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N-soliton solutions and dynamic property analysis of a generalized three-component Hirota–Satsuma coupled KdV equation

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ABSTRACT

In this paper, a generalized three-component Hirota–Satsuma coupled KdV equation describing the interactions of two long waves with different dispersion relations, is investigated. Applying Hirota bilinear operator theory, the bilinear form of the proposed model is first obtained, and then its N-soliton solutions are given in explicit forms. Finally, the analysis of the dynamic property shows that the collisions between two solitons are elastic.

1. Introduction

Soliton solutions of nonlinear partial differential equations (NPDEs) can describe many complicated and nonlinear wave phenomena. Therefore, finding analytic solutions, particularly N-soliton solutions, is now becoming more and more important in solitary wave theory [1–23]. During the research, a lot of effective methods have been proposed. For instance, the Bäcklund method [24–27], the F-expansion method [28], the inverse scattering transformation (IST) approach [29–31], the unified transformed method [32], the Darboux transformation method [33], the Hirota direct method [34], the algebra geometric method [35], and so on.

In Ref. [36], we presented and studied the following generalized two-component Hirota–Satsuma coupled KdV equation:

$$u_t = a(u_{xxx} - 6uu_x) + 3b(|v|^2)_x, v_t = -v_{xxx} + 3uv_x, \tag{1}$$

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which might be employed to investigate the interaction phenomena between long waves with different dispersive relations. In this letter, we further study the following generalized three-component Hirota–Satsuma coupled KdV equation as follows:

$$u_t = a(u_{xxx} - 6uu_x) + 3b(vw)_x, v_t = -v_{xxx} + 3uv_x, w_t = -w_{xxx} + 3uw_x, \tag{2}$$

where a and b are arbitrary real constants. $3b(vw)_x$ is the force term on the KdV wave system with dispersion relation $w = -k^3$. (2) can be reduced to the general KdV system without the effect of v and w. When $a = \frac{1}{2}, b = 1$, (2) becomes the well-known generalized Hirota–Satsuma coupled KdV equation firstly introduced [37] and studied by researchers [35–39]. We might hope to obtain N-soliton solutions of the complex coupled system (2) with $a = \frac{1}{2}$ and $b \neq 0$ and the asymptotic analysis. To the best of our knowledge, (2) has not been revealed and studied by using the Hirota direct approaching previous articles.

The structure of this paper is: In Section 2, the Hirota bilinear form of (2) is derived, and soliton solutions are expressed in explicit forms. In Section 3, the dynamic property analysis for two-soliton solutions is done, which might prove that the collisions between soliton solutions are elastic. Finally, some conclusions are given in Section 4.

2. The bilinear form and N-soliton solutions

It is well-known that the Hirota bilinear method is a powerful and effective tool for studying soliton solutions of many important equations. Now, we use the following two transformations to derive the bilinear form of (2)

$$u = -2(\ln f)_{xx}, v = \frac{g}{\sqrt{3}f}, w = \frac{h}{\sqrt{3}f}, \tag{3}$$

where $f = f(x,t)$ is a real function, $g = g(x,t)$ and $h = h(x,t)$ are complex functions. Substituting (3) into (2) and applying the properties of Hirota bilinear operators, we obtain bilinear form of (2) as follows:

$$(\tfrac{1}{2}D_x^4 - D_xD_t)f \cdot f = bgh, (D_t + D_x^3)g \cdot f = 0, (D_t + D_x^3)h \cdot f = 0, \tag{4}$$

where D_x and D_t are Hirota bilinear operators defined by

$$D_x^m D_y^n a \cdot b = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n a(x,y)b(x',y')|_{x'=x,y'=y}. \tag{5}$$

Using the perturbation technique, we expand f and g into power series :

$$\begin{aligned} f &= 1 + f^{(2)}\epsilon^2 + f^{(4)}\epsilon^4 + \cdots + f^{(2k)}\epsilon^{2k} + \cdots, \\ g &= g^{(1)}\epsilon + g^{(3)}\epsilon^3 + \cdots + g^{(2k+1)}\epsilon^{2k+1} + \cdots, \\ h &= h^{(1)}\epsilon + h^{(3)}\epsilon^3 + \cdots + h^{(2k+1)}\epsilon^{2k+1} + \cdots, \end{aligned} \tag{6}$$

where ϵ is a small parameter.

