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Abundant lump-type solutions of the Jimbo–Miwa equation in (3 + 1)-dimensions

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ABSTRACT

Based on the Hirota bilinear form of the (3+1)-dimensional Jimbo–Miwa equation, ten classes of its lump-type solutions are generated via Maple symbolic computations, whose analyticity can be easily achieved by taking special choices of the involved parameters. Those solutions supplement the existing lump-type solutions presented previously in the literature.

1. Introduction

Integrable equations possess Hirota bilinear forms, and among important examples of such equations are the Korteweg–de Vries (KdV) equation, the Boussinesq equation, the Kadomtsev–Petviashvili (KP) equation, the BKP equation and the Toda lattice equation [1]. All those integrable equations have exponentially localized solutions—soliton solutions [2]. It is the Hirota bilinear formulation that plays a key role in generating soliton solutions, but some intelligent guesswork is often required [3].

Besides soliton solutions, there exist rational solutions to nonlinear partial differential equations, certainly to integrable equations (see, e.g., [4,5]). Particularly important are rationally localized solutions in all directions of space, called lump solutions, and examples of lump solutions are found for many interesting nonlinear equations arising from physically relevant situations, which contain the KP equation I [6,7], the three-dimensional three-wave resonant interaction [8], the BKP equation [9,10], the Davey–Stewartson equation II [11] and the Ishimori-I equation [12]. In particular, the KP equation I of the following form:

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} = 0, \tag{1.1}$$

has the lump solution [6]:

$$u = 4\frac{-[x + ay + 3(a^2 - b^2)t]^2 + b^2(y + 6at)^2 + 1/b^2}{\{[x + ay + 3(a^2 - b^2)t]^2 + b^2(y + 6at)^2 + 1/b^2\}^2}, \tag{1.2}$$

where a and b are real free parameters. Rogue wave solutions, which draw a big attention of mathematicians and physicists in the international research community, are a particularly important kind of lump or lump-type solutions, and such

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solutions, usually with rational function amplitudes, could be used to describe interesting nonlinear wave phenomena in both oceanography [13] and nonlinear optics [14]. Lump or lump-type solutions to nonlinear partial differential equations present an interesting research question for us, and Hirota bilinear forms will be a good basis for carrying out research on such solutions.

General rational solutions to integrable equations have been considered within the Wronskian formulation, the Casoratian formulation and the Grammian or Pfaffian formulation (see [1,2]). Typical examples include the KdV equation and the Boussinesq equation in (1+1)-dimensions, the KP equation in (2+1)-dimensions, and the Toda lattice equation in (0+1)-dimensions (see, e.g., [15–18]). A few new attempts have also been made to look for rational solutions to the non-integrable (3+1)-dimensional KP I [19,20] and KP II [21] by direct analytical approaches, for example, the tanh-function method, the $\frac{G'}{G}$-expansion method and symbolic computations (see, e.g., [22–26]). There is a link of rational solutions between the (3+1)-dimensional KP II and the good Boussinesq equation [21], bilinear Bäcklund transformations are applied to rational solutions to (3+1)-dimensional generalized KP equations (see, e.g., [27]), and there exist some direct searches for rational solutions to generalized bilinear equations (see, e.g., [24,16,26]). Moreover, there are studies on rational solutions to nonlinear equations by the Exp-function method without using Hirota bilinear forms [28,29].

In this paper, we would like to consider the Jimbo–Miwa equation in (3+1)-dimensions [30] and generate ten classes of its lump-type solutions by Maple symbolic computations, based on the studies on lumps to (2+1)-dimensional equations (see, e.g., [7]). The resulting lump-type solutions supplement the existing lump-type solutions in the literature (see, e.g., [31]). The (3+1)-dimensional Jimbo–Miwa equation possesses a Hirota bilinear form, and thus, we will search for positive quadratic function solutions of the corresponding (3+1)-dimensional bilinear Jimbo–Miwa equation. A few concluding remarks will be given finally in Section 3.

