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Binary Darboux transformation and soliton solutions for the coupled complex modified Korteweg-de Vries equations

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This paper considers the coupled complex modified Korteweg-de Vries (mKdV) equations and presents a binary Darboux transformation for the equations. As a direct application, we give a classification of general soliton solutions derived from vanishing and non-vanishing backgrounds, on the basis of the dynamical behavior of the solutions. Special types of solutions in the presented solutions include breathers, bright-bright solitons, bright-dark solitons, bright-W-shaped solitons, and rogue wave solutions. Furthermore, dynamics and interactions of vector bright solitons are exhibited.

KEYWORDS
binary Darboux transformation, coupled complex mKdV equation, soliton, rogue wave

MSC CLASSIFICATION
35Q51; 37K10

1 | INTRODUCTION

As is well known, integrable nonlinear models play an important role in the study of nonlinear wave propagation. Most of such systems are often widely applicable in nonlinear optics, water waves, hydrodynamics, plasma physics, and molecular biology. Among them, the nonlinear Schrödinger equation (NLS), the Korteweg-de Vries (KdV) equation, and the sine-Gordon equation are celebrated examples. Since integrable systems have exceptional mathematical properties and numerous physical applications, their multi-component generalizations have attracted much attention of many researchers. One of the most famous examples are Manakov's two-component NLS equations, which has been studied by a few authors.[1-5]

Another interesting soliton equation is the complex modified Korteweg-de Vries (mKdV) equation

$$u_t + u_{xxx} + 6|u|^2 u_x = 0, \tag{1}$$

which is also the third non-trivial member of the NLS hierarchy, and its integrability and large class of exact solutions have been studied extensively, such as conservation laws, exact group invariant solutions,[6] periodic traveling waves solutions,[7] breather solutions, rogue wave solutions,[8,9] interaction complex solitons,[10] and numerical solutions.[11] Liu et al[12] gave the explicit formulas of smooth positon solutions and discuss the decomposition, trajectory, and phase shift of the positon solutions. Recently, Ablowitz and Musslimani[13] proposed the nonlocal integrable complex mKdV equation. In addition, in Ma et al,[14] the dark soliton solutions, W-type solitons, M-type solitons, and periodic solutions of the nonlocal complex mKdV equation were investigated by using the Darboux transformation (DT).

It is essential to note that generalizations of the mKdV equation to a multi-component system, such as a matrix equation, has been studied by many scholars. For example, a vector version of the mKdV equation was proposed by Yajima and Oikawa.[15] Sasa and Satsuma solved the initial value problem of the equation, and obtain multi-soliton solutions.[16] Iwao and Hirota considered a coupled version of the mKdV and constructed multi-soliton solutions.[17] Tsuchida and Wadati extended a matrix version of the mKdV equation.[18] Furthermore, on the basis of the matrix form inverse scattering formulation, Zhang et al investigated the multi-soliton solutions.[19]

The complex mKdV equation can be extended to the coupled case

$$\begin{aligned} u_t + u_{xxx} + 6(|u|^2 + |v|^2)u_x = 0, \\ v_t + v_{xxx} + 6(|u|^2 + |v|^2)v_x = 0, \end{aligned} \tag{2}$$

which can be used to describe the interaction of two orthogonally polarized transverse waves. Meanwhile, it has been proposed as a model for the nonlinear evolution of plasma waves and incorporates the propagation of transverse waves in a molecular chain model.

The binary DT is an efficient method to construct exact solutions for integrable systems.[20-25] In the present paper, by iterating a standard binary DT, a variety of soliton solutions of Equation (2) in terms of the quasideterminants are generated from the zero and nonzero backgrounds. Nice quasideterminants play a prominent role in presenting binary DT of nonlinear physical systems.

The outline of this paper is as follows. In Section 2, we first present the Lax pair and construct a 4×2 eigenfunction and corresponding constant 2×2 square matrix for the eigenvalue problem of Equation (2), and then we propose a binary DT, which can be considered as the dimensional reductions from $(2 + 1)$ to $(1 + 1)$, on the basis which an Nth fold iterative transformation in terms of quasideterminants will be derived. In Section 3, several special types of solutions from vanishing and non-vanishing backgrounds are derived and classified. These solutions include bright-dark solitons, bright-bright solitons, bright-W-shaped solitons, breather solutions, periodic solutions, and rogue wave solutions. Finally, Section 4 contains some discussions.

2 | LAX PAIR AND BINARY DARBOUX TRANSFORMATION

2.1 | Lax pair

The Lax pair for the coupled complex modified KdV equations is given by

$$\begin{aligned} \Phi_x = -(L - \partial_x)\Phi, \\ \Phi_t = -(M - \partial_t)\Phi, \end{aligned} \tag{3}$$

where

$$L = \partial_x + \lambda J + R, \quad M = \partial_t + 4\lambda^3 J + 4\lambda^2 R - 2\lambda Q + W,$$

with

$$J = i\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad R = \begin{pmatrix} 0 & -G \\ G^\dagger & 0 \end{pmatrix}, \quad G = \begin{pmatrix} u^* & -v \\ v^* & u \end{pmatrix},$$

$$Q = i\begin{pmatrix} GG^\dagger & G_x \\ G_x^\dagger & -GG^\dagger \end{pmatrix}, \quad W = \begin{pmatrix} G_xG^\dagger - GG_x^\dagger & G_{xx} + 2GG^\dagger G \\ -G_{xx}^\dagger - 2G^\dagger GG^\dagger & G_x^\dagger G - G^\dagger G_x \end{pmatrix},$$

where I and O denote the 2×2 identity matrix and 2×2 zero matrix, the * and the † represent the complex conjugate and the Hermitian conjugate, respectively. The compatibility condition $\Phi_{xt} = \Phi_{tx}$ generates Equation (2).

To obtain some interesting exact solutions for the system (2), it is essential to derive the corresponding DT. Before presenting the DT, we give the following symmetry property for the Lax pair (3).

Proposition 2.1. *If $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$ is an eigenfunction of the Lax pair (3) with an eigenvalue λ, then $\psi = (\phi_2^*, -\phi_1^*, \phi_4^*, -\phi_3^*)^T$ is also an eigenfunction of the Lax pair (3) with an eigenvalue $-\lambda^*$.*

One can check this by direct computations. Thus, we can define a 4×2 matrix eigenfunction θ with a 2×2 eigenvalue matrix Λ, where

$$\theta = \begin{pmatrix} \phi_1 & \phi_2^* \\ \phi_2 & -\phi_1^* \\ \phi_3 & \phi_4^* \\ \phi_4 & -\phi_3^* \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda^* \end{pmatrix} \tag{4}$$

satisfying

$$\theta_x + J\theta\Lambda + R\theta = 0, \tag{5a}$$

$$\theta_t + 4J\theta\Lambda^3 + 4R\theta\Lambda^2 - 2Q\theta\Lambda + W\theta = 0. \tag{5b}$$

2.2 | Binary Darboux transformation

From the knowledge of the binary DT, we consider the following linear differential operators

$$L = \partial_x + \sum_{i=0}^{N} u_i \partial_y^i, \quad M = \partial_t + \sum_{i=0}^{N} v_i \partial_y^i, \tag{6}$$

where u_i and v_i are $m\times m$ matrices.

Then, we can propose the following standard DT.

Theorem 2.1. *Let θ be a non-singular $m\times m$ matrix solution of the linear system*

$$L(\phi) = M(\phi) = 0. \tag{7}$$

Then, the Darboux transformation $\tilde{\phi} = G_\theta(\phi) = \theta\partial_y\theta^{-1}\phi$ keeps the linear system $L(\phi) = M(\phi) = 0$ invariant:

$$\tilde{L}(\tilde{\phi}) = \tilde{M}(\tilde{\phi}) = 0, \tag{8}$$

where the linear operators $\tilde{L} = G_\theta L G_\theta^{-1}$ and $\tilde{M} = G_\theta M G_\theta^{-1}$ have the same forms as L and M:

$$\tilde{L} = \partial_x + \sum_{i=0}^{N} \tilde{u}_i \partial_y^i, \quad \tilde{M} = \partial_t + \sum_{i=0}^{N} \tilde{v}_i \partial_y^i. \tag{9}$$

The corresponding binary DT is constructed by composing a DT with the inverse of another.

Theorem 2.2. *Let θ and ρ be $m\times k$ matrix solutions of the linear system $L(\phi) = M(\phi) = 0$ and its adjoint system $L^\dagger(\psi) = M^\dagger(\psi) = 0$, respectively. Then, a binary Darboux transformation*

$$B_{\theta,\rho} = I - \theta\Omega(\theta,\rho)^{-1}\Omega(\cdot,\rho) \tag{10}$$

and its adjoint

$$B_{\theta,\rho}^{-\dagger} = I - \rho\Omega(\theta,\rho)^{-\dagger}\Omega(\theta,\cdot)^\dagger, \tag{11}$$

where $\Omega(\theta,\rho)_y = \rho^\dagger\theta$, preserve the linear system $L(\phi) = M(\phi) = 0$ and its adjoint system $L^\dagger(\psi) = M^\dagger(\psi) = 0$, respectively.

