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Lump solutions to the Kadomtsev–Petviashvili I equation with a self-consistent source

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A B S T R A C T

Based on symbolic computations, lump solutions to the Kadomtsev–Petviashvili I (KPI) equation with a self-consistent source (KPIESCS) are constructed by using the Hirota bilinear method and an ansatz technique. In contrast with lower-order lump solutions of the Kadomtsev–Petviashvili (KP) equation, the presented lump solutions to the KPIESCS exhibit more diverse nonlinear phenomena. The method used here is more natural and simpler.

1. Introduction

Soliton equations with self-consistent sources are role models in many fields of physics, and important developments have been made in exploring their soliton phenomena [1–20]. The Kadomtsev–Petviashvili (KP) equation with a self-consistent source arose in describing the interaction of long and short waves and its soliton solutions were first found by Mel'nikov [21,22]. Later, N-soliton solutions of the KP equation with self-consistent sources were obtained through the Hirota method [23] and the generalized binary Darboux transformation [24]. Furthermore, the general high-order rogue waves which are given in terms of determinants whose matrix elements have plain algebraic expressions of the KP equation with self-consistent sources were derived via the Hirota method [25].

In contrast to soliton solutions, lump solutions are another kind of important exact solutions, which are rational analytical and localized in all directions in the space. General rational function solutions were presented for the Korteweg–de Vries (KdV) equation, the Boussinesq equation, the nonlinear Schrödinger (NLS) equation and the Toda lattice equation systematically through the Wronskian and Casoratian determinant techniques for integrable equations [26–30]. Special examples of lump solutions are also found for many integrable equations such as the Kadomtsev–Petviashvili I (KPI) equation [31,32], the three-dimensional three-wave resonant interaction [33], the B-Kadomtsev–Petviashvili (BKP) equation [34], the Davey–Stewartson-II equation [35,36] and the Ishimori-I equation [37].

In mathematical physics, the KP equation is usually written as

$$(u_t + 6uu_x + u_{xxx})_x - \sigma u_{yy} = 0, \sigma = \pm 1, \tag{1}$$

which is classified as the KPI equation when $\sigma = 1$ and the KPII equation when $\sigma = -1$. Recently, one of the authors (Ma) has proposed a symbolic computation method to search for positive quadratic function solutions, particularly for the

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(2 + 1)-dimensional bilinear KPI equation [38]. The obtained quadratic function solutions in the KPI case [38] contain a set of six free parameters, and taking special choices of the involved parameters covers a particular class of lump solutions generated from computing long wave limits of soliton solutions. The same idea was also adopted to derive lumps and interaction solutions between lumps and kinks for several equations such as KPI equation [39,40], BKP equation [41,42], (2 + 1)-dimensional Ito equation and (2 + 1)-dimensional Caudrey–Dodd–Gibbon–Sawada–Kotera (CDGKS) equation [43], (2+1)-dimensional Sawada–Kotera equation [44], dimensionally reduced gKP and gBKP equations [45], (3+1)-dimensional Jimbo–Miwa equation [46,47]. Especially, in the case of the (2 + 1)-dimensional Ito equation, an interesting characteristic that an arbitrary function is involved in the resulting interaction solutions was explored [48]. It should be noticed that the Hirota bilinear method plays an important role in these works and this technique has the advantage of being applicable directly upon the equations [49–51].

In this paper, we would like to discuss the KPI equation with a self-consistent source (KPIESCS)

$$\begin{cases}(u_t + 6uu_x + u_{xxx} + 8|\Phi|_x^2)_x - 3u_{yy} = 0,\\ i\Phi_y = \Phi_{xx} + u\Phi,\end{cases} \tag{2}$$

and want to determine its lump solutions and their dynamics. We begin with the Hirota bilinear form of the KPIESCS and make some ansatz by combination functions of quadratic functions to solve the corresponding bilinear counterpart equation. The resulting solutions exhibit more diverse nonlinear phenomena than lower-order lumps solutions to the KPI equation and provide supplements to the existing solutions in the literature. A few concluding remarks will be given in the last section.

