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Dynamics of lump solitary wave of Kadomtsev–Petviashvili–Boussinesq-like equation

Yong-Li Sun [a,c], Wen-Xiu Ma [b,d,e], Jian-Ping Yu [b,c,*], Chaudry Masood Khalique [e]

[a] Department of Mathematics, Beijing University of Chemical Technology, Beijing 100029, PR China
[b] Department of Applied Mathematics, University of Science and Technology Beijing, Beijing 100083, PR China
[c] Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA
[d] College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, 266590, Shandong, China
[e] International Institute for Symmetry Analysis and Mathematical Modeling, Department of Mathematical Sciences, North-West University, Mafikeng Campus, Private Bag X2046, Mmabatho 2735, South Africa

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ABSTRACT

We first introduce a (3+1)-dimensional Kadomtsev–Petviashvili–Boussinesq-like (KPB-like) equation. In order to study the dynamics of lump solutions of this new model, two dimensionally reduced cases are firstly investigated by using the generalized bilinear method. The quadratic functions are used to construct lump solutions to the aforementioned dimensionally reduced cases. Analyzing these lumps, we find the free parameters play an important role during the research on the dynamics of lump solutions, which are utilized to find the sufficient and necessary conditions for guaranteeing the existence, the analyticity and the rational localization of lump solitary waves. The triple sums of quadratic function solutions are further studied. To show the dynamics, we present some graphical analyses of the resulting solutions, which can be applied to the study of nonlinear phenomena in physics, such as nonlinear optics, and oceanography.

1. Introduction

In the recent years, the research of nonlinear partial differential equations is flourishing since they can provide a description of the features in various fields, for example, oceanography and optics, and so on [1–43]. we noticed that many vital nonlinear partial differential equations describing the nonlinear phenomena can be generated from Hirota bilinear equations [1] and generalized bilinear equations [12], such as the KdV equation and the KP equation. In the meantime, their solutions can also be constructed by using the Hirota bilinear forms and generalized bilinear forms, for instance, constructing the rational solutions [12–15]. Additionally, the rational solutions of some nonintegrable have also been investigated [16].

The lump solution is a special kind of rational solution and is localized in all directions in the space. Recently, the study of lump solution has attracted a lot more attentions [15]. In this research, we aimed at using the Hirota bilinear forms to generate some new generalized KP equation in (3+1)-dimensions and the corresponding (2+1)-dimensional reduced cases.

It is well-known that the Kadomtsev–Petviashvili equation is applied to describe the evolution of the nonlinear and long waves with small and slow dependence on the transverse coordinate. The restriction, which is the waves must be

* Corresponding author at: Department of Applied Mathematics, University of Science and Technology Beijing, Beijing 100083, PR China.
E-mail address: jpyu@ustb.edu.cn (J.-P. Yu).

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one dimensional, was relaxed by Kadomtsev and Petviashvili [17–43], so that the completely integrable KP equation was obtained as follows:

$$(u_t + 6uu_x + u_{xxx})_x + au_{yy} = 0$$

A lot of generalized KP equations have been studied in some literatures [20–23]. Particularly, one generalized KP equation was investigated in [20]

$$(u_t + \alpha u^n u_x + \beta u_{xxx})_x + \nu(u_{xx} + u_{yy} + u_{zz}) + r(u_{xy} + u_{xz} + u_{yz}),$$

where α, β, ν, r are parameters.

Additionally, one new KP-like equation in (2+1)-dimensions

$$u_{xt} + 3u_x u_{xx} + 3uu_x^2 + \frac{3}{2}u^3 u_x + \frac{3}{2}u^2 u_{xx} + u_{yy} = 0, \tag{1}$$

was investigated in [19–24]. This model was introduced via generalized bilinear operators and some better results have been obtained.