In the following, we construct the Nth-iterated binary DT on the basis of the above theorem.

Theorem 2.3. *Let $\theta_1, \theta_2, \ldots, \theta_N$ be N linearly independent solutions of the linear system $L(\phi) = M(\phi) = 0$, and $\rho_1, \rho_2, \ldots, \rho_N$ be N linearly independent solutions of its adjoint system $L^\dagger(\psi) = M^\dagger(\psi) = 0$. Then, the N-fold binary Darboux transformation can be represented in terms of the quasideterminants*

$$\phi[N] = \begin{vmatrix} \Omega(\Theta, P) & \Omega(\phi, P) \\ \Theta & \boxed{\phi} \end{vmatrix}, \tag{12a}$$

$$\psi[N] = \begin{vmatrix} \Omega(\Theta, P)^{\dagger} & \Omega(\Theta, \psi)^{\dagger} \\ P & \boxed{\psi} \end{vmatrix}, \tag{12b}$$

where $\Theta = (\theta_1, \theta_2, \ldots, \theta_N)$, $P = (\rho_1, \rho_2, \ldots, \rho_N)$, $\Omega(\Theta, P) = \Omega((\theta_i, \rho_j))_{i,j=1,2,\ldots,N}$ *is an* $N \times N$ *matrix,* $\Omega(\phi, P) = (\Omega(\phi, \rho_j))_{j=1,2,\ldots,N}$ *is an* $N \times 1$ *vector, and* $\Omega(\Theta, \psi) = (\Omega(\psi, \theta_i))_{i=1,2,\ldots,N}$ *is a* $1 \times N$ *vector.*

Now, we consider the reduction of the binary DT from $(2+1)$ to $(1+1)$ dimensions by the separation of variables

$$\begin{aligned} \phi = \phi^r(x,t)e^{\lambda y}, \quad \theta = \theta^r(x,t)e^{\Lambda y}, \\ \psi = \psi^r(x,t)e^{\mu y}, \quad \rho = \rho^r(x,t)e^{\Pi y}, \end{aligned} \tag{13}$$

where λ, μ are constant scalars, and Λ, Π are $N \times N$ constant matrices. Hence, the matrix operator L and M become

$$L^r = \partial_x + \sum_{i=0}^{N} u_i \lambda^i, \quad M^r = \partial_t + \sum_{i=0}^{N} v_i \lambda^i. \tag{14}$$

From this, it follows that the y dependence of the potential Ω can be also made explicitly as follows:

$$\Omega(\theta, \rho) = e^{\Pi^{\dagger} y}\Omega^r(\theta^r, \rho^r)e^{\Lambda y}, \quad \Omega(\phi, \rho) = e^{(\Pi^{\dagger} + \lambda I)y}\Omega^r(\phi^r, \rho^r), \tag{15a}$$

where Π and Λ satisfy

$$\Pi^{\dagger}\Omega^r(\theta^r, \rho^r) + \Omega^r(\theta^r, \rho^r)\Lambda = \rho^{r\dagger}\theta^r, \quad (\Pi^{\dagger} + \lambda I)\Omega^r(\phi^r, \rho^r) = \rho^{r\dagger}\phi^r. \tag{15b}$$

Then, for notational simplicity, we omit the superscript r denoting reduced objects and discuss the binary DT in this reduced case. For example, if we choose the involved constant matrices to be diagonal, $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_N)$ and $\Pi = diag(\mu_1, \mu_2, \ldots, \mu_N)$, we could obtain the explicit expressions

$$\Omega(\theta, \rho) = \frac{(\rho^{\dagger}\theta)_{ij}}{\lambda_j + \mu_i^*}, \quad \Omega(\phi, \rho) = \frac{(\rho^{\dagger}\phi)_{ij}}{\lambda + \mu_i^*}, \quad (i, j = 1, 2, \ldots, N). \tag{16}$$

In the Lax operators L and M, the matrix coefficients J and R are both skew-Hermitian. So we can easily check $L + L^{\dagger} = 0, M + M^{\dagger} = 0$. On the basis of this property, we choose $\rho = \theta$ and $\Pi = -\Lambda$ to keep constraints among the potentials in the matrix R. In addition, L is form invariant under the binary DT

$$L \to \tilde{L} = B_{\theta,\rho} L B_{\theta,\rho}^{-1} = \partial_x + \lambda J + \tilde{R}, \tag{17}$$

where

$$\tilde{R} = R + [J, \theta\Omega(\theta, \theta)^{-1}\theta^{\dagger}]. \tag{18}$$

For notational convenience, we introduce the following 4×4 matrix

$$S = \frac{1}{2i}\begin{pmatrix} 0 & G \\ G^{\dagger} & 0 \end{pmatrix}, \tag{19}$$

and then we have $R = [S, J]$. Further, it follows that

$$\tilde{S} = S + \begin{vmatrix} \Omega(\theta, \theta) & \theta^{\dagger} \\ \theta & \boxed{0} \end{vmatrix} = S - \theta\Omega(\theta, \theta)^{-1}\theta^{\dagger}, \tag{20}$$

where $\Omega(\theta, \theta)$ satisfies

$$\Omega(\theta, \theta)\Lambda - \Lambda^{\dagger}\Omega(\theta, \theta) = \theta^{\dagger}\theta. \tag{21}$$

Theorem 2.4. *Suppose there are N different linearly independent solutions* $\theta_1, \theta_2, \ldots, \theta_N$ *for the system* $L(\phi) = M(\phi) = 0$ *corresponding to* $\lambda_1, \lambda_2, \ldots, \lambda_N$*, respectively. Then, we have the following N-fold iterative potential transformation:*

$$S[N] = S + \begin{vmatrix} \Omega(\Theta, \Theta) & \Theta^{\dagger} \\ \Theta & \boxed{O} \end{vmatrix}, \tag{22}$$

where O denote the 4×4 *zero matrix, and*

$$\Omega(\Theta,\Theta)=\begin{pmatrix}\Omega(\theta_1,\theta_1) & \Omega(\theta_2,\theta_1) & \dots & \Omega(\theta_N,\theta_1)\\ \Omega(\theta_1,\theta_2) & \Omega(\theta_2,\theta_2) & \dots & \Omega(\theta_N,\theta_2)\\ \vdots & \vdots & \ddots & \vdots\\ \Omega(\theta_1,\theta_N) & \Omega(\theta_2,\theta_N) & \dots & \Omega(\theta_N,\theta_N)\end{pmatrix},$$

$$\Theta=(\theta_1,\theta_2,\ \dots,\theta_N),\quad \theta_k=\begin{pmatrix}\phi_{4k-3} & \phi^*_{4k-2}\\ \phi_{4k-2} & -\phi^*_{4k-3}\\ \phi_{4k-1} & \phi^*_{4k}\\ \phi_{4k} & -\phi^*_{4k-1}\end{pmatrix},\quad (k=1,2,...N),$$

where $\Omega(\theta_i,\theta_j)$ *satisfies the relation*

$$\Omega(\theta_i,\theta_j)\Lambda_i-\Lambda_j^\dagger\Omega(\theta_i,\theta_j)=\theta_j^\dagger\theta_i,\quad \Lambda_i=diag(\lambda_i,-\lambda_i^*),\quad (i,j=1,2,\ \dots,N). \tag{23}$$

It follows from this relation that the potential Ω can be written explicitly as

$$\Omega(\theta_i,\theta_j)=\begin{pmatrix}F_{ij} & G^*_{ij}\\ G_{ij} & -F^*_{ij}\end{pmatrix}, \tag{24}$$

with

$$\begin{aligned}F_{ij}=&\frac{1}{\lambda_i-\lambda_j^*}(\phi_{4i}\phi^*_{4j}+\phi_{4i-1}\phi^*_{4j-1}+\phi_{4i-2}\phi^*_{4j-2}+\phi_{4i-3}\phi^*_{4j-3}),\\ G_{ij}=&\frac{1}{\lambda_i+\lambda_j}(\phi_{4i-3}\phi_{4j-2}-\phi_{4i-2}\phi_{4j-3}+\phi_{4i-1}\phi_{4j}-\phi_{4i}\phi_{4j-1}).\end{aligned} \tag{25}$$

By substituting Equation (24) into Equation (22), we can get the transformations between the fields

$$u[N]=u+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_1^\dagger\\ \Phi_3 & \boxed{0}\end{vmatrix}=u+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_4^\dagger\\ \Phi_2 & \boxed{0}\end{vmatrix}, \tag{26a}$$

$$v[N]=v+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_2^\dagger\\ \Phi_3 & \boxed{0}\end{vmatrix}=v-2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_4^\dagger\\ \Phi_1 & \boxed{0}\end{vmatrix}, \tag{26b}$$

$$u[N]^*=u^*+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_3^\dagger\\ \Phi_1 & \boxed{0}\end{vmatrix}=u^*+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_2^\dagger\\ \Phi_4 & \boxed{0}\end{vmatrix}, \tag{27a}$$

$$v[N]^*=v^*+2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_3^\dagger\\ \Phi_2 & \boxed{0}\end{vmatrix}=v^*-2i\begin{vmatrix}\Omega(\Theta,\Theta) & \Phi_1^\dagger\\ \Phi_4 & \boxed{0}\end{vmatrix}, \tag{27b}$$

where Φ_j $(j=1,2,3,4)$ is the jth row vector of the matrix Θ. By using the properties of quasideterminants, it is easy to check that the above relations are consistent.