Substituting these expansions into (4) and collecting terms of each order of small parameter ϵ yield the following three sets of relations

$$g_t^{(1)} + g_{xxx}^{(1)} = 0, \tag{7}$$
$$g_t^{(3)} + g_{xxx}^{(3)} = -(D_t + D_x^3)g^{(1)} \cdot f^{(2)}, \tag{8}$$
$$g_t^{(5)} + g_{xxx}^{(5)} = -(D_t + D_x^3)(g^{(1)} \cdot f^{(4)} + g^{(3)} \cdot f^{(2)}), \tag{9}$$
$$\cdots\cdots\cdots$$

$$h_t^{(1)} + h_{xxx}^{(1)} = 0, \tag{10}$$
$$h_t^{(3)} + h_{xxx}^{(3)} = -(D_t + D_x^3)h^{(1)} \cdot f^{(2)}, \tag{11}$$
$$h_t^{(5)} + h_{xxx}^{(5)} = -(D_t + D_x^3)(h^{(1)} \cdot f^{(4)} + h^{(3)} \cdot f^{(2)}), \tag{12}$$
$$\cdots\cdots\cdots$$

$$2(\frac{1}{2}f_{xxxx}^{(2)} - f_{xt}^{(2)}) = bg^{(1)}h^{(1)} \tag{13}$$
$$2(\frac{1}{2}f_{xxxx}^{(4)} - f_{xt}^{(4)}) = -(\frac{1}{2}D_x^4 - D_xD_t)f^{(2)} \cdot f^{(2)} + b(g^{(1)}h^{(3)} + g^{(3)}h^{(1)}) \tag{14}$$
$$2(\frac{1}{2}f_{xxxx}^{(6)} - f_{xt}^{(6)}) = -(\frac{1}{2}D_x^4 - D_xD_t)f^{(2)} \cdot f^{(4)} + b(g^{(1)}h^{(5)} + g^{(3)}h^{(3)} + g^{(5)}h^{(1)}) \tag{15}$$
$$\cdots\cdots\cdots$$

2.1. One-soliton solutions

According to (1), we suppose

$$g^{(1)} = e^{\xi_1}, h^{(1)} = e^{\eta_1}, \tag{16}$$

where

$$\xi_1 = k_1x + p_1t + \xi_1^0, p_1 = -k_1^3, \eta_1 = \kappa_1x + \omega_1t + \eta_1^0, \omega_1 = -\kappa_1^3, \tag{17}$$

with $k_1, p_1, \kappa_1, \omega_1$ real constants and ξ_1^0, η_1^0 complex numbers. Plugging (16) into (13), we have

$$2(\frac{1}{2}f_{xxxx}^{(2)} - f_{xt}^{(2)}) = be^{\xi_1+\eta_1}. \tag{18}$$

One solution of which is easily obtained

$$f^{(2)} = \frac{be^{\xi_1+\eta_1}}{24k_1^4} \tag{19}$$

Plugging (19) into (8) and (11) generates two equations

$$g_t^{(3)} + g_{xxx}^{(3)} = 0, h_t^{(3)} + h_{xxx}^{(3)} = 0, \tag{20}$$

which implies that $g^{(3)} = 0, h^{(3)} = 0$ are solutions. Similarly, we are able to take $f^{(4)} = g^{(5)} = h^{(5)} = f^{(6)} = \cdots = 0$, which means that the expansions of f, g and h could be truncated into the following finite sums

$$f = 1 + f^{(2)} = 1 + A_1e^{\xi_1+\eta_1}, g = g^{(1)} = e^{\xi_1}, h = h^{(1)} = e^{\eta_1} \tag{21}$$

with

$$A_1 = \frac{b}{3(k_1+\kappa_1)^2(k_1^2+\kappa_1^2)}. \tag{22}$$

It is noted the small perturbation parameter ϵ might be absorbed into the phase constant ξ_1^0 in the exponent ξ_1, so taking $\epsilon = 1$, we obtain the solution of (2)

$$u = -2(\ln(1 + A_1e^{\xi_1+\eta_1}))_{xx}, v = \frac{e^{\xi_1}}{\sqrt{3}(1+A_1e^{\xi_1+\eta_1})}, w = \frac{e^{\eta_1}}{\sqrt{3}(1+A_1e^{\xi_1+\eta_1})}. \tag{23}$$