2. Abundant lump-type solutions

The (3+1)-dimensional Jimbo–Miwa equation reads [30]

$$P_{JM}(u) := u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{2.1}$$

called the Jimbo–Miwa equation in [32]. The equation is among the entire KP hierarchy [30] and completely defined by a Hirota bilinear equation

$$\begin{aligned} B_{JM}(f) &:= (D_x^3 D_y + 2D_t D_y - 3D_x D_z) f \cdot f \\ &= 2(f_{xxxy} f - f_y f_{xxx} - 3f_x f_{xxy} + 3f_{xx} f_{xy} + 2f_{yt} f - 2f_y f_t - 3f_{xz} f + 3f_x f_z) = 0, \end{aligned} \tag{2.2}$$

under the transformation between f and u:

$$u = 2(\ln f)_x. \tag{2.3}$$

This is also a characteristic transformation adopted in Bell polynomial theories of soliton equations (see, e.g., [33,34]) and actually we have

$$P_{JM}(u) = \frac{f(B_{JM}(f))_x - 2f_x B_{JM}(f)}{f^3}. \tag{2.4}$$

Therefore, when f solves the bilinear Jimbo–Miwa equation (2.2), $u = 2(\ln f)_x$ will present a solution to the (3+1)-dimensional Jimbo–Miwa equation (2.1).

The Hirota perturbation technique allows us to present one- and two-soliton solutions [32] and dromion-type solutions [35], and the Exp-function method and the transformed rational function method generates many traveling wave solutions [36,37]. A direct computation also shows that the Jimbo–Miwa equation (2.1) has the following polynomial solutions [37]:

$$u = a_0 + a_1 x + a_2 y + a_3 z + a_4 t + a_5 xz + a_6 xt + a_7 yz + \frac{3}{2} a_5 yt + a_8 zt, \tag{2.5}$$

where $a_i,\ 0 \le i \le 8$, are arbitrary parameters.

In what follows, we focus on computing lump-type solutions to the (3+1)-dimensional Jimbo–Miwa equation (2.1) through carefully searching for positive quadratic function solutions to the bilinear Jimbo–Miwa equation (2.2) with symbolic computations.

We apply the computer algebra system Maple to look for quadratic function solutions to the (3+1)-dimensional bilinear Jimbo–Miwa equation (2.2). A direct Maple symbolic computation starting with

$$\begin{cases} f = g^2 + h^2 + a_{11}, \\ g = a_1 x + a_2 y + a_3 z + a_4 t + a_5, \\ h = a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \end{cases} \tag{2.6}$$

yields the following ten sets of solutions for the parameters $a_i,\ 1 \le i \le 11$:

$$\left\{a_1 = \frac{2}{3}\frac{a_2a_3a_4 + a_2a_8a_9 - a_3a_7a_9 + a_4a_7a_8}{{a_3}^2 + {a_8}^2},\ a_2 = a_2,\ a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\right.$$
$$a_6 = \frac{2}{3}\frac{a_2a_3a_9 - a_2a_4a_8 + a_3a_4a_7 + a_7a_8a_9}{{a_3}^2 + {a_8}^2},\ a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},$$
$$\left. a_{11} = \frac{4}{9}\frac{({a_2}^2 + {a_7}^2)^2({a_4}^2 + {a_9}^2)(a_3a_4 + a_8a_9)}{({a_3}^2 + {a_8}^2)(a_2a_8 - a_3a_7)(a_3a_9 - a_4a_8)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = \frac{3}{2}\frac{a_1a_3a_4 + a_1a_8a_9 + a_3a_6a_9 - a_4a_6a_8}{{a_4}^2 + {a_9}^2},\ a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\right.$$
$$a_6 = a_6,\ a_7 = -\frac{3}{2}\frac{a_1a_3a_9 - a_1a_4a_8 - a_3a_4a_6 - a_6a_8a_9}{{a_4}^2 + {a_9}^2},\ a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},$$
$$\left. a_{11} = \frac{3}{2}\frac{({a_1}^2 + {a_6}^2)^2(a_3a_4 + a_8a_9)}{(a_1a_9 - a_4a_6)(a_3a_9 - a_4a_8)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = a_2,\ a_3 = a_3,\ a_4 = \frac{3}{2}\frac{a_1a_2a_3 + a_1a_7a_8 - a_2a_6a_8 + a_3a_6a_7}{{a_2}^2 + {a_7}^2},\ a_5 = a_5,\right.$$
$$a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = \frac{3}{2}\frac{a_1a_2a_8 - a_1a_3a_7 + a_2a_3a_6 + a_6a_7a_8}{{a_2}^2 + {a_7}^2},\ a_{10} = a_{10},$$
$$\left. a_{11} = -\frac{({a_1}^2 + {a_6}^2)({a_2}^2 + {a_7}^2)(a_1a_2 + a_6a_7)}{(a_1a_7 - a_2a_6)(a_2a_8 - a_3a_7)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = \frac{1}{2}\frac{3\,a_1{a_3}^2 + 3\,a_1{a_8}^2 + 2\,a_3a_7a_9 - 2\,a_4a_7a_8}{a_3a_4 + a_8a_9},\ a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\right.$$
$$a_6 = \frac{1}{3}\frac{3\,a_1a_3a_9 - 3\,a_1a_4a_8 + 2\,{a_4}^2a_7 + 2\,a_7{a_9}^2}{a_3a_4 + a_8a_9},\ a_7 = a_7,\ ,\ a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},$$
$$\left. a_{11} = \frac{1}{18}\frac{({a_4}^2 + {a_9}^2)[(3\,a_1a_3 + 2\,a_7a_9)^2 + (3\,a_1a_8 - 2\,a_4a_7)^2]^2}{(a_3a_4 + a_8a_9)^2(a_3a_9 - a_4a_8)(3\,a_1a_8 - 2\,a_4a_7)}\right\},$$
$$\left\{a_1 = \frac{1}{3}\frac{2\,{a_2}^2a_4 + 3\,a_2a_6a_8 - 3\,a_3a_6a_7 + 2\,a_4{a_7}^2}{a_2a_3 + a_7a_8},\ a_2 = a_2,\ a_3 = a_3,\ a_4 = a_4,\ a_5 = a_5,\right.$$
$$a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = \frac{1}{2}\frac{2\,a_2a_4a_8 + 3\,{a_3}^2a_6 - 2\,a_3a_4a_7 + 3\,a_6{a_8}^2}{a_2a_3 + a_7a_8},\ a_{10} = a_{10},$$
$$\left. a_{11} = \frac{1}{9}\frac{({a_2}^2 + {a_7}^2)^2(2\,a_2a_4 + 3\,a_6a_8)[(2\,a_2a_4 + 3\,a_6a_8)^2 + (3\,a_3a_6 - 2\,a_4a_7)^2]}{(a_2a_3 + a_7a_8)^2(a_2a_8 - a_3a_7)(3\,a_3a_6 - 2\,a_4a_7)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = a_2,\ a_3 = -\frac{1}{3}\frac{3\,a_1a_7a_8 - 2\,{a_2}^2a_4 - 3\,a_2a_6a_8 - 2\,a_4{a_7}^2}{a_1a_2 + a_6a_7},\ a_4 = a_4,\ a_5 = a_5,\right.$$
$$a_6 = a_6,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = \frac{1}{2}\frac{3\,{a_1}^2a_8 - 2\,a_1a_4a_7 + 2\,a_2a_4a_6 + 3\,{a_6}^2a_8}{a_1a_2 + a_6a_7},\ a_{10} = a_{10},$$
$$\left. a_{11} = -\frac{3\,[({a_1}^2 + {a_6}^2)(a_1a_2 + a_6a_7)^2]}{(a_1a_7 - a_2a_6)(3\,a_1a_8 - 2\,a_4a_7)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = \frac{1}{2}\frac{3\,{a_1}^2a_8 - 2\,a_1a_4a_7 + 3\,{a_6}^2a_8 - 2\,a_6a_7a_9}{a_1a_9 - a_4a_6},\right.$$
$$a_3 = \frac{1}{3}\frac{3\,a_1a_4a_8 - 2\,{a_4}^2a_7 + 3\,a_6a_8a_9 - 2\,a_7{a_9}^2}{a_1a_9 - a_4a_6},\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,$$
$$\left. a_7 = a_7,\ a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},\ a_{11} = \frac{3}{2}\frac{({a_1}^2 + {a_6}^2)^2(3\,a_1a_8 - 2\,a_4a_7)}{(a_1a_9 - a_4a_6)(3\,a_6a_8 - 2\,a_7a_9)}\right\},$$
$$\left\{a_1 = a_1,\ a_2 = a_2,\ a_3 = \frac{1}{3}\frac{3\,a_1a_2a_8 - 2\,{a_2}^2a_9 + 3\,a_6a_7a_8 - 2\,{a_7}^2a_9}{a_1a_7 - a_2a_6},\right.$$
$$a_4 = \frac{1}{2}\frac{3\,{a_1}^2a_8 - 2\,a_1a_2a_9 + 3\,{a_6}^2a_8 - 2\,a_6a_7a_9}{a_1a_7 - a_2a_6},\ a_5 = a_5,\ a_6 = a_6,\ a_7 = a_7,$$
$$\left. a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},\ a_{11} = \frac{3\,({a_1}^3a_2 + {a_1}^2a_6a_7 + a_1a_2{a_6}^2 + {a_6}^3a_7)}{3\,a_6a_8 - 2\,a_7a_9}\right\},$$