3 | SOLUTIONS OF THE COUPLED COMPLEX MKDV EQUATIONS

In this section, our concern is to construct different types of exact solutions of Equations (2) from vanishing and non-vanishing backgrounds.

3.1 | Solutions with vanishing background

Firstly, let us consider the once-iterated potential transformation

$$u[1]=u-2i\frac{(\lambda_1-\lambda_1^*)}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2}(\phi_1^*\phi_3+\phi_2\phi_4^*), \tag{28a}$$

$$v[1]=v-2i\frac{(\lambda_1-\lambda_1^*)}{|\phi_1|^2+|\phi_2|^2+|\phi_3|^2+|\phi_4|^2}(\phi_2^*\phi_3-\phi_1\phi_4^*). \tag{28b}$$

For the zero seed solutions $u = v = 0$ of Equation (2), the linear system $L(\phi)=M(\phi)=0$ becomes

$$\phi_x + \lambda J\phi = 0, \tag{29a}$$

$$\phi_t + 4\lambda^3 J\phi = 0, \tag{29b}$$

which has the solution

$$\phi = (\alpha e^{-i\xi}, \beta e^{-i\xi}, \gamma e^{i\xi}, \delta e^{i\xi})^T, \quad \xi = \lambda(x + 4\lambda^2 t), \tag{30}$$

where α, β, γ, and δ are all complex constants.

Case 1($N = 1$) In order to obtain the one-soliton, by substituting the solution $\phi^{[1]} = (\phi_1, \phi_2, \phi_3, \phi_4)^T = (\alpha_1 e^{-i\xi_1}, \beta_1 e^{-i\xi_1}, \gamma_1 e^{i\xi_1}, \delta_1 e^{i\xi_1})^T$ associated with $\xi_1 = \lambda_1(x + 4\lambda_1^2 t)$, $\lambda_1 = a_1 + ib_1 (b_1 \neq 0)$ into Equation (28), we can obtain the following breather solution of Equation (2):

$$u[1] = \frac{4b_1[(\alpha_1^*\gamma_1 + \beta_1\delta_1^*)cos(\tau) + i(\beta_1\delta_1^* - \alpha_1^*\gamma_1)sin(\tau)]}{m_1 cosh(\eta) + m_2 sinh(\eta)}, \tag{31a}$$

$$v[1] = -\frac{4b_1[(\alpha_1\delta_1^* - \beta_1^*\gamma_1)cos(\tau) + i(\alpha_1\delta_1^* + \beta_1^*\gamma_1)sin(\tau)]}{m_1 cosh(\eta) + m_2 sinh(\eta)}, \tag{31b}$$

where

$$\begin{gathered} \tau = -2a_1[x + 4(a_1^2 - 3b_1^2)t)], \quad \eta = -2b_1[x + 4(3a_1^2 - b_1^2)t], \\ m_1 = |\alpha_1|^2 + |\beta_1|^2 + |\gamma_1|^2 + |\delta_1|^2, \quad m_2 = |\gamma_1|^2 + |\delta_1|^2 - |\alpha_1|^2 - |\beta_1|^2. \end{gathered} \tag{32}$$

Through the above expression of solutions, we can see this kind of breather solutions is characterized by six involved parameters of α_1, β_1, γ_1, δ_1, a_1, and b_1. Notably, these breather solutions degenerate to bright vector solitons when either $\alpha_1 = 0$, or $\beta_1 = 0$, or $\gamma_1 = 0$, or $\delta_1 = 0$. Without loss of generality, we can take $\alpha_1 = 0$, and then we get the following vector bright one-soliton solutions:

$$\begin{aligned} u[1] &= \frac{4b_1\beta_1\delta_1^*}{m_3 cosh(\eta) + m_4 sinh(\eta)} e^{i\tau}, \\ v[1] &= \frac{4b_1\beta_1^*\gamma_1}{m_3 cosh(\eta) + m_4 sinh(\eta)} e^{-i\tau}, \end{aligned} \tag{33}$$

where

$$m_3 = |\beta_1|^2 + |\gamma_1|^2 + |\delta_1|^2, \quad m_4 = |\gamma_1|^2 + |\delta_1|^2 - |\beta_1|^2. \tag{34}$$

In order to avoid $u[1]$ (or $v[1]$) being zero, we require $\beta_1\delta_1\gamma_1 \neq 0$. On the basis of solution (33), we find that the velocities of $u[1]$ and $v[1]$ components are both $12a_1^2 - 4b_1^2$. In addition, we see that the $u[1]$ and $v[1]$ are not proportional to each other but $u[1]$ and $v[1]^*$ are proportional to each other. Namely, the two intensity distributions of $|u[1]|$ and $|v[1]|$ are proportional to each other.

To give us a clear understanding of the solution, we exhibit the dynamics of solutions of $|u[1]|^2$ and $|v[1]|^2$ by plotting the solution. Figure 1 depicts the evolution plot of a vector breather solution and it is seen that $|v[1]|^2$ possesses the feature of breather and bright solitons. Figure 2 shows that the $|u[1]|^2$ and $|v[1]|^2$ are both bright solitons of the bell profiles. From the density plot, we can see that the trajectory of $|u[1]|^2$ and $|v[1]|^2$ are along the same line.

Case 2 ($N = 2$) To derive two-soliton solutions, we need to choose another solutions of the linear system $L(\phi) = M(\phi) = 0$ as $\phi^{[2]} = (\phi_5, \phi_6, \phi_7, \phi_8)^T = (\alpha_2 e^{-i\xi_2}, \beta_2 e^{-i\xi_2}, \gamma_2 e^{i\xi_2}, \delta_2 e^{i\xi_2})^T$ associated with $\xi_2 = \lambda_2(x + 4\lambda_2^2 t)$, $\lambda_2 = a_2 + ib_2 (b_2 \neq 0)$. Then, we obtain the two-soliton solutions for Equation (2) as

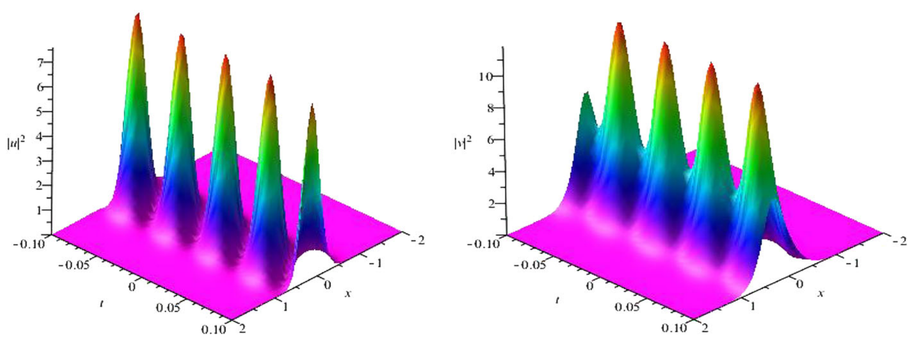

FIGURE 1 The Ma breather solutions: Parameters $\alpha_1 = \frac{1}{2}, \beta_1 = 1, \gamma_1 = 2, \delta_1 = 1, \lambda_1 = \sqrt{3}i{+}1$ [Colour figure can be viewed at wileyonlinelibrary.com]