Recently, the research of the lump solutions of some generalized KP equations has been concerned by many people (e.g., see [19–36]). It is noticed that the generalized bilinear equations play an important role during the investigation of the lump solutions [12] given by

$$\begin{aligned}
&D_{p,x}^m D_{p,t}^n f \cdot f \\
&= (\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'})^m (\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'})^n f(x,t) f(x',t')|_{x'=x,t'=t} \\
&= \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^{(i)}} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^{(j)}} \\
&f(x,t) f(x',t')|_{x'=x,t'=t}, \\
&= \sum_{i=0}^{m}\sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} f(x,t)}{\partial x^{m-i} t^{n-j}} \frac{\partial^{i+j} f(x,t)}{\partial x^i t^j}, \quad m, n \geq 0,
\end{aligned}$$

where $\alpha_p^s = (-1)^{r_p(s)}, s = r_p(s) \mod p$.

Now, let $p = 2$, then we have the following

$$\begin{aligned}
&D_{2,t} D_{2,x} f \cdot f = 2f_{xt} f - 2f_x f_t, D_{2,x}^4 f \cdot f = 2f_{xxxx} f - 8f_{xxx} f_x + 6f_{xx}^2, \\
&\quad D_{2,y}^2 f \cdot f = 2f_{yy} f - 2f_y^2, D_{2,z}^2 f \cdot f = 2f_{zz} f - 2f_z^2,
\end{aligned}$$

which can generate the standard generalized bilinear KP equation in (3+1)-dimensions in the form

$$\begin{aligned}
&(D_{2,t} D_{2,x} + D_{2,x}^4 + D_{2,y}^2 + D_{2,z}^2) f \cdot f \\
&= 2f_{xt} f - 2f_x f_t + 2f_{xxxx} f - 8f_{xxx} f_x + 6f_{xx}^2 + 2f_{yy} f - 2f_y^2 + 2f_{zz} f - 2f_z^2 = 0,
\end{aligned}$$

which is exactly corresponding to the standard Hirota case.

Such as the generalized Boussinesq equation written as

$$u_{tt} + 3u_x u_{xx} + 3uu_x^2 + \frac{3}{2}u^3 u_x + \frac{3}{2}u^2 u_{xx} = 0, \tag{2}$$

and the generalized Kadomtsev–Petviashvili equation

$$u_{xt} + 3u_x u_{xx} + 3uu_x^2 + \frac{3}{2}u^3 u_x + \frac{3}{2}u^2 u_{xx} + u_{yy} = 0, \tag{3}$$

they have the generalized bilinear forms, through $u = 2(\ln f)_x$, respectively,

$$(D_{3,t}^2 + D_{3,x}^4) f \cdot f = 2f_{tt} f - 2f_t^2 + 6f_{xx}^2 = 0, \tag{4}$$

and

$$(D_{3,x} D_{3,t} + D_{3,x}^4 + D_{3,y}^2) f \cdot f = 2f_{xt} f - 2f_x f_t + 6f_{xx}^2 + 2f_{yy} f - 2f_y^2 = 0. \tag{5}$$

So that we can get one combination of Eqs. (4) and (5) as follows

$$\begin{aligned}
&(c_1 D_{3,x} D_{3,t} + c_2 D_{3,t}^2 + c_3 D_{3,x}^4 + c_4 D_{3,y}^2) f \cdot f \\
&= 2[c_1(f_{xt} f - f_x f_t) + c_2(f_{tt} f - f_t^2) + 3c_3 f_{xx}^2 + c_4(f_{yy} f - 2f_y^2)] = 0,
\end{aligned} \tag{6}$$

which is correspondent to the following nonlinear partial differential equation

$$c_1 u_{xt} + c_2 u_{tt} + c_3(3u_x u_{xx} + 3uu_x^2 + \frac{3}{2}u^3 u_x + \frac{3}{2}u^2 u_{xx}) + c_4 u_{yy} = 0. \tag{7}$$

We begin this paper with introducing a (3+1)-dimensional KPB-like equation in Section 2; then the lump solutions to two dimensionally reduced cases will be constructed in Section 3 and Section 4, respectively. In the meantime, the triple sum of quadratic function solutions are also discussed. To illustrate the dynamics of the resulting solutions vividly, some important figures are provided. We conclude this paper with Section 5.

