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N-fold Darboux transformation and conservation laws of the modified Volterra lattice

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In this work, we study the modified Volterra lattice. Applying the gauge transformation of the associated 2×2 matrix spectral problems, we establish the N-fold Darboux transformation (DT), and then construct a few explicit solutions in terms of determinants upon using the obtained DT. Moreover, all the results are illustrated by the graphs of the solitonic evolution profiles of the aforementioned solutions. Finally, infinitely many conservation laws for the modified Volterra lattice are proposed. The obtained results of this research might be applied to the research on nonlinear phenomena in physics or engineering areas.

Keywords: Lax pair; Darboux transformation; explicit solution; modified Volterra lattice; conversation law.

1. Introduction

Nonlinear integrable systems have been applied to many fields,[1] such as nonlinear optics and chaos. Moreover, their explicit solutions have been playing a key role in many research areas, for example, descriptions of different kinds of waves. Therefore, many researchers pay much attention to this important research. Until now, some excellent methods have been established, which are inverse scattering

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transformation,[2] Hirota bilinear method and generalized bilinear method,[3–8] the homogeneous method,[9–13] the hyperbolic function method,[14] the F-expansion method,[15] the Bäcklund transformation method,[16] the extended tanh-function method,[17] similarity transformation,[18,19] algebra-geometric approach,[20] variable-detached method,[21] Painleve analysis,[22] Darboux transformation,[23–27] and so forth.[28–33] Among the aforementioned methods, the Darboux transformation is one of the powerful and direct methods to investigate explicit solutions.[34]

In this paper, we will study the DT and explicit solutions of the modified Volterra lattice[35]:

$$p_{n,t} = (p_n^2 + c)(p_{n+1} - p_{n-1})\,, \tag{1}$$

where $p_n = p(n,t)$ is a function of discrete variable n and time variable t, and $p_{n,t} = \frac{dp_n}{dt}$. From Ref. 35, the Lax pair for Eq. (1) is

$$E\varphi_n = U_n\varphi_n = \begin{pmatrix} c\lambda^{-1} & p_n \\ -p_n & \lambda \end{pmatrix}\varphi_n\,, \tag{2}$$

$$\varphi_{n,t} = V_n\varphi_n = \begin{pmatrix} \dfrac{c^2}{\lambda^2} + p_n p_{n-1} & \dfrac{cp_n}{\lambda} + \lambda p_{n-1} \\ -\dfrac{cp_{n-1}}{\lambda} - \lambda p_n & \lambda^2 + p_n p_{n-1} \end{pmatrix}\varphi_n\,, \tag{3}$$

where $\varphi_n = (\varphi_{1,n}, \varphi_{2,n})^T$ is an eigenfunction vector, and T denotes the transpose of a vector or a matrix, and E is the shift operator defined by $Ef(n,t) = f(n+1,t)$, $E^{-1}f(n,t) = f(n-1,t)$, and λ stands for the spectral parameter which is independent of time variable t. Naturally, the condition of compatibility between Eqs. (2) and (3) leads to a zero-curvature equation

$$U_{n,t} = V_{n+1}U_n - U_nV_n\,. \tag{4}$$

This paper is organized as follows. In Sec. 2, the N-fold DT for Eq. (1) is constructed by using the AKNS procedure; then some explicit solutions described by determinants are obtained according to the aforementioned N-fold DT. Moreover, the interactions of those solutions are illustrated in Sec. 3. In Sec. 4, infinitely many conservation laws of Eq. (1) are given. Finally, some applications in physics and other conclusions are given in Sec. 5.

