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Rational solutions and their interaction solutions of the (2+1)-dimensional modified dispersive water wave equation

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ABSTRACT

A bilinear form for the modified dispersive water wave (mDWW) equation is presented by the truncated Painlevé series, which does not lead to lump solutions. In order to get lump solutions, a pair of quartic–linear forms for the mDWW equation is constructed by selecting a suitable seed solution of the mDWW equation in the truncated Painlevé series. Rational solutions are then computed by searching for positive quadratic function solutions. A regular nonsingular rational solution can describe a lump in this model. By combining quadratic functions with exponential functions, some novel interaction solutions are founded, including interaction solutions between a lump and a one-kink soliton, a bi-lump and a one-stripe soliton, and a bi-lump and a two-stripe soliton. Concrete lumps and their interaction solutions are illustrated by 3d-plots and contour plots.

1. Introduction

Solitary wave solutions of nonlinear partial differential equations (PDEs) play an important role in a variety of science and engineering applications. Various efficient methods have been used to study soliton solutions of PDEs, such as the inverse scatter method [1], the Lie group method [2], the Darboux transformation [3], the multi-variable separation method [4] and the Hirota bilinear method [5] and so on [6,7]. Recently, lump solutions, another kind of rational solutions, have attracted much attention in nonlinear science fields. Lump solutions are found to be localized in all directions of the space. Lump solutions have been investigated in fluids [8,9], plasmas [10], and optic media [11]. The Darboux transformation [12–14] and the Hirota bilinear method [15–28] are the effective direct methods to construct lump solutions. Particular examples of lump solutions are given for many integrable equations, such as the Kadomtsev–Petviashvili (KP) equation [15,16], the KP–Boussinesq equation [17,18] and the generalized KP equation [19]. Besides, interaction solutions among solitons and other kinds of complicated waves are studied by the localization procedure related with the nonlocal symmetry and the consistent tanh expansion method [29–31]. Like these interaction solutions, mixed lump-kink [26–28] and lump-soliton [32–34] solutions to nonlinear evolution equations have been studied by combining a positive quadratic function with an exponential

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function. Lumps and their interaction solutions are rarely given in multi-component nonlinear systems. The examination of interaction solutions for multi-component equations is an especially intriguing topic. In this paper, we shall focus on lumps and their interaction solutions of the two-component modified dispersive water wave (mDWW) equation.

The (2+1)-dimensional mDWW equation reads [35]

$$u_{yt} + u_{xxy} - 2w_{xx} - 2(uu_y)_x = 0,\quad w_t - w_{xx} - 2(uw)_x = 0. \tag{1}$$

It can describe the nonlinear and dispersive long gravity waves traveling in two horizontal directions on shallow waters of uniform depth. Abundant localized excitations are obtained with the help of the Painlevé – Bäcklund transformation and the multi-linear variable separation approach [36]. The multiple soliton solutions and fusion interaction phenomena are derived by means of the Bäcklund transformation and the Hirota bilinear method [37].

This paper is organized as follows. In Section 2, we try to get lump solutions of the mDWW equation by the standard Hirota bilinear method. It cannot get lump solutions from a bilinear form of the mDWW equation that we will present. In Section 3, we introduce a pair of quartic–linear forms of the mDWW equation to get lump solutions. Some novel lump solutions are derived by solving a pair of quartic–linear forms of the mDWW equation. In Section 4, by adding an exponential and two exponential terms to the quadratic function, interaction solutions between a bi-lump and one line-soliton solutions, and a bi-lump and two line-soliton solutions are obtained, respectively. Section 5 is a simple summary and discussion.

2. Study on lump solutions based on a bilinear form

Based on Painlevé analysis, the Painlevé – Bäcklund transformation of the mDWW equation reads

$$u = \frac{u_0}{\phi} + u_1, \qquad w = \frac{w_0}{\phi^2} + \frac{w_1}{\phi} + w_2, \tag{2}$$

where ϕ is an arbitrary function of variables x, y and t, and the pair of functions u_1 and w_2 is also a solution of the mDWW equation. By substituting (2) into (1) and balancing the coefficients ϕ^{-4} and ϕ^{-4}, we get

$$u_0 = \phi_x, \qquad w_0 = -\phi_x\phi_y, \tag{3}$$

Gathering the coefficients ϕ^{-3} and ϕ^{-3}, we obtain

$$u_1 = \frac{\phi_t + \phi_{xx}}{2\phi_x}, \qquad w_1 = -\phi_{xy}. \tag{4}$$

By substituting (3) and (4), the transformation (2) and the seed solution $u_1 = w_2 = 0$, the following equation is yielded

$$2\partial_y\left(\frac{\phi\phi_{xt} - \phi_t\phi_x - \phi\phi_{xxx} + \phi_x\phi_{xx}}{\phi^2}\right) = 0. \tag{5}$$

