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Abundant solutions of an extended KPII equation combined with a new fourth-order term

Liqin Zhang[*,‡,††], Wen Xiu Ma[†,‡,§,¶,‖,‡‡] and Yehui Huang[§,**]

[*] *College of Information Science and Artificial Intelligence, Xiamen Institute of Technology, Xiamen 361021, Fujian, China*
[†] *Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, China*
[‡] *Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia*
[§] *Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA*
[¶] *School of Mathematics, South China University of Technology, Guangzhou 510640, China*
[‖] *International Institute for Symmetry Analysis and Mathematical Modelling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735, South Africa*
[**] *School of Mathematics and Physics, North China Electric Power University, Beijing, 102206, China*
[††] *aqin98@163.com*
[‡‡] *mawx@cas.usf.edu*

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An extension of the KPII equation is studied. Adding a new fourth-order derivative term and some second-order derivative terms, we formulate an extended KPII equation. Different types of solutions of the extended equation are obtained by the Hirota bilinear method, and the presented solutions include soliton solutions, lump solutions and interaction solutions. Their dynamical behaviors are analyzed through plots.

Keywords: Hirota's bilinear form; KPII equation; solitary solution; lump solution; interaction solution.

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[††,‡‡]Corresponding authors.

1. Introduction

In mathematical physics, the Kadomtsev-Petviashvili (KP) equation

$$\left(u_t + \frac{3}{2}uu_x + \frac{1}{4}u_{xxx}\right)_x + \sigma^2 u_{yy} = 0\,. \tag{1.1}$$

describes many important nonlinear physical situations, such as the evolution of nonlinear long waves of small amplitude with slow dependence on the transverse coordinate.[1] When $\sigma = i$, (1.1) is the KPI equation, which is used to model waves in thin films with high surface tension.[2] And when $\sigma = 1$, (1.1) is known as the KPII equation, which describes water waves with small surface tension.[2]

The KP equation was accepted as a natural extension of the classical KdV equation to two spatial dimensions which has captured imagination of many scientists. The KP equation was derived as a model for surface and internal water waves by Ablowitz and Segur,[1] and in nonlinear optics by Pelinovsky, Stepanyants and Kivshar,[3] as well as in other physical settings.[4] The research of the KPI equation lies on mathematical structure, lax pair and equivalent formulations,[2,5] bilinear form[6] and wronskian representations,[7] connection with sato theory,[8] exact solutions and two-dimensional wave phenomena, line solitons,[1] existence and stability of two-dimensional solitary waves[9] and so on, while transverse stability of one-dimensional solitary waves,[10] resonant interactions of line solitons,[11] and finite-genus and quasi-periodic solutions were studied for the KPII equation from then on. In the last couple of decades, many extensions of the KP equation was studied in.[12–14] This inspires us to explore more extensions of the KP equation.

One of the most exciting and extremely active areas of research investigation arises on the study of exact solution and the related issue of the construction of solutions to a wide class of nonlinear equations. Exact solutions of partial differential equations describe significant mathematical and physical phenomena. A soliton solution is an exact solution determined by exponentially localized functons, which localized in all directions both in time and in space. Lump solutions are also a kind of exact solutions of partial differential equations, obtained from soliton theory by taking long wave limits.[1] Nevertheless, a lump solution is localized in all directions just in space. In addition, it is well known that interaction solutions between lump solutions and soliton solutions allow to describe more nonlinear phenomena.[15] However, the interaction properties are rarely discussed because the involved mathematical computation is much more complicated.

Generally, through a depended variable transformation, a partial differential equation can be mapped into a Hirota's bilinear form

$$P(D_x, D_y, D_t) f \cdot f = 0\,, \tag{1.2}$$

where P is a polynomial, and D_x, D_y, D_t are the Hirota bilinear dirivatives,[6] defined by

$$D_x^l D_y^n D_t^m f(x,y,t)\cdot g(x,y,t)$$
$$= \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^m f(x,y,t)\cdot g(x',y',t')|_{x'=x,y'=y,t'=t}\,. \tag{1.3}$$

When f solves (1.2), it presents the N-soliton solution in (2+1)-dimensions to the corresponding PDE under the transformation $u = 2(\ln f)_{xx}$:

$$f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right)\,, \tag{1.4}$$

where $\sum_{\mu=0,1}$ denotes the sum over all possibilities for $\mu_1, \ldots, \mu_N$ in $0, 1$, and

$$\begin{cases} \xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, \quad 1 \le i \le N\,, \\ e^{a_{ij}} = -\dfrac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, \quad 1 \le i < j \le N\,, \end{cases} \tag{1.5}$$

with k_i, l_i, ω_i satisfying the dispersion relation, and $\xi_{i,0}$ being arbitrary shifts.