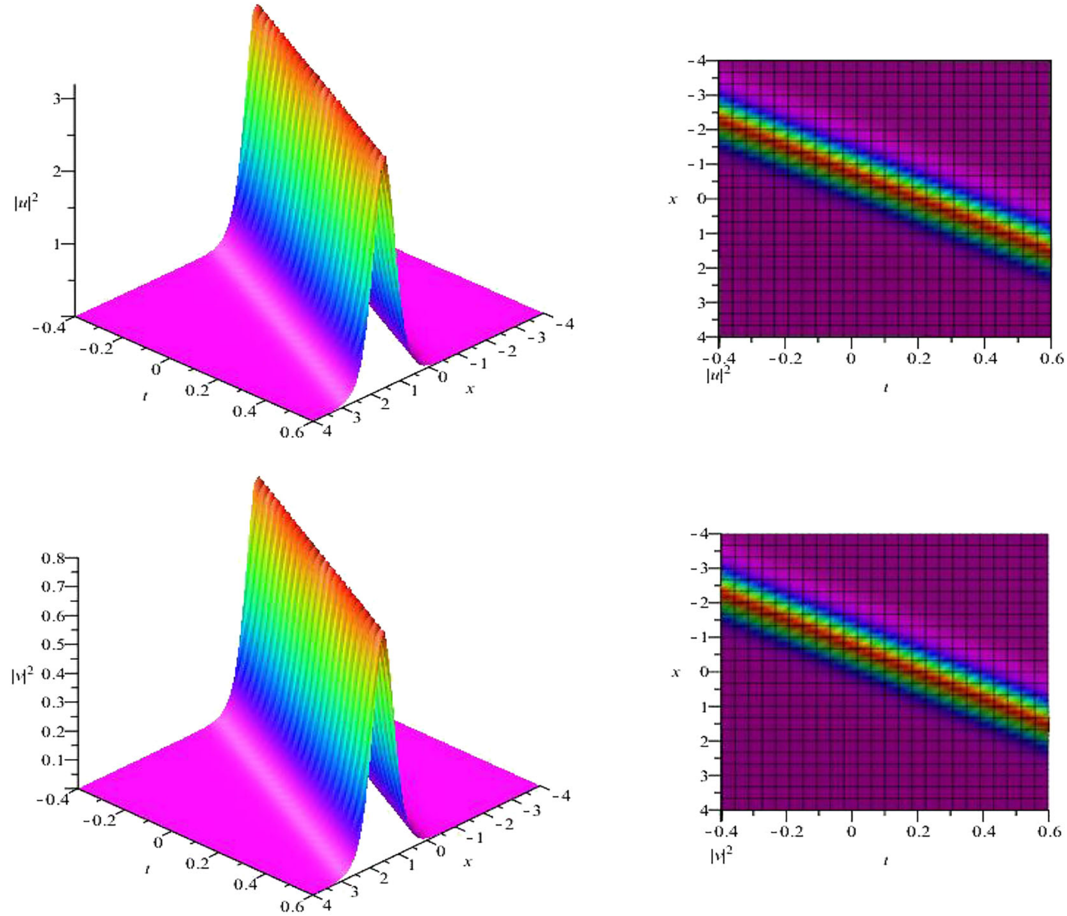

FIGURE 2 Bright-bright solitons: Parameters $\alpha_1 = 2, \beta_1 = 1, \gamma_1 = \frac{1}{2}, \delta_1 = 0, \lambda_1 = i + \frac{1}{8}$ [Colour figure can be viewed at wileyonlinelibrary.com]

$$u[2] = 2i \begin{vmatrix} F_{11} & 0 & F_{21} & G_{21}^* & \phi_1^* \\ 0 & -F_{11}^* & G_{21} & -F_{21}^* & \phi_2 \\ F_{12} & G_{12}^* & F_{22} & 0 & \phi_5^* \\ G_{12} & -F_{12}^* & 0 & -F_{22}^* & \phi_6 \\ \phi_3 & \phi_4^* & \phi_7 & \phi_8^* & \boxed{0} \end{vmatrix}, \tag{35a}$$

$$v[2] = 2i \begin{vmatrix} F_{11} & 0 & F_{21} & G_{21}^* & \phi_2^* \\ 0 & -F_{11}^* & G_{21} & -F_{21}^* & -\phi_1 \\ F_{12} & G_{12}^* & F_{22} & 0 & \phi_6^* \\ G_{12} & -F_{12}^* & 0 & -F_{22}^* & -\phi_5 \\ \phi_3 & \phi_4^* & \phi_7 & \phi_8^* & \boxed{0} \end{vmatrix}. \tag{35b}$$

On the basis of Equation (35), we only investigate the asymptotic behavior between two vector bright solitons for the coupled complex mKdV system. In the first place, we take parameters $\alpha_1 = \beta_1 = \gamma_2 = \delta_2 = 1, \gamma_1 = -i + \frac{1}{6}, \delta_1 = \alpha_2 = 0, \beta_2 = -3, \lambda_1 = i + 1, \lambda_2 = \frac{1}{2}i$. The corresponding two bright-bright solitons are shown in Figure 3, which displays the elastic collision of two bell-shaped bright solitons. We can see that after collision, the two bright solitons pass through each other without any change of shape and amplitude.

In studies of the elastic collision between two bright solitons, multi-bright-soliton bound states are an interesting subject. To obtain bright-bright-soliton bound states, the two bright-bright solitons in the solution should have the same velocity. In Figure 4, we choose $b_1 = b_2, a_1 = a_2 = 0$ to keep the same velocity so that the two constituent bright solitons can stay together for all times.

In Figure 5, the bright vector solitons between two components undergo the partial energy exchange inelastic collision. In fact, the complete energy exchange of two bright vector solitons also occurs in coupling models for suitable choice of

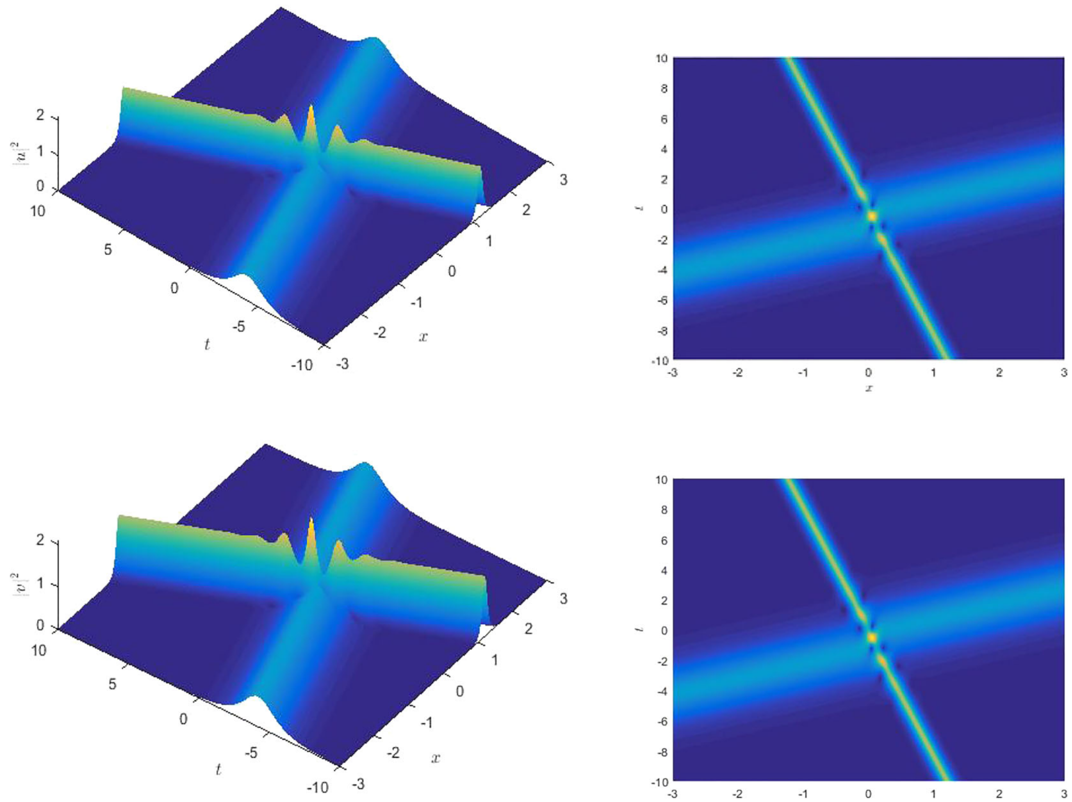

FIGURE 3 Elastic collision between two bright vector solitons: Parameters $\alpha_1 = \beta_1 = \gamma_2 = \delta_2 = 1, \gamma_1 = -i + \frac{1}{6}, \delta_1 = \alpha_2 = 0, \beta_2 = -3, \lambda_1 = i + 1, \lambda_2 = \frac{1}{2}i$ [Colour figure can be viewed at wileyonlinelibrary.com]

parameters. Moreover, with the vanishing boundary conditions $u[1]|_{x\to\pm\infty} \to 0, v[1]|_{x\to\pm\infty} \to 0$, it is clear that the total energy of Equation (2) is conserved from the integral of motion for Equation (2), which is

$$\int_{-\infty}^{+\infty} (|u|^2 + |v|^2)dx = \text{constant}. \tag{36}$$

3.2 | Solutions with nonvanishing background

Case 1 For nonzero seed solution u, v with c_j $(j = 1, 2)$ as complex constants. Substituting u, v into the Lax pair $L(\phi) = M(\phi) = 0$, we obtain the general solution $\phi = (\phi_1, \phi_2, \phi_3, \phi_4)^T$, where

$$\phi_1 = \beta e^{\nu} + \gamma e^{-\nu}, \tag{37a}$$

$$\phi_2 = \delta e^{\nu} + \alpha e^{-\nu}, \tag{37b}$$

$$\phi_3 = e^{\nu}\chi^{+}(c_2\delta + c_1\beta) + e^{-\nu}\chi^{-}(c_2\alpha + c_1\gamma), \tag{37c}$$

$$\phi_4 = e^{\nu}\chi^{+}(c_1^*\delta - c_2^*\beta) + e^{-\nu}\chi^{-}(c_1^*\alpha - c_2^*\gamma), \tag{37d}$$

in which $\beta, \gamma, \delta, \alpha$ are all complex constants, and

$$\nu = i\sqrt{|c_1|^2 + |c_2|^2 + \lambda^2}[x + 2(2\lambda^2 - (|c_1|^2 + |c_2|^2))t], \quad \chi^{\pm} = \frac{i\lambda \pm i\sqrt{\lambda^2 + |c_1|^2 + |c_2|^2}}{|c_1|^2 + |c_2|^2}.$$