2. The Kadomtsev–Petviashvili-Boussinesq-like equation

If we assume that p be a prime number, then the generalized bilinear operators can be defined as [12]:

$$\begin{aligned} &D_{p,x_1}^{n_1}\cdots D_{p,x_M}^{n_M}(f\cdot g)=\prod_{i=1}^{M}\Big(\frac{\partial}{\partial x_i}+\alpha_p\frac{\partial}{\partial x_i'}\Big)^i \\ &\times f(x_1,\ldots,x_M)f(x_1',\ldots,x_M')|_{x_1'=x_1,\ldots,x_M'=x_M}, \end{aligned} \tag{8}$$

where arbitrary parameters $n_1,\ldots,n_M$ are all nonnegative integers, and $\alpha_p^s=(-1)^{r_p(s)}$, $s=r_p(s) \mod p$.

Under the assumption $p=3$, Eqs. (4) and (5) can be converted into the following forms

$$(D_{3,t}^2+D_{3,x}^4+D_{3,z}^2)f\cdot f=2f_{tt}f-2f_t^2+6f_{xx}^2+2f_{zz}f-2f_z^2=0, \tag{9}$$

and

$$\begin{aligned} &(D_{3,x}D_{3,t}++D_{3,y}D_{3,t}+D_{3,x}^4+D_{3,y}^2+D_{3,z}^2)f\cdot f \\ &=2f_{xt}f-2f_xf_t+2f_{yt}f-2f_yf_t+6f_{xx}^2+2f_{yy}f-2f_y^2+2f_{zz}f-2f_z^2=0, \end{aligned} \tag{10}$$

respectively. Thus, we can get a (3+1)-dimensional KPB-like equation by adding two more terms $c_2D_{3,y}D_{3,t}$ and $c_6D_{3,z}^2$ as follows:

$$\begin{aligned} &(c_1D_{3,x}D_{3,t}+c_2D_{3,y}D_{3,t}+c_3D_{3,t}^2+c_4D_{3,x}^4+c_5D_{3,y}^2+c_6D_{3,z}^2)f\cdot f \\ &=2[c_1(f_{xt}f-f_xf_t)+c_2(f_{yt}f-f_yf_t)+c_3(f_{tt}f-f_t^2) \\ &+3c_4f_{xx}^2+c_5(f_{yy}f-2f_y^2)+c_6(f_{zz}f-2f_z^2)]=0, \end{aligned} \tag{11}$$

the scalar form of which is

$$c_1u_{xt}+c_2u_{yt}+c_3u_{tt}+c_4(3u_xu_{xx}+3uu_x^2+\frac{3}{2}u^3u_x+\frac{3}{2}u^2u_{xx})+c_5u_{yy}+c_6u_{zz}=0, \tag{12}$$

through $u=2(\ln f)_x$ (see [44]).

As matter of a fact, the relation between f and u can be obtained as

$$\begin{aligned} &\left[\frac{(c_1D_{3,x}D_{3,t}+c_2D_yD_t+c_3D_{3,t}^2+c_4D_{3,x}^4+c_5D_{3,y}^2+c_6D_{3,z}^2)f\cdot f}{f^2}\right]_x \\ &=c_1u_{xt}+c_2u_{yt}+c_3u_{tt}+c_4(3u_xu_{xx}+3uu_x^2+\frac{3}{2}u^3u_x+\frac{3}{2}u^2u_{xx})+c_5u_{yy}+c_6u_{zz}, \end{aligned} \tag{13}$$

from which it follows that if f solve Eq. (11) then $u=2(\ln f)_x$ is a solution to Eq. (12) as well.

3. Lump solutions of case I

Suppose $z=t$, then Eq. (11) is reduced to a (2+1)-dimensional KPB-like equation

$$\begin{aligned} &(c_1D_{3,x}D_{3,t}+c_2D_yD_t+c_3D_{3,t}^2+c_4D_{3,x}^4+c_5D_{3,y}^2+c_6D_{3,z}^2)f\cdot f \\ &=2[c_1(f_{xt}f-f_xf_t)+c_2(f_{yt}f-f_yf_t)+(c_3+c_6)(f_{tt}f-f_t^2) \\ &+3c_4f_{xx}^2+c_5(f_{yy}f-2f_y^2)]=0. \end{aligned} \tag{14}$$

