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Diversity of exact solutions to a (3+1)-dimensional nonlinear evolution equation and its reduction

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ABSTRACT

In this paper, a (3+1)-dimensional nonlinear evolution equation and its reduction is studied by use of the Hirota bilinear method and the test function method. With symbolic computation, diversity of exact solutions is obtained by solving the under-determined nonlinear system of algebraic equations for the associated parameters. Finally, analysis and graphical simulation are given to reveal the propagation and dynamical behavior of the solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) play an important role in mathematical physics [1–7]. Various mechanical features in fluid dynamics, optical communications and nonlinear vibration are described by NLEEs [8–11]. Generally speaking, it is very difficult to find exact solutions to NLEEs [12–14]. With the development of symbolic computation, it is reasonable to employ test function method in constructing exact solutions to NLEEs [15,16]. Further, it is of importance to solve high-dimensional NLEEs to study the associated spatiotemporal features [17–23].

In this paper, we will study a $(3+1)$-dimensional NLEE [24–30] as

$$3\,u_{xz} - (2\,u_t + u_{xxx} - 2\,u\,u_x)_y + 2\,(u_x\,\partial_x^{-1}\,u_y)_x = 0, \tag{1}$$

where ∂_x^{-1} stands for an inverse operator of $\partial_x = \frac{\partial}{\partial x}$. Eq. (1) was originally introduced as a model for the study of algebraic-geometrical solutions [24], and its integrability and large classes of exact solutions have been studied, e.g., the soliton, positon, negaton and rational solutions [25–29]. Further, two types of resonant solutions are obtained by the parameterization for wave numbers and frequencies for linear combinations of exponential traveling waves [30].

Through the following transformation

$$u = -3\,[\ln f(x,y,z,t)]_{xx}, \tag{2}$$

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Eq. (1) can be cast into the bilinear form as

$$(3\,D_x\,D_z - 2\,D_y\,D_t - D_x^3\,D_y)f\cdot f = 0, \tag{3}$$

where $D_x\,D_z$, $D_y\,D_t$ and $D_x^3\,D_y$ are bilinear operators [12] defined by

$$D_x^\alpha D_y^\beta D_z^\gamma D_t^\delta(f\cdot g) = \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^\alpha\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^\beta\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^\gamma\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^\delta \times f(x,y,z,t)g(x',y',z',t')\Big|_{x'=x,y'=y,z'=z,t'=t.}$$

We assume that the solution to Eq. (3) is in the form of

$$f = e^{-\xi} + \delta_1\,\cos(\eta) + \delta_2\,\cosh(\gamma) + \delta_3\,e^{\xi}, \tag{4}$$

or

$$f = e^{-\xi} + \delta_1\,\sin(\eta) + \delta_2\,\sinh(\gamma) + \delta_3\,e^{\xi}, \tag{5}$$

where $\xi = a_1x + b_1y + c_1z + d_1t$, $\eta = a_2x + b_2y + c_2z + d_2t$, $\gamma = a_3x + b_3y + c_3z + d_3t$ and a_i, b_i, c_i, d_i, and δ_i ($i = 1,2,3$) are some constants to be determined later. Based on Eq. (4) or Eq. (5), we can derive exact solutions to Eq. (1).

The structure of this paper is as follows: In Section 2, we will solve Eq. (3) and obtain the exact solutions to Eq. (1). In Section 3, we will give analysis and discussion on our solutions. Some figures describing the characteristics of our solutions will be presented. In Section 4, we will conclude our results.

2. Diversity of exact solutions

2.1. Case I: based on the test function (4)

Substituting Eq. (4) into Eq. (3), we can obtain a large expression in terms of $\cos(\eta)e^{\xi}$, $\cos(\eta)e^{-\xi}$, $\sin(\eta)e^{\xi}$, $\sin(\eta)e^{-\xi}$, $\cosh(\gamma)e^{\xi}$, $\sinh(\gamma)e^{\xi}$, $\cosh(\gamma)e^{-\xi}$, $\sinh(\gamma)e^{-\xi}$, $\cos(\eta)\cosh(\gamma)$, $\sin(\eta)\sinh(\gamma)$, etc., which generate a list of algebraic equations as