$$\Big\{a_1 = a_1,\ a_2 = -\frac{3}{2}\frac{b_1}{b_4},\ a_3 = -\frac{b_2}{b_4},\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,$$
$$a_7 = \frac{3}{2}\frac{b_3}{b_4},\ a_8 = a_8,\ a_9 = a_9,\ a_{10} = a_{10},\ a_{11} = a_{11}\Big\},$$
$$\Big\{a_1 = a_1,\ a_2 = a_2,\ a_3 = \frac{1}{3}\frac{c_1}{a_1 a_{11}},\ a_4 = a_4,\ a_5 = a_5,\ a_6 = a_6,$$
$$a_7 = a_7,\ a_8 = -\frac{1}{3}\frac{c_2}{c_4},\ a_9 = -\frac{1}{2}\frac{c_3}{c_4},\ a_{10} = a_{10},\ a_{11} = a_{11}\Big\},$$

in the last two sets of which the constants b_i and c_i, $1 \le i \le 4$, are defined by

$$\begin{cases} b_1 = (3\,{a_1}^4 a_6 + 6\,{a_1}^2{a_6}^3 + 3\,{a_6}^5 + 2\,{a_1}^2 a_9 a_{11} - 2\,a_1 a_4 a_6 a_{11})a_8, \\ b_2 = (3\,{a_1}^4 a_9 + 6\,{a_1}^2{a_6}^2 a_9 + 3\,{a_6}^4 a_9 + 2\,a_1 a_4 a_9 a_{11} - 2\,{a_4}^2 a_6 a_{11})a_8, \\ b_3 = (3\,{a_1}^5 + 6\,{a_1}^3{a_6}^2 + 3\,a_1{a_6}^4 - 2\,a_1 a_6 a_9 a_{11} + 2\,a_4{a_6}^2 a_{11})a_8, \\ b_4 = 3\,{a_1}^4 a_4 + 6\,{a_1}^2 a_4{a_6}^2 + 3\,a_4{a_6}^4 - 2\,a_1{a_9}^2 a_{11} + 2\,a_4 a_6 a_9 a_{11}, \end{cases}$$