As is well known, the KPI equation possesses lump solutions:[17] $u = 2(\ln f)_{xx}$, where

$$f = \left(a_1 x + a_2 y + \frac{a_1 a_2^2 - a_1 a_6^2 + 2a_2 a_5 a_6}{a_1^2 + a_5^2} t + a_4\right)^2$$
$$+ \left(a_5 x + a_6 y + \frac{2a_1 a_2 a_6 - a_2^2 a_5 + a_5 a_6^2}{a_1^2 + a_5^2} t + a_8\right)^2 + \frac{3(a_1^2 + a_5^2)^3}{(a_1 a_6 - a_2 a_5)^2}\,.$$

The condition $a_1 a_6 - a_2 a_5 \neq 0$ guarantees the rational localisation in all directions in the (x, y)-plane.

In the past few decades, many researchers have studied soliton solutions, lump solutions and other classes of solutions to integrable equations, such as the Ishimori-I equation,[18] the Davey-stewarton equation II,[1] the BKP equation,[19,20] the three-dimensional three-wave resonant interaction,[21] the KP equation with a self-consistent source[22] and so on.[23–26] Some non-integrable equations have lump solutions as well, such as the generalized KP, Sawada-Kotera equations.[27–29] Moreover, various studies show the existence of interaction solutions between lumps and other kind of exact solutions to nonlinear integrable equation.[30–36]

This paper is concerned with the following extended KPII equation combined with new fourth-order terms:

$$\left(u_t + \frac{3}{2} u u_x + \frac{1}{4} u_{xxx}\right)_x + u_{yy} + \delta_1 u_{yt} + \delta_2 u_{xy} + \delta_3 u_{xx}$$
$$+ \frac{\delta_4}{2}(u_{xxxt} + 3u_x u_{xt} + 3u_{xx} u_t) = 0\,. \tag{1.6}$$

where $\delta_i, i = 1, 2, 3, 4$, are arbitrary constants. It reduces to the KPII equation by choosing $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$. Based on a bilinear transformation, the equation possesses a Hirota's bilinear form. Solitary solutions, lump solutions and interaction solutions are obtained through symbolic computation with Maple. We exhibit three-dimensional plot and contour plot profiles of these solutions and study their dynamic behaviors. Some concluding remarks are given in the final section.

2. Bilinear Form

We substitute the logarithmic transformation $u = 2(\ln f)_{xx}$ into (1.6), and then the equation has the following Hirota's bilinear form:

$$\left(D_x D_t + \frac{1}{4}D_x^4 + D_y^2 + \delta_1 D_y D_t + \delta_2 D_x D_y + \delta_3 D_x^2 + \frac{\delta_4}{2}D_x^3 D_t\right) f \cdot f = 0\,, \tag{2.1}$$

equivalently,

$$\begin{aligned} &2ff_{xt} - 2f_t f_x + \frac{1}{2}ff_{xxxx} - 2f_x f_{xxx} + \frac{3}{2}f_{xx}^2 + 2(ff_{yy} - f_y^2) \\ &+ 2\delta_1(ff_{yt} - f_y f_t) + 2\delta_2(ff_{xy} - f_x f_y) + 2\delta_3(ff_{xx} - f_x^2) \\ &+ \delta_4(ff_{xxxt} - f_{xxx}f_t - 3f_x f_{xxt} + 3f_{xx}f_{xt}) = 0\,. \end{aligned} \tag{2.2}$$

Therefore, if f solves the bilinear equation (2.2), then $u = 2(\ln f)_{xx}$ will solve Eq. (1.6).

