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Resonance of solitons in a coupled higher-order Ito equation

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ABSTRACT

In this Letter, we propose a new coupled higher-order Ito equation and present its N-soliton solutions in Pfaffian form. Furthermore, some interesting examples of soliton resonance related to two solitons near the resonant state are pointed out.

1. Introduction

It is important in mathematical physics to look for exact solutions to soliton equations and to search for new soliton equations. In order to achieve these two objectives, several approaches have been developed. One of them is the Hirota bilinear approach [1–3], which provides a direct powerful approach to nonlinear integrable equations, and it is widely used in constructing N-soliton solutions. It is known that the Korteweg–de Vries (KdV) equation

$$u_t + 6uu_x + u_{xxx} = 0 \tag{1.1}$$

can be transformed into the bilinear form

$$D_x(D_t + D_x^3)f \cdot f = 0 \tag{1.2}$$

by the dependent variable transformation

$$u = 2(\ln f)_{xx}, \tag{1.3}$$

where the bilinear operators $D_x^m D_t^n$ are defined by

$$D_x^m D_t^n a \cdot b = (\partial_x - \partial_{x'})^m (\partial_t - \partial_{t'})^n a(x,t) b(x',t')|_{x'=x,t'=t}. \tag{1.4}$$

In [4], Ito investigated the following new type of bilinear equation:

$$D_t(D_t + D_x^3)f \cdot f = 0, \tag{1.5}$$

and noted that Eq. (1.5) and the KdV equation, Eq. (1.2), have the same 1-soliton solution but that their N-soliton solutions differ in the phase shift. Besides, using the same dependent variable transformation as Eq. (1.3), Eq. (1.5) is transformed into the nonlinear form:

$$u_{tt} + u_{xxxt} + 6u_x u_t + 3uu_{xt} + 3u_{xx}\partial_x^{-1} u_t = 0. \tag{1.6}$$

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Moreover, its Bäcklund transformation, conservation laws, and Hamiltonian structures are studied in [5,6]. In [4,7], the authors also presented a higher-order version of (1.5):

$$D_y(D_y + D_x^3)f \cdot f = 0, \tag{1.7a}$$

$$(6D_tD_y + D_x^5D_y - 5D_x^2D_y^2)f \cdot f = 0, \tag{1.7b}$$

where y is only an auxiliary variable. By the following transformations,

$$u = \ln f, \qquad p = (\ln f)_y, \qquad w = (\ln f)_{yy}, \tag{1.8}$$

we can derive a system with respect to u, w, p as follows:

$$w + p_{xxx} + 6u_{xx}p_x = 0, \tag{1.9a}$$

and

$$6p_t + p_{xxxxx} + 10p_xu_{xxxx} + 60u_{xx}^2p_x + 20u_{xx}p_{xxx} - 5w_{xx} - 20p_x^2 - 10u_{xx}w = 0, \tag{1.9b}$$

$$\begin{aligned} &w_t + w_{xxxxx} + \frac{20}{3}(w_xu_{xxxx} + p_xp_{xxxx}) + 20(2u_{xx}p_{xx}p_x + u_{xx}^2w_x) \\ &\quad + 10(p_{xx}p_{xxx} + u_{xx}w_{xxx}) + 10(p_{xxx}p_{xx} + u_{xxx}w_{xx}) - \frac{20}{3}p_xw_x = 0, \end{aligned} \tag{1.9c}$$

where the auxiliary variable y has vanished. The N-soliton solutions in Pfaffian form were obtained [8]. Meanwhile, bilinear Bäcklund transformation and nonlinear superposition formulas for Eq. (1.7) have also been presented [9].

Recently, the resonance of solitons has been studied theoretically and experimentally in many real physical models [10,11], where the interactions between solitons may be completely non-elastic. That is to say, the amplitude, velocity, and wave shape of a soliton may change after the nonlinear interaction. For instance, at a specific time, one soliton can undergo fission into two or more solitons; or, contrarily, two or more solitons may fuse into one soliton [12–15]. These types of phenomena are also called soliton fission and soliton fusion, respectively [16,17]. Furthermore, fission and fusion phenomena of the dromion, peakon, and compacton solutions have also be observed [18,19]. To describe the intermediate patterns of the resonant solitons of the shallow wave equation, Kodama considered N-soliton solutions including all possible interactions, and classified those N-soliton solutions by a chord diagram method [20,21].

In this paper, we propose a new coupled higher-order Ito equation,

$$(D_y + D_x^3)f \cdot g = 0, \tag{1.10a}$$

$$D_y(D_y + D_x^3)f \cdot g = 0, \tag{1.10b}$$

$$(6D_t + D_x^5 - 5D_x^2D_y)f \cdot g = 0, \tag{1.10c}$$

$$(6D_tD_y + D_x^5D_y - 5D_x^2D_y^2)f \cdot g = 0, \tag{1.10d}$$

where y is an auxiliary variable. It is obvious that Eq. (1.10) may lead to a higher-order Ito equation, Eq. (1.7), with $g = f$. Using the dependent variable transformation,

$$\phi = \frac{g}{f}, \qquad u = 2(\ln f)_{xx}, \tag{1.11}$$

the coupled higher-order Ito equation, Eq. (1.10), can be rewritten in the following nonlinear form:

$$\phi_y + \phi_{xxx} + 3u\phi_x = 0, \tag{1.12a}$$

$$\begin{aligned} &u_{xy} + \partial_x^{-1}u_{yy} + 3u_x\partial_x^{-1}u_y + 3uu_y = \{[3(u_y + 6uu_x + 2u_{xxx})\phi_x + 6u\phi_{xy} \\ &\quad - 3(\partial_x^{-1}u_y - 6u^2 - 6u_{xx})\phi_{xx} + 18u_x\phi_{xxx} + 2\phi_{xxxy} + 12u\phi_{xxxx} + 2\phi_{xxxxxx}]/\phi\}_x, \end{aligned} \tag{1.12b}$$

$$\begin{aligned} &6\phi_t + [\phi_{xxxxx} + 10\partial_y^{-1}u_x\phi_{xxx} + 5(\partial_y^{-1}u_{xxx} + 3(\partial_y^{-1}u_x)^2)\phi_x] \\ &\quad - 5(\phi_{xxy} + \phi_y\partial_y^{-1}u_x + 2u\phi_x) = 0, \end{aligned} \tag{1.12c}$$

$$\begin{aligned} &u_t - \frac{10}{3}uu_x - \frac{5}{6}u_{xxy} + \frac{5}{2}(\partial_y^{-1}u_x)^2u_x - \frac{5}{6}[\partial_y^{-1}u_{xx}\partial_x^{-1}u_y + \partial_y^{-1}u_xu_y \\ &\quad - \partial_y^{-1}u_{xxxx}u - \partial_y^{-1}u_{xxx}u_x] + \frac{5}{3}[\partial_y^{-1}u_{xx}u_{xx} + \partial_y^{-1}u_xu_{xxx}] + \frac{1}{6}u_{xxxxx} \\ &\quad + 5\partial_y^{-1}u_x\partial_y^{-1}u_{xx}u = \frac{5}{6}\{[u_y\phi_x - 6\partial_y^{-1}u_xu_x\phi_x - u_{xxx}\phi_x + u_x\phi_y \\ &\quad + 6\partial_y^{-1}u_xu\phi_{xx} + 2u_{xx}\phi_{xx} + u\phi_{xxxx} - 2u_x\phi_{xxx} - 2u\phi_{xy} - \partial_x^{-1}u_y\phi_{xx}]/\phi\}_x. \end{aligned} \tag{1.12d}$$

The purpose of this paper is to give the N-soliton solutions and to analyze the resonant soliton phenomena for this coupled higher-order Ito equation. Using the perturbation and Pfaffian technique, we present N-soliton solutions to Eq. (1.10), and give a strict proof. In addition, we discuss the resonance of solitons, described by a 2-soliton solutions of the coupled higher-order Ito equation.

2. *N*-soliton solutions to the coupled higher-order Ito equation

It is known that Eq. (1.9) belongs to the DKP hierarchy (dispersionless KP hierarchy) and the solutions of the DKP hierarchy can be written in Pfaffian form [22]. In the following, in relation to the D-type Lie algebraic structure of the solutions, we present the N-soliton solutions to Eq. (1.10) by virtue of Pfaffians and give a strict proof by the Pfaffian identity.

Using the perturbational method, we obtain 2-soliton solutions and 3-soliton solutions to Eq. (1.9), expressed as follows:

$$f = 1 + a_1 e^{\eta_1} + a_2 e^{\eta_2} + a_{12} e^{\eta_1+\eta_2}, \tag{2.1a}$$

$$g = 1 + b_1 e^{\eta_1} + b_2 e^{\eta_2} + b_{12} e^{\eta_1+\eta_2}, \tag{2.1b}$$

and

$$f = 1 + a_1 e^{\eta_1} + a_2 e^{\eta_2} + a_3 e^{\eta_3} + a_{12} e^{\eta_1+\eta_2} + a_{13} e^{\eta_1+\eta_3} + a_{23} e^{\eta_2+\eta_3} + a_{123} e^{\eta_1+\eta_2+\eta_3}, \tag{2.2a}$$

$$g = 1 + b_1 e^{\eta_1} + b_2 e^{\eta_2} + b_3 e^{\eta_3} + b_{12} e^{\eta_1+\eta_2} + b_{13} e^{\eta_1+\eta_3} + b_{23} e^{\eta_2+\eta_3} + b_{123} e^{\eta_1+\eta_2+\eta_3}, \tag{2.2b}$$

with

$$\eta_j = p_j x - p_j^3 y - p_j^5 t, \tag{2.3a}$$

$$a_{ij} = \frac{(p_i - p_j)}{(p_i + p_j)(p_i^3 + p_j^3)} \alpha_{ij}, \tag{2.3b}$$

$$b_{ij} = \frac{(p_i - p_j)}{(p_i + p_j)(p_i^3 + p_j^3)} \beta_{ij}, \tag{2.3c}$$

$$\alpha_{ij} = a_i p_i^3 b_j - a_j p_j^3 b_i, \qquad \beta_{ij} = b_i p_i^3 a_j - b_j p_j^3 a_i \quad (i, j = 1, 2, 3), \tag{2.3d}$$