$$\begin{cases} 3a_1c_1 - 3a_2c_2 - 2b_1d_1 + 2b_2d_2 - a_1^3b_1 - a_2^3b_2 + 3a_1^2a_2b_2 + 3a_1a_2^2b_1 = 0,\\ 3a_1c_1 + 3a_3c_3 - 2b_1d_1 - 2b_3d_3 - a_1^3b_1 - a_3^3b_3 - 3a_1^2a_3b_3 - 3a_1a_3^2b_1 = 0,\\ 3a_1c_2 + 3a_2c_1 - 2b_1d_2 - 2b_2d_1 - a_1^3b_2 + a_2^3b_1 - 3a_1^2a_2b_1 + 3a_1a_2^2b_2 = 0,\\ 3a_1c_3 + 3a_3c_1 - 2b_1d_3 - 2b_3d_1 - a_1^3b_3 - a_3^3b_1 - 3a_1^2a_3b_1 - 3a_1a_3^2b_3 = 0,\\ 3a_3c_3 - 3a_2c_2 - 2b_3d_3 + 2_b2d_2 - a_2^3b_2 - a_3^3b_3 + 3a_2^2a_3b_3 + 3a_2a_3^2b_2 = 0,\\ 3a_2c_3 + 3a_3c_2 - 2b_2d_3 - 2b_3d_2 + a_2^3b_3 - a_3^3b_2 + 3a_2^2a_3b_2 - 3a_2a_3^2b_3 = 0,\\ \delta_3(12a_1c_1 - 8b_1d_1 - 16a_1^3b_1) + \delta_2^2(3a_3c_3 - 2b_3d_3 - 4a_3^3b_3) + \delta_1^2(2b_2d2 - 3a_2c_2 - 4a_2^3b_2) = 0. \end{cases} \tag{6}$$

With symbolic computation, we solve two sets of parameters from Eq. (6):

The first set in this case is

$$\Big\{a_1 = 1, a_2 = 0, b_1 = -\frac{4}{3a_3^2}, b_2 = -2, b_3 = -\frac{4}{3a_3}, c_1 = -\frac{8(2a_3^3+1)}{9a_3}, c_2 = -\frac{2(3a_3^4+6a_3-4)}{9a_3^2}, c_3 = -\frac{4(a_3^2+3a_3+2)}{9a_3^2}, d_1 = \frac{a_3^3-a_3+2}{2a_3}, d_2 = -1, d_3 = 1, \delta_3 = \frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4}\Big\}, \tag{7}$$

where a_3, δ_1 and δ_2 are real constants.

Substituting parameters of Eq. (7) into Eq. (4), we have $f = e^{-A_1} + \delta_1\,\cos(B_1) + \delta_2\,\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}$, which leads to the exact solutions to Eq. (1) as

$$u = -3\frac{e^{-A_1} + \delta_2a_3^2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}}{e^{-A_1} + \delta_1\cos(B_1) + \delta_2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1}} + 3\frac{(-e^{-A_1} + a_3\delta_2\sinh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1})^2}{(e^{-A_1} + \delta_1\cos(B_1) + \delta_2\cosh(C_1) + (\frac{a_3^2\delta_1^2}{4(a_3^2-1)} + \frac{a_3^2\delta_2^2}{4})e^{A_1})^2} \tag{8}$$

with

$$A_1 = x - \frac{4}{3a_3^2}y - \frac{8(2a_3^3+1)}{9a_3}z + \frac{a_3^3 - a_3 + 2}{2a_3}t,$$
$$B_1 = -2y - \frac{2(3a_3^4+6a_3-4)}{9a_3^2}z - t,$$
$$C_1 = a_3x - \frac{4}{3a_3}y - \frac{4(a_3^2+3a_3+2)}{9a_3^2}z + t.$$

The second set in this case is

$$\left\{ a_1 = 1, a_2 = 0, b_1 = \frac{9a_3^4 - 9a_3^2 - 16}{12a_3^2}, b_2 = -2, b_3 = \frac{-9a_3^4 + 9a_3^2 - 16}{12a_3}, \right.$$
$$c_1 = \frac{-9a_3^7 + 36a_3^5 + 36a_3^4 - 107a_3^3 - 36a_3^2 - 48a_3 - 64}{72a_3^3}, c_2 = \frac{-3a_3^4 - 9a_3^2 - 24a_3 + 16}{18a_3^2},$$
$$c_3 = \frac{-9a_3^7 + 9a_3^5 - 18a_3^4 - 16a_3^3 + 18a_3^2 - 48a_3 - 32}{36a_3^2},$$
$$\left. d_1 = \frac{-a_3^3 + a_3 + 4}{4a_3}, d_2 = -1, d_3 = 1, \delta_3 = \frac{4a_3^2\delta_1^2}{9a_3^4 + 7a_3^2 - 16} + \frac{(9a_3^8 - 9a_3^6 + 16a_3^4 - 16a_3^2)\delta_2^2}{4(9a_3^4 + 7a_3^2 - 16)} \right\}, \tag{9}$$

where a_3, δ_1 and δ_2 are real constants.