2. Lump solutions to KPIESCS

Through the dependent variable transformation $u = 2(\ln F)_{xx}$ and $\Phi = G/F$, the KPIESCS is rewritten as

$$\begin{cases}\left[\dfrac{2(F_{xt}F - F_xF_t + F_{xxxx}F - 4F_{xxx}F_x + 3F_{xx}^2 - 3F_{yy}F + 3F_y^2) + 8GG^*}{F^2}\right]_{xx} = 0,\\ i(G_yF - GF_y) - G_{xx}F + 2G_xF_x - GF_{xx} = 0,\end{cases} \tag{3}$$

from which we obtain its bilinear equation

$$\begin{cases}2(F_{xt}F - F_xF_t + F_{xxxx}F - 4F_{xxx}F_x + 3F_{xx}^2 - 3F_{yy}F + 3F_y^2) + 8GG^* = cF^2,\\ i(G_yF - GF_y) - G_{xx}F + 2G_xF_x - GF_{xx} = 0,\end{cases} \tag{4}$$

where c is a constant of integration. Here asterisk G^* means the complex conjugation and D is the famous Hirota bilinear operator [26]. Eq. (4), with $c = 8$, may also be written concisely in terms of the D-operators as

$$\begin{cases}(D_xD_t + D_x^4 - 3D_y^2)F \cdot F + 8(GG^* - F^2) = 0,\\ (iD_y - D_x^2)G \cdot F = 0.\end{cases} \tag{5}$$

To search for lump solutions to the above bilinear equation, putting $G = G_R + iG_I$ and supposing

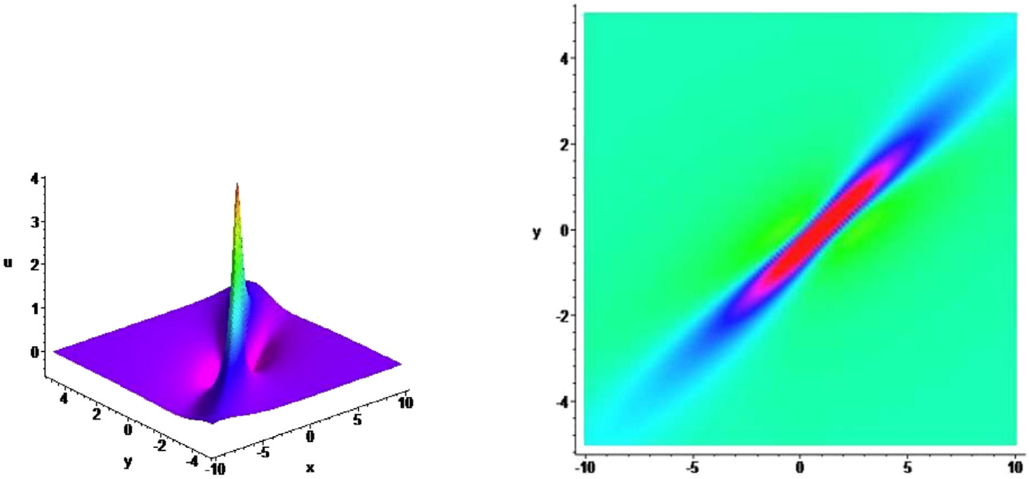
$$\begin{cases}F = 1 + \xi_1^2 + \xi_2^2,\\ G_R = b_0 + b_1\xi_1 + b_2\xi_2 + b_3\xi_1^2 + b_4\xi_2^2,\\ G_I = c_0 + c_1\xi_1 + c_2\xi_2 + c_3\xi_1^2 + c_4\xi_2^2,\end{cases} \tag{6}$$

with

$$\begin{cases}\xi_1 = a_1x + a_2y + a_3t + a_4,\\ \xi_2 = a_5x + a_6y + a_7t + a_8,\end{cases} \tag{7}$$

