}_{ij}^{(z)} = [\mathscr{I}^{\prime}_{ij}, \mathscr{I}^{\mho}_{ij}]$, and $0 \le \mathscr{T}^{\prime}_{ij}, \mathscr{T}^{\mho}_{ij}, \mathscr{I}^{\prime}_{ij}, \mathscr{I}^{\mho}_{ij} \le 1$ and $(\mathscr{T}^{\mho}_{ij})^2 + (\mathscr{I}^{\mho}_{ij})^2 \le 1$, $\forall i, j$. In essence, the above scenario capabilities a team of specialists' opinions on a set of alternatives according to particular features. Such ideas are conveyed as IVq-ROFSNs, which preserve interval values for membership and non-membership degrees. These features are important in comparing and evaluating the choices under debate. The stepwise algorithm of the proposed TOPSIS model is presented as follows:

Step 1. The development of decision matrices for $\{\mathfrak{L}_z : z = 1, 2, \ldots, s\}$ alternatives in the structure of IVq-ROFSNs under-considered attributes presented as:

$$(\mathfrak{L}_z, \varsigma)_{n\times m} = \begin{array}{c}\mathfrak{H}^1\\ \mathfrak{H}^2\\ \vdots\\ \mathfrak{H}^n\end{array}\begin{pmatrix} ([\mathscr{T}^{\prime}_{11}, \mathscr{T}^{\mho}_{11}], [\mathscr{I}^{\prime}_{11}, \mathscr{I}^{\mho}_{11}]) & ([\mathscr{T}^{\prime}_{12}, \mathscr{T}^{\mho}_{12}], [\mathscr{I}^{\prime}_{12}, \mathscr{I}^{\mho}_{12}]) & \cdots & ([\mathscr{T}^{\prime}_{1m}, \mathscr{T}^{\mho}_{1m}], [\mathscr{I}^{\prime}_{1m}, \mathscr{I}^{\mho}_{1m}]) \\ ([\mathscr{T}^{\prime}_{21}, \mathscr{T}^{\mho}_{21}], [\mathscr{I}^{\prime}_{21}, \mathscr{I}^{\mho}_{21}]) & ([\mathscr{T}^{\prime}_{22}, \mathscr{T}^{\mho}_{22}], [\mathscr{I}^{\prime}_{22}, \mathscr{I}^{\mho}_{22}]) & \cdots & ([\mathscr{T}^{\prime}_{2m}, \mathscr{T}^{\mho}_{2m}], [\mathscr{I}^{\prime}_{2m}, \mathscr{I}^{\mho}_{2m}]) \\ \vdots & \vdots & \vdots & \vdots \\ ([\mathscr{T}^{\prime}_{n1}, \mathscr{T}^{\mho}_{n1}], [\mathscr{I}^{\prime}_{n1}, \mathscr{I}^{\mho}_{n1}]) & ([\mathscr{T}^{\prime}_{n2}, \mathscr{T}^{\mho}_{n2}], [\mathscr{I}^{\prime}_{n2}, \mathscr{I}^{\mho}_{n2}]) & \cdots & ([\mathscr{T}^{\prime}_{nm}, \mathscr{T}^{\mho}_{nm}], [\mathscr{I}^{\prime}_{nm}, \mathscr{I}^{\mho}_{nm}]) \end{pmatrix}$$

Step 2. We commence examining the corresponding matrix $(\mathfrak{H}^{(z)}, \varsigma)_{n\times m}$ to get a typical interval-valued q-rung orthopair fuzzy soft decision matrix. The resulting matrix is assessed using two different types of attributes into consideration: benefit and cost attributes. The normalization process is essential if attributes possess the same type. But if both benefit and cost factors are given, the decision matrices should be normalized to ensure they are the same type. Normalization involves applying the rules to the matrices to bring them into a uniform structure. This way, we may effectively compare and analyze the decision matrices based on the stated attributes.

$$R_{ij}^{(z)} = \begin{cases} \Delta_{ij}^{c} = \left(\left[\mathscr{I}_{ij}^{\ell}, \mathscr{I}_{ij}^{\mho}\right], \left[\mathscr{T}_{ij}^{\ell}, \mathscr{T}_{ij}^{\mho}\right]\right); \text{cost type parameter} \\ \Delta_{ij} = \left(\left[\mathscr{T}_{ij}^{\ell}, \mathscr{T}_{ij}^{\mho}\right], \left[\mathscr{I}_{ij}^{\ell}, \mathscr{I}_{ij}^{\mho}\right]\right); \text{benefit type parameter} \end{cases} \tag{12}$$

Step 3. Design a weighted decision matrix for each alternate. $\overline{\mathfrak{Q}}^{(z)} = (\overline{\Delta}_{ij}^{(z)})_{n\times m}$, where

$$\overline{\mathfrak{Q}}^{(z)} = \gamma_j \Omega_i \Delta_{ij}^{(z)} = \left(\sqrt[q]{1-\left(\left(1-\left[\mathscr{T}_{ij}^{\ell}, \mathscr{T}_{ij}^{\mho}\right]^{q}\right)^{\Omega_i}\right)^{\gamma_j}}, \left(\left(\left[\mathscr{I}_{ij}^{\ell}, \mathscr{I}_{ij}^{\mho}\right]\right)^{\Omega_i}\right)^{\gamma_j}\right) = \left(\left[\overline{\mathscr{T}_{ij}^{\ell}}, \overline{\mathscr{T}_{ij}^{\mho}}\right], \left[\overline{\mathscr{I}_{ij}^{\ell}}, \overline{\mathscr{I}_{ij}^{\mho}}\right]\right) \tag{13}$$

Where Ω_i and γ_j be the weights of experts and parameters.

Step 4: Examine the indices $\hbar_{ij} = \arg max_z\{\theta_{ij}^{(z)}\}$ and $\mathfrak{g}_{ij} = \arg min_z\{\theta_{ij}^{(z)}\}$ to govern the PIA and NIA in the following manner:

$$\Delta^{+} = \left(\left[\overline{\mathscr{T}_{ij}^{+}}, \overline{\mathscr{T}_{ij}^{+}}\right], \left[\overline{\mathscr{I}_{ij}^{+}}, \overline{\mathscr{I}_{ij}^{+}}\right]\right)_{n\times m} = \left(\left[\overline{\mathscr{T}_{ij}^{\ell}}, \overline{\mathscr{T}_{ij}^{\mho}}\right]^{(\hbar_{ij})}, \left[\overline{\mathscr{I}_{ij}^{\ell}}, \overline{\mathscr{I}_{ij}^{\mho}}\right]^{(\hbar_{ij})}\right) \tag{14}$$

and

$$\Delta^{-} = \left(\left[\overline{\mathscr{T}_{ij}^{-}}, \overline{\mathscr{T}_{ij}^{-}}\right], \left[\overline{\mathscr{I}_{ij}^{-}}, \overline{\mathscr{I}_{ij}^{-}}\right]\right)_{n\times m} = \left(\left[\overline{\mathscr{T}_{ij}^{\ell}}, \overline{\mathscr{T}_{ij}^{\mho}}\right]^{(\mathfrak{g}_{ij})}, \left[\overline{\mathscr{I}_{ij}^{\ell}}, \overline{\mathscr{I}_{ij}^{\mho}}\right]^{(\mathfrak{g}_{ij})}\right) \tag{15}$$

Step 5. Compute the CC between $\overline{\mathfrak{Q}}^{(z)}$ and PIA Δ^{+} such as:

$$\kappa^{(z)} = \mathbb{C}_{IVq-ROFSS}(\overline{\mathfrak{Q}}^{(z)}, \Delta^{+}) = \frac{\mathscr{C}_{IVq-ROFSS}(\overline{\mathfrak{Q}}^{(z)}, \Delta^{+})}{\sqrt{\mathscr{E}_{IVq-ROFSS}\overline{\mathfrak{Q}}^{(z)}}\sqrt{\mathscr{E}_{IVq-ROFSS}\Delta^{+}}} = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\mathscr{T}}_{ij}^{(z)}\right)^{q} * \left(\mathscr{T}_{ij}^{+}\right)^{q} + \left(\overline{\mathscr{I}}_{ij}^{(z)}\right)^{q} * \left(\mathscr{I}_{ij}^{+}\right)^{q}\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\overline{\mathscr{T}}_{ij}^{(z)}\right)^{2}\right)^{q} + \left(\left(\overline{\mathscr{I}}_{ij}^{(z)}\right)^{2}\right)^{q}\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\mathscr{T}_{ij}^{+}\right)^{2}\right)^{q} + \left(\left(\mathscr{I}_{ij}^{+}\right)^{2}\right)^{q}\right)}} \tag{16}$$