Substituting parameters of Eq. (9) into Eq. (4), we have $f = e^{-A_2} + \delta_1\cos(B_2) + \delta_2\cosh(C_2) + (\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16} + \frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_2}$, which gives rise to the exact solutions to Eq. (1) as

$$\begin{aligned} u = &-3\frac{e^{-A_2} + \delta_2 a_3^2\cosh(C_2) + (\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16} + \frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_2}}{e^{-A_2} + \delta_1\cos(B_2) + \delta_2\cosh(C_2) + (\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16} + \frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_2}} \\ &+ 3\frac{(-e^{-A_2} + a_3\delta_2\sinh(C_2) + (\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16} + \frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_2})^2}{(e^{-A_2} + \delta_1\cos(B_2) + \delta_2\cosh(C_2) + (\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16} + \frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_2})^2} \end{aligned} \tag{10}$$

with

$$A_2 = x + \frac{9a_3^4 - 9a_3^2 - 16}{12a_3^2}y + \frac{-9a_3^7 + 36a_3^5 + 36a_3^4 - 107a_3^3 - 36a_3^2 - 48a_3 - 64}{72a_3^3}z + \frac{-a_3^3 + a_3 + 4}{4a_3}t,$$
$$B_2 = -2y + \frac{-3a_3^4 - 9a_3^2 - 24a_3 + 16}{18a_3^2}z - t,$$
$$C_2 = a_3x + \frac{-9a_3^4 + 9a_3^2 - 16}{12a_3}y + \frac{-9a_3^7 + 9a_3^5 - 18a_3^4 - 16a_3^3 + 18a_3^2 - 48a_3 - 32}{36a_3^2}z + t.$$

2.2. Case II: based on the test function (5)

Substituting Eq. (5) into Eq. (3), we can obtain a large expression in terms of $\cos(\eta)e^{\xi}$, $\cos(\eta)e^{-\xi}$, $\sin(\eta)e^{\xi}$, $\sin(\eta)e^{-\xi}$, $\cosh(\gamma)e^{\xi}$, $\sinh(\gamma)e^{\xi}$, $\cosh(\gamma)e^{-\xi}$, $\sinh(\gamma)e^{-\xi}$, $\cos(\eta)\cosh(\gamma)$, $\sin(\eta)\sinh(\gamma)$, etc., which generate a list of algebraic equations as

$$\begin{cases} 3a_1c_1 - 3a_2c_2 - 2b_1d_1 + 2b_2d_2 - a_1^3b_1 - a_2^3b_2 + 3a_1^2a_2b_2 + 3a_1a_2^2b_1 = 0, \\ 3a_1c_1 + 3a_3c_3 - 2b_1d_1 - 2b_3d_3 - a_1^3b_1 - a_3^3b_3 - 3a_1^2a_3b_3 - 3a_1a_3^2b_1 = 0, \\ 3a_1c_2 + 3a_2c_1 - 2b_1d_2 - 2b_2d_1 - a_1^3b_2 + a_2^3b_1 - 3a_1^2a_2b_1 + 3a_1a_2^2b_2 = 0, \\ 3a_1c_3 + 3a_3c_1 - 2b_1d_3 - 2b_3d_1 - a_1^3b_3 - a_3^3b_1 - 3a_1^2a_3b_1 - 3a_1a_3^2b_3 = 0, \\ 3a_3c_3 - 3a_2c_2 - 2b_3d_3 + 2b_2d_2 - a_2^3b_2 - a_3^3b_3 + 3a_2^2a_3b_3 + 3a_2a_3^2b_2 = 0, \\ 3a_2c_3 + 3a_3c_2 - 2b_2d_3 - 2b_3d_2 + a_2^3b_3 - a_3^3b_2 + 3a_2^2a_3b_2 - 3a_2a_3^2b_3 = 0, \\ \delta_3(12a_1c_1 - 8b_1d_1 - 16a_1^3b_1) - \delta_2^2(3a_3c_3 - 2b_3d_3 - 4a_3^3b_3) + \delta_1^2(2b_2d_2 - 3a_2c_2 - 4a_2^3b_2) = 0. \end{cases} \tag{11}$$

With symbolic computation, we solve two sets of parameters from Eq. (11):

The first set in this case is

$$\begin{Bmatrix} a_1=1, a_2=0, b_1=-\dfrac{4}{3a_3^2}, b_2=-2, b_3=-\dfrac{4}{3a_3}, \\ c_1=-\dfrac{8(2a_3^3+1)}{9a_3}, c_2=-\dfrac{2(3a_3^4+6a_3-4)}{9a_3^2}, c_3=-\dfrac{4(a_3^2+3a_3+2)}{9a_3^2}, \\ d_1=\dfrac{a_3^3-a_3+2}{2a_3}, d_2=-1, d_3=1, \delta_3=\dfrac{a_3^2\delta_1^2}{4(a_3^2-1)}-\dfrac{a_3^2\delta_2^2}{4} \end{Bmatrix}, \tag{12}$$

where a_3, δ_1 and δ_2 are real constants.