$$a_{123} = \frac{(p_1 - p_2)(p_1 - p_3)(p_2 - p_3)}{(p_1 + p_2)(p_1 + p_3)(p_2 + p_3)(p_1^3 + p_2^3)(p_1^3 + p_3^3)(p_2^3 + p_3^3)} \alpha_{123}, \tag{2.3e}$$

$$b_{123} = \frac{(p_1 - p_2)(p_1 - p_3)(p_2 - p_3)}{(p_1 + p_2)(p_1 + p_3)(p_2 + p_3)(p_1^3 + p_2^3)(p_1^3 + p_3^3)(p_2^3 + p_3^3)} \beta_{123}, \tag{2.3f}$$

$$\alpha_{123} = -[a_1 a_2 (p_1^6 - p_2^6) b_3 p_3^3 - a_1 a_3 (p_1^6 - p_3^6) b_2 p_2^3 + a_2 a_3 (p_2^6 - p_3^6) b_1 p_1^3], \tag{2.3g}$$

$$\beta_{123} = -[b_1 b_2 (p_1^6 - p_2^6) a_3 p_3^3 - b_1 b_3 (p_1^6 - p_3^6) a_2 p_2^3 + b_2 b_3 (p_2^6 - p_3^6) a_1 p_1^3], \tag{2.3h}$$

where p_j, a_j, b_j are free parameters.

These expressions suggest that N-soliton solutions to Eq. (1.10) are expressed by Pfaffians. In fact, we find that

$$f = pf(d_0, a, r_1, r_2, \ldots, r_N, c_N, \ldots, c_2, c_1) = pf(d_0, a, \bullet), \tag{2.4a}$$

$$g = pf(a, b, r_1, r_2, \ldots, r_N, c_N, \ldots, c_2, c_1) = pf(a, b, \bullet), \tag{2.4b}$$

where the entries of the Pfaffians are defined as

$$\begin{aligned}
&pf(d_m, r_j) = p_j^m e^{\eta_j}, \quad (m \geq 0,\ j = 1, 2, \ldots, N),\\
&pf(d_0, a) = 1, \qquad pf(a, b) = 1, \qquad pf(d_m, a) = 0, \quad (m \geq 1),\\
&pf(a, r_j) = -e^{\eta_j}, \qquad pf(a, c_j) = -a_j, \qquad pf(b, c_j) = b_j, \quad (j = 1, 2, \ldots, N),\\
&pf(r_j, r_k) = a_{j,k} e^{\eta_j+\eta_k}, \qquad pf(r_j, c_k) = \delta_{j,k}, \qquad pf(c_j, c_k) = -c_{j,k}, \quad (j, k = 1, 2, \ldots, N),\\
&pf(d_m, c_j) = pf(d_m, b) = pf(d_m, d_n) = pf(b, r_j) = 0, \quad (m, n \geq 0,\ j = 1, 2, \ldots, N),
\end{aligned} \tag{2.4c}$$

where

$$\begin{aligned}
&\delta_{j,k} = \begin{cases} 1, & j = k, \\ 0, & j \neq k, \end{cases}\\
&\eta_j = p_j x - p_j^3 y - p_j^5 t,\\
&a_{j,k} = \frac{p_j - p_k}{p_j + p_k}, \qquad c_{j,k} = \frac{a_j p_j^3 b_k - a_k p_k^3 b_j}{p_j^3 + p_k^3}.
\end{aligned} \tag{2.4d}$$

The above Pfaffians have $3N$ parameters, p_j, a_j, b_j, for $j = 1, 2, \ldots, N$.

In what follows, we show that f and g given by Eq. (2.4) are N-soliton solutions to Eq. (1.10). By virtue of the above Pfaffians, we come up with the following differential formulae for f and g:

$$f_x = pf(d_1, a, \bullet), \qquad f_{xx} = pf(d_2, a, \bullet), \tag{2.5a}$$

$$f_{xxx} = pf(d_3, a, \bullet) + pf(d_0, d_1, d_2, a, \bullet), \tag{2.5b}$$

$$f_{xxxx} = pf(d_4, a, \bullet) + 2pf(d_0, d_1, d_3, a, \bullet), \tag{2.5c}$$

$$f_{xxxxx} = pf(d_5, a, \bullet) + 3pf(d_0, d_1, d_4, a, \bullet) + 2pf(d_0, d_2, d_3, a, \bullet), \tag{2.5d}$$

$$f_y = -pf(d_3, a, \bullet) + 2pf(d_0, d_1, d_2, a, \bullet), \tag{2.5e}$$

$$f_{xy} = -pf(d_4, a, \bullet) + pf(d_0, d_1, d_3, a, \bullet), \tag{2.5f}$$

$$f_{xxy} = -pf(d_5, a, \bullet) + pf(d_0, d_2, d_3, a, \bullet), \tag{2.5g}$$

$$f_t = -pf(d_5, a, \bullet) + 2pf(d_0, d_1, d_4, a, \bullet) + 3pf(d_0, d_2, d_3, a, \bullet), \tag{2.5h}$$

$$g_x = pf(d_0, d_1, a, b, \bullet), \qquad g_{xx} = pf(d_0, d_2, a, b, \bullet), \tag{2.5i}$$

$$g_{xxx} = pf(d_0, d_3, a, b, \bullet) + pf(d_1, d_2, a, b, \bullet), \tag{2.5j}$$