Step 6. Compute the CC between $\overline{\mathfrak{Q}}^{(z)}$ and PIA Δ^{-} such as:

$$\tau^{(z)} = \mathbb{C}_{IVq-ROFSS}(\overline{\mathfrak{Q}}^{(z)}, \Delta^{-}) = \frac{\mathscr{C}_{IVq-ROFSS}(\overline{\mathfrak{Q}}^{(z)}, \Delta^{-})}{\sqrt{\mathscr{E}_{IVq-ROFSS}\overline{\mathfrak{Q}}^{(z)}}\sqrt{\mathscr{E}_{IVq-ROFSS}\Delta^{-}}} = \frac{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\overline{\mathscr{T}}_{ij}^{(z)}\right)^{q} * \left(\mathscr{T}_{ij}^{-}\right)^{q} + \left(\overline{\mathscr{I}}_{ij}^{(z)}\right)^{q} * \left(\mathscr{I}_{ij}^{-}\right)^{q}\right)}{\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\overline{\mathscr{T}}_{ij}^{(z)}\right)^{2}\right)^{q} + \left(\left(\overline{\mathscr{I}}_{ij}^{(z)}\right)^{2}\right)^{q}\right)}\sqrt{\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left(\left(\mathscr{T}_{ij}^{-}\right)^{2}\right)^{q} + \left(\left(\mathscr{I}_{ij}^{-}\right)^{2}\right)^{q}\right)}} \tag{17}$$

Step 7. Analyze the closeness coefficient:

$$\beth^{(z)} = \frac{\daleth(\overline{\mathfrak{Q}}^{(z)}, \Delta^{-})}{\daleth(\overline{\mathfrak{Q}}^{(z)}, \Delta^{+}) + \daleth(\overline{\mathfrak{Q}}^{(z)}, \Delta^{-})} \tag{18}$$

Where $\daleth(\overline{\mathfrak{Q}}^{(z)}, \Delta^{-}) = 1 - \kappa^{(z)}$ and $\daleth(\overline{\mathfrak{Q}}^{(z)}, \Delta^{+}) = 1 - \tau^{(z)}$.

Step 8: The closest alternative with the most significant value of the closeness coefficient is assigned.
Step 9: Evaluate the alternative categorization.

The developed TOPSIS procedure flow diagram is below (See Fig. 1).

6. Application of proposed technique for selection of cloud service provider

In this section, we demonstrate the pragmatic applicability of the proposed TOPSIS approach in decision-making by doing numerical computations.

6.1. Fundamental aspects of cloud service management

Cloud service management (CSM) is the procedure of dealing with and providing cloud services, containing Infrastructure as a Service (IaaS), Platform as a Service (PaaS), and Software as a Service (SaaS). A centralized repository of computer resources, such as servers, storage, applications, and software services, that may be quickly specified and de-allocated as needed is made available to users on demand. The core objective of CMS is to deliver software applications and infrastructure at a cost that is affordable to multiple users with minimal managerial duties. Due to the massive growth in online-based companies and services, CMS and cloud-based services have grown significantly in the past few decades. On the one hand, this field is experiencing significant technological advances, but on the contrary, experts like (Mukherjee et al., 2019; Büyüközkan et al., 2018; Youssef, 2020) concentrate on various methods of decision-making related to cloud management. The requirement for CMS has consequently increased significantly. The purpose of the cloud broker has become significant in determining the best solution to offer consumers due to the complexity of cloud services. Because there are so many services in this industry, evaluating the cloud service management issue is frequently tricky. MADM techniques can help cloud users select the best service to solve this issue. When there are several aspects to think about, MADM techniques aid in decision-making optimization, intending to choose the best option or options from a range of alternatives. MADM strategies must be applied to deliver users the most enjoyable experience possible in a challenging and dynamic environment for cloud service management. It is essential to distribute and remove computing resources in the cloud effectively. The role of cloud brokers in optimizing the supply of cloud services to multiple customers rises as the demand for cloud-based services grows. In a complicated and constantly evolving context of cloud service management, using MADM strategies can aid with improving decision-making and identifying the most suitable network to employ. IT departments or external cloud service providers can internally manage cloud services through specific software platforms. Cloud service management seeks to balance the needs of users and businesses by delivering cloud services safely, effectively, and economically. The administration of cloud services, including infrastructure, applications, and data, as well as assuring the provision of high-quality services to end users, constitute cloud service management. Among the crucial elements of cloud service management are the following.

6.1.1. Service level agreements (SLAs)

A service provider and a customer enter into a service level agreement (SLA), a contract or agreement. It represents the kind and caliber of services the supplier promises to offer the client. In the IT sector, where service providers offer their services to customers, service-level agreements are typical. They outline the specific services the supplier will give the client, including uptime, response speed, security, and support. SLAs are the foundation for evaluating the service provider's performance and ensuring the client receives the agreed-upon services. Additionally, it guarantees that the consumer will receive high-quality service from the supplier and that the service level expectations will be met. SLAs also offer both parties to the contract legal protection. Some of the fundamental components of an SLA include the following.

- ❖ Service description: The particulars of the provider's services, such as features or functionality and any drawbacks or exclusions.
- ❖ Service level objectives: These outline the service's predicted quality measures, such as uptime, customer query response times, and elimination times.
- ❖ Service availability: The time when service will remain visible to the consumers.
- ❖ Service credits: Service credits are monetary rewards provided to the client for any violations of the specified indicators.
- ❖ Problem management: The procedure the provider follows when problems or complications arise.
- ❖ Reporting and communication: Recommendations for reflecting on service level effectiveness measures and how regularly consumers consider service deliveries.
- ❖ Roles and responsibilities: Strong description of the provider and customer's roles and tasks in confirming service delivery.

SLAs ensure the provider's high-quality service delivery and confirm customer satisfaction agreements. Clear communication, thorough descriptions of services, objective performance measures, and service level advances for deviations are critical components of a good SLA to guarantee the supplier meets expectations. Service Exception management (issue administration) must be defined in the SLA to ensure transparent implementation.

6.1.2. Governance and compliance

Governance and compliance are two key ideas in business, ensuring organizations conform to legal and ethical norms to work properly.

- ❖ The regulations and procedures governance. The administration of finances, conformity to regulations, risk management, and social responsibility belong to those subjects regulated by these specifications. The purpose of governance is to assure that the corporation operates within established parameters, appropriately handles risks, and delivers user benefits.
- ❖ Whereas compliance signifies a system of regulations, standards, and norms that a company must comply with. Compliance generally involves environmental, safety, labor laws, industry-specific regulations, and other legal duties. Corporations must follow every law and regulation that applies to their activities to ensure that their operations are permitted.

Governance and compliance cooperate to ensure companies execute responsibly and ethically. Organizations that establish adequate governance mechanisms to handle and reduce threats are deemed compliant when implementing strategies to comply with ethical and constitutional requirements. Companies must take a preventive approach to governance and compliance, considering legislative and market developments that may necessitate modifications to protocols and procedures. Executing an effective governance and compliance structure is occasionally difficult since it necessitates the documentation of proper guidelines and policies and training and allocating resources to track and alleviate risks. These are required for companies to fulfill compliance with legal and ethical standards, which help with continued development and achievement. Corporations that execute efficient regulatory and compliance structures gain from the favorable impact of their operations on many stakeholders, such as staff, shareholders, and the general public.

6.1.3. Security and privacy

In the digital age, security and privacy are critical principles protecting confidential data from unintentional acquisition, use, and revelation. Protecting information from risks that could cause harm or damage to an organization or individual is called security. On the other hand, individuals' right to keep personal information private is called privacy. Given the increasing prevalence of digital technology to store and communicate confidential data, ensuring information safety and confidentiality is vital in today's society. Sensitive data, such as numbers for social security, monetary data, and health care records, should be secured from illegal access. Access control, firewalls, data encryption, and detection systems for intrusions are examples of security-related mechanisms. A reliable safety program is designed to discover, recognize, and react immediately to security problems and breaches. It also assures information is accessible and publicly available to the appropriate individuals at the proper time, preventing illegal access. Privacy, along with security, is critical in securing sensitive data. Organizations that collect and keep sensitive data must follow privacy laws and regulations to safeguard the protection of individuals' personal information. Consumers must be informed about their legal rights to privacy and take adequate steps to protect personal data. Data reduction, permission, and transparency are all examples of privacy safeguards. The reduction of information intends to reduce enterprises' contracting and retention of sensitive information to only what is required. According to consent laws, individuals must consent before collecting, using, or sharing their personal information. Consumers can learn which data has been assembled, how it is being used, and with whom it is being shared if there is transparency. Safety and confidentiality have become essential components of today's electronic interactions. Businesses must verify that their privacy and safety policies are current and meet the latest technical and legal requirements. Organizations can secure sensitive data, establish customer trust, and avoid costly information breaches that can result in reputational and financial losses by employing robust safety protocols and incorporating consumer privacy rights. During the age of technology, sensitive information is from illegal access and use. Organizations must deploy comprehensive security measures while protecting individual privacy rights to ensure their activities conform to current legal and technological standards.