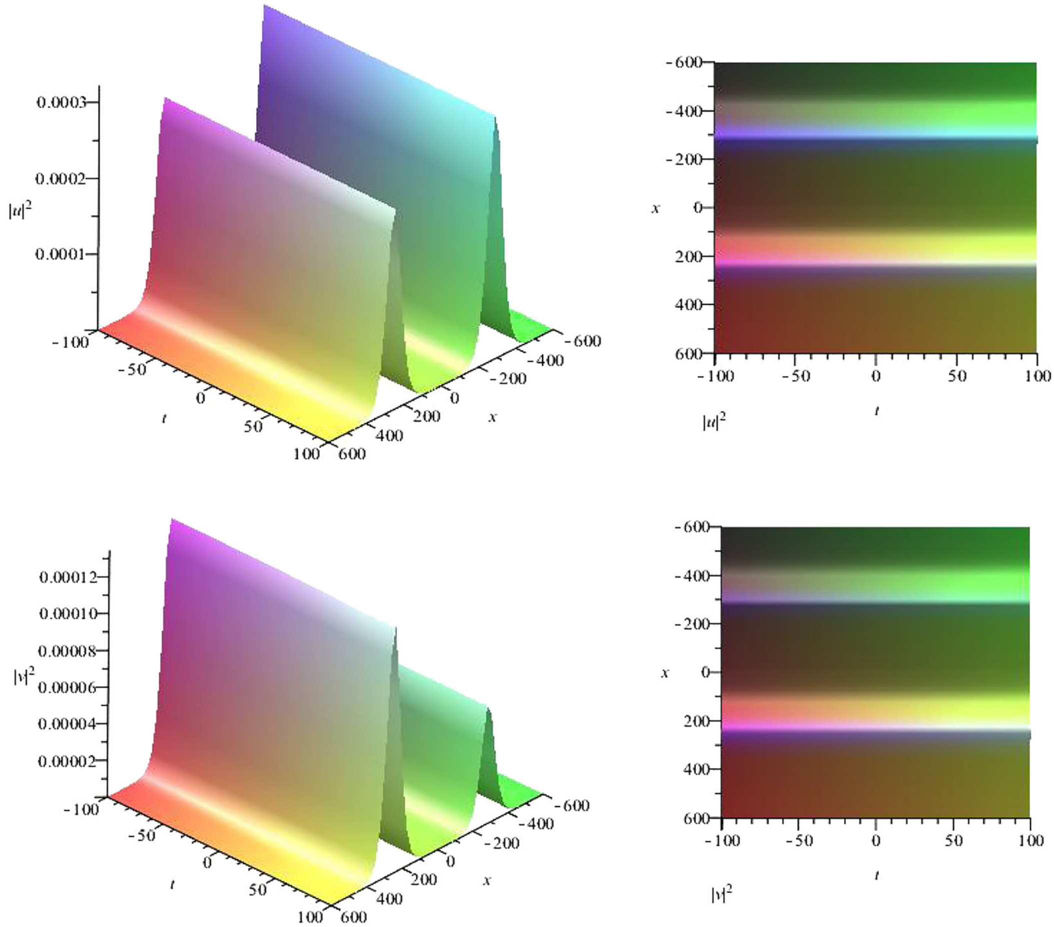

FIGURE 4 Bright-bright soliton bound states: Parameters $\alpha_1 = 2, \beta_1 = \gamma_1 = \gamma_2 = \delta_2 = i, \alpha_2 = -1, \delta_1 = \beta_2 = 0, \lambda_1 = \frac{1}{101}i, \lambda_2 = \frac{1}{99}i$ [Colour figure can be viewed at wileyonlinelibrary.com]

By inserting Equation (37) into Equation (28), we can obtain explicit solutions of Equation (2). For convenience, we have chosen $\lambda_1 = ih(h \neq 0)$ as a pure imaginary parameter. In what follows, there are two distinct cases according to the different values of the parameters.

(1) If $h > \sqrt{|c_1|^2 + |c_2|^2}$, we have

$$v = -\sqrt{h^2 - (|c_1|^2 + |c_2|^2)}[x - 2(2h^2 + (|c_1|^2 + |c_2|^2))t], \quad \chi^{\pm} = \frac{-h \mp \sqrt{h^2 - (|c_1|^2 + |c_2|^2)}}{|c_1|^2 + |c_2|^2}.$$

Then, by a direct calculation, we get

$$u[1] = c_1 \left(1 - \frac{4h}{c_1 \kappa} A_1\right), \tag{38a}$$

$$v[1] = c_2 \left(1 - \frac{4h}{c_2 \kappa} A_2\right), \tag{38b}$$

with

$$\begin{aligned}
\kappa = {} & e^{2v}(|\beta|^2 + |\delta|^2)[1 + \chi^+\chi^+(|c_1|^2 + |c_2|^2)] + e^{-2v}(|\gamma|^2 + |\alpha|^2)[1 + \chi^-\chi^-(|c_1|^2 + |c_2|^2)] \\
& + (\beta\gamma^* + \beta^*\gamma + \delta\alpha^* + \delta^*\alpha)[1 + \chi^+\chi^-(|c_1|^2 + |c_2|^2)], \\
A_1 = {} & -e^{2v}\chi^+ c_1(|\beta|^2 + |\delta|^2) - e^{-2v}\chi^- c_1(|\alpha|^2 + |\gamma|^2) - \chi^+[c_1(\beta\gamma^* + \delta^*\alpha) + c_2(\delta\gamma^* - \beta^*\alpha)] \\
& - \chi^-[c_1(\gamma\beta^* + \alpha^*\delta) + c_2(\alpha\beta^* - \gamma^*\delta)], \\
A_2 = {} & -e^{2v}\chi^+ c_2(|\beta|^2 + |\delta|^2) - e^{-2v}\chi^- c_2(|\alpha|^2 + |\gamma|^2) - \chi^+[c_1(\beta\alpha^* - \delta^*\gamma) + c_2(\delta\alpha^* + \beta^*\gamma)] \\
& - \chi^-[c_1(\gamma\delta^* - \alpha^*\beta) + c_2(\alpha\delta^* + \gamma^*\beta)].
\end{aligned}$$

From Equation (38), we can see that $|u[1]|^2 \to |c_1|^2$ and $|v[1]|^2 \to |c_2|^2$ as $t \to \pm\infty$. Below, we consider the asymptotic property of the solution. We only do this for the solution $u[1]$, since the asymptotic analysis for the solution $v[1]$ is similar.