Actually, (21) might be rewritten in a convenient form

$$f_1 = 1 + f^{(2)} = 1 + e^{\xi_1+\eta_1+\theta_{13}}, g_1 = g^{(1)} = e^{\xi_1}, h_1 = h^{(1)} = e^{\eta_1} \tag{24}$$

with

$$e^{\theta_{13}} = \frac{b}{3(k_1+\kappa_1)^2(k_1^2+\kappa_1^2)}. \tag{25}$$

Then, one-soliton solution could be expressed explicitly as

$$\begin{aligned} u &= -2(\ln(1+e^{\xi_1+\eta_1+\theta_{13}}))_{xx} = -\frac{(k_1+\kappa_1)^2}{2} sech^2(\frac{\xi_1+\eta_1+\theta_{13}}{2}),\\ v &= \frac{e^{\xi_1}}{\sqrt{3}(1+e^{\xi_1+\eta_1+\theta_{13}})} = \frac{\sqrt{(k_1+\kappa_1)^2(k_1^2+\kappa_1^2)}}{2} e^{\xi_1-\eta_1} sech(\frac{\xi_1+\eta_1+\theta_{13}}{2}),\\ w &== \frac{e^{\eta_1}}{\sqrt{3}(1+e^{\xi_1+\eta_1+\theta_{13}})} = \frac{\sqrt{(k_1+\kappa_1)^2(k_1^2+\kappa_1^2)}}{2} e^{\eta_1-\xi_1} sech(\frac{\xi_1+\eta_1+\theta_{13}}{2}), \end{aligned} \tag{26}$$

Particularly, if $a = \frac{1}{2}, b = 1$, the solution (26) coincides with that in [39].

2.2. Two-soliton solutions

It is obvious that Eq. (7) also admits the following solutions

$$g^{(1)} = e^{\xi_1} + e^{\xi_2}, h^{(1)} = e^{\eta_1} + e^{\eta_2}, \tag{27}$$

where

$$\xi_j = k_j x + p_j t + \xi_j^0, \eta_j = \kappa_j x + \omega_j t + \eta_j^0, (j = 1, 2), \tag{28}$$

with dispersive relations

$$p_j = -k_j^3, (j = 1, 2), \omega_j = -\kappa_j^3, (j = 1, 2), \tag{29}$$

with $k_j, p_j, \xi_j^0, \kappa_j$ real numbers and $, \omega_j, \eta_j^0$ complex numbers. Plugging (27) into (13) results in

$$2(\frac{1}{2} f^{(2)}_{xxxx} - f^{(2)}_{xt}) = b(e^{\xi_1+\eta_1} + e^{\xi_1+\eta_2} + e^{\xi_2+\eta_1} + e^{\xi_2+\eta_2}), \tag{30}$$

whose solution is

$$f^{(2)} = A_{13} e^{\xi_1+\eta_1} + A_{14} e^{\xi_1+\eta_2} + A_{23} e^{\xi_2+\eta_1} + A_{24} e^{\xi_2+\eta_2}, \tag{31}$$

where

$$\begin{aligned} A_{13} &= \frac{b}{3(k_1+\kappa_1)^2(k_1^2+\kappa_1^2)}, A_{14} = \frac{b}{3(k_1+\kappa_2)^2(k_1^2+\kappa_2^2)}, A_{23} = \frac{b}{3(k_2+\kappa_1)^2(k_2^2+\kappa_1^2)},\\ A_{24} &= \frac{b}{3(k_2+\kappa_2)^2(k_2^2+\kappa_2^2)}, B_{13} = A_{13}, B_{14} = A_{23}, B_{23} = A_{14}, B_{24} = A_{24}. \end{aligned} \tag{32}$$

Using (31), (27), (8) and (11), we get the solutions of (8) and (11), respectively,

$$g^{(3)} = A_{12}A_{13}A_{23} e^{\xi_1+\xi_2+\eta_1} + A_{12}A_{14}A_{24} e^{\xi_1+\xi_2+\eta_2}, h^{(3)} = B_{12}B_{13}B_{23} e^{\eta_1+\eta_2+\xi_1} + B_{12}B_{14}B_{24} e^{\eta_1+\eta_2+\xi_2} \tag{33}$$

where

$$A_{12} = \frac{((k_1+k_2)^3 + 2(k_1^3+k_2^3))(k_1-k_2)^2}{b(k_1+k_2)}, B_{12} = \frac{((\kappa_1+\kappa_2)^3 + 2(\kappa_1^3+k_2^3))(\kappa_1-\kappa_2)^2}{b(\kappa_1+\kappa_2)}. \tag{34}$$