6.1.4. Resource provisioning and management

The provisioning and management of resources is an integral feature of modern IT infrastructures. It entails assigning and handling capabilities such as CPU, memory, disk space, and network bandwidth to satisfy the specifications of activities and customers. Depending on the company's requirements, this can be achieved by human or automatic processes. The process of selecting, executing, and maintaining software and hardware resources required by applications is known as resource provisioning. This method ensures sufficient resources are available to meet the applications' performance and scalability requirements. Provisioning entails balancing computing, storage, and network capacity to fulfill defined performance requirements efficiently. Organizations can employ tools and processes that automate tasks, such as deploying and configuring new resources, to manage resources more effectively. Automation can assist in ensuring that new resources are available as soon as possible and reduce the time it takes to deploy new infrastructure. Cloud infrastructure systems like Azure, Amazon Web Services, and Google Cloud are some of the most frequently used. Such platforms enable businesses to offer content accessible to users' web portals or APIs. Resource management and monitoring also as programs perform smoothly. IT teams must check networks regularly to ensure adequate resources are available, identify bottlenecks, and take remedial action. They can also add new resources to the infrastructure to maintain ideal performance levels if there is a substantial need for resources. Modern computing systems rely heavily on resource provisioning and management. Organizations may manage their resources more effectively thanks to cloud-based infrastructure platforms and automation technologies. Organizations can keep their operations operating smoothly while avoiding downtime and optimizing performance by ensuring sufficient resources are available to satisfy the demands of apps and users.

6.1.5. Monitoring and reporting

Monitoring and reporting are two crucial facets of assessments, particularly in managing programs and execution. Monitoring is the process of collecting and analyzing data on an ongoing basis, whereas reporting is the process of presenting that knowledge to stakeholders. Competent monitoring and reporting are necessary to verify that initiatives fulfill their objectives and benefit the targeted population. Organizations can enhance program performance by monitoring progress over time and finding areas for development. It enables them to adapt their objectives and take corrective action as required.

❖ Monitoring entails several actions, such as data collecting, assessment, and response. This information will be utilized to discover movements, track progress, and make informed decisions. Effective monitoring must be executed from both quantitative and qualitative sources. Surveys, screenings, group discussions, and informal observation all constitute common ways of surveillance.
❖ The method of communicating and analyzing outcomes to stakeholders is called reporting. Reports should be clear and concise, focusing on major outcomes and suggestions for the next steps. Reports can also include graphs, charts, or additional visuals to help understandably explain complex facts. Effective reporting should be customized to the needs and preferences of the target audience. The information must be accurate, current, and readily available, and they must provide stakeholders with the details they require to make enlightened project recommendations. Periodic reporting also helps develop credibility and openness by allowing stakeholders to monitor achievement around common objectives.

As a result, monitoring and reporting are essential aspects of good program administration. Corporations can assess progress, know results and difficulties, and initiate corrective measures when appropriate to improve the program results by frequently gathering, analyzing, and disclosing program data with stakeholders. Proper monitoring and reporting help ensure that programs benefit the target population and produce the desired outcomes.

6.1.6. Change and release management

Change and release management are two key components of IT service management. Change management guarantees that modifications to the IT system, structures, and programs are scheduled, confirmed, authorized, and carried out in an organized and systematic way. Release management involves conceptualizing, organizing, arranging, and implementing software releases in different settings, from development to production.

❖ Effective change management is critical for limiting concerns and reducing disturbance to company business as changes are implemented. The procedure is usually divided into four stages: request, review, approval, and implementation. The request for modification is submitted and reported at the request stage, including facts such as the nature of the change, potential impacts, and required resources. A change advisory board or designated authority reviews, assesses and authorizes the change request to ensure that it satisfies business objectives and does not damage the existing system. Once approved, the modification is executed in a monitored, planned manner to reduce any potential impact on the IT infrastructure and the business's operations.
❖ Effective release management ensures that software releases are distributed, controlled, and scheduled, causing as little disturbance to company operations as possible. The process is frequently separated into five stages: design, construction, testing, deployment, and monitoring. Release managers define the release scope, build a strategy, and establish release criteria during the planning stage. The building phase entails developing and verifying the release package, whereas the testing stage includes ensuring the set standards. The release is released into a real-world setting in the deployment process, and release performances and opinions are recorded in the monitoring stage to identify areas for growth.

As a result, change and release management are vital aspects of managing IT services. Effective change and release management processes assist enterprises in reducing risks, ensuring business continuity, and delivering software and service products that carry out company goals. Communication, partnership, and sufficient records are essential for achieving change and release management through the procedure.

Cloud service management ensures that end customers get exceptional cloud services. Cloud service management can ensure that cloud services carry out the specifications and standards of consumers by monitoring SLAs, governance and compliance, security and privacy, resource provisioning, monitoring and reporting, and change and release management. Fig. 2 shows how to choose criteria for a cloud service provider.

6.2. Selection of cloud service provider

This sub-section studies a problem for the best cloud service provider under the IVq-ROFSS environment. Here, we have considered a problem in a scenario where four cloud service providers are available. Let these cloud service providers be denoted as $\mathfrak{L} = \{\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3, \mathfrak{L}_4\}$. We want to select the best cloud service provider logically among these cloud service providers. Cloud services depend entirely on attributes like accessibility, performance, reliability, management skills, cost efficiency, security, etc. Let us categorize different characteristics as follows: $\varsigma_1 =$

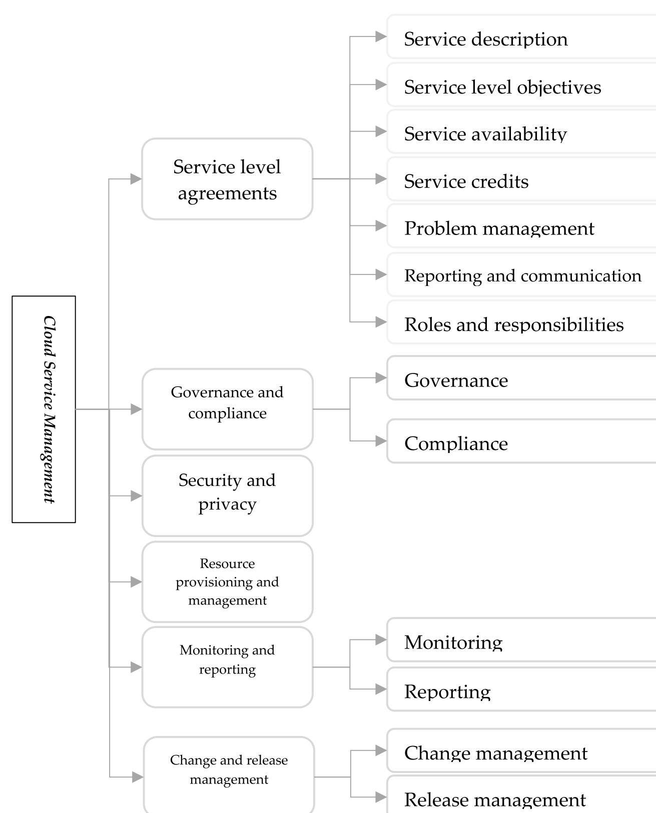

Fig. 2. Flowchart of the cloud service management.

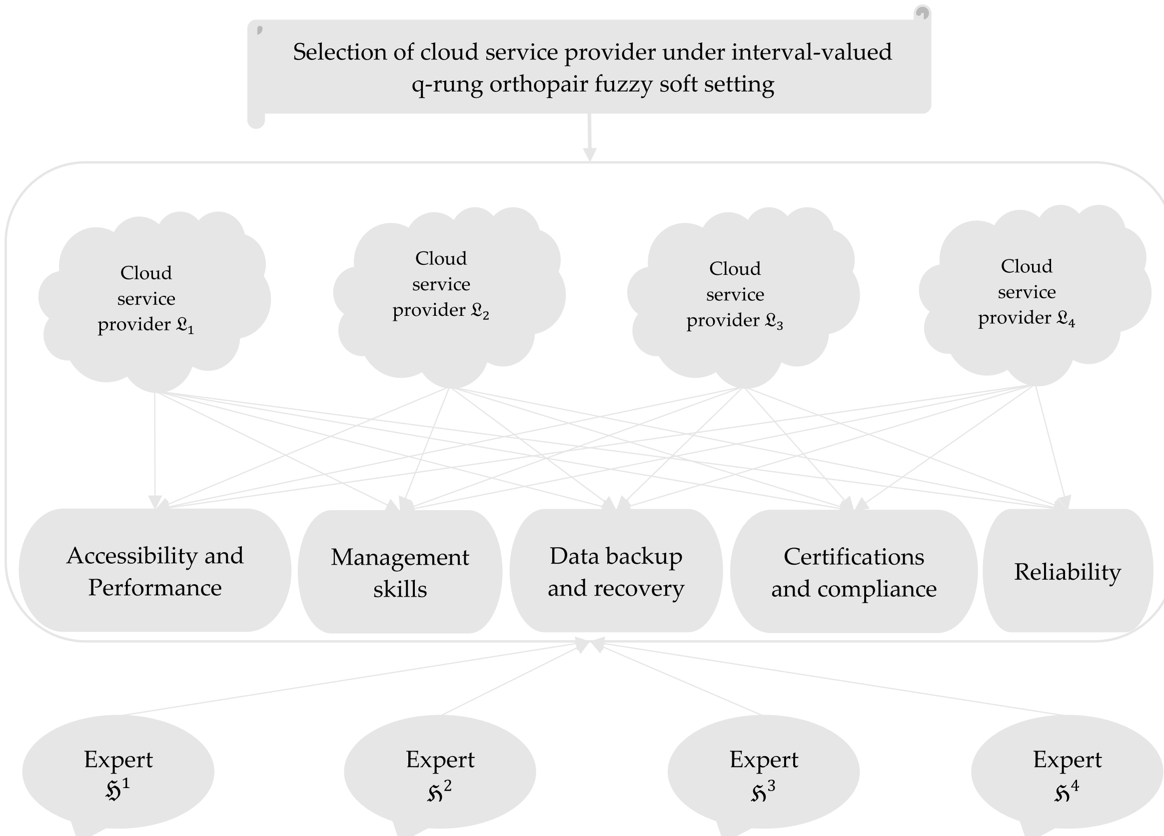

Fig. 3. Selection of cloud service provider under interval-valued q-rung orthopair fuzzy soft environment.