It can be easily seen that ϕ satisfies the bilinear form

$$\phi\phi_{xt} - \phi_t\phi_x - \phi\phi_{xxx} + \phi_x\phi_{xx} = 0. \tag{6}$$

To get lump solutions of the mDWW equation, we take a quadratic function for ϕ

$$\phi = g^2 + h^2 + a_9,\quad g = a_1x + a_2y + a_3t + a_4,\quad h = a_5x + a_6y + a_7t + a_8. \tag{7}$$

By substituting (7) into (6) and balancing the different powers of x, y and t, we get the solutions for the parameters

$$a_1 = 0, \quad a_5 = 0, \qquad or \qquad a_2 = 0, \quad a_6 = 0. \tag{8}$$

Therefore, we cannot obtain any non-trivial lump solution by using the above standard bilinear form (6).

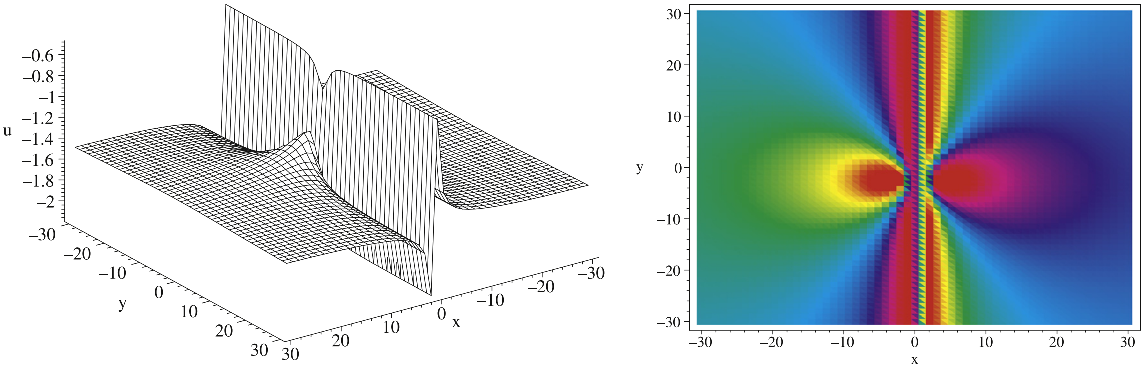
3. Rational solutions form a pair of quartic–linear forms

In order to get non-trivial quadratic function solutions, we try to select a different seed solution for u_1 and w_1. We assume

$$u = \frac{\phi_x}{\phi} + \frac{\phi_t + \phi_{xx}}{2\phi_x}, \qquad w = \frac{\phi_x\phi_y}{\phi^2} + \frac{\phi_{xy}}{\phi}, \tag{9}$$

with the seed solution

$$u_1 = \frac{\phi_t + \phi_{xx}}{2\phi_x}, \qquad w_2 = 0. \tag{10}$$



(a) (b)

Fig. 1. Profile of the solution u in (16). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

Substituting (9) into (1), we get the following two quartic–linear equations

$$\begin{aligned}&\phi\phi_t\phi_{xx}\phi_{xy} + \phi_t\phi_x^2\phi_{xy} - \phi_x^3\phi_{yt} + \phi\phi_{xyt}\phi_x^2 - \phi\phi_t\phi_x\phi_{xxy} - \phi\phi_x^2\phi_{xxy}\\&+ \phi\phi_x\phi_{xx}\phi_{xxy} + \phi_x^3\phi_{xxy} + \phi_x^3\phi_{xxy} - \phi\phi_{xx}^2\phi_{xy} - \phi_x^2\phi_{xx}\phi_{xy} + \phi\phi_x\phi_{xy}\phi_{xxx} = 0,\end{aligned} \tag{11}$$