2. *N*-Fold Darboux Transformation

In order to construct the N-fold Darboux transformation, we firstly introduce the following gauge transformation:

$$\hat{\varphi}_n = T\varphi_n\,, \tag{5}$$

where $\hat{\varphi}_n$ satisfies Eqs. (2) and (3), and

$$E\hat{\varphi}_n = \hat{U}_n\hat{\varphi}_n, \quad \hat{U}_n = T_{n+1}U_nT_n^{-1}\,, \tag{6}$$

$$\hat{\varphi}_{n,t} = \hat{V}_n \hat{\varphi}_n, \quad \hat{V} = (T_{n,t} + T_n V_n) T_n^{-1}\,. \tag{7}$$

The Darboux matrix T_n will play a vital role in establishing the DT for Eq. (1) since it can guarantee the validity of the N-fold DT. We assume

$$\begin{aligned} T_n &= \begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix} \\ &= \begin{pmatrix} \lambda^{2N+1} + \displaystyle\sum_{j=-N-1}^{N-1} a_n^{(2j+1)} \lambda^{2j+1} & \displaystyle\sum_{j=-N}^{N} b_n^{(2j)} \lambda^{2j} \\ -\displaystyle\sum_{j=-N}^{N} b_n^{(-2j)} \lambda^{2j} & \lambda^{-2N-1} + \displaystyle\sum_{j=-N}^{N} a_n^{(-2j-1)} \lambda^{2j+1} \end{pmatrix}, \end{aligned} \tag{8}$$

where λ_i's ($\lambda_i \neq 0, \lambda_i \neq \lambda_j, i \neq j, i, j = 1, 2, \ldots, 2N+1$) are roots of the polynomial $x^{4N+2}\det(T_n)$ of degree $8N+4$. It is easy to prove that

$$\lambda^{4N+2} \det(T_n) = a_n^{(-2N-1)} \prod_{i=1}^{2N+1} (\lambda^2 - \lambda_i^2)^2\,, \tag{9}$$

which implies that $\det(T_n(\lambda_i)) = 0$. Then we have

$$\begin{aligned} &\sum_{j=-N-1}^{N-1} a_n^{(2j+1)} \lambda_i^{2j+1} + \sum_{j=-N}^{N} b_n^{(2j)} \lambda_i^{2j} \delta_{i,n} = -\lambda_i^{2N+1}\,, \\ -&\sum_{j=-N}^{N} b_n^{(-2j)} \lambda_i^{2j} + \sum_{j=-N}^{N} a_n^{(-2j-1)} \lambda_i^{2j+1} \delta_{i,n} = -\delta_{i,n} \lambda_i^{-2N-1}\,, \end{aligned} \tag{10}$$

where

$$\delta_{i,n} = \frac{\varphi_{2,n}(\lambda_i)}{\varphi_{1,n}(\lambda_i)}, \quad \delta_{i,n+1} = \frac{(\lambda_i \delta_{i,n} - p_n)\lambda_i}{\lambda_i p_n \delta_{i,n} + c}\,, \tag{11}$$

where $\varphi_n = (\varphi_{1,n}, \varphi_{2,n})^T$ is a solution of the Lax pairs (2) and (3). We will choose suitable $\lambda_i, r_i (\lambda_i \neq 0, r_i \neq 0, i = 1, 2, \ldots, 2N+1)$ to ensure that the coefficient determinant for Eqs. (10) is not equal to zero. Thus, $a_n^{(i)}, b_n^{(i)}, c_n^{(i)}, d_n^{(i)}$ can be uniquely determined by Eqs. (10). Based on the aforementioned facts, the following theorem is true.

Theorem 1. *The matrices $\hat{U}_n, \hat{V}_n$ defined in* (6) *and* (7) *have the same forms as U_n, V_n, respectively, where the transformations from the old potential p to the new one $\hat{p}$ is given by*

$$\hat{p} = p_n a_{n+1}^{(-2N-1)} - c b_{n+1}^{(-2N)}\,. \tag{12}$$

Proof. Let the adjoint matrix of T_n be T_n^*, then $T_n^* = (\det T_n) T_n^{-1}$. Then

$$\lambda^{4N+3} T_{n+1} U_n T_n^* = \begin{pmatrix} f_{11}(\lambda, n) & f_{12}(\lambda, n) \\ f_{21}(\lambda, n) & f_{22}(\lambda, n) \end{pmatrix}. \tag{13}$$