Table 1
Significant aspects of parameters in cloud service provider selection.

Parameters	Essential aspects of considered attributes for cloud service provider selection
Accessibility and Performance	Data accessibility, network performance, latency and response time, scalability and elasticity, load balancing, data caching, bandwidth optimization, continuous monitoring, and optimization.
Management skills	Resource allocation and optimization, security and compliance, data governance and lifecycle management, change and release management, performance monitoring and troubleshooting, vendor management, continuous improvement, and innovation.
Data backup and recovery	Data protection, backup strategies, redundancy and replication, recovery point objective and recovery time objective, testing and validation, automation and monitoring, data encryption and security, and disaster recovery planning.
Certifications and compliance	Regulatory compliance, certification standards, data privacy, security audits, data residency and sovereignty, vendor due diligence, data breach response, and ongoing compliance monitoring.
Reliability	Service uptime, redundancy, replication, scalability, SLA commitments, proactive maintenance, fault tolerance and load balancing, performance optimization, regular audits, and assessments.

Accessibility and Performance, $\varsigma_2 =$ Management skills, $\varsigma_3 =$ Data backup and recovery, $\varsigma_4 =$ Certifications and compliance and $\varsigma_5 =$ Reliability. Experts (software engineers) provide different opinions based on the underlying attributes. Let us select four distinct decision-makers $\mathfrak{H}^1 =$ Engineer without any experience, $\mathfrak{H}^2 =$ Engineer (5 years experience), $\mathfrak{H}^3 =$ Engineer (10 years experience), and $\mathfrak{H}^4 =$ Engineer (Highly experienced) with a computer science background in the weight vector $(0.1, 0.2, 0.4, 0.3)^T$ and the weight corresponding to the

Table 2
Expert's evaluation for $\mathfrak{L}_1$ in the form of IVq-ROFSN.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	$([0.4, 0.6], [0.2, 0.7])$	$([0.7, 0.8], [0.5, 0.7])$	$([0.4, 0.6], [0.2, 0.5])$	$([0.2, 0.5], [0.2, 0.6])$	$([0.2, 0.7], [0.5, 0.6])$
$\mathbf{H}^2$	$([0.2, 0.7], [0.2, 0.6])$	$([0.3, 0.6], [0.2, 0.5])$	$([0.2, 0.7], [0.4, 0.8])$	$([0.6, 0.9], [0.4, 0.7])$	$([0.4, 0.6], [0.2, 0.5])$
$\mathbf{H}^3$	$([0.3, 0.7], [0.6, 0.8])$	$([0.3, 0.8], [0.2, 0.5])$	$([0.4, 0.8], [0.3, 0.7])$	$([0.5, 0.7], [0.2, 0.4])$	$([0.3, 0.5], [0.2, 0.8])$
$\mathbf{H}^4$	$([0.4, 0.6], [0.3, 0.7])$	$([0.4, 0.9], [0.3, 0.5])$	$([0.3, 0.6], [0.3, 0.5])$	$([0.3, 0.6], [0.4, 0.5])$	$([0.3, 0.7], [0.4, 0.6])$

Table 3
Expert's evaluation for $\mathfrak{L}_2$ in the form of IVq-ROFSN.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	$([0.2, 0.5], [0.5, 0.6])$	$([0.4, 0.8], [0.5, 0.7])$	$([0.5, 0.7], [0.6, 0.8])$	$([0.6, 0.7], [0.5, 0.8])$	$([0.4, 0.6], [0.4, 0.8])$
$\mathbf{H}^2$	$([0.5, 0.7], [0.5, 0.8])$	$([0.3, 0.4], [0.4, 0.5])$	$([0.2, 0.5], [0.3, 0.7])$	$([0.3, 0.5], [0.4, 0.6])$	$([0.2, 0.5], [0.3, 0.7])$
$\mathbf{H}^3$	$([0.4, 0.6], [0.1, 0.4])$	$([0.2, 0.5], [0.2, 0.9])$	$([0.4, 0.8], [0.3, 0.7])$	$([0.5, 0.8], [0.2, 0.9])$	$([0.3, 0.6], [0.2, 0.5])$
$\mathbf{H}^4$	$([0.2, 0.5], [0.3, 0.8])$	$([0.3, 0.5], [0.2, 0.8])$	$([0.4, 0.7], [0.3, 0.6])$	$([0.4, 0.7], [0.3, 0.6])$	$([0.5, 0.8], [0.3, 0.8])$

Table 4
Expert's evaluation for $\mathfrak{L}_3$ in the form of IVq-ROFSN.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	([0.6, 0.7], [0.5, 0.9])	([0.2, 0.4], [0.4, 0.6])	([0.5, 0.6], [0.4, 0.5])	([0.3, 0.4], [0.3, 0.6])	([0.4, 0.8], [0.2, 0.7])
$\mathbf{H}^2$	([0.3, 0.6], [0.4, 0.7])	([0.3, 0.5], [0.5, 0.9])	([0.3, 0.5], [0.5, 0.8])	([0.2, 0.6], [0.6, 0.9])	([0.3, 0.5], [0.1, 0.6])
$\mathbf{H}^3$	([0.1, 0.4], [0.3, 0.4])	([0.3, 0.5], [0.3, 0.7])	([0.3, 0.7], [0.3, 0.8])	([0.1, 0.3], [0.5, 0.6])	([0.5, 0.7], [0.4, 0.8])
$\mathbf{H}^4$	([0.5, 0.7], [0.3, 0.7])	([0.2, 0.6], [0.1, 0.4])	([0.2, 0.5], [0.3, 0.6])	([0.3, 0.4], [0.3, 0.7])	([0.2, 0.7], [0.3, 0.6])

Table 5
Expert's evaluation for $\mathfrak{L}_4$ in the form of IVq-ROFSN.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	([0.3, 0.5], [0.2, 0.6])	([0.4, 0.8], [0.4, 0.7])	([0.5, 0.9], [0.3, 0.6])	([0.4, 0.7], [0.6, 0.8])	([0.4, 0.7], [0.3, 0.6])
$\mathbf{H}^2$	([0.2, 0.6], [0.3, 0.7])	([0.1, 0.5], [0.4, 0.7])	([0.5, 0.8], [0.4, 0.7])	([0.2, 0.5], [0.3, 0.4])	([0.1, 0.5], [0.2, 0.6])
$\mathbf{H}^3$	([0.2, 0.5], [0.1, 0.6])	([0.2, 0.5], [0.1, 0.5])	([0.2, 0.4], [0.4, 0.7])	([0.6, 0.9], [0.1, 0.5])	([0.3, 0.6], [0.2, 0.6])
$\mathbf{H}^4$	([0.2, 0.6], [0.5, 0.8])	([0.2, 0.6], [0.5, 0.8])	([0.2, 0.7], [0.3, 0.8])	([0.2, 0.5], [0.4, 0.5])	([0.2, 0.7], [0.4, 0.6])

attribute function is taken as $(0.2, 0.15, 0.25, 0.1, 0.3)^T$. The four alternatives will be evaluated under these five attributes and given their preferences in IVq-ROFSNs by the decision-makers. The procedure is presented in the subsequent flowchart (See Fig. 3).

We also constructed substantive aspects related to the attribute functions shown in Table 1 for readability and clarity.

To choose the most effective cloud service provider, experts used the MADM methodology based on TOPSIS, as stated in section 5.1. They reported their recommendations as IVq-ROFSNs for each cloud provider specified in Tables 2–5.

Step 1: Construction of decision matrices for alternates $\mathfrak{L} = \{\mathfrak{L}_1, \mathfrak{L}_2, \mathfrak{L}_3, \mathfrak{L}_4\}$ in a configuration of IVq-ROFSNs with the attributes that follow.