$$\begin{aligned}&\phi_x\phi_t^2\phi_{xxy} + \phi_{tty}\phi_x^3 - \phi_x^2\phi_{tt}\phi_{xy} - 2\phi_x^2\phi_{xt}\phi_{yt} + 2\phi_x\phi_{xx}\phi_t\phi_{yt}\\&+ 4\phi_x\phi_{xy}\phi_t\phi_{xt} - 2\phi_t\phi_{xyt}\phi_x^2 - 3\phi_t^2\phi_{xx}\phi_{xy} + 2\phi_x^2\phi_{xxx}\phi_{xxy} - 3\phi_x\phi_{xx}^2\phi_{xxy}\\&- 4\phi_x\phi_{xx}\phi_{xy}\phi_{xxx} + 2\phi_x^2\phi_{xx}\phi_{xxxy} + \phi_x^2\phi_{xy}\phi_{xxxx} - \phi_x^3\phi_{xxxxy} + 3\phi_{xx}^3\phi_{xy} = 0.\end{aligned} \tag{12}$$

We want to find a solution ϕ which satisfies (11) and (12) simultaneously. It seems more complicated to solve (11) and (12) than one equation (6). Actually, we can get some kinds of lump solutions by solving (11) and (12). To obtain lump solutions, a quadratic function solution to (11) and (12) is similarly defined by

$$\begin{aligned}&\phi = g^2 + h^2 + a_9,\\&g = a_1x + a_2y + a_3t + a_4,\\&h = a_5x + a_6y + a_7t + a_8,\end{aligned} \tag{13}$$

where $a_i\,(1 \le i \le 9)$ are constant parameters to be determined. Substituting the expression (13) into Eqs. (11) and (12) and vanishing the coefficients of different powers of x, y and t, we can get the relationship among the parameters which satisfy (11) and (12) simultaneously. The relationship for the parameters reads

$$a_1 = -\frac{a_5a_6}{a_2}, \qquad a_3 = -\frac{a_6a_7}{a_2}, \tag{14}$$

which needs to satisfy $a_2 \neq 0$, $a_9 > 0$ to guarantee the well-definedness, the positiveness and the localization of the resulting solution. The parameters in the set (14) yield the positive quadratic function solution (13) as

$$\phi = (-\frac{a_5a_6}{a_2}x + a_2y - \frac{a_6a_7}{a_2}t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9. \tag{15}$$

The rational solution of (11) and (12) can be generated through the transformation: (9)

$$\begin{aligned}&u = \frac{2a_5h - \frac{2a_5a_6}{a_2}g}{\phi} + \frac{\frac{a_5^2a_6^2}{a_2^2} + a_5^2}{2a_5h - \frac{2a_6}{a_2}g} + \frac{-\frac{a_6a_7}{a_2}g + a_7h}{2a_5h - \frac{2a_5a_6}{a_2}g},\\&w = \frac{4(-\frac{a_5a_6}{a_2}g + a_5h)(a_2g + a_6h)}{\phi^2},\end{aligned} \tag{16}$$

where $g = -\frac{a_5a_6}{a_2}x + a_2y - \frac{a_6a_7}{a_2}t + a_4$ and $h = a_5x + a_6y + a_7t + a_8$. To describe this kind of rational solutions, we select the parameters $a_2 = 1$, $a_4 = 3$, $a_5 = 1$, $a_6 = 1$, $a_7 = -3$, $a_8 = 2$, $a_9 = 6$. The solution for u and w is shown in Figs. 1 and 2. It exits a singularity for the solution u. The kind of the solution u is different from usual lump solution. A special rational solution of u and a bi-lump of w are given in Figs. 1 and 2, respectively. The characteristics of these rational solutions are different from ones of lumps by calculations of the bilinear form [15–19].