3. Solitary Solutions

Based on Hirota's bilinear method, we would like to construct soliton solutions and take a choice with a combination of exponential functions for f:

$$f = 1 + e^{a_1 x + a_2 y + a_3 t} + e^{a_4 x + a_5 y + a_6 t}\,, \tag{3.1}$$

where $a_i, i = 1, 2, 3, \ldots, 6$, are arbitrary real constants to be determined. We insert (3.1) into (2.2), and then solve the resulting algebraic system by Maple symbolic computations to obtain the following parameters:

$$\begin{aligned} a_3 &= \frac{a_1 b_1 \delta_1 \delta_3 + a_1 b_2 \delta_3 \delta_4 + a_1 b_3 \delta_1 + a_1 b_4 \delta_4 + a_2 b_5}{A_1 \delta_4^2 + A_2 \delta_1 \delta_4 + A_3 \delta_1 + A_4 \delta_4}\,, \\ a_6 &= \frac{a_4 b_1 \delta_1 \delta_3 - a_4 b_2 \delta_3 \delta_4 + a_4 b_3 \delta_1 + a_4 b_4 \delta_4 + a_5 b_5}{A_1 \delta_4^2 + A_2 \delta_1 \delta_4 + A_3 \delta_1 + A_4 \delta_4}\,. \end{aligned} \tag{3.2}$$

where $b_i, i = 1, 2, 3, \ldots, 5, A_i, i = 1, 2, 3, 4$, satisfy:

$$\begin{cases} b_1 = -8(a_1a_5 - a_2a_4)^2\,, \\ b_2 = -4a_1a_4(a_1 - a_4)(2a_1 - a_4)(a_1a_5 - a_2a_4)\,, \\ b_3 = -2(a_1^4a_5^2 - 4a_1^3a_2a_4a_5 + 6a_1^2a_2a_4^2a_5 - 4a_1a_2a_4^3a_5 + a_2^2a_4^4)\,, \\ b_4 = {a_1}^7a_4a_5 - 3{a_1}^6{a_4}^2a_5 + 2{a_1}^5{a_4}^3a_5 + 2{a_1}^4a_2{a_4}^4 - 3{a_1}^3a_2{a_4}^5 \\ \qquad + {a_1}^2a_2{a_4}^6 - 4{a_1}^4a_2{a_5}^2 + 4{a_1}^3{a_2}^2a_4a_5 + 12{a_1}^3a_2a_4{a_5}^2 \\ \qquad - 12{a_1}^2{a_2}^2{a_4}^2a_5 - 12{a_1}^2a_2{a_4}^2{a_5}^2 + 16a_1{a_2}^2{a_4}^3a_5 - 4{a_2}^3{a_4}^4\,, \\ b_5 = 3{a_1}^4{a_4}^2 - 6{a_1}^3{a_4}^3 + 3{a_1}^2{a_4}^4 - 4{a_1}^2{a_5}^2 + 8a_1a_2a_4a_5 - 4{a_2}^2{a_4}^2\,, \\ A_1 = a_1a_4\left(a_1 - a_4\right)\left({a_1}^4a_5 - 2{a_1}^3a_4a_5 + 2a_1a_2{a_4}^3 - a_2{a_4}^4\right), \\ A_2 = -2{a_1}^4{a_5}^2 + 8{a_1}^3a_2a_4a_5 - 12{a_1}^2a_2{a_4}^2a_5 + 8a_1a_2{a_4}^3a_5 - 2{a_2}^2{a_4}^4, \\ A_3 = -4(a_1a_5 - a_2a_4)^2\,, \\ A_4 = 2a_1a_4\left(a_1 - a_4\right)\left({a_1}^2a_5 + 2a_1a_2a_4 - 2a_1a_4a_5 - a_2{a_4}^2\right). \end{cases} \tag{3.3}$$

And δ_2 satisfy:

$$\delta_2 = \frac{c_1\delta_1\delta_3\delta_4 + c_2\delta_1^2 + c_3\delta_1^2\delta_3 + c_4\delta_4^2 + c_5\delta_4^2\delta_3 + c_6\delta_1\delta_4 + c_7\delta_3\delta_4 + c_8\delta_1 + c_9\delta_4 + c_{10}}{A_1\delta_4^2 + A_2\delta_1\delta_4 + A_3\delta_1 + A_4\delta_4}\,, \tag{3.4}$$

where $c_i, i = 1, 2, 3, \ldots, 10$, satisfy:

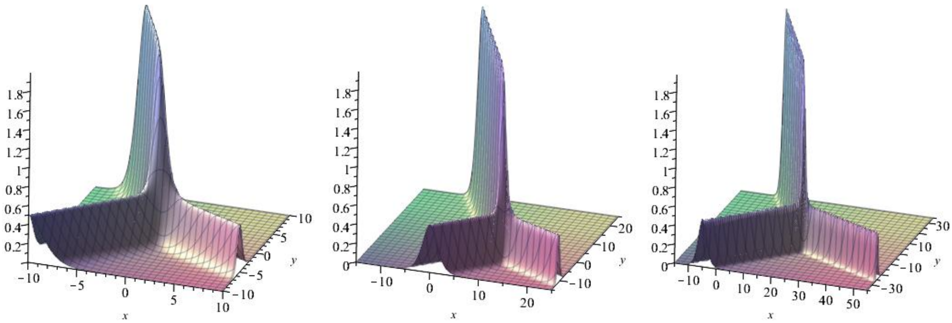
$$\begin{cases} c_1 = 4a_1a_4\left(a_1 - a_4\right)\left({a_1}^2a_5 + 2a_1a_2a_4 - 2a_1a_4a_5 - a_2{a_4}^2\right), \\ c_2 = 2{a_1}^4{a_5}^2 - 8{a_1}^3a_2a_4a_5 + 12{a_1}^2a_2{a_4}^2a_5 - 8a_1a_2{a_4}^3a_5 + 2{a_2}^2{a_4}^4\,, \\ c_3 = 8\left(a_1a_5 - a_2a_4\right)^2\,, \\ c_4 = 2{a_1}^6{a_5}^2 - 6{a_1}^5a_4{a_5}^2 + 6{a_1}^4{a_4}^2{a_5}^2 - 4{a_1}^3a_2{a_4}^3a_5 \\ \qquad + 6{a_1}^2{a_2}^2{a_4}^4 - 6a_1{a_2}^2{a_4}^5 + 2{a_2}^2{a_4}^6\,, \\ c_5 = 2{a_1}^2{a_4}^2\left({a_1}^2 - a_1a_4 + {a_4}^2\right)\left(a_1 - a_4\right)^2\,, \\ c_6 = -a_1a_4(a_1 - a_4)({a_1}^4a_5 - 2{a_1}^3a_4a_5 + 2a_1a_2{a_4}^3 \\ \qquad - a_2{a_4}^4 - 12{a_2}^2a_5 + 12a_2{a_5}^2), \\ c_7 = 12{a_1}^2{a_4}^2\left(a_1 - a_4\right)^2, \\ c_8 = -4a_1a_4\left(a_1 - a_4\right)\left({a_1}^2a_5 + 2a_1a_2a_4 - 2a_1a_4a_5 - a_2{a_4}^2\right), \\ c_9 = -{a_1}^6{a_4}^2 + 3{a_1}^5{a_4}^3 - 4{a_1}^4{a_4}^4 + 3{a_1}^3{a_4}^5 - {a_1}^2{a_4}^6 + 8{a_1}^4{a_5}^2 \\ \qquad - 8{a_1}^3a_2a_4a_5 - 12{a_1}^3a_4{a_5}^2 + 12{a_1}^2{a_2}^2{a_4}^2 + 12{a_1}^2{a_4}^2{a_5}^2 \\ \qquad - 12a_1{a_2}^2{a_4}^3 - 8a_1a_2{a_4}^3a_5 + 8{a_2}^2{a_4}^4\,, \\ c_{10} = -6{a_1}^4{a_4}^2 + 12{a_1}^3{a_4}^3 - 6{a_1}^2{a_4}^4 + 8{a_1}^2{a_5}^2 - 16a_1a_2a_4a_5 + 8{a_2}^2{a_4}^2\,. \end{cases} \tag{3.5}$$