It is easy to show that $f_{11}(\lambda, n)$ is a polynomial of degree $8N+4$ in λ and $f_{12}(\lambda, n)$ and $f_{21}(\lambda, n)$ are polynomials of degree $8N+5$ and $f_{22}(\lambda, n)$ is a polynomial of degree $8N+6$. Therefore, we have $f_{st}(\lambda_i, n) = 0 (s, t = 1, 2)$. Now, (13) can be rewritten as follows:

$$\begin{aligned}
&\lambda^{4N+3}(T_{n+1}U_n)T_n^* \\
&\quad = \lambda^{4N+2}\det(T_n)Q_n(\lambda) \\
&\quad = \lambda^{4N+2}\det(T_n)\begin{pmatrix} q_{11}^{(0)} & q_{12}^{(1)}\lambda + q_{12}^{(0)} \\ q_{21}^{(1)}\lambda + q_{21}^{(0)} & q_{22}^{(2)}\lambda^2 + q_{22}^{(1)}\lambda + q_{22}^{(0)} \end{pmatrix},
\end{aligned} \tag{14}$$

where $q_{ij}^{(m)}$ are functions to be determined and independent of λ. Then we have

$$\lambda(T_{n+1}U_n) = Q_n T_n\,. \tag{15}$$

Comparing the coefficients of $\lambda^{4N+3}, \lambda^{4N+1}$ in Eq. (16), we get

$$\begin{aligned}
&q_{11}^{(0)} = c, \quad q_{12}^{(0)} = 0, \quad q_{12}^{(1)} = p_n a_{n+1}^{(-2N-1)} - c b_{n+1}^{(-2N)} = \hat{p_n}, \quad q_{21}^{(0)} = 0\,, \\
&q_{21}^{(1)} = -(p_n a_{n+1}^{(-2N-1)} - c b_{n+1}^{(-2N)}) = -\hat{u}, \quad q_{22}^{(0)} = q_{22}^{(1)} = 0, \quad q_{22}^{(2)} = 1\,,
\end{aligned} \tag{16}$$

which implies $P = \hat{U}$.

Similarly, we can prove that the matrix $\hat{V}$ has the same form as V. □

It follows from Theorem 1 that the transformations (5) and (12) can transform the Lax pairs (2) and (3) into the Lax pairs (6) and (7) of the same type. Thus, they can both result in Eq. (1). The transformations (5) and (12) are called an N-fold DT of Eq. (1).

3. Explicit Solutions

In this section, the transformations (5) and (12) will be applied to constructing the explicit solutions of Eq. (1). Substituting the trivial solution $p = 0$ into (3) and (4), we got a solution of (2) and (3):

$$\varphi = \begin{pmatrix} \left(\dfrac{\lambda}{c}\right)^{-n} e^{\frac{c^2}{\lambda^2}t} \\ \lambda^n e^{\lambda^2 t} \end{pmatrix}. \tag{17}$$

Moreover, we have

$$\delta_{i,n} = \frac{\lambda_i^{2n}}{c^n} e^{(\lambda_i^2 - \frac{c^2}{\lambda_i^2})t}, \quad \delta_{i,n+1} = \frac{\lambda_i^2}{c}\delta_{i,n}\,. \tag{18}$$

Solving the linear algebraic system (10) yields to

$$a_n^{(-2N-1)} = \frac{\Delta a_n^{(-2N-1)}}{\Delta}, \quad bn^{(-2N)} = \frac{\Delta b_n^{(-2N)}}{\Delta}\,, \tag{19}$$