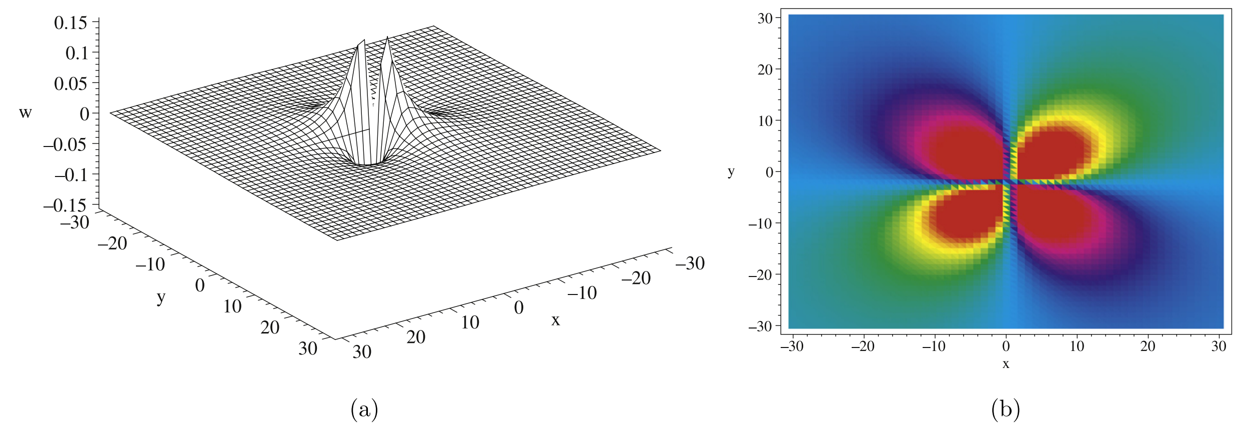


Fig. 2. Profile of the solution w in (16). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

4. Interaction solutions between lumps and soliton solutions

4.1. Between lumps and one line-soliton solutions

Interaction solutions between lumps and other type solutions can be obtained by combining the quadratic function with other type functions. In order to find interaction solutions between lump solutions and one line-soliton, we assume an interaction solution as a sum of a quadratic function and an exponential function

$$\begin{aligned} &\phi = g^2 + h^2 + a_9 + k_1 \exp(k_2 x + k_3 y + k_4 t + k_5), \\ &g = a_1 x + a_2 y + a_3 t + a_4, \\ &h = a_5 x + a_6 y + a_7 t + a_8, \end{aligned} \tag{17}$$

with $k_i\,(i = 1, 2, \ldots, 5)$ being five undetermined real parameters. By substituting (17) into Eqs. (11) and (12) and vanishing the different powers of x, y and t, we obtain the following four sets of constraining relations for the parameters.

Case I.

$$a_1 = \frac{a_3 a_5}{a_7}, \qquad a_8 = \frac{3a_3^2 a_5 + 3a_5 a_7^2 - 2a_3 a_4 a_7 k_2}{2k_2 a_7^2}, \qquad k_3 = 0, \qquad k_4 = \frac{k_2(a_7 \pm k_2 a_5)}{a_5}, \tag{18}$$

which should satisfy the constraint conditions

$$k_2 a_5 a_7 \neq 0, \qquad k_1 > 0, \qquad a_9 > 0, \tag{19}$$

to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. Substituting (17) into (9) and combining the parameters relations (18), we get a class of interaction solutions of the mDWW equation (1):

$$\begin{aligned} u^{\mathrm{I}} &= \frac{\frac{2a_3a_5}{a_7} g + 2a_5 h + k_1 k_2 \exp(f)}{\phi} + \frac{a_3 g + a_7 h + \frac{k_1 k_2}{2}\left(\frac{a_7}{a_5} \pm k_2\right)\exp(f) + \frac{a_3^2 a_5^2}{a_7^2} + a_5^2}{\frac{2a_3a_5}{a_7} g + 2a_5 h + k_1 k_2 \exp(f)}, \\ w^{\mathrm{I}} &= \frac{2\left[\frac{2a_3a_5}{a_7} g + 2a_5 h + k_1 k_2 \exp(f)\right](a_2 g + a_6 h)}{\phi^2} + \frac{\frac{2a_2a_3a_5}{a_7} + 2a_5 a_6}{\phi}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \phi &= g^2 + h^2 + a_9 + k_1 \exp(f), \\ g &= \frac{a_3 a_5}{a_7} x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + \frac{3a_3^2 a_5 + 3a_5 a_7^2 - 2a_3 a_4 a_7 k_2}{2k_2 a_7^2}, \\ f &= k_2 x + \frac{k_2(a_7 \pm k_2 a_5)}{a_5} t + k_5. \end{aligned} \tag{21}$$

Case II.