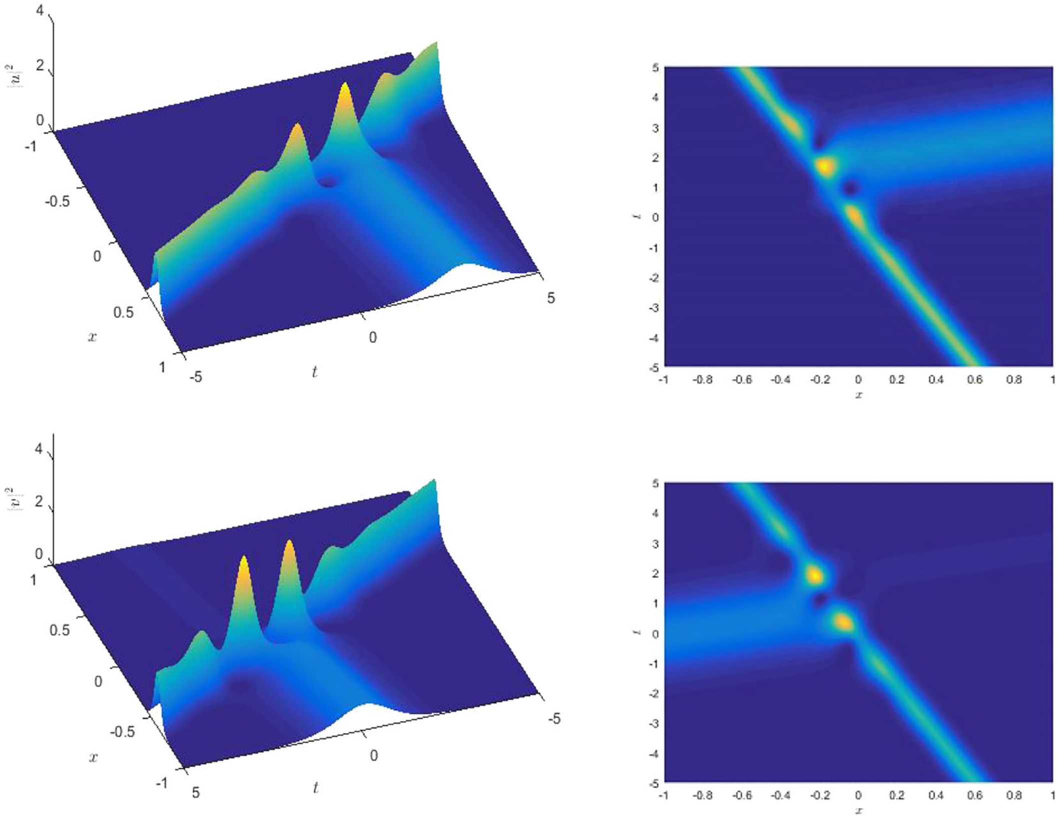

FIGURE 5 Inelastic collision between two bright vector solitons: Parameters $\alpha_1 = -1, \beta_1 = 1, \gamma_1 = i - 1, \delta_1 = \beta_2 = 0, \alpha_2 = \frac{1}{2}, \gamma_2 = \frac{1}{3}i, \delta_2 = 2, \lambda_1 = i + 1, \lambda_2 = \frac{1}{2}i$ [Colour figure can be viewed at wileyonlinelibrary.com]

For simplicity, letting $e^{2\nu} = Y > 0$, then $d|u[1]|^2/dY = 0$ gives

$$F(Y) = F_1Y^4 + F_2Y^3 + F_3Y^2 + F_4Y + F_5 = 0, \tag{39}$$

where

$$F_1 = [2|c_1|^2 d_3 g_1^2 g_3 \chi^+ - d_1 g_1^2 (c_1 \varrho_2 + c_1^* \varrho_1)](d_1 + 4h\chi^+),$$

$$F_2 = 4|c_1|^2 g_1^2 g_2 (d_2\chi^+ - d_1\chi^-)(d_1 + 4h\chi^+) - d_3 g_1 g_3 (c_1\varrho_2 + c_1^*\varrho_1)(d_1 - 4h\chi^+) - 2g_1(4hd_1\varrho_1\varrho_2 - |c_1|^2 d_3^2 g_3^2 \chi^+),$$

$$F_3 = 6g_1g_2(d_2\chi^+ - d_1\chi^-)[2h(c_1\varrho_2 + c_1^*\varrho_1) + |c_1|^2 d_3 g_3],$$

$$F_4 = 4|c_1|^2 g_1 g_2^2 (d_2\chi^+ - d_1\chi^-)(d_2 + 4h\chi^-) + d_3 g_2 g_3 (c_1\varrho_2 + c_1^*\varrho_1)(d_2 - 4h\chi^-) + 2g_2(4hd_2\varrho_1\varrho_2 - |c_1|^2 d_3^2 g_3^2 \chi^-),$$

$$F_5 = g_2^2[d_2(c_1\varrho_2 + c_1^*\varrho_1) - 2|c_1|^2 d_3 g_3 \chi^-](d_2 + 4h\chi^-),$$

with

$$d_1 = 1 + \chi^+\chi^+(|c_1|^2 + |c_2|^2), \quad g_1 = |\beta|^2 + |\delta|^2,$$

$$d_2 = 1 + \chi^-\chi^-(|c_1|^2 + |c_2|^2), \quad g_2 = |\alpha|^2 + |\gamma|^2,$$

$$d_3 = 1 + \chi^+\chi^-(|c_1|^2 + |c_2|^2), \quad g_3 = \beta\gamma^* + \beta^*\gamma + \delta\alpha^* + \delta^*\alpha,$$

$$\varrho_1 = \chi^+[c_1(\beta\gamma^* + \delta^*\alpha) + c_2(\delta\gamma^* - \beta^*\alpha)] + \chi^-[c_1(\gamma\beta^* + \alpha^*\delta) + c_2(\alpha\beta^* - \gamma^*\delta)],$$

$$\varrho_2 = \chi^+[c_1^*(\beta^*\gamma + \delta\alpha^*) + c_2^*(\delta^*\gamma - \beta\alpha^*)] + \chi^-[c_1^*(\gamma^*\beta + \alpha\delta^*) + c_2^*(\alpha^*\beta - \gamma\delta^*)].$$

It is obvious that the shape of the solution depends on the parameters which determine the number of positive real root of Equation (39). As a result, we find that there exist three different kinds of nontrivial solutions: the bright-bright solitons, the bright-dark solitons, and the bright-W-shaped solitons. To make the solutions more concise, we take specific values for the parameters.

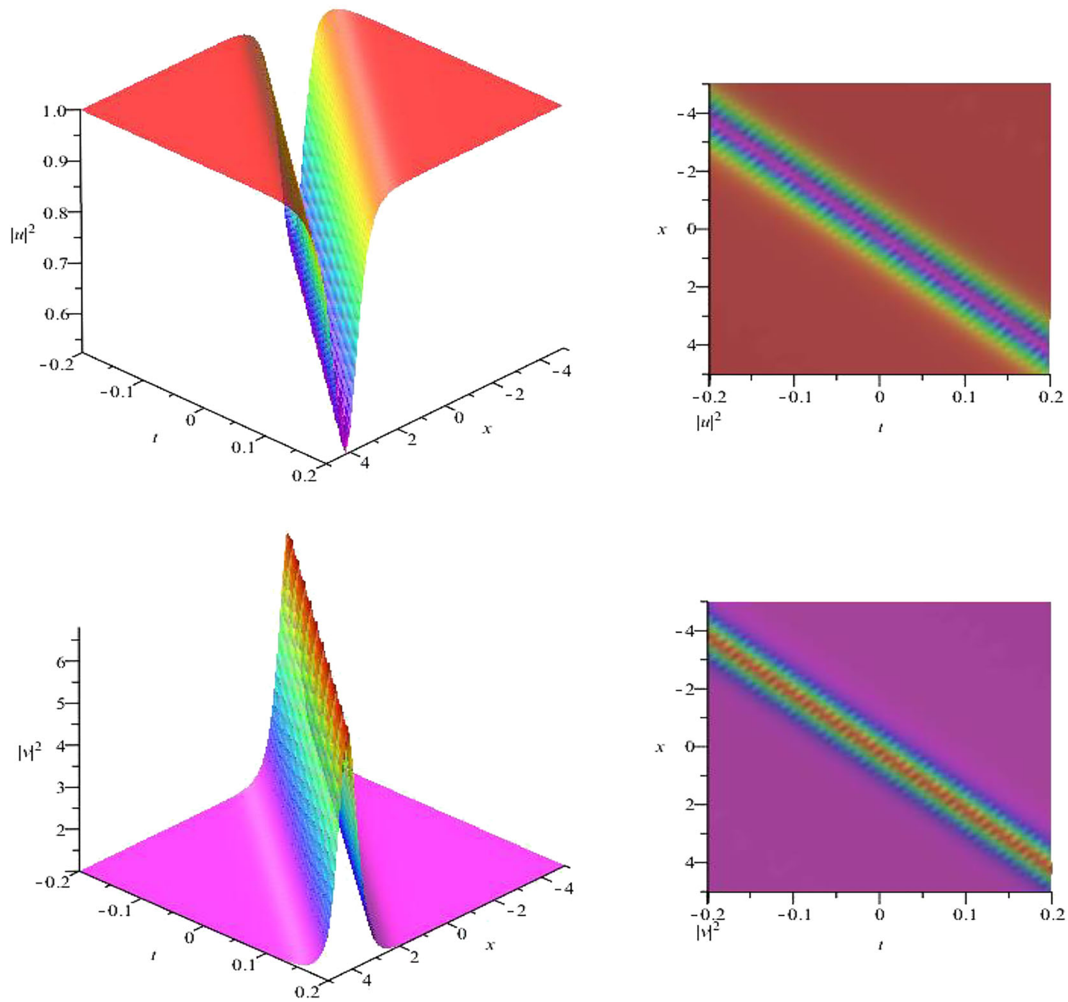

FIGURE 6 Bright-dark solitons: Parameters $\beta = 0, \gamma = \delta = \alpha = c_1 = 1, c_2 = -1, \lambda_1 = 2i$. The peaks values for $|u|^2$ is 0.52397 and for $|v|^2$ is 6.80936 [Colour figure can be viewed at wileyonlinelibrary.com]

- If we take $\beta = 1, \gamma = 1, \delta = 1, \alpha = 1, c_1 = 1, c_2 = -i, \lambda_1 = 2i$, the solution $|u[1]|^2$ and $|v[1]|^2$ are both bright solitons. In this condition, we find $F(\sqrt{2}-1) = 0$ and $F'(\sqrt{2}-1) < 0$ and Equation (39) has one positive real root.
- If we choose $\beta = 0, \gamma = 1, \delta = 1, \alpha = 1, c_1 = 1, c_2 = -1, \lambda_1 = 2i$, the solution $|u[1]|^2$ is a dark soliton and the $|v[1]|^2$ is a bright soliton. The condition involves $F(2-\sqrt{2}) = 0$ and $F'(2-\sqrt{2}) > 0$. The bright-dark soliton is described in Figure 6. The number of positive real roots is the same as the above.
- If we choose $\beta = 1, \gamma = 0, \delta = 1, \alpha = 1, c_1 = 1, c_2 = 2, \lambda_1 = \sqrt{15}i$, the solution $|v[1]|^2$ is a bright soliton and the solution $|u[1]|^2$ is a W-shaped soliton which has one hump and two valleys on the hump's two sides (see Figure 7). It is noted that Equation (39) has three different positive real roots. In addition, the hump's value increases when h does under this condition.