Therefore, applying $u=2(\ln f)_x$, Eq. (14) is converted into

$$c_1u_{xt}+c_2u_{yt}+(c_3+c_6)u_{tt}+c_4(3u_xu_{xx}+3uu_x^2+\frac{3}{2}u^3u_x+\frac{3}{2}u^2u_{xx})+c_5u_{yy}=0. \tag{15}$$

To construct the lump solitary waves of Eq. (14), we start with

$$f=g^2+h^2+a_9,\ g=a_1x+a_2y+a_3t+a_4,\ h=a_5x+a_6y+a_7t+a_8, \tag{16}$$

where $a_i's$ are real to be determined. Substituting (16) into Eq. (14), we got the relations among $a_i's$:

$$\begin{aligned} a_1 &= -\frac{a_2^2a_3c_5 + a_2a_3^2c_2 + 2a_2a_6a_7c_5 + a_2a_7^2c_2 + a_3^3c_3 + a_3^3c_6 - a_3a_6^2c_5 + a_3a_7^2c_3 + a_3a_7^2c_6}{c_1(a_3^2 + a_7^2)} \\ a_5 &= \frac{a_2^2a_7c_5 - 2a_2a_3a_6c_5 - a_3^2a_6c_2 - a_3^2a_7c_3 - a_3^2a_7c_6 - a_6^2a_7c_5 - a_6a_7^2c_2 - a_7^3c_3 - a_7^3c_6}{c_1(a_3^2 + a_7^2)} \\ a_9 &= -\frac{3c_4q}{c_1^4c_5(a_3^2 + a_7^2)(a_2a_7 - a_3a_6)^2}, \end{aligned} \tag{17}$$

where q is a polynomial of degree 12, for simplicity, we do not give it here. It is obvious that the set (17) has to satisfy the following conditions

$$c_1c_4c_5 \neq 0,\ c_4c_5q < 0,\ \triangle = \begin{vmatrix} a_2 & a_3 \\ a_6 & a_7 \end{vmatrix} \neq 0, \tag{18}$$

which can guarantee not only f is well defined but also it is positive. Therefore, (17) gives rises to a class of positive quadratic function solution of Eq. (14):

$$f = g^2 + h^2 - \frac{3c_4q}{c_1^4c_5(a_3^2 + a_7^2)(a_2a_7 - a_3a_6)^2}, \tag{19}$$

which yields lump solutions to Eq. (15):

$$u = 2(\ln f)_x = 4\frac{a_1g + a_5h}{f}, \tag{20}$$

with f defined by (20), and functions g and h given by

$$\begin{aligned} g &= -\frac{a_2^2a_3c_5 + a_2a_3^2c_2 + 2a_2a_6a_7c_5 + a_2a_7^2c_2 + a_3^3c_3 + a_3^3c_6 - a_3a_6^2c_5 + a_3a_7^2c_3 + a_3a_7^2c_6}{c_1(a_3^2 + a_7^2)}x + a_2y + a_3t + a_4, \\ h &= \frac{a_2^2a_7c_5 - 2a_2a_3a_6c_5 - a_3^2a_6c_2 - a_3^2a_7c_3 - a_3^2a_7c_6 - a_6^2a_7c_5 - a_6a_7^2c_2 - a_7^3c_3 - a_7^3c_6}{c_1(a_3^2 + a_7^2)}x + a_6y + a_7t + a_8. \end{aligned} \tag{21}$$

It is found that there are 6 arbitrary parameters in this class of lump solutions: $a_2, a_3, a_4, a_6, a_7, a_8$, provided that the solutions are all well defined, i.e., the determinant condition (18) is satisfied, which precisely implies that two directions (a_2, a_3) and (a_6, a_7) are not parallel in the xy-plane . Moreover, analyzing (8), we observed that (20) is analytical in the xy-plane if and only if $a_9 > 0$ and $u \to 0$ if and only if $g^2 + h^2 \to \infty$, or $x^2 + y^2 \to \infty$ at any time. So that (18) guarantees the analyticity and localization of (20).