$$a_1 = -\frac{a_5 a_6}{a_2}, \qquad a_3 = -\frac{a_2 a_8}{a_2}, \qquad a_4 = \pm\frac{a_2 a_8}{a_6}, \qquad k_2 = k_4 = 0, \tag{22}$$

which should satisfy the constraint conditions

$$a_2 a_6 \neq 0, \qquad k_1 > 0, \qquad a_9 > 0, \tag{23}$$

to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. By using the parameters relations (22) and the transformation (9), the second class of interaction solutions between rational solutions and one line-soliton to the mDWW equation (1) reads

$$u^{\mathrm{II}} = \frac{-\frac{2a_5 a_6}{a_2} g + 2a_5 h}{\phi} + \frac{-\frac{a_2 a_8}{a_2} g + a_7 h + \frac{a_5^2 a_6^2}{a_2^2} + a_5^2}{-\frac{2a_5 a_6}{a_2} g + 2a_5 h},$$
$$w^{\mathrm{II}} = \frac{2(-\frac{a_5 a_6}{a_2} g + a_5 h)[2a_2 g + 2a_6 h + k_1 k_3 \exp(f)]}{\phi^2}, \tag{24}$$

where

$$\phi = g^2 + h^2 + a_9 + k_1 \exp(f),$$
$$g = -\frac{a_5 a_6}{a_2} x + a_2 y - \frac{a_2 a_8}{a_2} t \pm \frac{a_2 a_8}{a_6},$$
$$h = a_5 x + a_6 y + a_7 t + \frac{3a_3^2 a_5 + 3a_5 a_7^2 - 2a_3 a_4 a_7 k_2}{2k_2 a_7^2},$$
$$f = k_3 y + k_5, \tag{25}$$

Case III.

$$a_1 = -\frac{a_5 a_6}{a_2}, \qquad a_3 = -\frac{a_2 a_8}{a_2}, \qquad a_4 = \pm\frac{a_2 a_8}{a_6}, \qquad k_3 = 0, \tag{26}$$

which should satisfy the constraint conditions

$$a_2 a_6 \neq 0, \qquad k_1 > 0, \qquad a_9 > 0, \tag{27}$$

to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. By using Eqs. (17) and (26) and the transformation (9), the third class of interaction solutions is

$$u^{\mathrm{III}} = \frac{-\frac{2a_5 a_6}{a_2} g + 2a_5 h + k_1 k_2 \exp(f)}{\phi} + \frac{-\frac{a_2 a_8}{a_2} g + a_7 h + \frac{k_1}{2}(k_4 + k_2^2)\exp(f) + \frac{a_5^2 a_6^2}{a_2^2} + a_5^2}{-\frac{2a_5 a_6}{a_2} g + 2a_5 h + k_1 k_2 \exp(f)},$$
$$w^{\mathrm{III}} = \frac{2[-\frac{2a_5 a_6}{a_2} g + 2a_5 h + k_1 k_2 \exp(f)](a_2 g + a_6 h)}{\phi^2}, \tag{28}$$

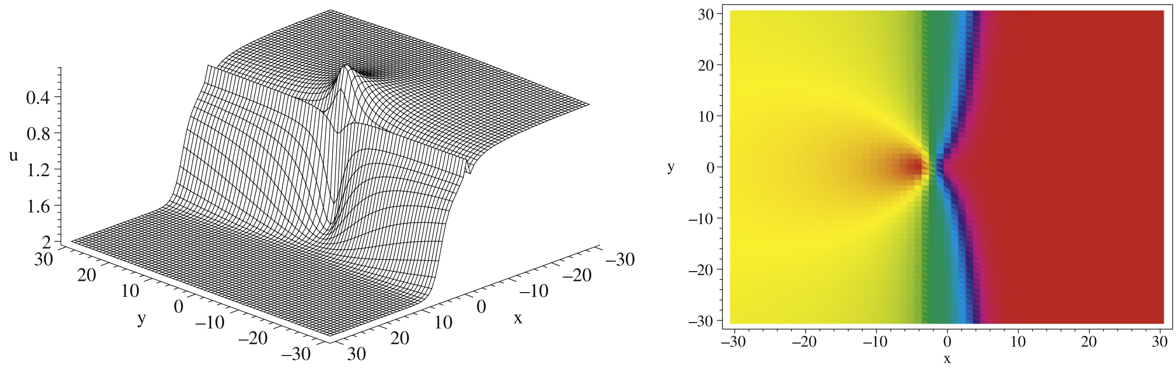
where

$$\phi = g^2 + h^2 + a_9 + k_1 \exp(f),$$
$$g = -\frac{a_5 a_6}{a_2} x + a_2 y - \frac{a_2 a_8}{a_2} t \pm \frac{a_2 a_8}{a_6},$$
$$h = a_5 x + a_6 y + a_7 t + a_8,$$
$$f = k_2 x + k_4 t + k_5. \tag{29}$$

The parameters are selected to be $a_2 = 1, a_5 = 1, a_6 = 1, a_7 = 1, a_8 = 1, a_9 = 3, k_1 = 3, k_2 = 1, k_4 = 3, k_5 = 2$ in case III. The symbol of " $\pm$ " in (26) takes as " $-$ " to plot Figs. 3 and 4. The interaction solution between a lump and a one-kink soliton of u is presented in Fig. 3. The interaction solution between a bi-lump and a stripe soliton of w is plotted in Fig. 4.