with

$$\Delta = \begin{vmatrix} \lambda_1^{2N-1} & \cdots & \lambda_1^{-2N-1} & \lambda_1^{-2N}\delta_{1,n} & \cdots & \lambda_1^{2N}\delta_{1,n} \\ \lambda_2^{2N-1} & \cdots & \lambda_2^{-2N-1} & \lambda_2^{-2N}\delta_{2,n} & \cdots & \lambda_2^{2N}\delta_{1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_{2N+1}^{2N-1} & \cdots & \lambda_{2N+1}^{-2N-1} & \lambda_{2N+1}^{-2N}\delta_{2N+1,n} & \cdots & \lambda_{2N+1}^{2N}\delta_{2N+1,n} \\ \lambda_1^{-2N+1}\delta_{1,n} & \cdots & \lambda_1^{2N+1}\delta_{1,n} & -\lambda_1^{-2N} & \cdots & -\lambda_1^{2N} \\ \lambda_2^{-2N+1}\delta_{2,n} & \cdots & \lambda_2^{2N+1}\delta_{2,n} & -\lambda_2^{-2N} & \cdots & -\lambda_2^{2N} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_{2N+1}^{-2N+1}\delta_{1,n} & \cdots & \lambda_{2N+1}^{2N+1}\delta_{1,n} & -\lambda_{2N+1}^{-2N} & \cdots & -\lambda_{2N+1}^{2N} \end{vmatrix}. \tag{20}$$

$\Delta a_n^{(-2N-1)}$ is generated from Δ by replacing its $(2N+1)$th column with $(-\lambda_1^{2N+1}, -\lambda_2^{2N+1}, \ldots, -\lambda_{2N+1}^{2N+1}, -\lambda_1^{2N+1}\delta_{1,n}, -\lambda_2^{-2N-1}\delta_{2,n}, \ldots, -\lambda_{2N+1}^{-2N-1}\delta_{2N+1,n})$, and $\Delta b_n^{(-2N)}$ is generated from Δ by replacing its $(4N+2)$th column with $(-\lambda_1^{2N+1}, -\lambda_2^{2N+1}, \ldots, -\lambda_{2N+1}^{2N+1}, -\lambda_1^{2N+1}\delta_{1,n}, -\lambda_2^{-2N-1}\delta_{2,n}, \ldots, -\lambda_{2N+1}^{-2N-1}\delta_{2N+1,n})$. The suitable λ_i are selected to guarantee $\Delta \neq 0$. According to (12), a solution of Eq. (1) is obtained

$$\hat{p}_n = -b_{n+1}^{(-2N)}\,. \tag{21}$$

In order to better understand (21), the following cases of $N = 0$ and $N = 1$ are studied:

(I) When $N = 0$, solving Eqs. (10) for $n+1$ leads to

$$a_{n+1}^{(-1)} = \frac{\Delta a_{n+1}^{(-1)}}{\Delta}, \quad b_{n+1}^{(0)} = \frac{\Delta b_{n+1}^{(0)}}{\Delta}\,, \tag{22}$$

$$\begin{aligned} \Delta &= \begin{vmatrix} \lambda_1^{-1} & \delta_{1,n+1} \\ \lambda_1\delta_{1,n+1} & -1 \end{vmatrix}, \quad \Delta a_{n+1}^{(-1)} = \begin{vmatrix} -\lambda_1 & \delta_{1,n+1} \\ -\delta_{1,n+1}\lambda_1^{-1} & -1 \end{vmatrix}, \\ \Delta b_{n+1}^{(0)} &= \begin{vmatrix} \lambda_1^{-1} & -\lambda_1 \\ \delta_{1,n+1}\lambda_1 & -\delta_{1,n+1}\lambda_1^{-1} \end{vmatrix}. \end{aligned} \tag{23}$$

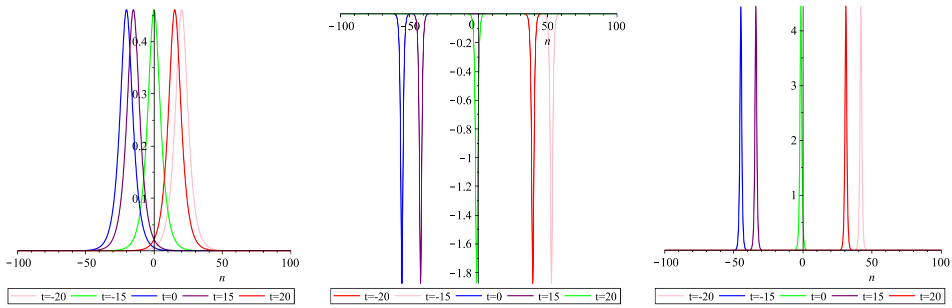
Applying the DT, we got the solution of Eq. (1) as follows:

$$\hat{p}_n = -b_{n+1}^{(0)}\,, \tag{24}$$

the evolution profiles of which at different times are shown in Fig. 1.