(2) If $h < \sqrt{|c_1|^2 + |c_2|^2}$, we have

$$v = i\sqrt{(|c_1|^2 + |c_2|^2) - h^2}[x - 2(2h^2 + (|c_1|^2 + |c_2|^2))t],$$

$$\chi^{\pm} = \frac{-h \pm i\sqrt{|c_1|^2 + |c_2|^2 - h^2}}{|c_1|^2 + |c_2|^2}.$$

After some calculations, one can derive the periodic solutions for the system (2)

$$u[1] = c_1 \left(1 - \frac{4h}{c_1 \Delta} D_1\right), \tag{40a}$$

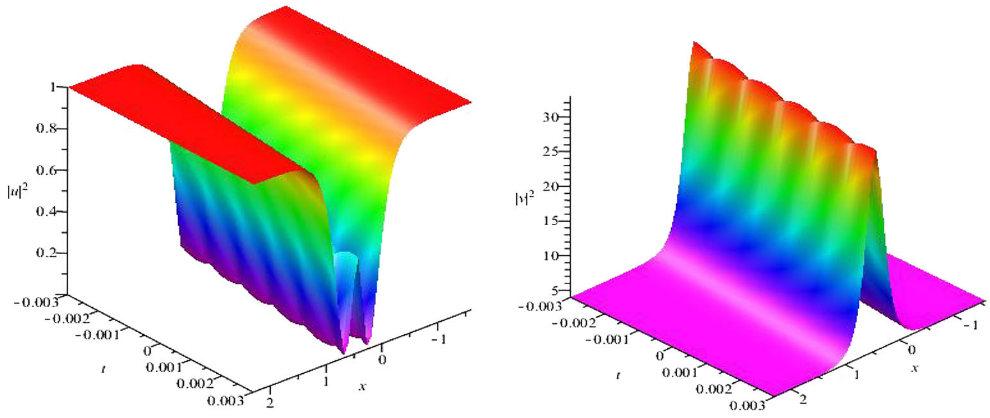

FIGURE 7 The propagation dynamics of W-shaped soliton and bright soliton: Parameters $\beta = \delta = \alpha = c_1 = 1, \gamma = 0, c_2 = 2, \lambda_1 = \sqrt{15}i$ [Colour figure can be viewed at wileyonlinelibrary.com]

$$v[1] = c_2\left(1 - \frac{4h}{c_2\Delta}D_2\right), \tag{40b}$$

where

$$\begin{aligned}\Delta =& e^{2\nu}(\delta\alpha^* + \beta\gamma^*)[1 + \chi^+\chi^+(|c_1|^2 + |c_2|^2)] + e^{-2\nu}(\delta^*\alpha + \beta^*\gamma)[1 + \chi^-\chi^-(|c_1|^2 + |c_2|^2)]\\ &+ (|\alpha|^2 + |\gamma|^2 + |\delta|^2 + |\beta|^2)[1 + \chi^+\chi^-(|c_1|^2 + |c_2|^2)],\\ D_1 =& - e^{2\nu}\chi^+c_1(\gamma^*\beta + \alpha^*\delta) - e^{-2\nu}\chi^-c_1(\gamma\beta^* + \alpha\delta^*) - \chi^+[c_1(|\beta|^2 + |\alpha|^2) + c_2(\beta^*\delta - \gamma^*\alpha)]\\ &- \chi^-[c_1(|\gamma|^2 + |\delta|^2) + c_2(\alpha\gamma^* - \beta^*\delta)],\\ D_2 =& - e^{2\nu}\chi^+c_2(\gamma^*\beta + \alpha^*\delta) - e^{-2\nu}\chi^-c_2(\gamma\beta^* + \alpha\delta^*) - \chi^+[c_2(|\gamma|^2 + |\delta|^2) + c_1(\beta\delta^* - \gamma\alpha^*)]\\ &- \chi^-[c_2(|\beta|^2 + |\alpha|^2) + c_1(\alpha^*\gamma - \beta\delta^*)].\end{aligned}$$

Under the conditions: $\beta = 1, \gamma = 0, \delta = 1, \alpha = 1, c_1 = c_2 = 1, \lambda_1 = i$, we get periodic solutions with the period $T_{time} = \frac{\pi}{8}, T_{space} = \pi$ as shown in Figure 8. Moreover, when $\mathrm{Re}(\lambda_1) \neq 0$, we can obtain the kinds of complicated breathers, which change periodically and are lower or higher than the background, or sometimes lower, sometimes higher than the background.

Case 2 In this case, we try to obtain explicit rogue wave solutions for Equation (2) by using the generalized DT. Here, we begin with the following plane wave seed solution

$$\begin{aligned}u = c_1e^{i\theta_1}, \quad \theta_1 = a_1x + [a_1^3 - 6(c_1^2 + c_2^2)a_1]t,\\ v = c_2e^{i\theta_2}, \quad \theta_2 = a_2x + [a_2^3 - 6(c_1^2 + c_2^2)a_2]t,\end{aligned} \tag{41}$$

where a_i and c_i are all real parameters. After substitution, we obtain the following special solution

$$\Phi(\lambda) = \begin{pmatrix} \alpha e^{i\omega_1} + \beta e^{i\omega_2}\\ (\alpha\delta e^{i\omega_1} + \beta\delta e^{i\omega_2})e^{i(\theta_1-\theta_2)}\\ (\alpha\frac{-c_1-c_2\delta}{ia_1+i\xi_1-i\lambda}e^{i\omega_1} + \beta\frac{-c_1-c_2\delta}{ia_1+i\xi_2-i\lambda}e^{i\omega_2})e^{i\theta_1}\\ (\alpha\frac{c_2-c_1\delta}{-ia_2+i\xi_1-i\lambda}e^{i\omega_1} + \beta\frac{c_2-c_1\delta}{-ia_2+i\xi_2-i\lambda}e^{i\omega_2})e^{-i\theta_2} \end{pmatrix}, \tag{42}$$

with

$$\begin{aligned}\omega_i =& \xi_ix + [\xi_i^3 - 3\lambda\xi_i^2 + (3\lambda^2 - 3c_1^2 - 3c_2^2)\xi_i + 3\lambda^3 + 3(c_1^2 + c_2^2)\lambda + 3(a_1c_1^2 - a_2c_2^2)\\ &+ 3\delta c_1c_2(a_1 + a_2)]t, \quad (i = 1, 2),\end{aligned}$$

where α, β, δ are free complex constants and ξ_i satisfies the following equations

$$\begin{aligned}&\xi_i + \lambda - c_1\frac{c_1 + c_2\delta}{a_1 + \xi_i - \lambda} - c_2\frac{c_2 - c_1\delta}{-a_2 + \xi_i - \lambda} = 0,\\ &\delta(a_1 - a_2 + \xi_i + \lambda) - c_2\frac{c_1 + c_2\delta}{a_1 + \xi_i - \lambda} + c_1\frac{c_2 - c_1\delta}{-a_2 + \xi_i - \lambda} = 0.\end{aligned} \tag{43}$$