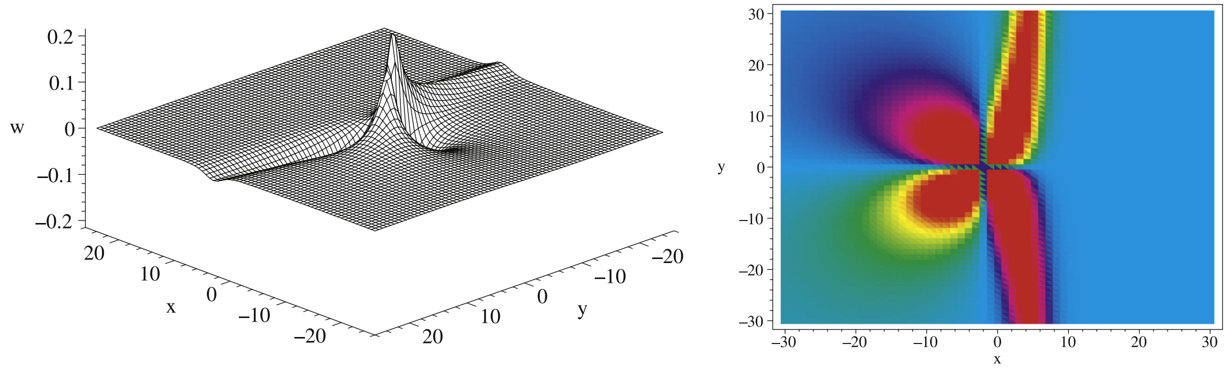
Case IV.

$$a_1 = -\frac{a_5 a_6}{a_2}, \quad a_4 = \pm\frac{a_2 a_8}{a_6}, \quad a_7 = \frac{k_4 a_5}{k_2} + \frac{a_3 a_6}{a_2} + \frac{a_5 a_6^2 k_4}{a_2^2 k_2}, \quad k_3 = 0, \quad k_4 = -\frac{a_2 k_2 k_3}{a_5 a_6}, \tag{30}$$



(a) (b)

Fig. 3. Profile of the solution (28). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.



(a) (b)

Fig. 4. Profile of the solution (28). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

which should satisfy the constraint conditions

$$k_2a_2a_5a_6 \neq 0, \qquad k_1 > 0, \qquad a_9 > 0, \tag{31}$$

to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. By using Eqs. (17) and (30) and the transformation (9), the fourth class of interaction solutions is

$$u^{\mathrm{IV}} = \frac{-\frac{2a_5a_6}{a_2}g + 2a_5h + k_1k_2\exp(f)}{\phi} + \frac{M + \frac{a_5^2a_6^2}{a_2^2} + a_5^2}{-\frac{2a_5a_6}{a_2}g + 2a_5h + k_1k_2\exp(f)},$$

$$w^{\mathrm{IV}} = \frac{(-\frac{2a_5a_6}{a_2}g + 2a_5h + k_1k_2\exp(f))(2a_2g + 2a_6h)}{\phi^2}, \tag{32}$$