In the same way, we get one solution of Eq. (14) as follows:

$$f^{(4)} = A_{12}A_{13}A_{14}A_{23}A_{24}A_{34} e^{\xi_1+\xi_2+\eta_1+\eta_2}, \tag{35}$$

where

$$A_{34} = B_{12}. \tag{36}$$

If we further substitute $g^{(1)}, g^{(3)}, h^{(1)}, h^{(3)}$ and $f^{(2)}, f^{(4)}$ into (9), (12) and (15), we conclude that $g^{(5)} = 0$ is a solution of (9), $h^{(5)} = 0$ is a solution of (12) and $f^{(6)} = 0$ is a solution of (15). Similarly, we get $g^{(7)} = h^{(7)} = f^{(8)} = \cdots = 0$. Now, we take $\epsilon = 1$, functions f and g, h are truncated to the following forms:

$$\begin{aligned} f_2 &= 1 + A_{13} e^{\xi_1+\eta_1} + A_{14} e^{\xi_1+\eta_2} + A_{23} e^{\xi_2+\eta_1} + A_{24} e^{\xi_2+\eta_2} + A_{12}A_{13}A_{14}A_{23}A_{24}A_{34} e^{\xi_1+\xi_2+\eta_1+\eta_2},\\ g_2 &= e^{\xi_1} + e^{\xi_2} + A_{12}A_{13}A_{23} e^{\xi_1+\xi_2+\eta_1} + A_{12}A_{14}A_{24} e^{\xi_1+\xi_2+\eta_2},\\ h_2 &= e^{\eta_1} + e^{\eta_2} + B_{12}B_{13}B_{23} e^{\eta_1+\eta_2+\xi_1} + B_{12}B_{14}B_{24} e^{\eta_1+\eta_2+\xi_2}. \end{aligned} \tag{37}$$

Plugging(37) into (3) yields a solution of Eq. (2).

To express two-soliton solutions of Eq. (2) explicitly, we rewrite A_{ij}, B_{ij} as

$$A_{ij} = e^{\theta_{ij}}(1 \le i < j \le 4), B_{ij} = e^{\beta_{ij}}, (1 \le i < j \le 4, i = 1, 2). \tag{38}$$

Hence, functions f_2 and g_2 might be rewritten

$$\begin{aligned} &f_2 = 1 + e^{\xi_1+\eta_1+\theta_{13}} + e^{\xi_1+\eta_2+\theta_{14}} + e^{\xi_2+\eta_1+\theta_{23}} + e^{\xi_2+\eta_2+\theta_{24}} \\ &+e^{\xi_1+\xi_2+\eta_1+\eta_2+\theta_{12}+\theta_{13}+\theta_{14}+\theta_{23}+\theta_{24}+\theta_{34}} \\ &g_2 = e^{\xi_1} + e^{\xi_2} + e^{\xi_1+\xi_2+\eta_1+\theta_{12}+\theta_{13}+\theta_{23}} + e^{\xi_1+\xi_2+\eta_2+\theta_{12}+\theta_{14}+\theta_{24}}, \\ &h_2 = e^{\eta_1} + e^{\eta_2} + e^{\eta_1+\eta_2+\xi_1+\beta_{12}+\beta_{13}+\beta_{23}} + e^{\eta_1+\eta_2+\xi_2+\beta_{12}+\beta_{14}+\beta_{24}}. \end{aligned} \tag{39}$$

Plugging (37)) into (3) yields a two-soliton solution of Eq. (2) as

$$u = -2(\ln(f_2))_{xx}, v = \frac{g_2}{\sqrt{3}f_2}, v = \frac{h_2}{\sqrt{3}f_2}. \tag{40}$$