$$g_{xxxx} = pf(d_0, d_4, a, b, \bullet) + 2pf(d_1, d_3, a, b, \bullet), \tag{2.5k}$$

$$g_{xxxxx} = pf(d_0, d_5, a, b, \bullet) + 3pf(d_1, d_4, a, b, \bullet) + 2pf(d_2, d_3, a, b, \bullet), \tag{2.5l}$$

$$g_y = -pf(d_0, d_3, a, b, \bullet) + 2pf(d_1, d_2, a, b, \bullet), \tag{2.5m}$$

$$g_{xy} = -pf(d_0, d_4, a, b, \bullet) + pf(d_1, d_3, a, b, \bullet), \tag{2.5n}$$

$$g_{xxy} = -pf(d_0, d_5, a, b, \bullet) + pf(d_2, d_3, a, b, \bullet), \tag{2.5o}$$

$$g_t = -pf(d_0, d_5, a, b, \bullet) + 2pf(d_1, d_4, a, b, \bullet) + 3pf(d_2, d_3, a, b, \bullet). \tag{2.5p}$$

Substituting these relations into Eq. (1.10a), Eq. (1.10c), we find that Eq. (1.10a) is reduced to the Pfaffian identity

$$\begin{aligned} &pf(d_0, d_1, d_2, a, \bullet)pf(a, b, \bullet) - pf(d_0, d_1, a, b, \bullet)pf(d_2, a, \bullet) \\ &\quad + pf(d_0, d_2, a, b, \bullet)pf(d_1, a, \bullet) - pf(d_1, d_2, a, b, \bullet)pf(d_0, a, \bullet) \equiv 0, \end{aligned} \tag{2.6a}$$

and Eq. (1.10c) is reduced to Pfaffian identity

$$\begin{aligned} &15[pf(a, b, \bullet)pf(d_0, d_1, d_4, a, \bullet) - pf(d_0, d_1, a, b, \bullet)pf(d_4, a, \bullet) \\ &\quad - pf(d_0, a, \bullet)pf(d_1, d_4, a, b, \bullet) + pf(d_0, d_4, a, b, \bullet)pf(d_1, a, \bullet)] \\ &\quad \times 15[pf(a, b, \bullet)pf(d_0, d_2, d_3, a, \bullet) + pf(d_0, d_2, a, b, \bullet)pf(d_3, a, \bullet) \\ &\quad - pf(d_0, a, \bullet)pf(d_2, d_3, a, b, \bullet) - pf(d_0, d_3, a, b, \bullet)pf(d_2, a, \bullet)] \equiv 0. \end{aligned} \tag{2.6b}$$

Furthermore, in order to prove that (2.4) also satisfies Eq. (1.10b) and Eq. (1.10d), we have to use the second expression for g, which is equal to (2.4b):

$$g = pf(e_0, a, r_1, r_2, \ldots, r_N, c_N, \ldots, c_2, c_1) = pf(e_0, a, \bullet), \tag{2.7a}$$

where the new entries are defined by

$$\begin{aligned} &pf(e_0, a) = 1, \qquad pf(e_1, a) = 0, \qquad pf(e_m, c_j) = -b_j p_j^{3m}, \quad (m = 0, 1), \\ &pf(e_m, r_j) = pf(d_m, e_1) = 0, \quad (m \geq 0,\ j = 1, 2, \ldots, N). \end{aligned}$$

Using the properties of the Pfaffian [1], we obtain the following differential formulas:

$$g_y = pf(e_1, a, \bullet), \qquad g_{xy} = pf(d_0, d_1, e_1, a, \bullet), \qquad g_{xxy} = pf(d_0, d_2, e_1, a, \bullet), \tag{2.7b}$$

$$g_{xxxy} = pf(d_0, d_3, e_1, a, \bullet) + pf(d_1, d_2, e_1, a, \bullet), \tag{2.7c}$$

$$g_{xxxxy} = pf(d_0, d_4, e_1, a, \bullet) + 2pf(d_1, d_3, e_1, a, \bullet), \tag{2.7d}$$

$$g_{xxxxxy} = pf(d_0, d_5, e_1, a, \bullet) + 3pf(d_1, d_4, e_1, a, \bullet) + 2pf(d_2, d_3, e_1, a, \bullet), \tag{2.7e}$$

$$g_{yy} = -pf(d_0, d_3, e_1, a, \bullet) + 2pf(d_1, d_2, e_1, a, \bullet), \tag{2.7f}$$

$$g_{xyy} = -pf(d_0, d_4, e_1, a, \bullet) + pf(d_1, d_3, e_1, a, \bullet), \tag{2.7g}$$

$$g_{xxyy} = -pf(d_0, d_5, e_1, a, \bullet) + pf(d_2, d_3, e_1, a, \bullet), \tag{2.7h}$$

$$g_{ty} = -pf(d_0, d_5, e_1, a, \bullet) + 2pf(d_1, d_4, e_1, a, \bullet) + 3pf(d_2, d_3, e_1, a, \bullet). \tag{2.7i}$$