One special example is given in the following

$$\begin{aligned} &c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, c_5 = 1, c_6 = 1, \\ &a_2 = 2, a_3 = 1, a_4 = 0, a_6 = 1, a_7 = 1, a_8 = 0, \end{aligned} \tag{22}$$

which tells us that

$$f = 2t^2 - 22tx + 6ty + \frac{137}{2}x^2 - 37xy + 5y^2 + \frac{56307}{2}, \tag{23}$$

$$u = -\frac{4(22t - 137x + 37y)}{4t^2 - 44tx + 12ty + 137x^2 - 74xy + 10y^2 + 56307}. \tag{24}$$

The contour plot of u is drawn in Fig. 1 with $t = 1$.

It is found that (21) generally goes to zero while $\triangle \to 0$. We illustrate this by choosing

$$\begin{aligned} &c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, c_5 = 1, c_6 = 1, \\ &a_2 = 1, a_3 = 1, a_4 = 0, a_6 = 1, a_7 = 1 + \varepsilon, a_8 = 0, \end{aligned} \tag{25}$$

which leads to $\triangle = \varepsilon$, then (20) becomes

$$u = -\frac{4\varepsilon^2p(\varepsilon)}{q(\varepsilon)} \tag{26}$$

where $p(x, y, t, \varepsilon)$ is a polynomial of degree 5, and $q(x, y, t, \varepsilon) = \varepsilon s(x, y, t, \varepsilon) + 12288$ is also a polynomial of degree 8. So that it is obvious (26) goes to zero while $\varepsilon \to 0$.

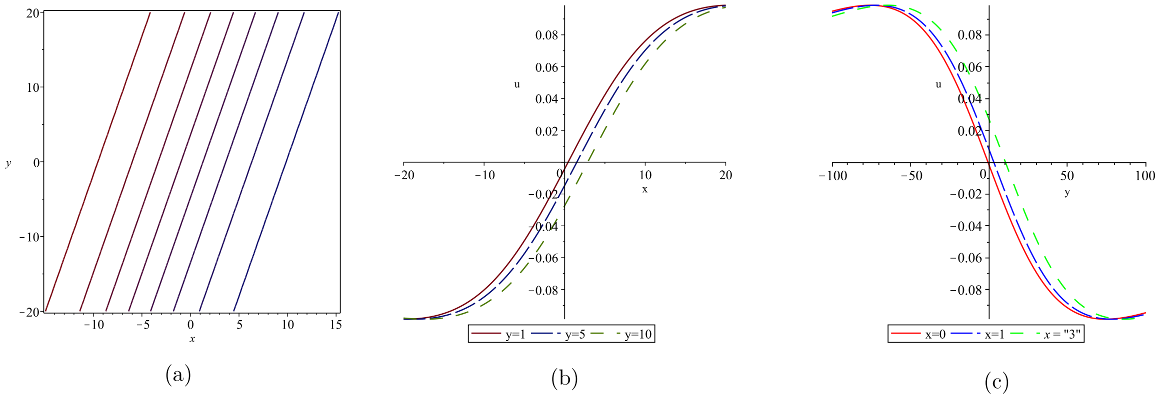


Fig. 1. Profiles of (24) with $t = 1$: (a) contour plot (b) x-curve (c) y-curve.

4. Lump solutions of case II

Under the assumption $z = y$, Eq. (11) can be reduced to the following (2+1)-dimensional case

$$\begin{aligned}&(c_1D_{3,x}D_{3,t} + c_2D_yD_t + c_3D_{3,t}^2 + c_4D_{3,x}^4 + (c_5 + c_6)D_{3,y}^2)f \cdot f\\&= 2[c_1(f_{xt}f - f_xf_t) + c_2(f_{yt}f - f_yf_t) + c_3(f_{tt}f - f_t^2)\\&+3c_4f_{xx}^2 + (c_5 + c_6)(f_{yy}f - 2f_y^2)] = 0.\end{aligned} \tag{27}$$

Now, let $u = 2(\ln f)_x$, Eq. (12) turns into

$$c_1u_{xt} + c_2u_{yt} + c_3u_{tt} + c_4(3u_xu_{xx} + 3uu_x^2 + \frac{3}{2}u^3u_x + \frac{3}{2}u^2u_{xx}) + (c_5 + c_6)u_{yy} = 0. \tag{28}$$