Substituting (3.1) into (2.2) yields soliton solutions:

$$u = 2\frac{a_1^2\mathrm{e}^{a_1x+a_2y+a_3t} + a_4^2\mathrm{e}^{a_4x+a_5y+a_6t} + (a_1 - a_4)^2\mathrm{e}^{a_1x+a_2y+a_3t}\mathrm{e}^{a_4x+a_5y+a_6t}}{(1 + \mathrm{e}^{a_1x+a_2y+a_3t} + \mathrm{e}^{a_4x+a_5y+a_6t})^2}\,.$$

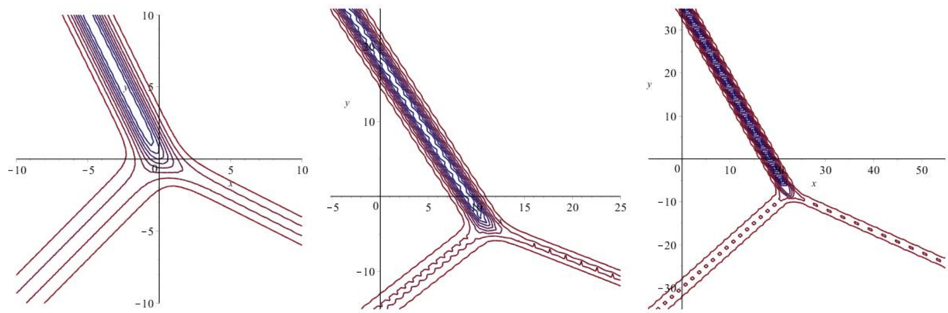
where $a_3,\ a_6,\ \delta_2$ satisfy (3.2) and (3.4). When we substitute the following

$$f = 1 + e^{a_1x+a_2y+a_3t} + e^{a_4x+a_5y+a_6t} + Ke^{a_1x+a_2y+a_3t}e^{a_4x+a_5y+a_6t}\,, \tag{3.6}$$



(a) (b) (c)

Fig. 1. (Color online) soliton solution u with $t = 0, 15, 30$: 3d plots.



(a) (b) (c)

Fig. 2. (Color online) soliton solution u with $t = 0, 15, 30$: contour plots.

into (2.2), we solve that $K = 0$ under the condition $a_3,\ a_6,\ \delta_2$ satisfy (3.2) and (3.4). And we prove that this solution we find is a resonant soliton solution of (1.6).

We take a special choice for the parameters: $a_1 = 1,\ a_2 = -1,\ a_3 = -1,\ a_4 = 2,\ a_5 = 1,\ a_6 = \frac{-17}{4},\ \delta_1 = 1,\ \delta_2 = \frac{11}{4},\ \delta_3 = 1,\ \delta_4 = 1$, and plot the graphs of the solution in Figs. 1 and 2, where we found that the soliton solution u is shown as "Y" shaped. The characteristics of the soliton which are caused by dispersive effects can help us to understand the dynamical behaviors of solutions.

4. Lump Solutions

A search for positive quadratic solutions to the bilinear equation (2.2) generates a class of lump solutions to the equation (1.6):

$$f = (a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9\,. \tag{4.1}$$

where $a_i, i = 1, 2, 3, \ldots, 9$, are arbitrary real constants to be determined. We insert (4.1) into (2.2), and then solve the resulting algebraic system by Maple symbolic computations to obtain the following parameters:

$$\begin{cases} a_3 = -\dfrac{a_1b_1\delta_1\delta_2 + b_2\delta_1\delta_3 + a_2b_1\delta_1 + a_2b_3\delta_2 + a_1b_3\delta_3 + b_4}{(a_1+\delta_1a_2)^2 + (a_5+\delta_1a_6)^2}\,, \\ a_7 = -\dfrac{a_5b_1\delta_1\delta_2 + b_5\delta_1\delta_3 + a_6b_1\delta_1 + a_6b_3\delta_2 + a_5b_3\delta_3 + b_6}{(a_1+\delta_1a_2)^2 + (a_5+\delta_1a_6)^2}\,, \\ a_9 = \dfrac{3}{4}\,\dfrac{2b_1b_3^2\delta_1\delta_2\delta_4 + 2b_7b_3^2\delta_1\delta_3\delta_4 - b_1b_3^2\delta_1^2 + 2b_1b_3b_7\delta_1\delta_4 + 2b_7b_3^2\delta_2\delta_4}{b_8^2(\delta_1^2\delta_3 - \delta_1\delta_2 + 1)} \\ \qquad + \dfrac{2b_3^3\delta_3\delta_4 + 2b_7b_3^2\delta_1 + 2b_3(b_7^2 - b_8^2)\delta_4 - b_3^3}{b_8^2(\delta_1^2\delta_3 - \delta_1\delta_2 + 1)}\,. \end{cases} \tag{4.2}$$