Table 6
Weighted decision matrix for $\overline{\mathfrak{L}}_1$.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	([0.0567, 0.0745], [0.4725, 0.5173])	([0.0122, 0.0448], [0.5129, 0.5343])	([0.0251, 0.0642], [0.4832, 0.5463])	([0.0338, 0.0477], [0.5134, 0.5351])	([0.0227, 0.0443], [0.5376, 0.5658])
$\mathbf{H}^2$	([0.0178, 0.0543], [0.6950, 0.7965])	([0.0238, 0.0499], [0.5868, 0.6776])	([0.0191, 0.0540], [0.6746, 0.6372])	([0.0572, 0.0676], [0.5861, 0.5960])	([0.0368, 0.0385], [0.5373, 0.6852])
$\mathbf{H}^3$	([0.0467, 0.0696], [0.5704, 0.6381])	([0.0231, 0.0578], [0.6249, 0.7129])	([0.0142, 0.0549], [0.5862, 0.6872])	([0.0136, 0.0561], [0.6352, 0.6532])	([0.0136, 0.0561], [0.6256, 0.7135])
$\mathbf{H}^4$	([0.0341, 0.0476], [0.5236, 0.5752])	([0.0461, 0.0565], [0.5767, 0.5964])	([0.0136, 0.0561], [0.6322, 0.6432])	([0.0356, 0.0545], [0.4735, 0.5267])	([0.0294, 0.0537], [0.6149, 0.6578])

Table 7
Weighted decision matrix for $\overline{\mathfrak{L}}_2$.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	([0.0254, 0.0451], [0.4621, 0.5279])	([0.0265, 0.0345], [0.4945, 0.5567])	([0.0247, 0.0379], [0.5264, 0.5458])	([0.0561, 0.0746], [0.9728, 0.9863])	([0.0227, 0.0283], [0.5124, 0.5648])
$\mathbf{H}^2$	([0.0534, 0.0613], [0.6345, 0.7637])	([0.0334, 0.0367], [0.6549, 0.6878])	([0.0532, 0.0596], [0.6563, 0.6860])	([0.0175, 0.0534], [0.6941, 0.7981])	([0.0245, 0.0387], [0.5060, 0.6372])
$\mathbf{H}^3$	([0.0365, 0.0377], [0.5302, 0.5389])	([0.0172, 0.0239], [0.5287, 0.6162])	([0.0276, 0.0461], [0.6187, 0.6252])	([0.0471, 0.0642], [0.9139, 0.9382])	([0.0448, 0.0587], [0.6356, 0.6445])
$\mathbf{H}^4$	([0.0564, 0.0641], [0.4931, 0.5068])	([0.0227, 0.0453], [0.5874, 0.5948])	([0.0134, 0.0219], [0.7142, 0.7235])	([0.0375, 0.0379], [0.6373, 0.6852])	([0.0153, 0.0351], [0.5809, 0.6085])

Table 8
Weighted decision matrix for $\overline{\mathfrak{L}}_3$.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	([0.0207, 0.0568], [0.6527, 0.9249])	([0.0238, 0.0547], [0.9143, 0.9265])	([0.0272, 0.0291], [0.9451, 0.9733])	([0.0407, 0.0654], [0.8524, 0.9247])	([0.0578, 0.0772], [0.8086, 0.9074])
$\mathbf{H}^2$	([0.0454, 0.0821], [0.8612, 0.8975])	([0.0262, 0.0348], [0.9427, 0.9652])	([0.0071, 0.0105], [0.9783, 0.9962])	([0.0535, 0.0923], [0.8719, 0.9069])	([0.0237, 0.0478], [0.9379, 0.9526])
$\mathbf{H}^3$	([0.0263 0.0364], [0.9380, 0.9584])	([0.0139, 0.0179], [0.9424, 0.9640])	([0.0043, 0.0063], [0.9652, 0.9932])	([0.0159, 0.0227], [0.9421, 0.9573])	([0.0469 0.0613], [0.9289, 0.9547])
$\mathbf{H}^4$	([0.0227, 0.0272], [0.9713, 0.9728])	([0.0264, 0.0509], [0.9436, 0.9495])	([0.0218, 0.0327], [0.9549, 0.9683])	([0.0067, 0.0143], [0.9809, 0.9867])	([0.0203, 0.0231], [0.9661, 0.9761])

Table 9
Weighted decision matrix for $\overline{\mathfrak{L}}_4$.

	ς_1	ς_2	ς_3	ς_4	ς_5
$\mathbf{H}^1$	$\begin{pmatrix}[0.0057, 0.0097],\\ [0.9631, 0.9906]\end{pmatrix}$	$\begin{pmatrix}[0.0272, 0.0291],\\ [0.9451, 0.9733]\end{pmatrix}$	$\begin{pmatrix}[0.0218, 0.0327],\\ [0.9549, 0.9683]\end{pmatrix}$	$\begin{pmatrix}[0.0159, 0.0227],\\ [0.9421, 0.9573]\end{pmatrix}$	$\begin{pmatrix}[0.0227, 0.0272],\\ [0.9713, 0.9728]\end{pmatrix}$
$\mathbf{H}^2$	$\begin{pmatrix}[0.0237, 0.0478],\\ [0.9379, 0.9526]\end{pmatrix}$	$\begin{pmatrix}[0.0238, 0.0547],\\ [0.9143, 0.9265]\end{pmatrix}$	$\begin{pmatrix}[0.0254, 0.0451],\\ [0.4621, 0.5279]\end{pmatrix}$	$\begin{pmatrix}[0.0334, 0.0367],\\ [0.6549, 0.6878]\end{pmatrix}$	$\begin{pmatrix}[0.0134, 0.0219],\\ [0.7142, 0.7235]\end{pmatrix}$
$\mathbf{H}^3$	$\begin{pmatrix}[0.0448, 0.0587],\\ [0.6356, 0.6445]\end{pmatrix}$	$\begin{pmatrix}[0.0172, 0.0239],\\ [0.5287, 0.6162]\end{pmatrix}$	$\begin{pmatrix}[0.0472, 0.0476],\\ [0.5463, 0.5762]\end{pmatrix}$	$\begin{pmatrix}[0.0175, 0.0534],\\ [0.6941, 0.7981]\end{pmatrix}$	$\begin{pmatrix}[0.0142, 0.0549],\\ [0.5867, 0.6872]\end{pmatrix}$
$\mathbf{H}^4$	$\begin{pmatrix}[0.0136, 0.0561],\\ [0.6256, 0.7135]\end{pmatrix}$	$\begin{pmatrix}[0.0231, 0.0578],\\ [0.6249, 0.7129]\end{pmatrix}$	$\begin{pmatrix}[0.0191, 0.0540],\\ [0.6146, 0.6372]\end{pmatrix}$	$\begin{pmatrix}[0.0375, 0.0379],\\ [0.6373, 0.6852]\end{pmatrix}$	$\begin{pmatrix}[0.0153, 0.0351],\\ [0.5809, 0.6085]\end{pmatrix}$

Step 2: As all of the parameters that are under evaluation are of a similar type, no normalization is necessary.

Step 3: Utilizing Eq. (13), develop a weighted decision matrix for each alternate $\overline{\mathfrak{L}}_z$. Tables 6, 7, 8 and 9 provide the weighted decision matrices for each alternative.

Step 4. Using Eqs. (5.3) and (5.4) determine the PIA and NIA correspondingly.

$$\Delta^{(z)+}_{\varsigma_{ij}} = \begin{bmatrix} \begin{pmatrix}[0.0567, 0.0745],\\ [0.4725, 0.5173]\end{pmatrix} & \begin{pmatrix}[0.0272, 0.0291],\\ [0.9451, 0.9733]\end{pmatrix} & \begin{pmatrix}[0.0272, 0.0291],\\ [0.9451, 0.9733]\end{pmatrix} & \begin{pmatrix}[0.0159, 0.0227],\\ [0.9421, 0.9573]\end{pmatrix} & \begin{pmatrix}[0.0227, 0.0272],\\ [0.9713, 0.9728]\end{pmatrix} & \begin{pmatrix}[.0320, .0354],\\ [.9643, .9836]\end{pmatrix} \\ \begin{pmatrix}[0.0534, 0.0613],\\ [0.6345, 0.7637]\end{pmatrix} & \begin{pmatrix}[0.0334, 0.0367],\\ [0.6549, 0.6878]\end{pmatrix} & \begin{pmatrix}[0.0071, 0.0105],\\ [0.9783, 0.9962]\end{pmatrix} & \begin{pmatrix}[0.0334, 0.0367],\\ [0.6549, 0.6878]\end{pmatrix} & \begin{pmatrix}[0.0368, 0.0385],\\ [0.5373, 0.6852]\end{pmatrix} & \begin{pmatrix}[.0472, .0476],\\ [.5463, .5762]\end{pmatrix} \\ \begin{pmatrix}[0.0365, 0.0377],\\ [0.5302, 0.5389]\end{pmatrix} & \begin{pmatrix}[0.0139, 0.0179],\\ [0.9424, 0.9640]\end{pmatrix} & \begin{pmatrix}[0.0472, 0.0476],\\ [0.5463, 0.5762]\end{pmatrix} & \begin{pmatrix}[0.0159, 0.0227],\\ [0.9421, 0.9573]\end{pmatrix} & \begin{pmatrix}[0.0448, 0.0587],\\ [0.6356, 0.6445]\end{pmatrix} & \begin{pmatrix}[.0143, .0169],\\ [.9414, .9529]\end{pmatrix} \\ \begin{pmatrix}[0.0227, 0.0272],\\ [0.9713, 0.9728]\end{pmatrix} & \begin{pmatrix}[0.0461, 0.0565],\\ [0.5767, 0.5964]\end{pmatrix} & \begin{pmatrix}[0.0134, 0.0219],\\ [0.7142, 0.7235]\end{pmatrix} & \begin{pmatrix}[0.0375, 0.0379],\\ [0.6373, 0.6852]\end{pmatrix} & \begin{pmatrix}[0.0203, 0.0231],\\ [0.9661, 0.9761]\end{pmatrix} & \begin{pmatrix}[.0057, .0097],\\ [.9631, .9906]\end{pmatrix} \end{bmatrix}$$