Substituting the parameters of Eq. (12) into Eq. (5), we have $f=e^{-A_3}+\delta_1\sin(B_3)+\delta_2\sinh(C_3)+(\frac{a_3^2\delta_1^2}{4(a_3^2-1)}+\frac{a_3^2\delta_2^2}{4})e^{A_3}$, which generates the exact solutions to Eq. (1) as

$$\begin{aligned} u=&-3\frac{e^{-A_3}+\delta_2 a_3^2\sinh(C_3)+(\frac{a_3^2\delta_1^2}{4(a_3^2-1)}-\frac{a_3^2\delta_2^2}{4})e^{A_3}}{e^{-A_3}+\delta_1\sin(B_3)+\delta_2\sinh(C_3)+(\frac{a_3^2\delta_1^2}{4(a_3^2-1)}-\frac{a_3^2\delta_2^2}{4})e^{A_3}} \\ &+3\frac{(-e^{-A_3}+a_3\delta_2\cosh(C_3)+(\frac{a_3^2\delta_1^2}{4(a_3^2-1)}-\frac{a_3^2\delta_2^2}{4})e^{A_3})^2}{(e^{-A_3}+\delta_1\sin(B_3)+\delta_2\sinh(C_3)+(\frac{a_3^2\delta_1^2}{4(a_3^2-1)}-\frac{a_3^2\delta_2^2}{4})e^{A_3})^2} \end{aligned} \tag{13}$$

with

$$\begin{aligned} A_3&=x-\frac{4}{3a_3^2}y-\frac{8(2a_3^3+1)}{9a_3}z+\frac{a_3^3-a_3+2}{2a_3}t, \\ B_3&=-2y-\frac{2(3a_3^4+6a_3-4)}{9a3^2}z-t, \\ C_3&=a_3x-\frac{4}{3a_3}y-\frac{4(a_3^2+3a_3+2)}{9a_3^2}z+t. \end{aligned}$$

The second set in this case is

$$\begin{Bmatrix} a_1=1, a_2=0, b_1=\dfrac{9a_3^4-9a_3^2-16}{12a_3^2}, b_2=-2, b_3=\dfrac{-9a_3^4+9a_3^2-16}{12a_3}, \\ c_1=\dfrac{-9a_3^7+36a_3^5+36a_3^4-107a_3^3-36a_3^2-48a_3-64}{72a_3^3}, c_2=\dfrac{-3a_3^4-9a_3^2-24a_3+16}{18a_3^2}, \\ c_3=\dfrac{-9a_3^7+9a_3^5-18a_3^4-16a_3^3+18a_3^2-48a_3-32}{36a_3^2}, \\ d_1=\dfrac{-a_3^3+a_3+4}{4a_3}, d_2=-1, d_3=1, \delta_3=\dfrac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}-\dfrac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)} \end{Bmatrix}, \tag{14}$$

where a_3, δ_1 and δ_2 are real constants.

Substituting the parameters of Eq. (14) into Eq. (5), we have $f=e^{-A_4}+\delta_1\sin(B_4)+\delta_2\sinh(C_4)+(\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}+\frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_4}$, which gives rise to the exact solutions to Eq. (1) as

$$\begin{aligned} u=&-3\frac{e^{-A_4}+\delta_2 a_3^2\sinh(C_4)+(\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}-\frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_4}}{e^{-A_4}+\delta_1\sin(B_4)+\delta_2\sinh(C_4)+(\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}-\frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_4}} \\ &+3\frac{(-e^{-A_4}+a_3\delta_2\cosh(C_4)+(\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}-\frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_4})^2}{(e^{-A_4}+\delta_1\sin(B_4)+\delta_2\sinh(C_4)+(\frac{4a_3^2\delta_1^2}{9a_3^4+7a_3^2-16}-\frac{(9a_3^8-9a_3^6+16a_3^4-16a_3^2)\delta_2^2}{4(9a_3^4+7a_3^2-16)})e^{A_4})^2} \end{aligned} \tag{15}$$

with

$$A_4 = x + \frac{9a_3^4 - 9a_3^2 - 16}{12a_3^2}y + \frac{-9a_3^7 + 36a_3^5 + 36a_3^4 - 107a_3^3 - 36a_3^2 - 48a_3 - 64}{72a_3^3}z + \frac{-a_3^3 + a_3 + 4}{4a_3}t,$$

$$B_4 = -2y + \frac{-3a_3^4 - 9a_3^2 - 24a_3 + 16}{18a_3^2}z - t,$$

$$C_4 = a_3x + \frac{-9a_3^4 + 9a_3^2 - 16}{12a_3}y + \frac{-9a_3^7 + 9a_3^5 - 18a_3^4 - 16a_3^3 + 18a_3^2 - 48a_3 - 32}{36a_3^2}z + t.$$

2.3. Dimensionally reduced case with $z = x$

By taking $z = x$, Eq. (3) reduces to

$$(3\,D_x^2 - 2\,D_y\,D_t - D_x^3\,D_y)f\cdot f = 0, \tag{16}$$

which is assumed to possess the exact solutions in the form of

$$f = e^{-\xi} + \delta_1\,\cos(\eta) + \delta_2\,\cosh(\gamma) + \delta_3\,e^{\xi}, \tag{17}$$

where $\xi = a_1\,x + b_1\,y + c_1\,t$, $\eta = a_2\,x + b_2\,y + c_2\,t$, $\gamma = a_3\,x + b_3\,y + c_3\,t$ and a_i, b_i, c_i, δ_i $(i = 1, 2, 3)$ are some constants to be determined later.