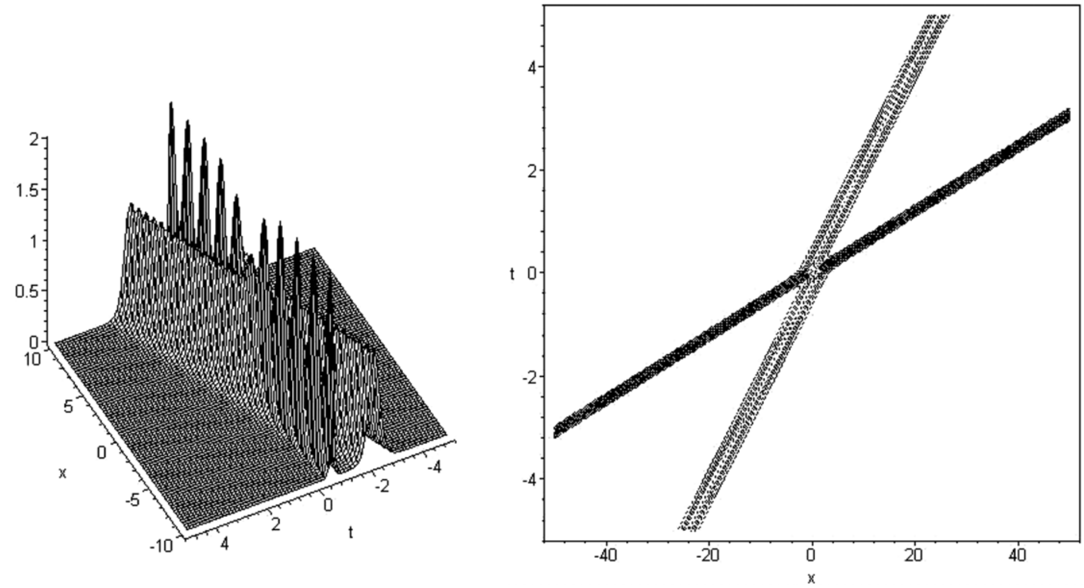

Fig. 1. The plot of the regular interaction of two solitons to the new coupled higher-order Ito equation. The parameters used are $a_1 = b_1$, $a_2 = b_2$, $p_1 = 1.5$, $p_2 = 2$, $b_1 = 3$, $b_2 = 4$, $y = 0$. The left figure shows a three-dimensional plot and the right figure shows a contour map.

Moreover, we have to rewrite Eq. (1.9b) and Eq. (1.9d) in the following form:

$$(D_y + D_x^3)f \cdot g_y = 0, \tag{2.8a}$$

$$(6D_t + D_x^5 - 5D_x^2 D_y)f \cdot g_y = 0, \tag{2.8b}$$

where we have used the y-derivative of Eqs. (1.9a) and (1.9c). Substituting (2.4a), (2.5a)–(2.5h), and (2.7) into (2.8), we find that Eq. (2.8a) is reduced to the Pfaffian identity

$$\begin{aligned} &pf(d_0, d_1, d_2, a, \bullet)pf(e_1, a, \bullet) - pf(d_0, d_1, e_1, a, \bullet)pf(d_2, a, \bullet) \\ &\quad + pf(d_0, d_2, e_1, a, \bullet)pf(d_1, a, \bullet) - pf(d_1, d_2, e_1, a, \bullet)pf(d_0, a, \bullet) \equiv 0, \end{aligned} \tag{2.9a}$$

and Eq. (2.8b) is reduced to the Pfaffian identity

$$\begin{aligned} &15[pf(e_1, a, \bullet)pf(d_0, d_1, d_4, a, \bullet) - pf(d_0, d_1, e_1, a, \bullet)pf(d_4, a, \bullet) \\ &\quad - pf(d_0, a, \bullet)pf(d_1, d_4, e_1, a, \bullet) + pf(d_0, d_4, e_1, a, \bullet)pf(d_1, a, \bullet)] \\ &\quad \times 15[pf(e_1, a, \bullet)pf(d_0, d_2, d_3, a, \bullet) + pf(d_0, d_2, e_1, a, \bullet)pf(d_3, a, \bullet) \\ &\quad - pf(d_0, a, \bullet)pf(d_2, d_3, e_1, a, \bullet) - pf(d_0, d_3, e_1, a, \bullet)pf(d_2, a, \bullet)] \equiv 0. \end{aligned} \tag{2.9b}$$

Thus we have proved that the N-soliton solutions given in Eq. (2.4) actually satisfy the bilinear equations Eq. (1.9).

3. Resonance phenomena of solitons

In this section, we discuss the details of interactions between two solitons using the 2-soliton solutions of the coupled higher-order Ito equation – amongst which are resonant solitons – and we compare their interaction properties with similar solutions for the higher-order Ito equation. The interactions are classified into four types, depending on the value of the phase shift a_{12}. In the accompanying figures, we plot only $u = 2(\ln f)_{xx}$ of the coupled higher-order Ito equation, Eq. (1.11).

For the case of the a_{12} being finite, the resulting solution, Eq. (2.1a), represents regular interaction of two solitons, in which the larger soliton takes over the smaller soliton (see Fig. 1).

For the case of the solution Eq. (2.1a) under a resonant condition, $a_{12} = 0$, we show that two solitons fuse into one soliton after colliding with each other (see Fig. 2), or that one soliton splits into two solitons in the resonant state (see Fig. 3).