(II) When $N = 1$, solving Eqs. (10), we got

$$\begin{aligned} a_{n+1}^{(-3)} &= \frac{\Delta a_{n+1}^{(-3)}}{\Delta}\,, \\ b_{n+1}^{(-2)} &= \frac{\Delta b_{n+1}^{(-2)}}{\Delta} \end{aligned} \tag{25}$$



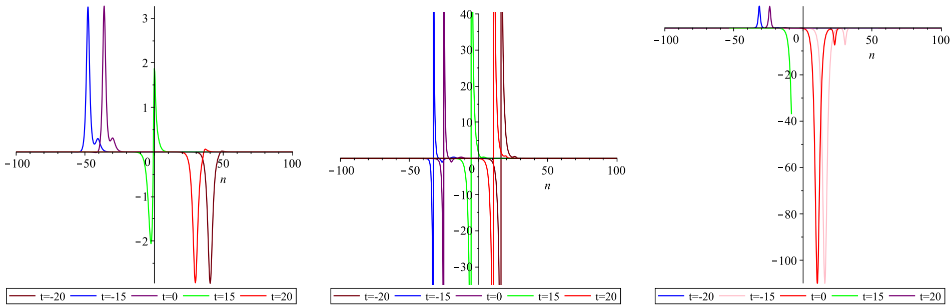
(a) (b) (c)

Fig. 1. (Color online) The evolution plot of solution (24) when $N = 0$ with specific parameter (a) $\lambda = \frac{4}{5}$, $c = \frac{1}{2}$, (b) $\lambda = 2$, $c = 1$ and (c) $\lambda = \frac{1}{3}$, $c = \frac{2}{3}$.

with

$$\Delta = \begin{vmatrix} \lambda_1 & \frac{1}{\lambda_1} & \frac{1}{\lambda_1^3} & \frac{\delta_{1,n+1}}{\lambda_1^2} & \delta_{1,n+1} & \lambda_1^2\delta_{1,n+1} \\ \lambda_2 & \frac{1}{\lambda_2} & \frac{1}{\lambda_2^3} & \frac{\delta_{2,n+1}}{\lambda_2^2} & \delta_{2,n+1} & \lambda_2^2\delta_{2,n+1} \\ \lambda_3 & \frac{1}{\lambda_3} & \frac{1}{\lambda_3^3} & \frac{\delta_{3,n+1}}{\lambda_3^2} & \delta_{3,n+1} & \lambda_3^2\delta_{3,n+1} \\ \frac{\delta_{1,n+1}}{\lambda_1} & \lambda_1\delta_{1,n+1} & \lambda_1^3\delta_{1,n+1} & -\lambda_1^2 & -1 & -\frac{1}{\lambda_1^2} \\ \frac{\delta_{2,n+1}}{\lambda_2} & \lambda_2\delta_{2,n+1} & \lambda_2^3\delta_{2,n+1} & -\lambda_2^2 & -1 & -\frac{1}{\lambda_2^2} \\ \frac{\delta_{3,n+1}}{\lambda_3} & \lambda_3\delta_{3,n+1} & \lambda_3^3\delta_{3,n+1} & -\lambda_3^2 & -1 & -\frac{1}{\lambda_3^2} \end{vmatrix},$$