and

$$\begin{cases} c_1 = 3\,{a_1}^3 a_2 + 3\,{a_1}^2 a_6 a_7 + 3\,a_1 a_2{a_6}^2 + 3\,{a_6}^3 a_7 + 2\,a_2 a_4 a_{11}, \\ c_2 = 3\,{a_1}^4{a_2}^2 + 6\,{a_1}^3 a_2 a_6 a_7 + 3\,{a_1}^2{a_2}^2{a_6}^2 + 3\,{a_1}^2{a_6}^2{a_7}^2 \\ \quad +6\,a_1 a_2{a_6}^3 a_7 + 3\,{a_6}^4{a_7}^2 - 2\,a_1 a_4{a_7}^2 a_{11} + 2\,a_2 a_4 a_6 a_7 a_{11}, \\ c_3 = 3\,{a_1}^5 a_2 + 3\,{a_1}^4 a_6 a_7 + 6\,{a_1}^3 a_2{a_6}^2 + 6\,{a_1}^2{a_6}^3 a_7 \\ \quad +3\,a_1 a_2{a_6}^4 + 3\,{a_6}^5 a_7 - 2\,a_1 a_4 a_6 a_7 a_{11} + 2\,a_2 a_4{a_6}^2 a_{11}, \\ c_4 = a_1 a_{11}(a_1 a_7 - a_2 a_6). \end{cases}$$

These sets of solutions for the parameters generate ten classes of quadratic function solutions f_i, $1 \le i \le 10$, defined by (2.6), to the bilinear Jimbo–Miwa equation (2.2); and further the resulting quadratic function solutions present ten classes of lump-type solutions u_i, $1 \le i \le 10$, under the transformation (2.3), to the (3+1)-dimensional Jimbo–Miwa equation (2.1).

The analyticity of those rational solutions can be achieved, if we choose the parameters guaranteeing $a_{11} > 0$. All the above rational function solutions u_i, $1 \le i \le 10$, go to zero, when the corresponding sum of squares $g^2 + h^2 \to \infty$, which can be easily satisfied. However, they do not approach zero in all directions in $\mathbb{R}^4$ due to the character of (3+1)-dimensions in the resulting solutions, and thus, they are lump-type solutions but not lump solutions.

For the first and fourth lump-type solutions, let us choose the following two special sets of parameters:

$$a_2 = 1,\quad a_3 = -2,\quad a_4 = 3,\quad a_5 = -1,\quad a_7 = 2,\quad a_8 = 4,\quad a_9 = -1,\quad a_{10} = 5,$$

and

$$a_1 = 1,\quad a_3 = 2,\quad a_4 = -2,\quad a_5 = -1,\quad a_7 = 3,\quad a_8 = 2,\quad a_9 = 1,\quad a_{10} = 5,$$

which all satisfy $a_{11} > 0$. The corresponding two special lump-type solutions read

$$u_1 = \frac{16\,(18\,t + 10\,x - 15\,y - 42\,z - 48)}{g_1} \tag{2.7}$$

with

$$g_1 = 360\,t^2 + 144\,tx + 72\,ty - 720\,tz + 40\,x^2 - 120\,xy - 336\,xz$$
$$+ 180\,y^2 + 432\,yz + 720\,z^2 - 576\,t - 384\,x + 648\,y + 1584\,z + 961, \tag{2.8}$$

and

$$u_4 = -\frac{24\,(10\,t - 65\,x + 39\,y + 14\,z + 41)}{g_2} \tag{2.9}$$

with

$$g_2 = 30\,t^2 - 120\,tx + 396\,ty - 24\,tz + 390\,x^2 - 468\,xy - 168\,xz$$
$$+ 1404\,y^2 - 288\,yz + 48\,z^2 + 84\,t - 492\,x + 360\,y + 96\,z + 1001, \tag{2.10}$$

respectively.

3. Concluding remarks

Through the Hirota formulation and symbolic computations with Maple, we constructed ten classes of lump-type solutions to the (3+1)-dimensional Jimbo–Miwa equation (2.1) explicitly, and the resulting classes of lump-type solutions are supplements to the existing lump-type solutions in the literature [31].

We point out that if we change the Hirota derivatives in (2.2) into generalized bilinear derivatives [38], all the quadratic function solutions presented in the previous section are true for the generalized (3+1)-dimensional bilinear Jimbo–Miwa equations. It is also interesting to find positive polynomial solutions to other generalized bilinear or even tri-linear differential equations, formulated in terms of general bilinear derivatives [38], as did for resonant solutions in terms of exponential functions [39,40]. This kind of polynomial solutions will present lump or lump-type solutions, including rogue wave solutions, to the corresponding nonlinear equations by $u = 2(\ln f)_x$ or $u = 2(\ln f)_{xx}$.