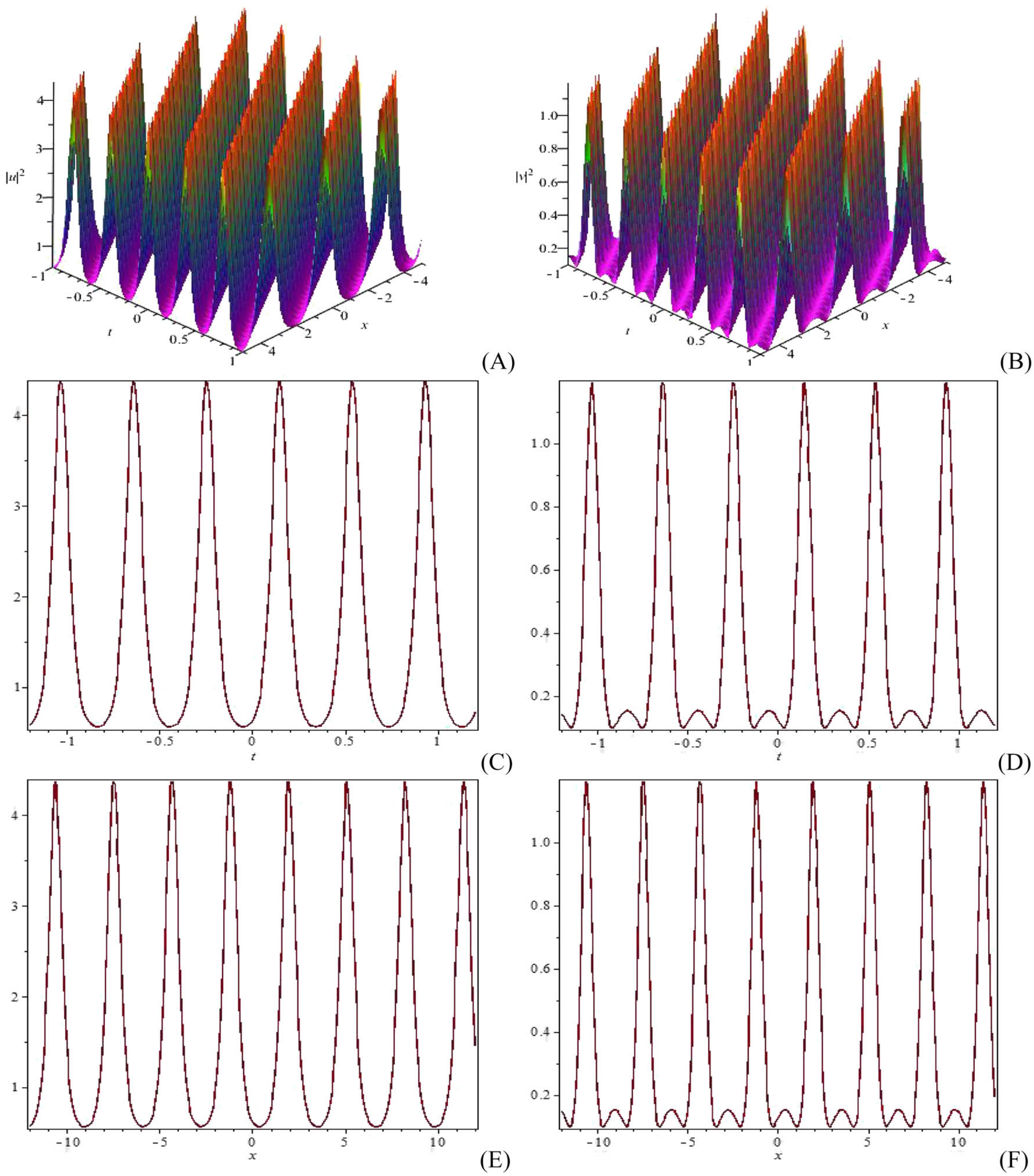

FIGURE 8 (A,B) The space-time periodic solutions $|u|^2$ and $|v|^2$ with $T_{time} = \frac{\pi}{8}$ and $T_{space} = \pi$ as parameters $\beta = \delta = \alpha = c_1 = c_2 = 1, \gamma = 0, \lambda = i$; (C,D) time period at x=0; (E,F) space period at t=0 [Colour figure can be viewed at wileyonlinelibrary.com]

For example, taking $c_1 = 1, c_2 = \frac{1}{2}, a_1 = \frac{3}{2}, a_2 = -\frac{3}{2}, \delta = 0$, we have the seed solutions $u = e^{\frac{3}{8}i(4x-21t)}, v = \frac{1}{2}e^{-\frac{3}{8}i(4x-21t)}$. Based on the above analysis, we can obtain the corresponding solution

$$\Phi(\lambda) = \begin{pmatrix} e^{-\frac{3}{16}i(4x-21t)}(\alpha e^{A} + \beta e^{-A}) \\ 0 \\ e^{\frac{3}{16}i(4x-21t)}(\alpha \xi_1 e^{A} + \beta \xi_2 e^{-A}) \\ -\frac{1}{2}e^{\frac{3}{16}i(4x-21t)}(\alpha \xi_1 e^{A} + \beta \xi_2 e^{-A}) \end{pmatrix}, \tag{44}$$

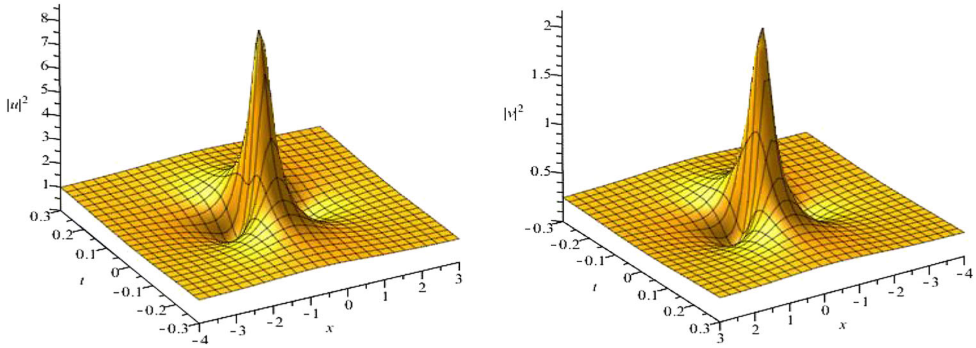

FIGURE 9 The temporal and spatial structure of first-order vector rogue waves: Parameters $c_1 = 1, c_2 = \frac{1}{2}, a_1 = -a_2 = \frac{3}{2}, \alpha = 1, \beta = \delta = 0$ [Colour figure can be viewed at wileyonlinelibrary.com]

where

$$A = \frac{1}{4}\sqrt{16\lambda^2 - 24\lambda + 29i}\left[x + \left(4\lambda^2 + 3\lambda - \frac{1}{4}\right)t\right],$$
$$\xi_1 = \frac{4i}{3 + \sqrt{16\lambda^2 - 24\lambda + 29} - 4\lambda},$$
$$\xi_2 = \frac{-4i}{-3 + \sqrt{16\lambda^2 - 24\lambda + 29} + 4\lambda}.$$

Now we manage to find a generalized DT. Let $\lambda_1 = \frac{3}{4} - \frac{\sqrt{5}}{2}i$, where ϵ is a small parameter. Expanding the vector $\Phi(\lambda_1 + \epsilon^2)$ in Equations (44) at $\epsilon = 0$, we obtain

$$\Phi = \Phi^{[0]} + \Phi^{[1]}\epsilon^2 + \Phi^{[2]}\epsilon^4 + \ldots + \Phi^{[N]}\epsilon^{2N} + \ldots, \tag{45}$$

where $\Phi^{[j]} = \frac{1}{(2j)!}\frac{\partial^{(2j)}}{\partial\epsilon^{(2j)}}|_{\epsilon\to 0}\Phi(\epsilon)$, $(j = 0, 1, 2, \ldots)$.

Through the following limit process

$$\Phi[1] = \lim_{\epsilon\to 0}\frac{B_{\theta,\rho}|_{\lambda=\lambda_1+\epsilon^2}\Phi}{\epsilon^2} = B_{\theta,\rho}|_{\lambda=\lambda_1}\Phi^{[1]} + \frac{\theta\Omega(\theta,\theta)^{-1}\Omega(\Phi^{[0]},\theta)}{\lambda_1 - \lambda_1^*}, \tag{46}$$

we find a solution to the Lax pair (3) with $u[1]$, $v[1]$, and $\lambda_1 = \frac{3}{4} - \frac{\sqrt{5}}{2}i$. This allows us to go to a first generalized DT, and substituting the above data into Equation (28), one can directly derive the form of the first-order vector rogue waves of the system (2). For a set of given values $\alpha = 1, \beta = 0$, the first-order vector rogue waves are of the form

$$u[1] = e^{\frac{3}{8}i(4x-21t)}\left(1 - \frac{-1440it + 64}{M}\right),$$
$$v[1] = \frac{1}{2}e^{-\frac{3}{8}i(4x-21t)}\left(1 - \frac{1440it + 64}{M}\right), \tag{47}$$

where

$$M = 8145t^2 - 24\sqrt{5}t - 120xt + 80x^2 + 32\sqrt{5}x + 32.$$

Figure 9 displays the temporal spatial structure of first-order vector rogue wave solutions.

It should be pointed out that high-order soliton solutions of the complex mKdV equation have been derived by DT systemically.[8,26]

4 | CONCLUSION

In this paper, we have investigated the coupled complex modified KdV equations. By constructing a binary Darboux transformation, we have obtained various kinds of exact solutions, including bright-bright solitons, bright-dark solitons, bright-W-shaped solitons, breather solitons, periodic solutions, and rogue waves solutions. It will be an interesting topic to study higher order rogue waves for the coupled complex mKdV equations. We plan to report our results in a separate publication in the future. The binary Darboux transformation can be also extended directly to the N-component complex mKdV equations and other equations of different types including integrable couplings (see, eg, Ma & Zhang[27] for DTs of integrable couplings).

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CONFLICT OF INTEREST

There are no conflicts of interest to this work.

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