Substituting Eq. (17) into Eq. (16) and equating all the coefficients of different powers of e^{ξ}, $e^{-\xi}$, $\cos(\eta)$, $\sin(\eta)$, $\cosh(\gamma)$, $\sinh(\gamma)$ and constant term to zero, we can obtain a set of algebraic equations for a_i, b_i, c_i, and δ_i $(i = 1, 2, 3)$. We solve this set of algebraic equations and obtain the parameters as

$$\left\{b_1 = \frac{1}{a_1}, b_2 = -\frac{1}{a_2}, b_3 = \frac{1}{a_3}, c_1 = -\frac{a_1^3 + 1}{2}, c_2 = \frac{a_2^3}{2}, c_3 = -\frac{a_3^3}{2}\right\}, \tag{18}$$

where a_1, a_2, a_3, δ_1, δ_2 and δ_3 are real constants.

Corresponding to Eq. (17), we have

$$f = e^{-(a_1x + \frac{1}{a_1}y - \frac{a_1^3}{2}t)} + \delta_1\,\cos(a_2x - \frac{1}{a_2}y + \frac{a_2^3}{2}t) + \delta_2\,\cosh(a_3x + \frac{1}{a_3}y - \frac{a_3^3}{2}t) + \delta_3\,e^{(a_1x + \frac{1}{a_1}y - \frac{a_1^3}{2}t)},$$

and the exact solutions to Eq. (1) with $z = x$ as

$$\begin{aligned} u = & -3\frac{a_1^2e^{-A} - \delta_1a_2^2\cos(B) + \delta_2a_3^2\cosh(C) + \delta_3a_1^2e^A}{e^{-A} + \delta_1\cos(B) + \delta_2\cosh(C) + \delta_3e^A} \\ & + 3\frac{(-a_1e^{-A} - \delta_1a_2\sin(B) + a_3\delta_2\sinh(C) + \delta_3a_1e^A)^2}{(e^{-A} + \delta_1\cos(B) + \delta_2\cosh(C) + \delta_3e^A)^2}, \end{aligned} \tag{19}$$

where

$$A = a_1x + \frac{1}{a_1}y - \frac{a_1^3}{2}t,$$

$$B = a_2x - \frac{1}{a_2}y + \frac{a_2^3}{2}t,$$

$$C = a_3x + \frac{1}{a_3}y - \frac{a_3^3}{2}t.$$

3. Analysis and discussion

In this section, we will study the propagation and dynamical behavior of the exact solutions with graphical simulation. It is clear that Eqs. (6) and (11) are under-determined nonlinear systems of algebraic equations, among which, there are fifteen unknown variables and seven equations. Generally speaking, it is very difficult to solve the under-determined nonlinear systems of algebraic equations. With symbolic computation, we have derived four sets of special solutions to Eq. (1), as seen as, Eqs. (8), (10), (13) and (15). The 3-dimensional plots and contour plots of these solutions can be found in Figs. 1 to 4. The plots of solution to the dimensionally reduced case of Eq. (1) with $z = x$ can be seen in Fig. 5.