2.3. N-soliton solutions

Generally, in order to construct N-soliton solutions, functions f_N, g_N can be written in the following forms:

$$\begin{aligned} f_N &= \textstyle\sum_{\mu=0,1} B_1(\mu) \Pi_{1\le j<l\le 2N}(A_{jl})^{\mu_j\mu_l}(\mu) e^{\sum_{j=1}^{2N}\mu_j\xi_j}, \\ g_N &= \textstyle\sum_{\mu=0,1} B_2(\mu) \Pi_{1\le j<l\le 2N}(A_{jl})^{\mu_j\mu_l}(\mu) e^{\sum_{j=1}^{2N}\mu_j\xi_j}, \\ h_N &= \textstyle\sum_{\mu=0,1} B_3(\mu) \Pi_{1\le j<l\le 2N}(B_{jl})^{\mu_j\mu_l}(\mu) e^{\sum_{j=1}^{2N}\mu_j\eta_j}, \end{aligned} \tag{41}$$

with

$$\begin{aligned} &\xi_{N+j} = \eta_j, \eta_{N+j} = \xi_j, (j = 1, 2, \ldots, N), \\ &A_{j,N+l} = \tfrac{b}{3(k_j+\kappa_l)^2(k_j^2+\kappa_l^2)}, (j, l = 1, 2, \ldots, N), A_{j,l} = \tfrac{3(k_j-k_l)^2(k_j^2+k_l^2)}{b}, (1 \le j < l \le N), \\ &B_{j,l} = A_{N+j,N+l} = \tfrac{3(\kappa_j-\kappa_l)^2(\kappa_j^2+\kappa_l^2)}{b}, (1 \le j < l \le N), B_{j,N+l} = A_{l,N+j}(j, l = 1, 2, \ldots, N), \end{aligned} \tag{42}$$

$B_j(\mu), (j = 1, 2, 3)$ take over all possible combinations of $\mu_j = 0, 1, (j = 1, 2, \ldots, 2N)$. Additionally, $B_1(\mu)$ satisfies the condition $\sum_{j=1}^{N}\mu_j = \sum_{j=1}^{N}\mu_{N+j}$, $B_2(\mu)$ and $B_3(\mu)$ both satisfy $\sum_{j=1}^{N}\mu_j = \sum_{j=1}^{N}\mu_{N+j} + 1$. Substituting (41) into (3) gives rise to some solutions of (2).

Now, we rewrite functions f_N, g_N, h_N as follows:

$$\begin{aligned} &f_N = \textstyle\sum_{\mu=0,1} B_1(\mu) e^{\sum_{j=1}^{2N}\mu_j\xi_j + \sum_{1\le j<l}^{2N}\mu_j\mu_l\theta_{jl}}, g_N = \sum_{\mu=0,1} B_2(\mu) e^{\sum_{j=1}^{2N}\mu_j\xi_j + \sum_{1\le j<l}^{2N}\mu_j\mu_l\theta_{jl}}, \\ &h_N = \textstyle\sum_{\mu=0,1} B_3(\mu) e^{\sum_{j=1}^{2N}\mu_j\eta_j + \sum_{1\le j<l}^{2N}\mu_j\mu_l\beta_{jl}}, \end{aligned} \tag{43}$$

with

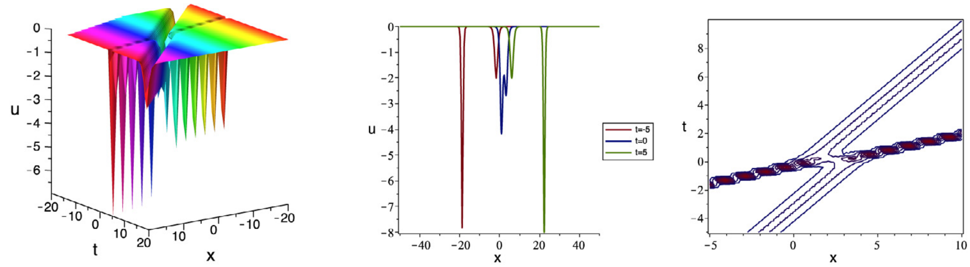
$$e^{\theta_{jl}} = A_{jl}, e^{\beta_{jl}} = B_{jl}, (1 \le j < l \le 2N). \tag{44}$$

Substituting (41) into (3) leads to some N-soliton solutions

$$u = -2(\ln f_N)_{xx}, v = \frac{g_N}{\sqrt{3}f_N}, w = \frac{h_N}{\sqrt{3}f_N}. \tag{45}$$