where

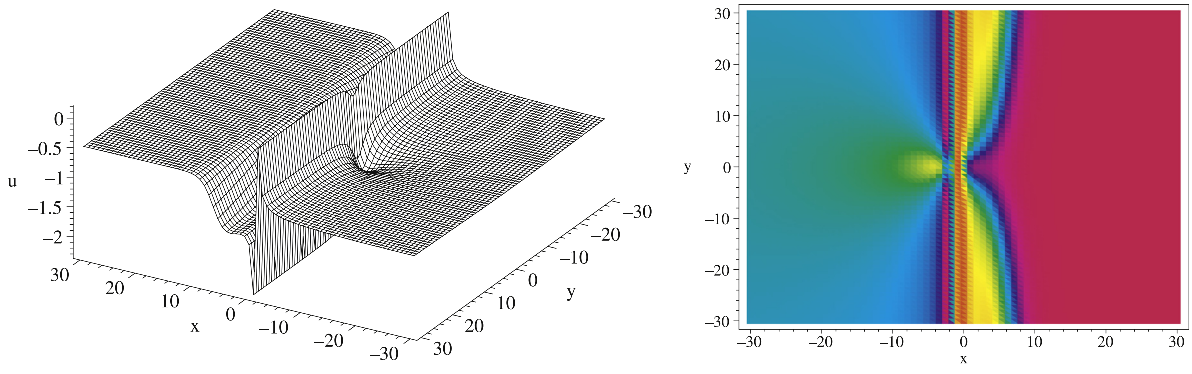
$$\phi = g^2 + h^2 + a_9 + k_1\exp(f), \tag{33}$$

$$g = -\frac{a_5a_6}{a_2}x + a_2y - \frac{a_2a_8}{a_2}t \pm \frac{a_2a_8}{a_6},$$

$$h = a_5x + a_6y + \left(\frac{k_4a_5}{k_2} + \frac{a_3a_6}{a_2} + \frac{a_5a_6^2k_4}{a_2^2k_2}\right)t + a_8,$$

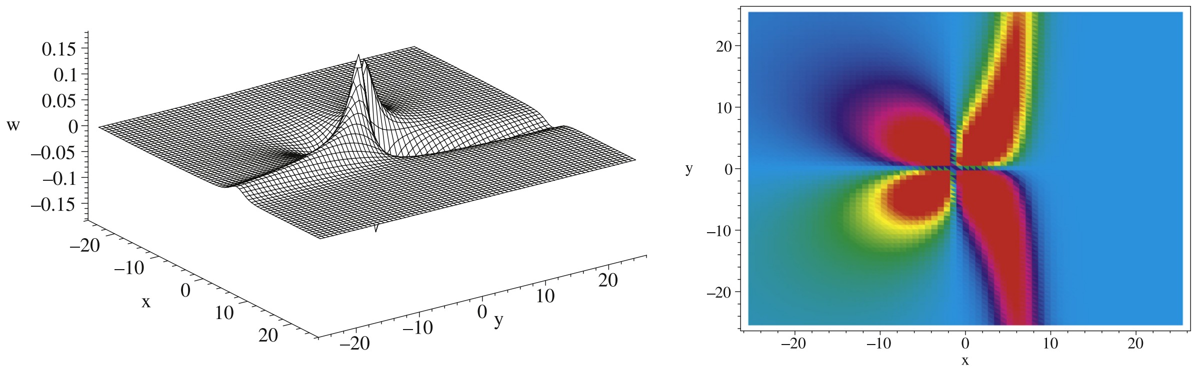
$$f = k_2x - \frac{a_2k_2k_3}{a_5a_6}t + k_5,$$

$$M = -\frac{a_2a_8}{a_2}g + \left(\frac{k_4a_5}{k_2} + \frac{a_3a_6}{a_2} + \frac{a_5a_6^2k_4}{a_2^2k_2}\right)h + \frac{k_1k_2}{2}\left(k_2 - \frac{a_2k_3}{a_5a_6}\right)\exp(f).$$



(a) (b)

Fig. 5. Profile of the solution (32). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.



(a) (b)

Fig. 6. Profile of the solution (32). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

The parameters are selected to be $a_2 = 1, a_3 = 2, a_5 = 1, a_6 = 1, a_7 = 1, a_8 = 1, a_9 = 3, k_1 = 3, k_2 = 1, k_4 = 3, k_5 = 2$ in case IV. The symbol of " $\pm$ " in (30) takes as " $-$ " to plot Figs. 5 and 6. The special interaction solution between a rational and a one-soliton of u is plotted in Fig. 5. The bi-lumps soliton catch up with a one-stripe soliton is given in Fig. 6.

4.2. Between lumps and a pair of line soliton solutions

For interaction solutions between lumps and a two-stripe solitary, we use a quadratic function with two exponential functions. Based on the quartic–linear form, the interaction solution of Eqs. (11) and (12) is defined by

$$\begin{aligned} \phi &= g^2 + h^2 + a_9 + k_1 \exp(f) + k_6 \exp(-f), \\ g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \\ f &= k_2 x + k_3 y + k_4 t + k_5. \end{aligned} \tag{34}$$

By substituting (34) into (11) and (12) and collecting the coefficients of x, y, t, the following two sets of constraining relations for the parameters are yielded by solving the algebraic equations.