$$\Delta^{(z)-}_{\varsigma_{ij}} = \begin{bmatrix} \begin{pmatrix}[0.0207, 0.0568],\\ [0.6527, 0.9249]\end{pmatrix} & \begin{pmatrix}[0.0122, 0.0448],\\ [0.5129, 0.5343]\end{pmatrix} & \begin{pmatrix}[0.0251, 0.0642],\\ [0.4832, 0.5463]\end{pmatrix} & \begin{pmatrix}[0.0407, 0.0654],\\ [0.8524, 0.9247]\end{pmatrix} & \begin{pmatrix}[0.0227, 0.0443],\\ [0.5376, 0.5658]\end{pmatrix} & \begin{pmatrix}[0.0454, 0.0818],\\ [0.8612, 0.8975]\end{pmatrix} \\ \begin{pmatrix}[0.0534, 0.0613],\\ [0.6345, 0.7637]\end{pmatrix} & \begin{pmatrix}[0.0238, 0.0547],\\ [0.9143, 0.9265]\end{pmatrix} & \begin{pmatrix}[0.0191, 0.0540],\\ [0.6746, 0.6372]\end{pmatrix} & \begin{pmatrix}[0.0535, 0.0923],\\ [0.8719, 0.9069]\end{pmatrix} & \begin{pmatrix}[0.0237, 0.0478],\\ [0.9379, 0.9526]\end{pmatrix} & \begin{pmatrix}[0.0175, 0.0534],\\ [0.6941, 0.7981]\end{pmatrix} \\ \begin{pmatrix}[0.0467, 0.0696],\\ [0.5704, 0.6381]\end{pmatrix} & \begin{pmatrix}[0.0231, 0.0578],\\ [0.6249, 0.7129]\end{pmatrix} & \begin{pmatrix}[0.0142, 0.0549],\\ [0.5862, 0.6872]\end{pmatrix} & \begin{pmatrix}[0.0136, 0.0561],\\ [0.6352, 0.6532]\end{pmatrix} & \begin{pmatrix}[0.0142, 0.0549],\\ [0.5867, 0.6872]\end{pmatrix} & \begin{pmatrix}[0.0153, 0.0357],\\ [0.5709, 0.6385]\end{pmatrix} \\ \begin{pmatrix}[0.0136, 0.0561],\\ [0.6256, 0.7135]\end{pmatrix} & \begin{pmatrix}[0.0231, 0.0578],\\ [0.6249, 0.7129]\end{pmatrix} & \begin{pmatrix}[0.0191, 0.0540],\\ [0.6146, 0.6372]\end{pmatrix} & \begin{pmatrix}[0.0356, 0.0545],\\ [0.4735, 0.5267]\end{pmatrix} & \begin{pmatrix}[0.0294, 0.0537],\\ [0.6149, 0.6578]\end{pmatrix} & \begin{pmatrix}[0.0136, 0.0561],\\ [0.6256, 0.7135]\end{pmatrix} \end{bmatrix}$$

Step 5. Determine the CC among $\overline{\mathfrak{L}}^{(z)}$ and PIA Δ^+ using Eq. (16), such as:

$$\kappa^{(1)} = 0.99457, \kappa^{(2)} = 0.99768, \kappa^{(3)} = 0.99573, \kappa^{(4)} = 0.99396.$$

Step 6. Determine the CC among $\overline{\mathfrak{L}}^{(z)}$ and NIA Δ^- using Eq. (17), such as:

$$\tau^{(1)} = 0.99683, \tau^{(2)} = 0.99615, \tau^{(3)} = 0.99562, \tau^{(4)} = 0.99697.$$

Step 7. Find the closeness coefficient using Eq. (18).

$$\beth^{(1)} = 0.63139, \beth^{(2)} = 0.37601, \beth^{(3)} = 0.49364, \text{and} \beth^{(4)} = 0.66593.$$

Step 8: The preceding computation demonstrates that the most significant value of the closeness coefficient is $\beth^{(4)} = 0.66593$. As a consequence, $\mathfrak{L}_4$ is the most effective cloud service provider for everyday use.

Step 9: Ranking of the alternatives $\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$.

7. Discussion and comparative analysis

The following section assesses the proposed strategy's feasibility by comparing it to currently employed methods.

7.1. The effect of the "q" variations on alternative categorization

The $\mathfrak{L}_4$ and $\mathfrak{L}_2$ are the most supportive and poor alternates, depending on their expectations. Table 10 demonstrates that there's not any variation among the scheduling of the various alternates when "q" falls between 3 and 10, i.e., $\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$. Moreover, the TOPSIS

Table 10
Influences of the parameter "q" on the decision results.

Parameter	Closeness coefficient	Ranking
$q = 3$	$\beth^{(1)} = 0.63139, \beth^{(2)} = 0.37601, \beth^{(3)} = 0.49364, \beth^{(4)} = 0.66593$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 4$	$\beth^{(1)} = 0.62951, \beth^{(2)} = 0.37257, \beth^{(3)} = 0.48946, \beth^{(4)} = 0.65937$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 5$	$\beth^{(1)} = 0.62072, \beth^{(2)} = 0.36852, \beth^{(3)} = 0.48302, \beth^{(4)} = 0.65438$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 6$	$\beth^{(1)} = 0.61218, \beth^{(2)} = 0.36148, \beth^{(3)} = 0.47843, \beth^{(4)} = 0.64879$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 7$	$\beth^{(1)} = 0.60869, \beth^{(2)} = 0.35716, \beth^{(3)} = 0.47346, \beth^{(4)} = 0.64349$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 8$	$\beth^{(1)} = 0.60183, \beth^{(2)} = 0.34915, \beth^{(3)} = 0.46981, \beth^{(4)} = 0.63935$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 9$	$\beth^{(1)} = 0.59782, \beth^{(2)} = 0.34407, \beth^{(3)} = 0.46038, \beth^{(4)} = 0.63247$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
$q = 10$	$\beth^{(1)} = 0.59427, \beth^{(2)} = 0.34063, \beth^{(3)} = 0.45871, \beth^{(4)} = 0.62874$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$

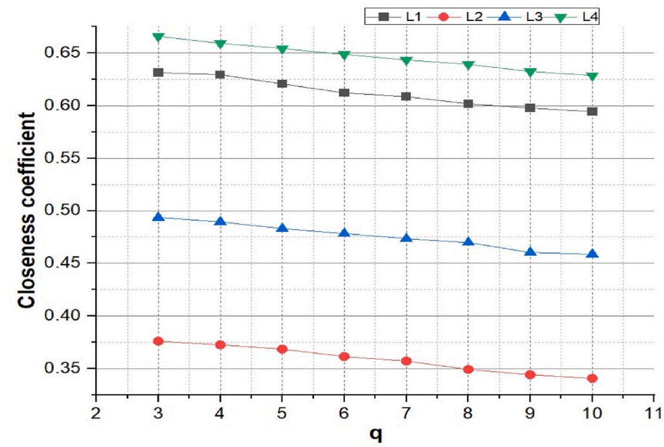

Fig. 4. Impact of parameter 'q' on final ranking of alternatives.

methods employed by IVIFSS (Zulqarnain et al., 2021a) and IVPFSS (Zulqarnain et al., 2022b) failed to cope with the information at hand if $(MD^{\mho})^q + (NMD^{\mho})^q > 1$, where $q > 2$. Meanwhile, it shows that this data extraction method becomes more responsive. The investigation demonstrated that the structure of a parameter could make it simpler for specialists to judge any information. They are instructed to decide on the value of each parameter determined by their demands.