$$\Delta a_{n+1}^{(-3)} = \begin{vmatrix} \lambda_1 & \frac{1}{\lambda_1} & -\lambda_1^3 & \frac{\delta_{1,n+1}}{\lambda_1^2} & \delta_{1,n+1} & \lambda_1^2\delta_{1,n+1} \\ \lambda_2 & \frac{1}{\lambda_2} & -\lambda_2^3 & \frac{\delta_{2,n+1}}{\lambda_2^2} & \delta_{2,n+1} & \lambda_2^2\delta_{2,n+1} \\ \lambda_3 & \frac{1}{\lambda_3} & -\lambda_3^3 & \frac{\delta_{3,n+1}}{\lambda_3^2} & \delta_{3,n+1} & \lambda_3^2\delta_{3,n+1} \\ \frac{\delta_{1,n+1}}{\lambda_1} & \lambda_1\delta_{1,n+1} & -\frac{\delta_{1,n+1}}{\lambda_1^3} & -\lambda_1^2 & -1 & -\frac{1}{\lambda_1^2} \\ \frac{\delta_{2,n+1}}{\lambda_2} & \lambda_2\delta_{2,n+1} & -\frac{\delta_{2,n+1}}{\lambda_2^3} & -\lambda_2^2 & -1 & -\frac{1}{\lambda_2^2} \\ \frac{\delta_{3,n+1}}{\lambda_3} & \lambda_3\delta_{3,n+1} & -\frac{\delta_{3,n+1}}{\lambda_3^3} & -\lambda_3^2 & -1 & -\frac{1}{\lambda_3^2} \end{vmatrix}, \quad (26)$$



(a) (b) (c)

Fig. 2. (Color online) The evolution profile of the solution (27) when $N = 1$ with specific parameter (a) $\lambda_1 = 1$, $\lambda_2 = \frac{7}{5}$, $\lambda_3 = \frac{8}{5}$, $c = 1$, (b) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{7}{5}$, $\lambda_3 = \frac{8}{5}$, $c = \frac{1}{2}$ and (c) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = -\frac{1}{5}$, $\lambda_3 = \frac{8}{5}$, $c = \frac{1}{2}$.

$$\Delta b_{n+1}^{(-2)} = \begin{vmatrix} \lambda_1 & \frac{1}{\lambda_1} & -\lambda_1^3 & \frac{\delta_{1,n+1}}{\lambda_1^2} & \delta_{1,n+1} & -\lambda_1^3 \\ \lambda_2 & \frac{1}{\lambda_2} & -\lambda_2^3 & \frac{\delta_{2,n+1}}{\lambda_2^2} & \delta_{2,n+1} & -\lambda_2^3 \\ \lambda_3 & \frac{1}{\lambda_3} & -\lambda_3^3 & \frac{\delta_{3,n+1}}{\lambda_3^2} & \delta_{3,n+1} & -\lambda_3^3 \\ \frac{\delta_{1,n+1}}{\lambda_1} & \lambda_1\delta_{1,n+1} & -\frac{\delta_{1,n+1}}{\lambda_1^3} & -\lambda_1^2 & -1 & -\frac{\delta_{1,n+1}}{\lambda_1^3} \\ \frac{\delta_{2,n+1}}{\lambda_2} & \lambda_2\delta_{2,n+1} & -\frac{\delta_{2,n+1}}{\lambda_2^3} & -\lambda_2^2 & -1 & -\frac{\delta_{2,n+1}}{\lambda_2^3} \\ \frac{\delta_{3,n+1}}{\lambda_3} & \lambda_3\delta_{3,n+1} & -\frac{\delta_{3,n+1}}{\lambda_3^3} & -\lambda_3^2 & -1 & -\frac{\delta_{3,n+1}}{\lambda_3^3} \end{vmatrix},$$

where $\Delta a_{n+1}^{(-3)}$ is generated from Δ by replacing its (3)th column with $(-\lambda_1^3, -\lambda_2^3, -\lambda_3^3, -\lambda_1^3\delta_{1,n+1}, -\lambda_2^{-3}\delta_{2,n+1}, -\lambda_3^{-3}\delta_{3,n+1})$, and $\Delta b_{n+1}^{(-2)}$ is generated from Δ by replacing its (6)th column with $(-\lambda_1^3, -\lambda_2^3, -\lambda_3^3, -\lambda_1^3\delta_{1,n+1}, -\lambda_2^{-3}\delta_{2,n+1}, -\lambda_3^{-3}\delta_{3,n+1})$. So that the solution of Eq. (1) is obtained by using the DT as

$$\hat{p}_n = -b_{n+1}^{(-2)}, \tag{27}$$

the evolution profiles of which at different times are illustrated by Fig. 2.