The above involved eight constants $b_i, i = 1, 2, 3, \ldots, 8$, are defined as follows:

$$\begin{cases} b_1 = \left({a_2}^2 + {a_6}^2\right), \quad b_5 = a_6({a_5}^2 - {a_1}^2) + 2a_1a_2a_5\,, \\ b_2 = a_2({a_1}^2 - {a_5}^2) + 2a_1a_5a_6, \quad b_6 = a_5({a_6}^2 - {a_2}^2) + 2a_1a_2a_6\,, \\ b_3 = \left({a_1}^2 + {a_5}^2\right), \quad b_7 = a_1a_2 + a_5a_6, \\ b_4 = a_1({a_2}^2 - {a_6}^2) + 2a_2a_5a_6, \quad b_8 = a_1a_6 - a_2a_5\,. \end{cases} \tag{4.3}$$

Therefore, besides $a_1a_6 - a_2a_5 \neq 0$, the condition guaranteeing the nonsingularity of the lump solution is $(\delta_1^2\delta_3 - \delta_1\delta_2 + 1) \neq 0$, and they should satisfy the following constraint conditions:

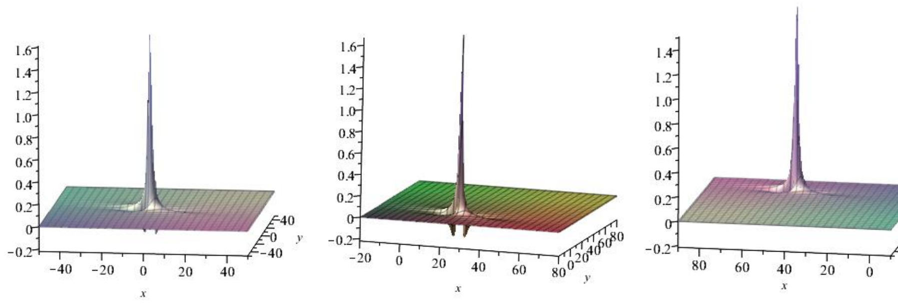
$$\frac{2b_1b_3^2\delta_1\delta_2\delta_4 + 2b_7b_3^2\delta_1\delta_3\delta_4 - b_1b_3^2\delta_1^2 + 2b_1b_3b_7\delta_1\delta_4}{(\delta_1^2\delta_3 - \delta_1\delta_2 + 1)} + \frac{2b_7b_3^2\delta_2\delta_4 + 2b_3^3\delta_3\delta_4 + 2b_7b_3^2\delta_1 + 2b_3(b_7^2 - b_8^2)\delta_4 - b_3^3}{(\delta_1^2\delta_3 - \delta_1\delta_2 + 1)} > 0\,. \tag{4.4}$$

we take a special choice for the parameters: $a_1 = 1,\ a_2 = -1,\ a_3 = -\frac{5}{4},\ a_4 = 2,\ a_5 = 1,\ a_6 = 3,\ a_7 = -3,\ a_8 = 6,\ a_9 = \frac{9}{2},\ \delta_1 = 1,\ \delta_2 = 1,\ \delta_3 = 1,\ \delta_4 = 1,$ under which f by (4.1), (4.2), (4.3) will present the lump solution to (1.6).

$$u = \frac{1024(15t^2 - 17tx - 3ty + 4x^2 + 8xy - 12y^2 - 54t + 32x + 39)}{(169t^2 - 136tx - 248ty + 32x^2 + 64xy + 160y^2 - 656t + 256x + 512y + 712)^2}\,. \tag{4.5}$$