It is obvious that f solves Eq. (27) implies $u = 2(\ln f)_x$ solves Eq. (28). To derive the lump solitary wave solution to Eq. (28), we begin with

$$f = g^2 + h^2 + a_9, g = a_1x + a_2y + a_3t + a_4, h = a_5x + a_6y + a_7t + a_8, \tag{29}$$

where the parameters $a_i's$ are all real to be determined. Substituting (29) into Eq. (27) leading to

$$\begin{aligned}a_1 &= -\frac{a_2^2a_3c_5 + a_2^2a_3c_6 + a_2a_3^2c_2 + 2a_2a_6a_7c_5 + 2a_2a_6a_7c_6 + a_2a_7^2c_2 + a_3^3c_3 - a_3a_6^2c_5 - a_3a_6^2c_6 + a_3a_7^2c_3}{c_1(a_3^2 + a_7^2)}\\a_5 &= \frac{a_2^2a_7c_5 + a_2^2a_7c_6 - 2a_2a_3a_6c_5 - 2a_2a_3a_6c_6 - a_3^2a_6c_2 - a_3^2a_7c_3 - a_6^2a_7c_5 - a_6^2a_7c_6 - a_6a_7^2c_2 - a_7^3c_3}{c_1(a_3^2 + a_7^2)}\\a_9 &= -\frac{3c_4q}{c_1^4(c_5 + c_6)(a_3^2 + a_7^2)(a_2a_7 - a_3a_6)^2},\end{aligned} \tag{30}$$

where q is a polynomial of degree 12. For simplicity, we do not present it here. (31) has to satisfy the following condition

$$c_1c_4(c_5 + c_6) \neq 0, c_4(c_5 + c_6)q < 0, \triangle = \begin{vmatrix}a_2 & a_3\\a_6 & a_7\end{vmatrix} \neq 0. \tag{31}$$

These conditions can guarantee that f is well defined and is positive.

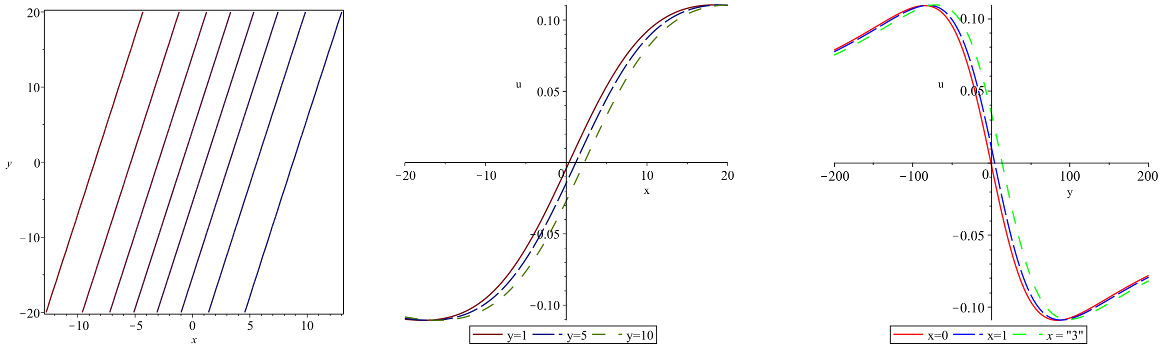
Thus, (31) gives a rise to a class of positive quadratic function solutions of Eq. (27) as follows:

$$f = g^2 + h^2 - \frac{3c_4q}{c_1^4(c_5 + c_6)(a_3^2 + a_7^2)(a_2a_7 - a_3a_6)^2}, \tag{32}$$

which gives a rise to a class of lump solutions of Eq. (28) via $u = 2(\ln f)_x$:

$$u = 2(\ln f)_x = 4\frac{a_1g + a_5h}{f}, \tag{33}$$

where g and h are defined as before. Via analyzing the computation, we observed the directions (a_2, a_3) and (a_6, a_7) are not parallel in the xy-plane at all. In the meantime, It is noted that (33) is analytic if and only if $a_9 > 0$, and at any given time, $u \to 0$ if and only if $g^2 + h^2 \to \infty$, or $x^2 + y^2 \to \infty$ at any time. Thus, (31) can guarantee both analyticity and localization of (33).