For the case of the solution Eq. (2.1a) under another resonant condition, $a_{12} \to \infty$, we show that two solitons fuse into one soliton after colliding with each other in the resonant state, and then this splits into two solitons at the end of the resonant state (see Fig. 4).

For the case of the solution Eq. (2.1a) under a quasi-resonant condition, $a_{12} \to 0$, the resulting solution, Eq. (2.1a), shows that a higher soliton splits into two solitons as it approaches a lower soliton, then one of the two solitons moves and collides with the lower soliton, and finally the lower soliton exchanges energy with the higher soliton by fusing one of the two solitons (see Fig. 5).

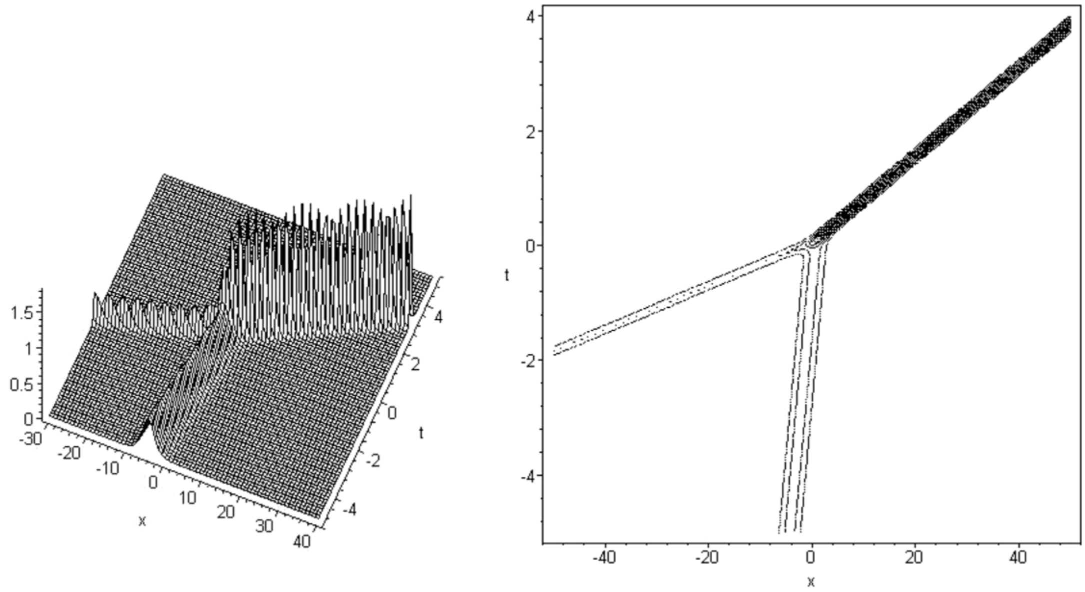

Fig. 2. The plot of two solitons fusing into a large soliton to the new coupled higher-order Ito equation. The parameters used are $a_1 = \frac{p_2}{p_1}$, $b_2 = \frac{a_2 p_2^2 b_1}{p_1^2}$, $p_1 = 1.1$, $p_2 = 1.9$, $a_2 = 2$, $y = 0$. The left figure shows a three-dimensional plot and the right figure shows a contour map.

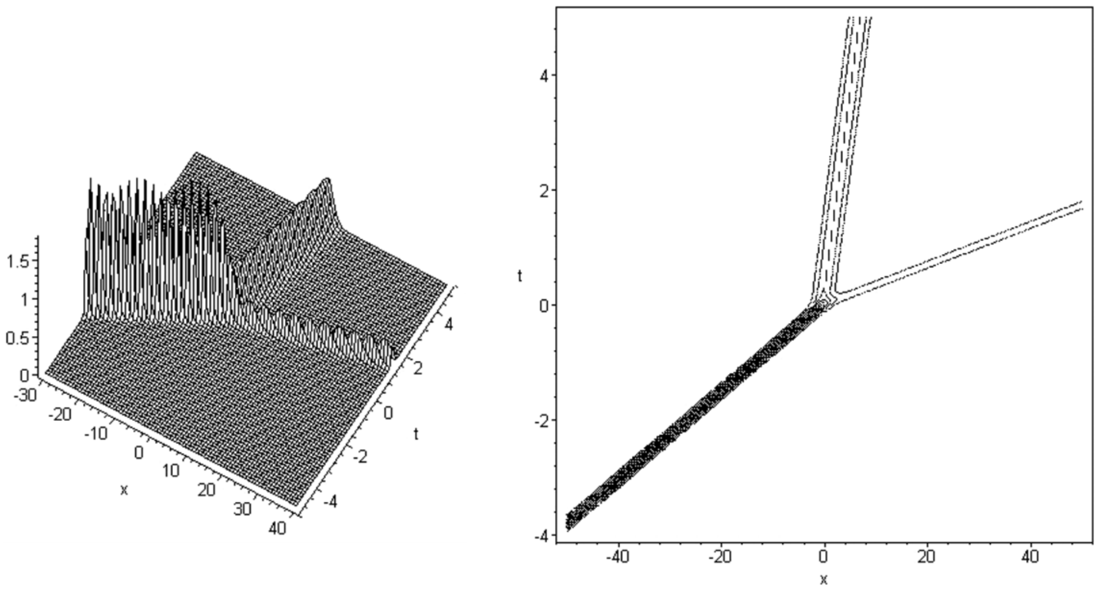

Fig. 3. The plot of one soliton splitting into two solitons to the new coupled higher-order Ito equation. The parameters used are $a_1 = \frac{p_2}{p_1}$, $b_2 = \frac{a_2 p_2^2 b_1}{p_1^2}$, $p_1 = -1.9$, $p_2 = -1$, $a_2 = 2$, $y = 0$. The left figure shows a three-dimensional plot and the right figure shows a contour map.