Through multiple components, the approach suggested in the present study indicates streamlining the visualization of fuzzy knowledge and promoting the synthesis of reality. Multiple FS amalgamation structures can be transformed into particular features of IVq-ROFSS by integrating specific patterns, as illustrated in Table 11. The parameter "q" plays a vital role in permitting experts to engage in a more thorough examination of a specific assignment. It helps a more comprehensive analysis and confirmation of the trends. Through the research and assessment conducted, it was concluded that the outcomes achieved through the proposed approach are superior to those obtained from alternative models. Fig. 4 shows a graphical illustration of the parameter "q" effect on the outcome. The following illustration assists in recognizing and comprehending the influence of modifying "q" values on the research's results.

7.2. Proposed Methodology's superiority

The method suggested makes utilization of TOPSIS, an improved MADM strategy. The significant benefits of this technique over conventional approaches are apparent, and it adequately addresses the challenges presented by MADM. It delivers enhanced inequality, is established flexibly and accurately, and provides precise and broadened outcomes. Despite other approaches adhering to specific systemic attitudes, this systematic approach significantly changes the pre-existing structure and supplies a unique perspective. According to the findings, the results of the proposed methodology's analytical research and evaluations are equivalent to composite techniques. Several FS mixtures created are transformed into IVq-ROFSS by including applicable conditions. This novel and excellent creativity incorporates infrequent and obscure data within the pragmatic scheme. This makes the ability to express rich and convoluted facts more comprehensively and precisely. In contrast to many hybrid FS circumstances, the proposed approach effectively integrates practical realities and challenging information into the decision-making procedure, making it more devoted, significantly fantastic, and professional. The disparity of features among the novel technique and frequently used methods is emphasized in Table 11, demonstrating the strengths and originality of the presented strategy.

Table 11
Qualitative assessment of the planned model with the prevalent models.

	Set	Parameters	Expert's opinions in interval form	Advantages	Limitations
Zadeh (Zadeh, 1965)	FS	×	×	Deals uncertainty using MD	Unable to handle NMD
Atanassov (Atanassov, 1986)	IFS	×	×	Deals uncertainty using MD and NMD	Unable to handle $MD + NMD > 1$
Yager (Yager, 2013)	PFS	×	×	Deals uncertainty using MD and NMD	Unable to handle $MD^2 + NMD^2 > 1$
Yager (Yager, 2016)	q-ROFS	×	✓	Deals uncertainty using MD and NMD	Deal with $MD^q + NMD^q \leq 1$
Turksen (Turksen, 1986)	IVFS	×	✓	Deals uncertainty using MD intervals	Unable to handle NMD interval
Atanassov (Atanassov, 1999)	IVIFS	×	✓	Deals uncertainty using MD and NMD intervals	Unable to handle $MD^u + NMD^u > 1$
Peng & Yang (Peng and Yang, 2016)	IVPFS	×	✓	Deals uncertainty using MD and NMD intervals	Unable to handle $(MD^u)^2 + (NMD^u)^2 > 1$
Joshi et al. (Joshi et al., 2018)	IVq-ROFS	×	✓	Deals uncertainty using MD and NMD intervals	Handle the $(MD^u)^q + (NMD^u)^q \leq 1$
Maji et al. (Maji et al., 2001a)	FSS	✓	×	Deals uncertainty using MD	Unable to handle NMD
Maji et al. (Maji et al., 2001b)	IFSS	✓	×	Deals uncertainty using MD and NMD	Unable to handle $MD + NMD > 1$
Peng et al. (Peng et al., 2015)	PFSS	✓	×	Deals uncertainty using MD and NMD	Unable to handle $MD^2 + NMD^2 > 1$
Hussain et al. (Hussain et al., 2020)	q-ROFSS	✓	×	Deals uncertainty using MD and NMD	Handle $MD^q + NMD^q \leq 1$
Jiang et al. (Jiang et al., 2010)	IVIFSS	✓	✓	Deals uncertainty using MD and NMD intervals	Unable to handle $MD^u + NMD^u > 1$
Zulqarnain et al. (Zulqarnain et al., 2022b)	IVPFSS	✓	✓	Deals uncertainty using MD and NMD intervals	Unable to handle $(MD^u)^2 + (NMD^u)^2 > 1$
Proposed approach	IVq-ROFSS	✓	✓	Deals uncertainty using MD and NMD intervals	Handle $(MD^u)^q + (NMD^u)^q \leq 1$

Table 12
Comparative analysis of the proposed model with existing models under the considered data set.

Structure	Alternatives score values or closeness coefficient				Ranking
Fuzzy TOPSIS (Ansari et al., 2020)	n/a				n/a
IVFS TOPSIS (Ashtiani et al., 2009)	n/a				n/a
IFS TOPSIS (Rouyendegh et al., 2020)	n/a				n/a
IVIFS TOPSIS (Zhang and Yu, 2012)	n/a				n/a
PFS TOPSIS (Hajiaghaei-Keshteli et al., 2023)	n/a				n/a
IVPFS TOPSIS (Wang, 2018)	n/a				n/a
IVIFSS TOPSIS (Zulqarnain et al., 2021a)	n/a				n/a
PFSS TOPSIS (Zulqarnain et al., 2021c)	n/a				n/a
q-ROFSS TOPSIS (Hamid et al., 2020)	n/a				n/a
IVq-ROFSWA (Yang et al., 2022)	$Sc(0.59521)$	$Sc(0.56512)$	$Sc(0.60351)$	$Sc(0.61712)$	$\mathfrak{L}_4 > \mathfrak{L}_3 > \mathfrak{L}_1 > \mathfrak{L}_2$
IVq-ROFSWG (Yang et al., 2022)	$Sc(-0.09114)$	$Sc(-0.01327)$	$Sc(0.01551)$	$Sc(0.05096)$	$\mathfrak{L}_4 > \mathfrak{L}_3 > \mathfrak{L}_2 > \mathfrak{L}_1$
IVq-ROFSIWA (Yang et al., 2022)	$Sc(0.43215)$	$Sc(0.37846)$	$Sc(0.40529)$	$Sc(0.46459)$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
IVq-ROFSWIG (Yang et al., 2022)	$Sc(0.28845)$	$Sc(0.23730)$	$Sc(0.26094)$	$Sc(0.34297)$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
GGIVq-ROFSWA (Hayat et al., 2023)	$Sc(0.15491)$	$Sc(0.11583)$	$Sc(0.12916)$	$Sc(0.18369)$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
GGIVq-ROFSWG (Hayat et al., 2023)	$Sc(-0.22721)$	$Sc(-0.34862)$	$Sc(-0.30557)$	$Sc(-0.05952)$	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$
Proposed TOPSIS	0.63139	0.37601	0.49364	0.66593	$\mathfrak{L}_4 > \mathfrak{L}_1 > \mathfrak{L}_3 > \mathfrak{L}_2$

It seems that an unfamiliar issue has merely recently shown up, justifying the implementation of a unique MADM model customized to the particular specifications of an individual company. Although the reality of the fact that there are a lot of other current methods, the particular method which is provided distinguishes remarkable because it employs a distinctive hybrid model which incorporates numerous fuzzy set models, namely FS, IVFS, IFS, IVIFS, PFS, IVPFS, q-ROFS, IVq-ROFS, FSS, IVFSS, IFSS, IVIFSS, PFSS, IVPFSS, and q-ROFSS. These hybrid models are still having challenges accurately assessing specific circumstances, although. With several existing aggregation operators, we have developed a MADM framework for IVq-ROFSS, which can deal with attributes that include both membership degrees (MD) and non-membership degrees (NMD) in intervals, complying with a condition of $0 \leq (MD^{\mho})^q + (NMD^{\mho})^q \leq 1$. This newly presented strategy facilitates an in-depth examination of the available information compared with previous hybrid structures. Table 11 demonstrates that our designed hybrid fuzzy set system exceeds other previous hybrid fuzzy set designs. The achievement of any company depends upon determining the most effective MADM approach, and our revolutionary methodology presents a more comprehensive examination of the given problem and enhanced decision-making. By utilizing this strategy, businesses can improve their decision-making processes and make better decisions to meet their goals.