It is recognized that higher-order rogue wave solutions are linked to different mathematical topics including generalized Wronskian solutions [41] and generalized Darboux transformations [42]. Higher-order generalizations of lump solutions and rogue waves, which exhibit more diverse soliton phenomena, can also be presented by the Fredholm determinant [43].

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References

[1] R. Hirota, The Direct Method in Soliton Theory, Cambridge University Press, New York, 2004.
[2] M.J. Ablowitz, P.A. Clarkson, Solitons, Nonlinear Evolution Equations and Inverse Scattering, Cambridge University Press, Cambridge, 1991.
[3] P.J. Caudrey, Memories of Hirota's method: application to the reduced Maxwell-Bloch system in the early 1970s, Phil. Trans. R. Soc. A 369 (2011) 1215–1227.
[4] A. Tacchella, On rational solutions of multicomponent and matrix KP hierarchies, J. Geom. Phys. 61 (2011) 1319–1328.
[5] A.-M. Wazwaz, Two B-type Kadomtsev–Petviashvili equations of (2+1) and (3+1) dimensions: multiple soliton solutions, rational solutions and periodic solutions, Comput. & Fluids 86 (2013) 357–362.
[6] S.V. Manakov, V.E. Zakharov, L.A. Bordag, V.B. Matveev, Two-dimensional solitons of the Kadomtsev–Petviashvili equation and their interaction, Phys. Lett. A 63 (1977) 205–206.
[7] W.X. Ma, Lump solutions to the Kadomtsev–Petviashvili equation, Phys. Lett. A 379 (2015) 1975–1978.
[8] D.J. Kaup, The lump solutions and the Bäcklund transformation for the three-dimensional three-wave resonant interaction, J. Math. Phys. 22 (1981) 1176–1181.
[9] C.R. Gilson, J.J.C. Nimmo, Lump solutions of the BKP equation, Phys. Lett. A 147 (1990) 472–476.
[10] J.Y. Yang, W.X. Ma, Lump solutions of the BKP equation by symbolic computation, Internat. J. Modern Phys. B 30 (2016) 1640028.
[11] J. Satsuma, M.J. Ablowitz, Two-dimensional lumps in nonlinear dispersive systems, J. Math. Phys. 20 (1979) 1496–1503.
[12] K. Imai, Dromion and lump solutions of the Ishimori-I equation, Progr. Theoret. Phys. 98 (1997) 1013–1023.
[13] P. Müller, C. Garrett, A. Osborne, Rogue waves, Oceanography 18 (2005) 66–75.
[14] D.R. Solli, C. Ropers, P. Koonath, B. Jalali, Optical rogue waves, Nature 450 (2007) 1054–1057.
[15] W.X. Ma, Y. You, Solving the Korteweg–de Vries equation by its bilinear form: Wronskian solutions, Trans. Amer. Math. Soc. 357 (2005) 1753–1778.
[16] W.X. Ma, C.X. Li, J.S. He, A second Wronskian formulation of the Boussinesq equation, Nonlinear Anal. TMA 70 (2009) 4245–4258.
[17] F. Gesztesy, W. Schweiger, Rational KP and mKP-solutions in Wronskian form, Rep. Math. Phys. 30 (1991) 205–222.
[18] W.X. Ma, Y. You, Rational solutions of the Toda lattice equation in Casoratian form, Chaos Solitons Fractals 22 (2004) 395–406.
[19] M. Khalfallah, New exact traveling wave solutions of the (3+1) dimensional Kadomtsev–Petviashvili (KP) equation, Commun. Nonlinear Sci. Numer. Simul. 14 (2009) 1169–1175.
[20] D.I. Sinelshchikov, Comment on: New exact traveling wave solutions of the (3+1)-dimensional Kadomtsev–Petviashvili (KP) equation, Commun. Nonlinear Sci. Numer. Simul. 15 (2010) 3235–3236.