(a) (b) (c)

Fig. 2. Profiles of (36) with $t = 1$: (a) contour plot (b) x-curve (c) y-curve.

One particular example is presented as

$$\begin{aligned} &c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, c_5 = 1, c_6 = 1, \\ &a_2 = 2, a_3 = 1, a_4 = 0, a_6 = 1, a_7 = 1, a_8 = 0, \end{aligned} \tag{34}$$

which means that

$$f = 2t^2 - 26tx + 6ty + 109x^2 - 46xy + 5y^2 + 35643, \tag{35}$$

$$u = -\frac{4(13t - 109x + 23y)}{2t^2 - 26tx + 6ty + 109x^2 - 46xy + 5y^2 + 35643}. \tag{36}$$

The contour plot, x-curve and y-curve of function u are given by Fig. 3 with $t = 1$ (see Fig. 2).

We observed that (33) generally goes to zero while $\triangle$ in (31) tends to zero. We can illustrate this by selecting

$$\begin{aligned} &c_1 = 1, c_2 = 1, c_3 = 1, c_4 = -1, c_5 = 1, c_6 = 1, \\ &a_2 = 1, a_3 = 1, a_4 = 0, a_6 = 1, a_7 = 1 + \varepsilon, a_8 = 0, \end{aligned} \tag{37}$$

which leads to $\triangle = \varepsilon$, then the lump solution (33) becomes

$$u = -\frac{8\varepsilon^2 p(\varepsilon)}{q(\varepsilon)} \tag{38}$$

where $p(x, y, t, \varepsilon)$ is a polynomial of degree 5 and $q(x, y, t, \varepsilon) = \varepsilon s(x, y, t, \varepsilon) + 12288$ is a polynomial of degree 8. it is clear that (38) goes to zero when $\varepsilon \to 0$.

5. Triple sum of quadratic function solutions

Within this section, we utilize the triple sum of quadratic functions to construct lump solutions. The definition of multiple sum of quadratic function solutions was firstly proposed in [34], all the references can be found therein. Assume that

$$f = \sum_{i\geq 1}^{M_1} g_i^2 + \sum_{j\geq 1}^{M_2} h_j^2 + c_{4(M_1+M_2)+1}, \tag{39}$$

where

$$g_i = a_{1i}x + a_{2i}y + a_{3i}t + a_{4i},\ h_j = b_{1j}x + b_{2j}y + b_{3j}t + b_{4j}$$

and $a_{1i}, a_{2i}, a_{3i}, a_{4i}, b_{1j}, b_{2j}, b_{3j}, b_{4j}, c_{4(M_1+M_2)+1}$ are arbitrary real constants and M_1, M_2 are integers. For the particular case of Eq. (11):

$$\begin{aligned} &(D_{3,x}D_{3,t} + D_{3,y}D_{3,t} - D_{3,t}^2 + D_{3,x}^4 - D_{3,y}^2)f \cdot f \\ &= 2[(f_{xt}f - f_xf_t) + (f_{yt}f - f_yf_t) - (f_{tt}f - f_t^2) \\ &+3f_{xx}^2 - (f_{yy}f - 2f_y^2)] = 0 \end{aligned} \tag{40}$$

Applying the transformation $u = 2(\ln f)_x$, we converted Eq. (40) into

$$u_{xt} + u_{yt} - u_{tt} + (3u_xu_{xx} + 3uu_x^2 + \frac{3}{2}u^3u_x + \frac{3}{2}u^2u_{xx}) - u_{yy} = 0. \tag{41}$$