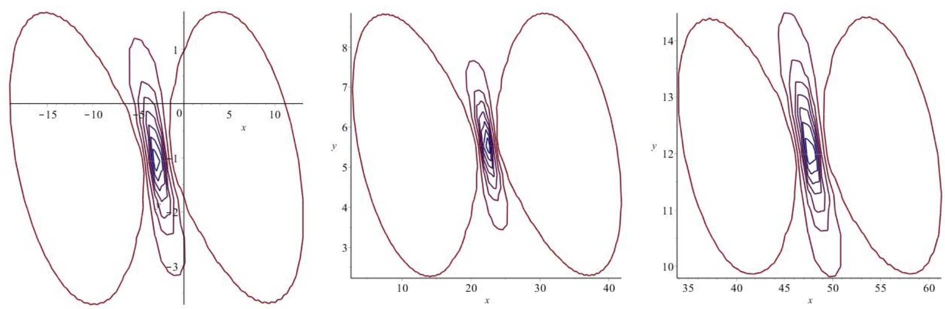
We plot the graphs of this solution in Figs. 3 and 4.

Though the KPII equation does not have any lump solution, we have presented lump solutions to its extended equation by means of the Hirota bilinear formulation. The solutions have been depicted for special values of the parameters and three different values of t. Figure 3 shows lump wave, which has a peak and two valleys and algebraically decays in all space directions. Because the height of the peak is larger than the depths of the valley bottoms, the solution (4.5) can be called the bright lump wave solution. By caculating, we find that the lump solution reaches the peak at $x = -3, y = -1$, when $t = 0$. As time goes on, the waves moving forward reach the same amplitude at other points.



(a) (b) (c)

Fig. 3. (Color online) lump solution u with $t = 0, 15, 30$: 3d plots.



(a) (b) (c)

Fig. 4. (Color online) lump solution u with $t = 0, 15, 30$: contour plots.

5. Interaction Solutions

We would like to consider interaction solutions of the extedned KPII equation. We mix a quadratic function with an exponential function. This would tell us interactions between lump waves and soliton solutions. Let

$$f = (a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9 + ke^{a_{10}x+a_{11}y+a_{12}t}, \tag{5.1}$$

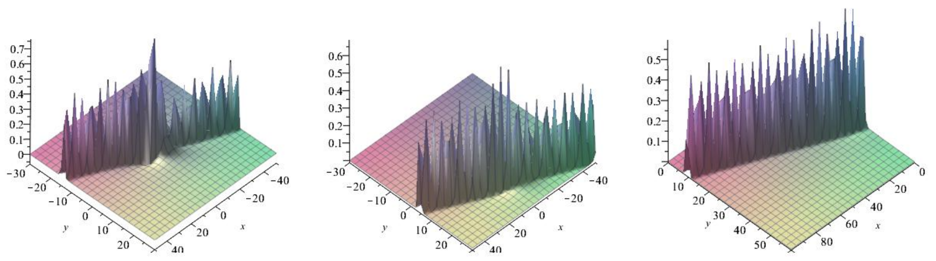
where $a_i, i = 1, 2, 3, \ldots, 12$, are arbitrary real constants to be determined. We insert (5.1) into (2.2), and then solve the resulting algebraic system by Maple symbolic computations to obtain the following parameters:

$$\begin{cases} a_3 = -\dfrac{a_2}{\delta_1}, \quad a_7 = -\dfrac{a_6}{\delta_1}, \\ a_{12} = -\dfrac{1}{2}\dfrac{{a_{10}}^3\delta_1 + 2{a_{10}}^2a_{11}\delta_4 + 4a_{10}\delta_1\delta_3 + 4a_{11}}{\delta_1\left({a_{10}}^2\delta_4 + 2\right)}, \\ \delta_2 = \dfrac{1}{2}\dfrac{{a_{10}}^4{\delta_4}^2 + {a_{10}}^2{\delta_1}^2 + 4{a_{10}}^2\delta_4 + 4{\delta_1}^2\delta_3 + 4}{\delta_1\left({a_{10}}^2\delta_4 + 2\right)}, \end{cases} \tag{5.2}$$