[21] W.X. Ma, Comment on the 3+1 dimensional Kadomtsev–Petviashvili equations, Commun. Nonlinear Sci. Numer. Simul. 16 (2011) 2663–2666.
[22] W.A. Malfliet, Solitary wave solutions of nonlinear wave equations, Amer. J. Phys. 60 (1992) 650–654.
[23] M.L. Wang, X.Z. Li, J.L. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A 372 (2008) 417–423.
[24] Y. Zhang, W.X. Ma, Rational solutions to a KdV-like equation, Appl. Math. Comput. 256 (2015) 252–256.
[25] Y.F. Zhang, W.X. Ma, A study on rational solutions to a KP-like equation, Z. Naturforsch. A 70 (2015) 263–268.
[26] C.G. Shi, B.Z. Zhao, W.X. Ma, Exact rational solutions to a Boussinesq-like equation in (1+1)-dimensions, Appl. Math. Lett. 48 (2015) 170–176.
[27] W.X. Ma, A. Abdeljabbar, A bilinear Bäcklund transformation of a (3+1)-dimensional generalized KP equation, Appl. Math. Lett. 12 (2012) 1500–1504.
[28] İ. Aslan, Constructing rational and multi-wave solutions to higher order NEEs via the Exp-function method, Math. Methods Appl. Sci. 34 (2011) 990–995.
[29] İ. Aslan, Rational and multi-wave solutions to some nonlinear physical models, Romanian J. Phys. 58 (2013) 893–903.
[30] M. Jimbo, T. Miwa, Solitons and infinite-dimensional Lie algebras, Publ. Res. Inst. Math. Sci. 19 (1983) 943–1001.
[31] W.X. Ma, Int. J. Nonlinear Sci. Numer. Simul., preprint, 2016.
[32] B. Dorizzi, B. Grammaticos, A. Ramani, P. Winternitz, Are all the equations of the KadomtsevPetviashvili hierarchy integrable? J. Math. Phys. 27 (1986) 2848–2852.
[33] C. Gilson, F. Lambert, J. Nimmo, R. Willox, On the combinatorics of the Hirota D-operators, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci. 452 (1996) 223–234.
[34] W.X. Ma, Bilinear equations, Bell polynomials and linear superposition principle, J. Phys. Conf. Ser. 411 (2013) 012021. 11pp.
[35] A.-M. Wazwaz, New solutions of distinct physical structures to high-dimensional nonlinear evolution equations, Appl. Math. Comput. 196 (2008) 363–370.
[36] T. Öziş, İ. Aslan, Exact and explicit solutions to the (3+1)-dimensional Jimbo–Miwa equation via the Exp-function method, Phys. Lett. A 372 (2008) 7011–7015.

[37] W.X. Ma, J.-H. Lee, A transformed rational function method and exact solutions to the 3+1 dimensional Jimbo–Miwa equation, Chaos Solitons Fractals 42 (2009) 1356–1363.
[38] W.X. Ma, Generalized bilinear differential equations, Stud. Nonlinear Sci. 2 (2011) 140–144.
[39] W.X. Ma, Bilinear equations and resonant solutions characterized by Bell polynomials, Rep. Math. Phys. 72 (2013) 41–56.
[40] W.X. Ma, Trilinear equations, Bell polynomials, and resonant solutions, Front. Math. China 8 (2013) 1139–1156.
[41] W.X. Ma, Wronskians, Generalized Wronskians and solutions to the Korteweg–de Vries equation, Chaos Solitons Fractals 19 (2004) 163–170.
[42] B.L. Guo, L.M. Ling, Q.P. Liu, Nonlinear Schrödinger equation: Generalized Darboux transformation and rogue wave solutions, Phys. Rev. E 85 (2012) 026607. 9pp.
[43] P. Gaillard, Fredholm and Wronskian representations of solutions to the KPI equation and multi-rogue waves, J. Math. Phys. 57 (2016) 063505.