4. Conclusions

A new type of coupled higher-order Ito equation has been given, and N-soliton solutions have been obtained in the form of Pfaffians. The result that the coupled higher-order Ito equation possesses N-soliton solutions suggests this system might be a candidate of integrable equations. In this aspect, we will present a Bäcklund transformation for Eq. (1.10) to confirm the integrability of the coupled higher-order Ito equation; details will be given elsewhere.

For more parameters than the higher-order Ito equation, Eq. (1.7), the 2-soliton solutions of Eq. (1.10) exhibit some special phenomena, such as one soliton undergoing fission into two solitons or two solitons fusing into one soliton at the resonant state after colliding with each other. Owing to only needing two solitons to analyze fission, fusion, and mixed

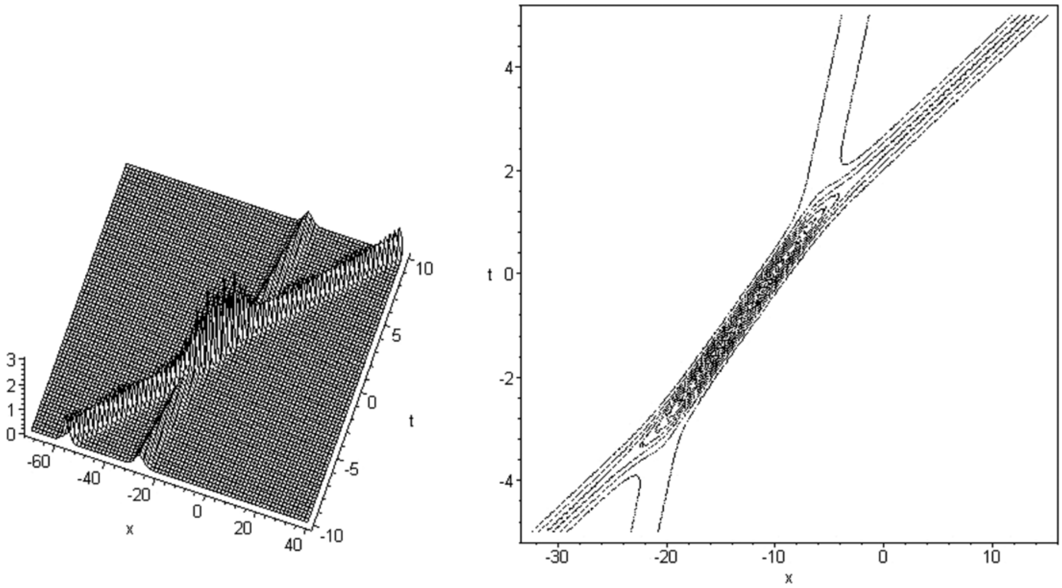

Fig. 4. The plot of two solitons fusing into one soliton and then splitting into two solitons to the new coupled higher-order Ito equation. The parameters used are $a_1 = 2 * 10^3$, $a_2 = 4 * 10^3$, $p_1 = 1$, $p_2 = 1.5$, $b_1 = 10^7$, $b_2 = 10^9$, $y = 0$. The left figure shows a three-dimensional plot and the right figure shows a contour map.