where the parameters $a_i (1 \le i \le 8)$, $b_j, c_j (0 \le j \le 4)$ are all real constants to be determined. It is easy to see that the assumption for F here guarantees analyticity and rational localization of solutions. Substituting Eqs. (6) and (7) into Eq. (5) and equating all the coefficients of different polynomials of x, y, t to zero, we obtain a set of algebraic equations on the undetermined parameters. After careful discussions, we can generate the following set of constraining equations for the parameters

$$\begin{aligned}&\{a_3 = \frac{(a_2^2 - a_1^4)[3(a_1^4 + a_2^2)^2 + 16a_1^4]}{a_1(a_1^4 + a_2^2)^2}, a_5 = 0, a_6 = a_1^2,\\ &a_7 = \frac{2a_1a_2[3(a_1^4 + a_2^2)^2 - 16a_1^4]}{(a_1^4 + a_2^2)^2}, b_0 = \frac{b_3(a_2^2 - 3a_1^4)}{a_1^4 + a_2^2},\\ &b_1 = kc_1, b_2 = kc_2, b_4 = b_3, c_0 = -kb_0, c_1 = -\frac{4a_1^2a_2b_3}{a_1^4 + a_2^2},\end{aligned} \tag{8}$$



(a) u at $t = 0$.

(b) Density plot of Fig. 1(a).

Fig. 1. (Color online) Plots of lump solution for u with $a_1 = 1, a_2 = -2, a_4 = -1, a_8 = 0, b_3 = \frac{3}{5}, k = -\frac{4}{3}$ at $t = 0$.

$$c_2 = \frac{4a_1^4 b_3}{a_1^4 + a_2^2}, c_3 = -kb_3, c_4 = c_3\},$$

where k is an auxiliary arbitrary real constant which needs to satisfy the following condition

$$b_3^2(1 + k^2) = 1, \tag{9}$$

and the other parameters not expressed in the set are arbitrary, provided that all corresponding formulas are well defined. Therefore, the non-zero determinant condition discussed before in [38] now becomes $a_1a_6 - a_2a_5 = a_1^3 \neq 0$, which guarantees the localization of the associated solutions.

Actually, it is readily observed that at any given time t, the above lump solution $u \to 0$ when $x^2 + y^2 \to \infty$. We can also get all critical points of the function F at a fixed time t by solving the first-order derivative equations $(F_x, F_y) = 0$ to obtain

$$\begin{cases} x(t) = \dfrac{3(a_1^4 + a_2^2)^3 + 16(a_1^8 - 3a_1^4a_2^2)}{a_1^2(a_1^4 + a_2^2)^2} t - \dfrac{a_1^2a_4 - a_2a_8}{a_1^3}, \\ y(t) = \dfrac{2a_2[16a_1^4 - 3(a_1^4 + a_2^2)^2]}{a_1(a_1^4 + a_2^2)^2} t - \dfrac{a_8}{a_1^2}, \end{cases} \tag{10}$$

and they are also critical points in the (x, y)-plane for the function $u = 2(\ln F)_{xx}$. Moreover, based on the second partial derivative test, we can know that the lump solution u has a peak at the critical point $(x(t), y(t))$ since

$$u_{xx} = -24a_1^4 < 0, u_{xx}u_{yy} - u_{xy}^2 = 192a_1^{10} > 0. \tag{11}$$

However, with respect to the consistent-source term $|\Phi|^2 = |G|^2/F^2 = (G_R^2 + G_I^2)/F^2$, it decays into $b_3^2 + c_3^2 = 1$ for $|x|, |y| \to \infty$. Its critical points do not always coincide with those functions F and G. For illustration, two special pairs of the lump solutions with specific values of the parameters are given as follows.