we obtain the interaction solution for (1.6). The parameters not determined above are arbitrary constants. Therefore, the condition for guaranteeing solution is $\delta_1 \neq 0, ({a_{10}}^2\delta_4+2) \neq 0,\ a_9 > 0$. Taking a special choice for the parameters: $a_1 = 1,\ a_2 = -1,\ a_3 = 1,\ a_4 = 2,\ a_5 = 0,\ a_6 = 3,\ a_7 = -3,\ a_8 = 4,\ a_9 = 2,\ a_{10} = -1,\ a_{11} = -4,\ a_{12} = \frac{29}{6},\ k = 8,\ \delta_1 = 1,\ \delta_2 = \frac{7}{3},\ \delta_3 = 1,\ \delta_4 = 1$, we can get the interaction solution between a lump solution and a one-kink soliton.

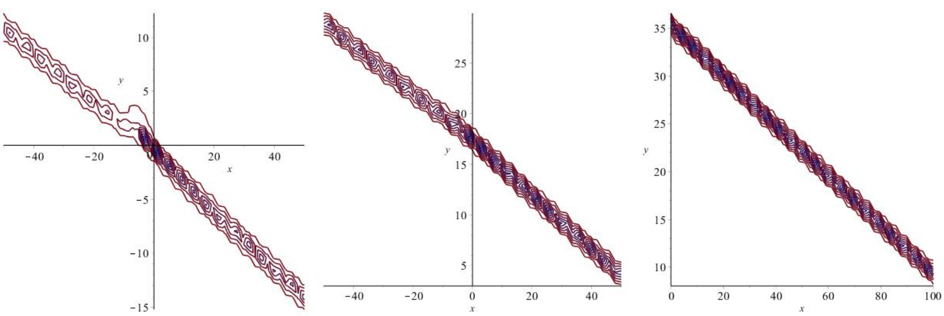
$$u = 4\frac{(40t^2 + 8tx - 80ty + 4x^2 - 8xy + 40y^2 - 64t + 32x + 64y + 128)\mathrm{e}^{-x-4y+29t/6}}{(10t^2 + 2tx - 20ty + x^2 - 2xy + 10y^2 + 8e^{-x-4y+29t/6} - 20t + 4x + 20y + 22)^2}$$
$$+ 4\frac{8t^2 - 2tx - 16ty - x^2 + 2xy + 8y^2 - 28t - 4x + 28y + 14}{(10t^2 + 2tx - 20ty + x^2 - 2xy + 10y^2 + 8e^{-x-4y+29t/6} - 20t + 4x + 20y + 22)^2}\,. \tag{5.3}$$

We plot the graphs of this solution in Figs. 5 and 6.



(a) (b) (c)

Fig. 5. (Color online) interaction solution u with $t = 0, 15, 30$: 3d plots.



(a) (b) (c)

Fig. 6. (Color online) interaction solution u with $t = 0, 15, 30$: contour plots.

6. Conclusion and Remarks

In this paper, we have considered an extension of the KPII equation. We have worked out a class of particular soliton solutions, lump solutions and interaction solutions by Hirota's bilinear method. It is important to remark that the KPII appears in [18] as a nonintegrable example to show the applicability of Hirota's method to nonintegrable systems. The extended terms added to the KPII equation enhance the integrability of KP models. The extended model was proved to possess the integrability and soliton solutions. The effect of the extensed terms was on the dispersion relation as shown. It is also worth mentioning that our interest in investigating the existence of lump solutions for the extended KPII equation stems from the fact that the KPII equation does not have lump solutions. Exact solutions of the extended KPII are abundant, which can explain more physical phenomena. All the above results offer us abundant new exact solutions, which enrich the existing theories of solutions[5,39–43] to equations, and add valuable insights into soliton solutions and dromion-type solutions, developed through various powerful solution techniques including the Hirota perturbation approach, the Riemann-Hilbert approach, the Wronskian technique, symmetry reductions, and symmetry constraints. It is also interesting to research for lump and interaction solutions to other generalized bilinear differential equations.[44,45] The research to establish a fundamental theory of lump and interaction solutions for PDEs deserve our further effort.

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