4. Conservation Laws

It follows from (2) that

$$\varphi_{1,n+1} = \frac{c}{\lambda}\varphi_{1,n} + p_n\varphi_{2,n}, \varphi_{2,n+1} = -p_n\varphi_{1,n} + \lambda\varphi_{2,n}. \tag{28}$$

If assume $\theta_n = \frac{\varphi_{2,n}}{\varphi_{1,n}}$, then (28) can be rewritten as

$$\frac{\varphi_{1,n+1}}{\varphi_{1,n}} = \frac{c}{\lambda} + p_n\theta_n\,, \quad \frac{\varphi_{2,n+1}}{\varphi_{2,n}} = -\frac{p_n}{\theta_n} + \lambda\,, \tag{29}$$

which implies

$$-\lambda^2\theta_n + p_n(\theta_n\theta_{n+1} + 1)\lambda + c\theta_{n+1} = 0\,. \tag{30}$$

Assuming $\theta_n = \sum_{j=1}^{+\infty} \frac{\theta_n^{(j)}}{\lambda^j}$ and plugging it into (30), then the recursion relation is obtained

$$\theta_n^{(1)} = p_n\,, \quad \theta_n^{(2)} = 0\,, \quad \theta_n^{(m+2)} = p_n \sum_{j=1}^{m} \theta_n^{(j)}\theta_{n+1}^{(m+1-j)} + c\theta_{n+1}^{(m)}\,, \tag{31}$$

where $m > 0$ is a positive integer. Moreover, from (3) and (29), we can get

$$[\ln(\varphi_{1,n})]_t = \frac{[\varphi_{1,n}]_t}{\varphi_{1,n}} = \left(\frac{c^2}{\lambda^2} + p_np_{n-1}\right) + \left(\frac{cp_n}{\lambda} + \lambda p_{n-1}\right)\theta_n \tag{32}$$

and

$$\left[\ln\left(\frac{c}{\lambda} + p_n\theta_n\right)\right]_t = (E-1)\left[\frac{-c^2}{\lambda^2} + p_np_{n-1} + \left(\frac{c}{\lambda}p_n + \lambda p_{n-1}\right)\theta_n\right]. \tag{33}$$

Equating the same powers of λ in (33), we obtain an infinite number of conservation laws for Eq. (1), and list the first two conservation laws:

$$\begin{aligned} &\left(-\frac{p_n^8}{4c^4} + \frac{p_n^6}{3c^3} - \frac{p_n^4}{2c^4} + \frac{p_n^2}{c}\right)_t = (E-1)[2p_np_{n-1}]\,, \\ &\left(p_np_{n+1} - \frac{p_n^9 p_{n+1}}{c^4}\right)_t = (E-1)[p_n^2p_{n-1}p_{n+1} + cp_n^2 + cp_{n-1}p_{n+1} - c^2]\,. \end{aligned} \tag{34}$$

5. Conclusions

This modified Volterra lattice not only behaves like a discrete version of the KdV equation but also is an integrable system related to the Toda lattice. It has been applied to solve complex nonlinear phenomenon in physics, biology and other engineering fields. For instance, it can be used to describe predator–prey interactions of a series of species preying on the next, and it can be also used as a model for Langmuir waves in plasmas.[36] In this work, the N-fold DT (5) and (12) of Eq. (1) were established, and the explicit solutions (24) and (27) described by determinants were also constructed. Furthermore, the solitonic evolution profiles were plotted, Figs. 1 and 2 show the solution of p_n when $N = 0$ and $N = 1$, respectively. Finally, the infinitely many conservation laws of Eq. (1) were derived using a standard way.

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