Firstly, if we take a special choice for the parameters

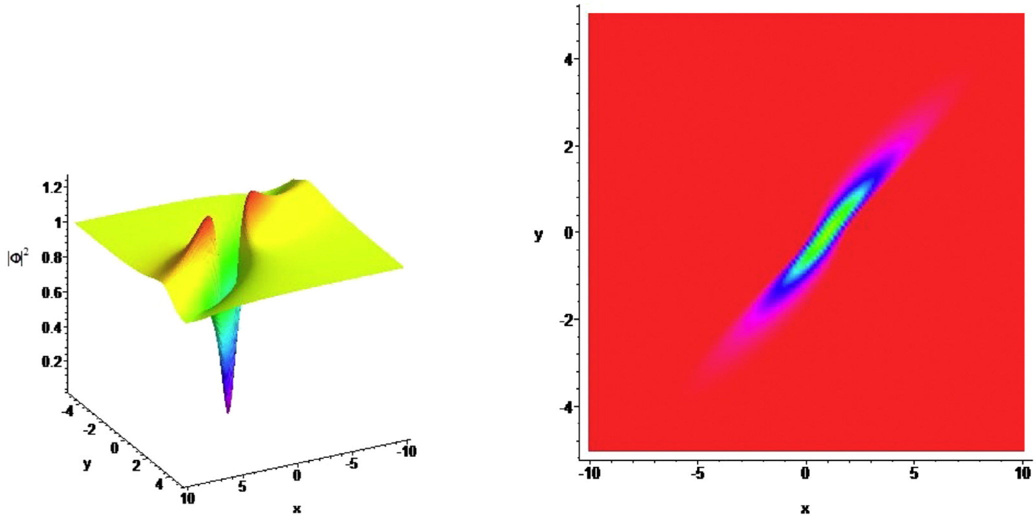
$$a_1 = 1, a_2 = -2, a_4 = -1, a_8 = 0, b_3 = \frac{3}{5}, k = -\frac{4}{3}, \tag{12}$$

we get the lump solution which was exactly presented in Ref. [24] as $\mu = 1/2$, $\nu = 1$. The corresponding three dimensional plots and density plots when $t = 0$ are shown in Figs. 1 and 2, respectively. The lump solutions are all centered at (1, 0), but the source term $|\Phi|^2$ owns two maximal points and one minimal point as shown in the figures. Meanwhile, we can see a twist in Fig. 2(b) for the source term.

Secondly, the plots for another selection of the parameters

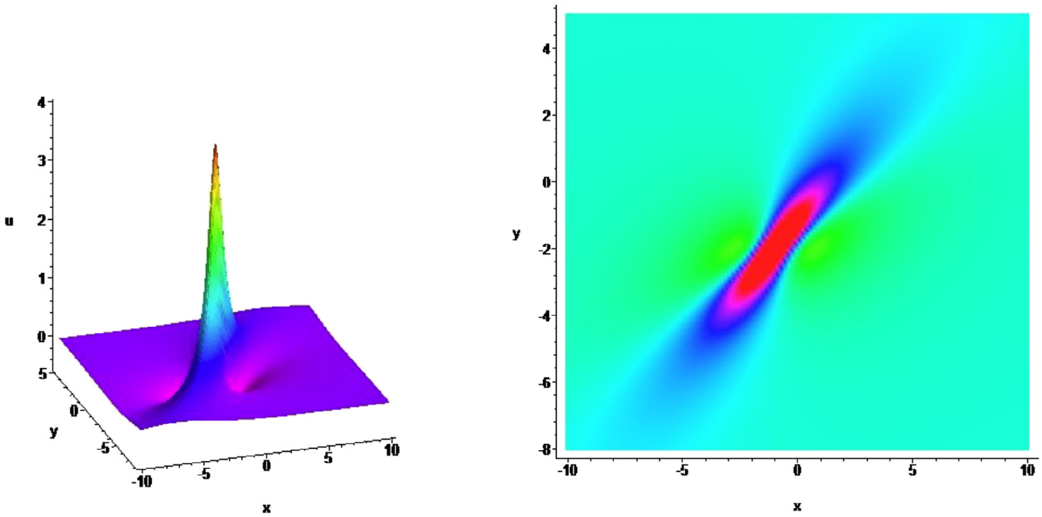
$$a_1 = 1, a_2 = -1, a_4 = -1, a_8 = 0, b_3 = 1, k = 0, \tag{13}$$