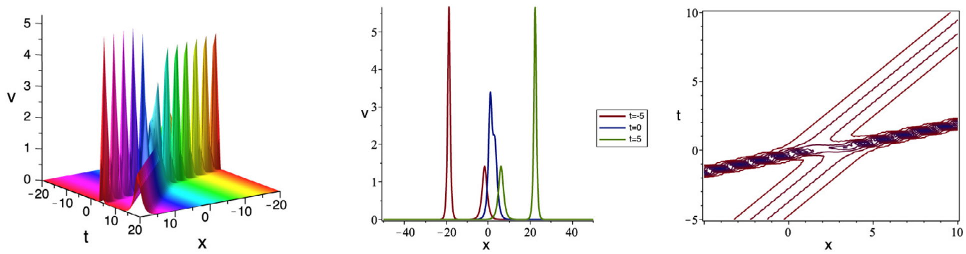
3. Dynamic property analysis

To better understand the dynamic properties of collisions between two solitons, the analysis sometimes needs to be performed using the technique in [40]. Therefore, some important physical quantities of solitons



(a) (b) (c)

Fig. 1. Evolution profile of the interaction between two solitons u expressed in (40) (a). Three-dimensional plot; (b) The interaction process plots; (c) Contour plot.



(a) (b) (c)

Fig. 2. Evolution profile of the interaction solution v in (40) (a). Three-dimensional plot; (b) The interaction process plots;2(c) Contour plot.

before and after collisions might be found from the following useful graphs. For simplicity, we graphically show the collisions between two solitons are elastic. given the following graphs with specific parameters $y = 1, b = 3, k_1 = 1, k_2 = 2, \kappa_1 = 2, \kappa_2 = 1, \xi_1^0 = \xi_2^0 = 0, \eta_1^0 = \eta_2^0 = 0$. It is obvious that the collisions between two solitons might be elastic. Since their respective amplitudes and velocities keep the same as those before collisions, only might some phase shifts be changed. Fig. 1 illustrates the collisions between two solitons. Meanwhile, it is also observed that the two solitons with u are dark solitons, which both travel from left to right. The soliton with larger amplitude propagates much faster than another one with smaller amplitude, the faster soliton travels cross the slower one. Moreover, during the collision process, the faster soliton is suppressed in its amplitude, but the slower one is enhanced temporarily. Fig. 1(b) shows that these two solitons almost merge into a single pulse at the moment of $t = 0$. Fig. 1(a)–(c) tell us that the two solitons travel forward and keep their original velocities, amplitudes and widths after the collisions.

Fig. 2(a)–(c) show us the collisions between two solitons with v and w being bright solitons, respectively. Moreover, the corresponding two solitons both travel from left to right. As aforementioned discuss, the soliton with larger amplitude propagates much faster than another one with smaller amplitude, the faster soliton moves passes the slower one. Moreover, during the collisions, the faster soliton is suppressed in its amplitude, but the slower one is enhanced temporarily. Figs. 2(b) and 3(b))show us that the two solitons almost merge into a single pulse at the moment of $t = 0$. Figs. 2 and 3 illustrate that their two solitons keep moving forward and keep their original velocities, amplitudes and widths after their collisions.

4. Conclusions

In this paper, we studied a generalized three-component Hirota–Satsuma coupled KdV equation and obtained the bilinear form (4) and N-soliton solutions (45). Additionally, the dynamic properties of two

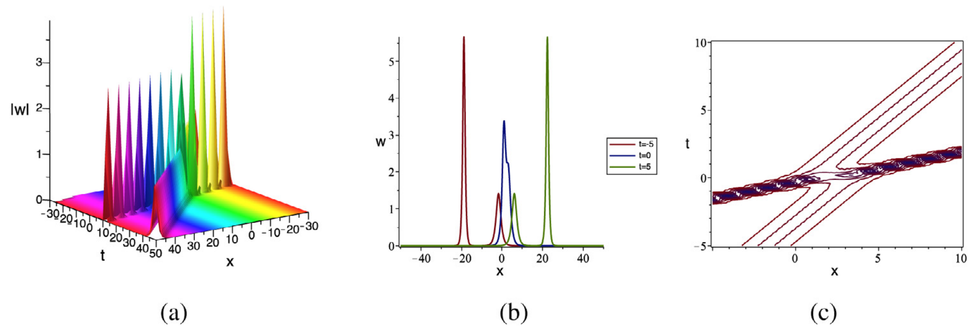


Fig. 3. Evolution profile of the interaction solution w in (40) (a) Three-dimensional plot; (b) The interaction process plots; (c) Contour plot.

solitons (40) is performed. According to the graph and analysis, we conclude that the collisions between two solitons are elastic. In the future, we will apply this method to study other PDEs and physical phenomena. As an interesting and meaningful and future work, we will study the lump-soliton solutions and the interaction solutions of the proposed model based on the results obtained in [41–44].

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