Based on the expressions of u, we have

$$\lim_{t\to\pm\infty} u = 0,$$

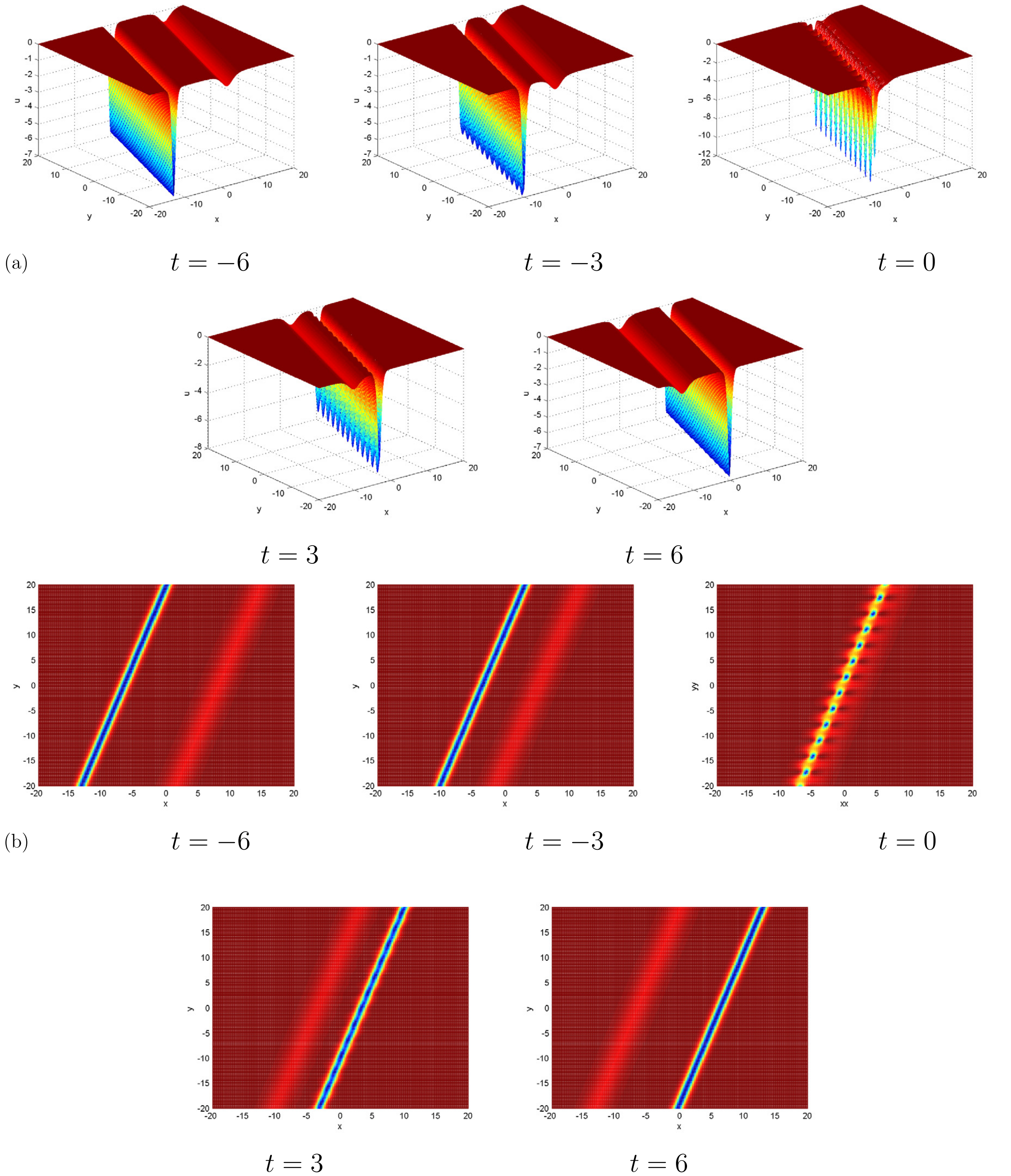

Fig. 1. (a) The 3-dimensional plots of u via Eq. (8) at the time $t=-6$, $t=-3$, $t=0$, $t=3$ and $t=6$ with $a_3=2$, $z=0$, $\delta_1=3$ and $\delta_2=1$; (b) The corresponding contour plots of (a).

where the solutions u are given by Eqs. (8), (10), (13), (15) and (19). From the 3-dimensional plots and contour plots, it can be seen that the peaks of waves via Eqs. (10), (13) and (15) rise and fall periodically along the vertical direction of the propagation. The solutions via Eqs. (8) and (19) describe the interaction between two waves, which can be seen respectively in Figs. 1 and 5. Parallel face-to-face interaction occurs between two dark-type waves in Fig. 1, while oblique interaction between two dark-type waves is given in Fig. 5 (the detailed interaction process is revealed with selection of different values of the time).

In Fig. 2, the waves are dark-type, propagate to the negative direction of the x-axis and the peaks periodically rise and fall, while the waves in Figs. 3 and 4 are bright-type.