when $t = 1$ are depicted in Figs. 3 and 4, respectively. It can be seen that the potential u is similar, but the source term $|\Phi|^2$ exhibits different and diverse dynamics from the first case. It owns two peak points and two minimal points compared with Fig. 2.



(a) $|\Phi|^2$ at $t = 0$. (b) Density plot of Fig. 2(a).

Fig. 2. (Color online) Plots of lump solution for $|\Phi|^2$ with $a_1 = 1, a_2 = -2, a_4 = -1, a_8 = 0, b_3 = \frac{3}{5}, k = -\frac{4}{3}$ at $t = 0$.



(a) u at $t = 1$. (b) Density plot of Fig. 3(a).

Fig. 3. (Color online) Plots of lump solution for u with $a_1 = 1, a_2 = -1, a_4 = -1, a_8 = 0, b_3 = 1, k = 0$ at $t = 1$.

3. Conclusion

In this paper, the lump solutions to the KPI equation with a self-consistent source (KPIESCS) were presented by using the direct ansatz and the Hirota bilinear method. It is shown that the direct ansatz is a powerful means for seeking lump solutions to nonlinear equations, when it is combined with the Hirota bilinear method. We can see that the solutions obtained are consistent with those presented before, and the approach used here is more natural and simpler. It is hoped that the presented results would be helpful in understanding the propagation processes of nonlinear waves, particularly in fluid mechanics. Recently, interaction solutions between lumps and solitons have attracted a lot of attention and various kinds of interaction solutions have been obtained [52]. It will be interesting to discuss the existence conditions and solution methods for getting new interaction solutions to nonlinear wave equations. Those problems are left for future research publications.

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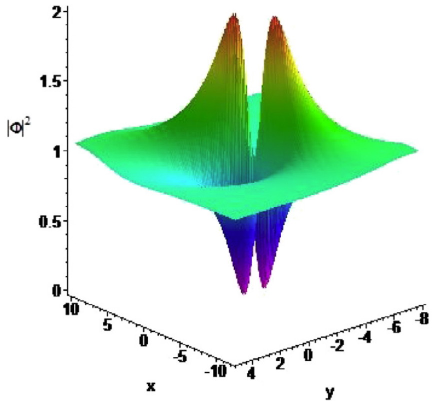
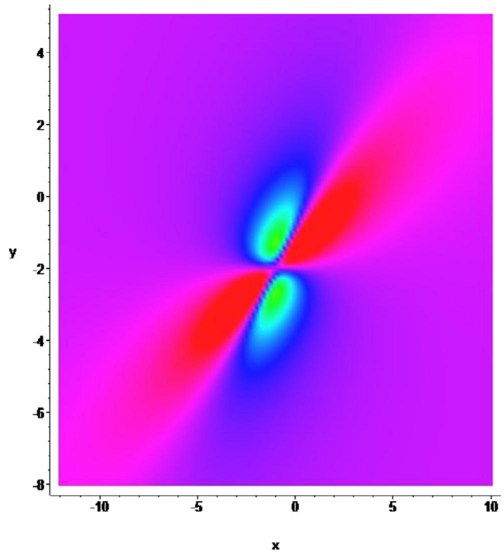


(a) $|\Phi|^2$ at $t = 1$.



(b) Density plot of Fig. 4(a).

Fig. 4. (Color online) Plots of lump solution for $|\Phi|^2$ with $a_1 = 1, a_2 = -1, a_4 = -1, a_8 = 0, b_3 = 1, k = 0$ at $t = 1$.

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