7.3. Comparative analysis

The correlation-based TOPSIS method has extensively been validated in earlier comparative research. These studies have continually shown that the findings of TOPSIS are equivalent to those of different methodologies. When applied in cooperation with various additional decision-making process (DM) strategies, this suggested TOPSIS model's capability for incorporating further information concerning the specifications of the alternatives is just one of its key benefits. This aspect enables weighing the impact of data imprecision, generating an improved solid and factual depiction of the reality regarding the things under review. As a result, TOPSIS transforms into a valuable tool for the DM procedure when addressing unclear or perplexing substances. The suggested approach is distinct from previous strategies. It addresses positive ideal alternatives (PIA) and negative ideal alternatives (NIA) based on correlation measures at a particular spatial level with the ground, depending only on distance and similarity measures, as demonstrated by a comparative analysis. This method avoids the potential knowledge loss that could result from assigning score values to particular parameters without considering how those parameters may affect other parameters. By analyzing the most beneficial findings and delivering correlations among them, the most suitable correlation measure for each parameter can be determined. The developed TOPSIS approach effectively conveys the extent of perceptions and similarity among explanations and has several benefits over current strategies and related measurements. The above approach prevents making inaccurate inferences. While all possibilities are considered seriously by existing TOPSIS approaches when selecting a cloud service provider (CSP). Some TOPSIS approaches, such as those developed by Ansari et al. (2020), Rouyendegh et al. (2020), and Hajiaghaei-Keshteli et al. (2023) have limitations when dealing with alternative parametric modeling and circumstances including membership degrees (MD) and non-membership degrees (NMD) intervals. Meanwhile, the TOPSIS techniques presented in (Ashtiani et al., 2009; Zhang and Yu, 2012; Wang, 2018) deal with the MD and NMD in interval form, but these structures can also not deal with the parametric values of alternatives. Different TOPSIS techniques that have been presented under the terms IFSS, PFSS, and q-ROFSS, respectively, Zulqarnain et al., 2021a, 2021c, have negative aspects whenever related to addressing scenarios where $MD + NMD > 1$, $MD^{\mho} + NMD^{\mho} > 1$, $(MD)^2 + (NMD)^2 > 1$. Hamid et al. (2020) established the TOPSIS technique in the q-ROFSS scenario to address the earlier limitations. Still, this approach is unable to handle the information in interval form. Whereas the aggregation operators presented in different research, such as (Yang et al., 2022; Hayat et al., 2023), address specific problems but do not calculate the closeness coefficient in certain situations. Alternatively, the TOPSIS approach proposed in other studies covers parametric data but excludes interval information. Our postulated TOPSIS model expertly eliminates such limitations and competes with existing methods given such issues. As stated in Table 12, the findings indicate similar consequences, illustrating the efficacy and repeatability of our approach in determining the best CSP. In Table 12, the phrase "n/a" stands for "not applicable," identifying instances where particular processes cannot fulfill the requirements stated.

The evaluation matrix was implemented to evaluate various TOPSIS methods and their associated outcomes, and the consequences for each option are shown in Table 12. The table demonstrates that alternative $\mathfrak{L}_4$ is the best cloud service provider (CSP) choice. This outcome indicates the efficacy of the presented approach for determining the best CSP alternative. Table 12 comprehensively reviews the suggested method and related investigation. A more comprehensive glance at the data presented demonstrates that the preceding literature, described in (Ansari et al., 2020; Ashtiani et al., 2009; Rouyendegh et al., 2020; Zhang and Yu, 2012; Hajiaghaei-Keshteli et al., 2023; Wang, 2018), lacks parameter analysis details. While numerous TOPSIS techniques, such as those stated in (Ma et al., 2020; Zulqarnain et al., 2021c; Hamid et al., 2020), tackle the modeled values of the alternatives efficiently, they fall short when dealing with particular features of the considered

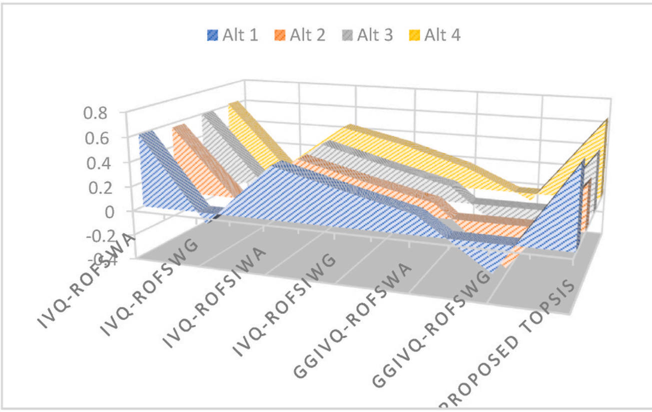

Fig. 5. Comparative s of the proposed model with existing models.

data set. On the other hand, the proposed approach provides a substantial advantage by adequately addressing the complexities of real-world scenarios by incorporating parametric features of the alternatives. This novel strategy solves a gap in the field of decision-making by offering an approach for issues that earlier operators accessible in the IVq-ROFSS framework have not adequately resolved. The indicated techniques' capability to cope with and conquer these DM complications represents an exciting, beneficial impact on the context. A graphical representation of comparative analysis is presented in the following Fig. 5.

7.4. Benefits and theoretical strengths of the proposed model

The proposed approach has multiple benefits and analytical advantages, making it a significant contribution to the field. The advantages mentioned earlier are discussed further as follows.

- ❖ The proposed strategy comprises an extensive assessment mechanism that examines various qualities and experts' opinions. The approach optimizes decision-making precision while taking in various factors and techniques. This leads to deeper and more accurate cloud service provider selection.
- ❖ Experts frequently confront challenges when picking attributes and arranging them using characteristics since these decisions are rarely precise. The proposed approach examines this issue using the interval-valued q-rung orthopair fuzzy parameterization theory, which includes attribute variation.
- ❖ The TOPSIS model designed by experts permits them to present assessments based on various criteria, which is more efficient than focusing on just one parameter. This capability is beneficial in the context of IVq-ROFSS.
- ❖ The study presents mathematical justifications for the correlation coefficients used in the suggested method, thereby establishing this method's validity. This symmetry structure promises impartiality and predictability in decision-making, making the algorithm suited for various solicitations involving stability and precision.
- ❖ This strategy entails fuzzy data properly, enabling an improved adaptive and realistic visualization of unpredictability. The approach allows imperfection and inconsistency in decision-making using fuzzy logic, which permits a less ideological and thorough investigation of the existing alternatives.
- ❖ The research's focus on IVq-ROFSS information is a significant speculated incursion since it is the most extensive IVq-ROFSS method. The growth of informational energies for IVq-ROFSS scenarios and the justification of their fundamental characteristics auxiliary extend the hypothetical structures for IVq-ROFSS, influencing more coherent and reliable DM protocols in the setting of IVq-ROFSS, a significant academic development in FS and DM.
- ❖ The algorithm is based on a solid foundation of theory, relying on well-known ideas such as the TOPSIS approach and the IVq-ROFSS. Using these theoretical frameworks, the model proves that decision-making development is rigorous, organized, and steady.

However, the recommended approach substantially benefits decision-making reliability, fuzzy knowledge integration, extended parametric analysis, real-world issues assessment, and fundamental integrity. As mentioned earlier, the benefits assist in its potency and practicality if it involves determining the most appropriate cloud service provider.

8. Conclusion

The main intention of this study is to conquer the obstacles induced by lack of information, invisibility, and unpredictability in the IVq-ROFSS. We propose a novel approach that adopts the advantages of each attribute's MD and NMD values under investigation. This research provides and effectively investigates new correlation measures, namely CC and WCC, designed explicitly for IVq-ROFSS. Moreover, this study demonstrates that by concentrating on one parameter, numerous prevailing correlation measures within the context of q-ROFS can be viewed as particular examples of the proposed measures. The TOPSIS approach is described, with attributes and the experts influencing the MADM challenges. The study employs correlation indices and the proximity coefficient to determine the Positive Ideal Alternative (PIA), Negative Ideal Alternative (NIA), and rank of alternatives. A numerical illustration is presented to support the benefit of the stated TOPSIS strategy in cloud service provider (CSP) selection. The comparison study also demonstrates the strategy's competency and integrity, reflecting its exceptional reliability and practicality in facilitating decision-makers in the DM procedure. Future study avenues can explore the incorporation of VIKOR and MABAC approaches to deal with DM complications and the implementation of other aggregation operators, such as Bonferroni Mean AOs, Einstein AOs, Einstein-ordered AOs, and Einstein hybrid AOs, with their usefulness in daily life problems. The suggested approach has tremendous potential in fields including management, the health sciences, network analysis (Zhou and Zhang, 2022), cyber-attack systems (Randles, 2022), and automated vehicles (Zhang et al., 2023a), where decisions under unpredictability and inaccurate facts are frequent. Its application may assist in streamlining decision-making methods and results in these fields. On the other hand, the presented study can be extended to diverse environments and their applications, such as supply chain with uncertain demands (Li et al., 2020b), Enterprise and cloud Management Systems (Dai et al., 2023; Zhang et al., 2023b), feature classification and extraction methods (Unogmu and Filali, 2023; Lu et al., 2023).

CRediT authorship contribution statement

Rana Muhammad Zulqarnain: Conceptualization, Methodology, Validation, Formal analysis, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Harish Garg:** Conceptualization, Methodology, Formal analysis, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Wen-Xiu Ma:** Methodology, Validation, Writing – original draft, Formal analysis. **Imran Siddique:** Methodology, Validation, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financialinterestsor personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

Acknowledgments

The work was supported in part by NSFC under grants 12271488, 11975145 and 11972291, the Ministry of Science and Technology of China (G2021016032L), and the Natural Science Foundation for Colleges and Universities in Jiangsu Province (17 KJB 110020) .

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