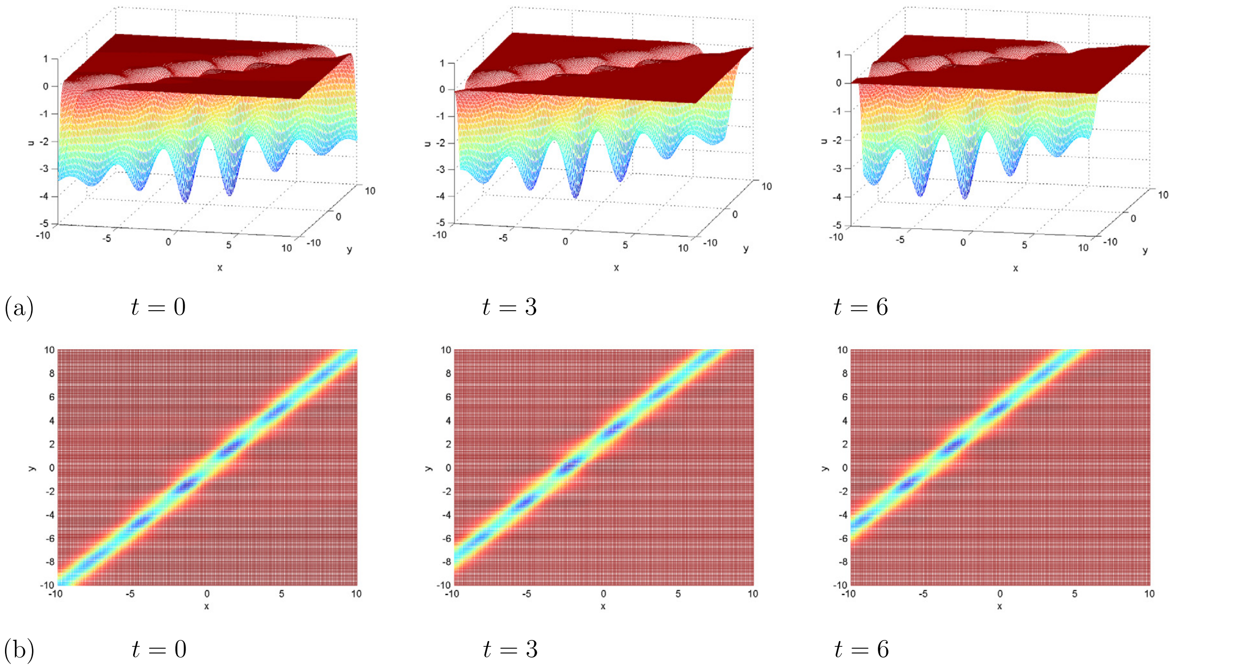

Fig. 2. (a) The 3-dimensional plots of u via Eq. (10) at the time $t = 0$, $t = 3$ and $t = 6$ with $a_3 = \frac{2}{\sqrt{3}}$, $z = 0$, $\delta_1 = 1$ and $\delta_2 = \frac{3}{2\sqrt{2}}$; (b) The corresponding contour plots of (a).

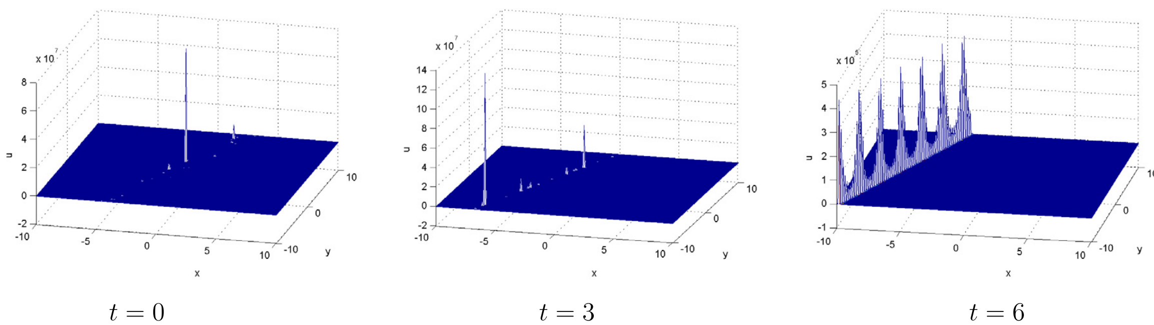

Fig. 3. The 3-dimensional plots of u via Eq. (13) at the time $t = 0$, $t = 3$ and $t = 6$ with $a_3 = 2$, $z = 0$, $\delta_1 = 3$ and $\delta_2 = 1$.

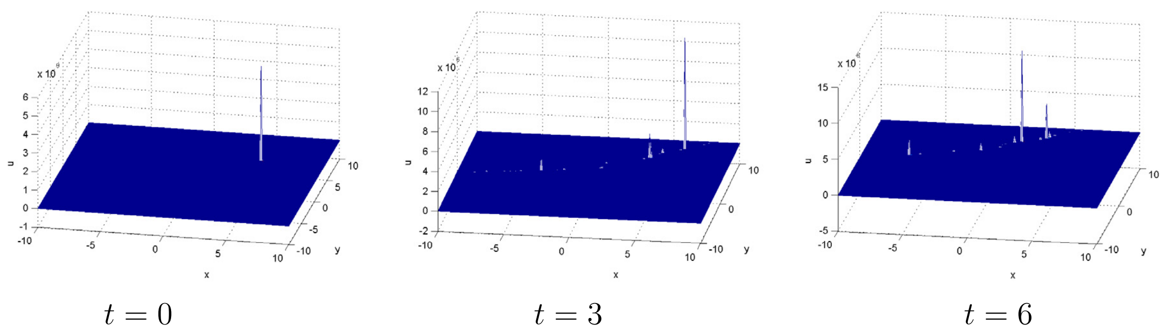

Fig. 4. The 3-dimensional plots of u via Eq. (15) at the time $t = 0$, $t = 3$ and $t = 6$ with $a_3 = \frac{2}{\sqrt{3}}$, $z = 0$, $\delta_1 = 1$ and $\delta_2 = 1$.