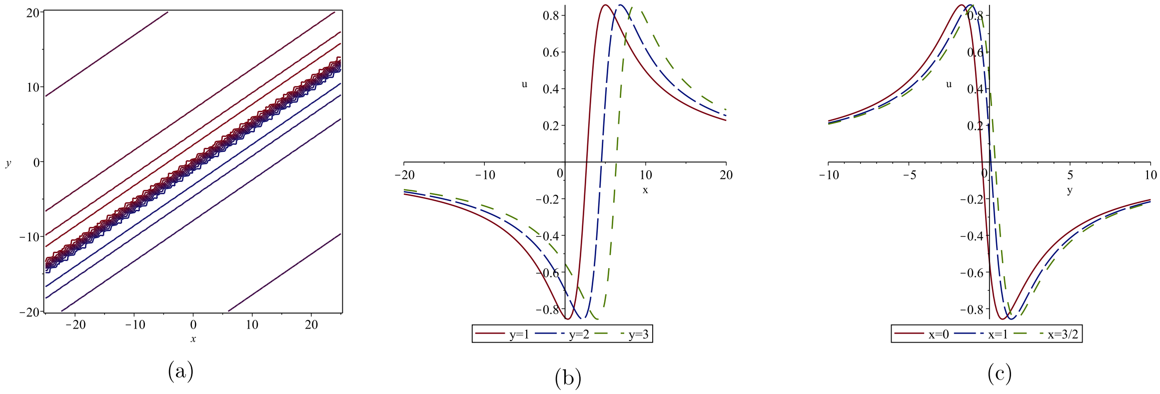


Fig. 3. Profiles of (45) with $t=1$: (a) contour plot (b) x-curve (c) y-curve.

To find the triple sum of positive quadratic function solutions of Eq. (40), we start with

$$\begin{aligned} &f=(ax+t+y+1)^2+(x+by)^2+(ct)^2+d\\ &g=(ax+t+y+1)^2, h=(x+by)^2, w=(ct)^2, \end{aligned} \tag{42}$$

where the real parameters a, b, c, d are all to be determined. After substituting (42) into Eq. (40), we obtained

$$a_1=-\alpha^2-\alpha+1, b=\alpha, c=\alpha, d=6\alpha^2+8\alpha-5, \tag{43}$$

where α is the real root of $Z^3+Z^2-Z+1=0$ presented by

$$\alpha=-\frac{1}{3}(19+3\sqrt{33})^{\frac{1}{3}}-\frac{4}{3(19+3\sqrt{33})^{\frac{1}{3}}}-1/3$$

and $d>0$. All have been validated in Maple.

Therefore, (42) gives a rise to a triple sum of quadratic function solution to Eq. (40):

$$f=((-\alpha^2-\alpha+1)x+y+t+1)^2+(x+\alpha y)^2+(\alpha t)^2+6\alpha^2+8\alpha-5 \tag{44}$$

and the resulting quadratic function solution turns out some lump solutions of Eq. (41) via $u=2(\ln f)_x$:

$$u=2(\ln f)_x=4\frac{(-\alpha^2-\alpha+1)((-\alpha^2-\alpha+1)x+y+t+1)+(x+\alpha y)}{((-\alpha^2-\alpha+1)x+t+y+1)^2+(x+\alpha y)^2+(\alpha t)^2+6\alpha^2+8\alpha-5}, \tag{45}$$

where the functions f, g, h are defined as above. The contour plots, x-curve and y-curve of u are presented by Fig. 3 with $t=1$.

6. Conclusions

A lot of researchers have paid much more attention to the research of lump solutions to the KP equation or KP-like equations recently. Applying the generalized bilinear method, we analyzed the dynamics of the introduced (3+1)-dimensional KPB-like equation (12) through investigating lump solutions of two special cases. So that we got the conditions for the analyticity and localization of the lump solitary waves. We observed lump solution goes to zero when $\triangle\to 0$ illustrated by the corresponding graphs.

For the decreasing solutions dependent rationally on x are related to Hamiltonian flow from Moser's theory of Toda system, and the nondecreasing solutions dependent rationally on x and y are connected with a generalized multiple particle Moser system, so that people have paid many concerns to investigate the dynamics of lumps [13], which can be efficiently obtained via the generalized bilinear method [12]. The higher order extensions and multiple component of lumps display very different soliton phenomenon, hence, they are very important and interesting research topics now. [34] firstly introduced the definition and form of sums of higher-order even function solutions or multiple sums of quadratic function solutions to study the dynamics of lumps. We studied the triple sum of quadratic function solutions of Eq. (12). Additionally, since rogue wave solutions and lumps can be derived from positive polynomial solutions, as a future research, we will also study the existence of positive polynomial solutions, which has attracted many mathematicians and physicists [18,44].

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