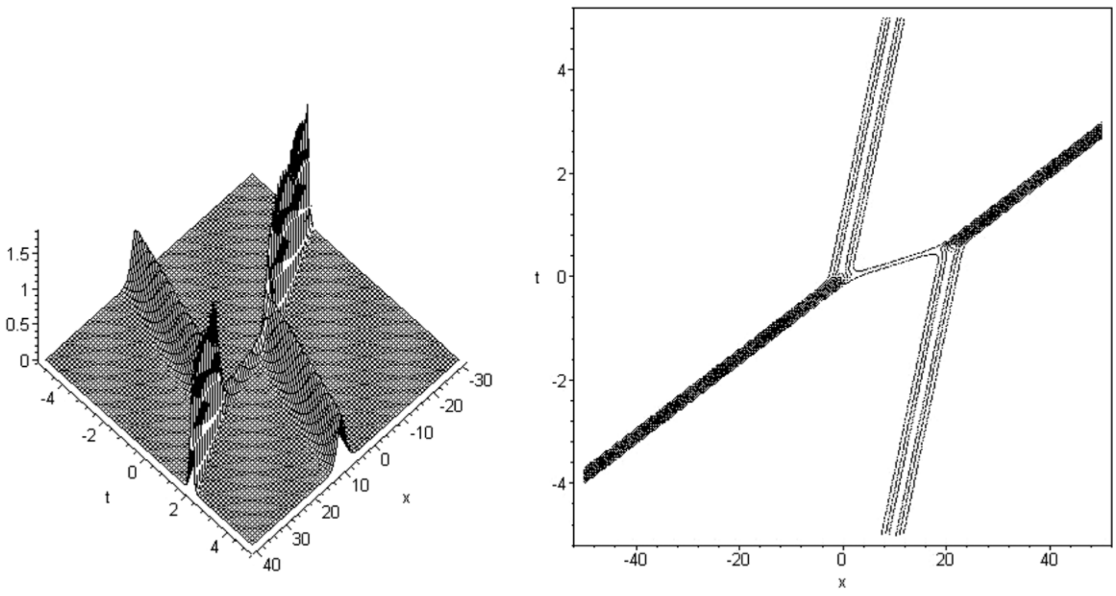

Fig. 5. The plot of two solitons splitting into three solitons and then fusing into two solitons to the new coupled higher-order Ito equation. The parameters used are $a_1 = \frac{p_2}{p_1} + 10^{-10}$, $b_2 = \frac{a_2 p_2^2 b_1}{p_1^2}$, $p_1 = 1.9$, $p_2 = 1.2$, $a_2 = 2$, $b_1 = 1$, $b_2 = 3$, $y = 0$. The left figure shows a three-dimensional plot and the right figure shows a contour map.

collision phenomena, we believe that three or more solitons must exhibit more special phenomena which have not observed before.

We also have not considered the positive and negative nature of a_1, a_1, a_{12}. In fact, for the case of a_{12} being negative and finite, the resulting solution, Eq. (2.1a), could represent one particular phenomenon, in which two regular solitons transmute into two singular solitons after colliding with each other (see Fig. 6). Although we have obtained a novel coupled higher-order Ito system in bilinear form, it is an interesting problem to derive a coupled higher-order Ito system without the auxiliary variable y. Detailed studies of these problems are left for the future.

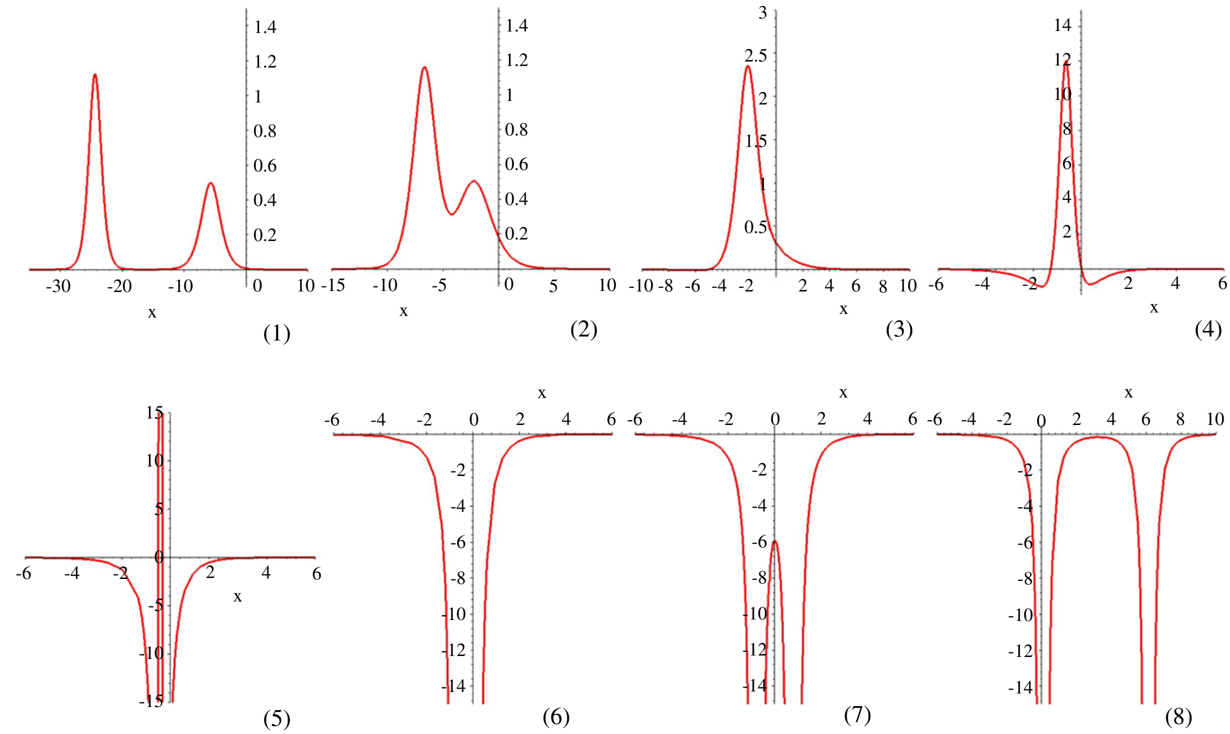

Fig. 6. The plot of two solitons transmuting into two singular solitons after colliding with each other to the new coupled higher-order Ito equation. The parameters used are $a_1 = 2$, $a_2 = 4$, $p_1 = 1$, $p_2 = -1.5$, $b_1 = 10$, $b_2 = 70$, $y = 0$, where $(1) t = -5$, $(2) t = -1.5$, $(3) t = -0.6$, $(4) t = -0.2$, $(5) t = -0.127$, $(2) t = -0.1$, $(3) t = 0$, $(4) t = 1$.

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