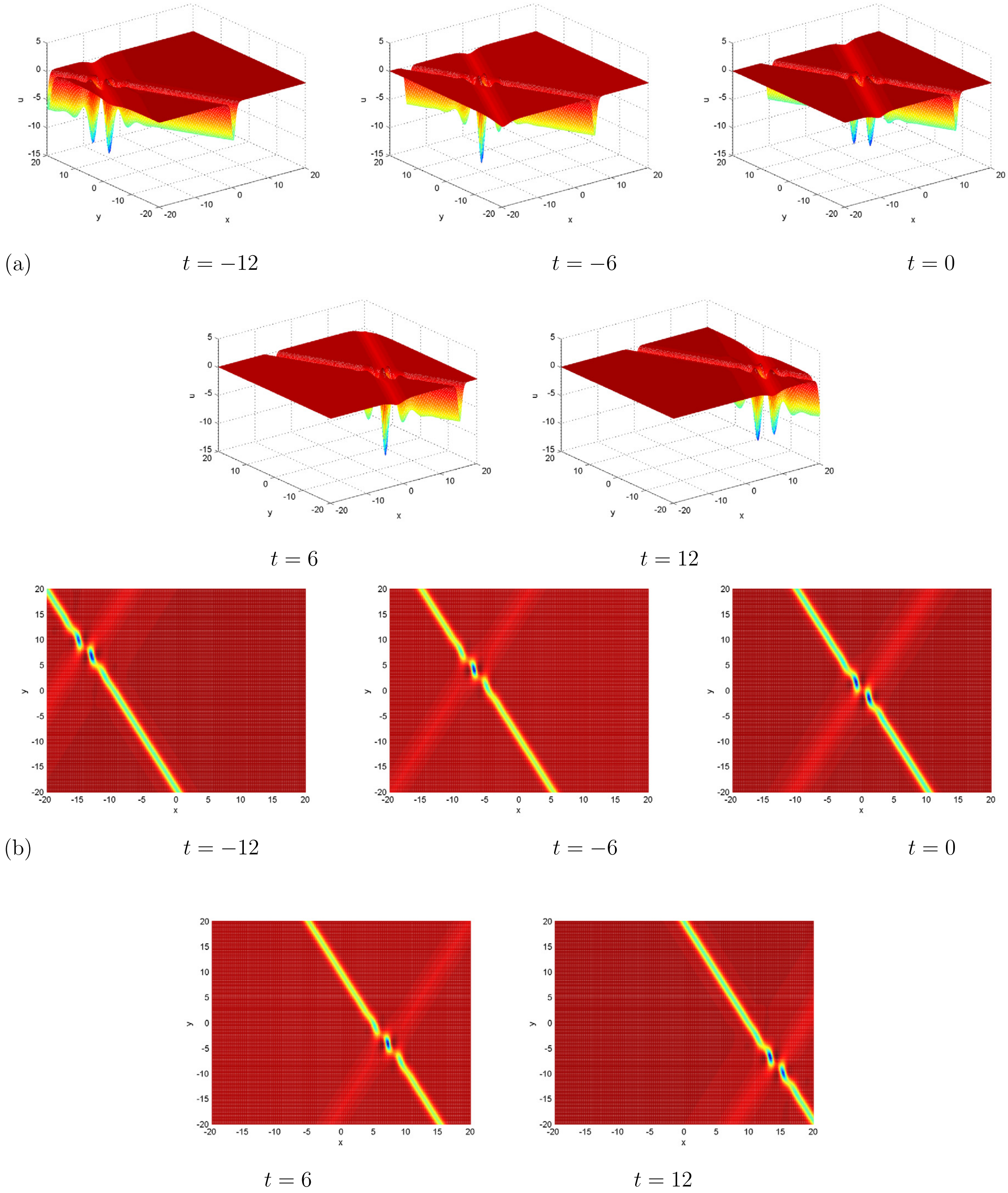

Fig. 5. (a) The 3-dimensional plots of u via Eq. (19) at the time $t = -12$, $t = -6$, $t = 0$, $t = 6$ and $t = 12$ with $a_1 = 1$, $a_2 = 2$, $a_3 = 2$, $\delta_1 = 1$, $\delta_2 = 1$ and $\delta_3 = 1$; (b) The corresponding contour plots of (a).

4. Concluding remarks

Based on the Hirota bilinear method, the test function method is powerful in finding exact solutions to nonlinear evolution equations. In this paper, a (3 + 1)-dimensional nonlinear evolution equation and its reduction have been studied as an example [see Eq. (1)]. With symbolic computation, diversity of exact solutions has been obtained by solving the under-determined nonlinear system of algebraic equations [see Eqs. (6) and (11)] for the associated parameters. In our future work, other types of test function will be considered to find exact solutions to some higher-dimensional nonlinear evolution equations.

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