Case I.

$$a_1 = -\frac{a_5 a_6}{a_2}, \qquad a_3 = -\frac{a_6 a_7}{a_2}, \qquad k_2 = k_4 = 0, \tag{35}$$

which should satisfy the constraint conditions

$$a_2 \neq 0, \qquad k_1 > 0, \qquad a_9 > 0, \qquad k_6 > 0, \tag{36}$$

to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. By using Eqs. (34) and (35) and the transformation (9), the first class of interaction solutions is

$$
\begin{aligned}
u^{\mathrm{I}} &= \frac{-\frac{2a_5a_6}{a_2}g + 2a_5h}{\phi} + \frac{-\frac{a_6a_7}{a_2}g + a_7h + \frac{a_5^2a_6^2}{a_2^2} + a_5^2}{-\frac{a_5a_6}{a_2}g + a_5h}, \\
w^{\mathrm{I}} &= \frac{4(-\frac{a_5a_6}{a_2}g + a_5h)(a_2g + a_6h)}{\phi^2},
\end{aligned} \tag{37}
$$

where

$$
\begin{aligned}
\phi &= g^2 + h^2 + a_9 + k_1 \exp(f) + k_6 \exp(-f), \\
g &= -\frac{a_5a_6}{a_2}x + a_2y - \frac{a_6a_7}{a_2}t + a_4, \\
h &= a_5x + a_6y + a_7t + a_8, \\
f &= k_3y + k_5.
\end{aligned} \tag{38}
$$

Case II.

$$
a_1 = -\frac{a_5a_6}{a_2}, \qquad a_3 = -\frac{a_6a_7}{a_2}, \qquad k_3 = 0, \tag{39}
$$

which satisfy the same constraint conditions (36) to guarantee that the corresponding solution ϕ is positive, analytical and localization in all directions in the (x, y)-plane. By using Eqs. (34) and (39) and the transformation (9), the second class of interaction solutions reads

$$
\begin{aligned}
u^{\mathrm{II}} &= \frac{N}{\phi} + \frac{M + \frac{a_6^2a_7^2}{a_2^2} + a_7^2}{N}, \\
w^{\mathrm{II}} &= \frac{2N(a_2g + a_6h)}{\phi^2},
\end{aligned} \tag{40}
$$

where

$$
\begin{aligned}
\phi &= g^2 + h^2 + a_9 + k_1 \exp(f) + k_6 \exp(-f), \\
g &= -\frac{a_5a_6}{a_2}x + a_2y - \frac{a_6a_7}{a_2}t + a_4, \\
h &= a_5x + a_6y + a_7t + a_8, \\
f &= k_2x + k_4t + k_5, \\
N &= -\frac{2a_5a_6}{a_2}g + 2a_5h + k_1k_2 \exp(f) - k_2k_6 \exp(-f), \\
M &= -\frac{a_6a_7}{a_2}g + a_7h + \frac{k_1}{2}(k_4 + k_2^2) \exp(f) - \frac{k_6}{2}(k_4 + k_2^2) \exp(-f).
\end{aligned} \tag{41}
$$

The parameters are selected as $a_2 = 2, a_4 = 3, a_5 = 1, a_6 = -2, a_7 = 2, a_8 = -2, a_9 = 6, k_1 = 2, k_2 = -1, k_4 = -3, k_5 = -2, k_6 = 2$. The special interaction solution between a rational and a soliton solution of u is shown in Fig. 7. The bi-lumps catch up with a two-stripe soliton is given in Fig. 8.

5. Conclusion

In summary, some novel interaction solutions between lumps and stripe solitons of the mDWW equation are considered in this paper. First, we construct a bilinear form of the mDWW equation by the truncated Painlevé series. The positive quadratic function is used for finding lump solutions. However, we fail to obtain lump solutions due to the trivial parameters in (7). Then, we construct a pair of quartic–linear forms of the mDWW equation by selecting a different seed solution in (10). Some novel interaction solutions between lumps and soliton solutions are studied by solving the pair of quartic–linear forms of the mDWW equation. A bi-lump, a lump in a one-kink soliton background, the interaction solution between a lump and a one-stripe soliton, the interaction solutions between a bi-lump and a two-stripe soliton are studied in detail. In this paper, we select the seed solution (10) to get the quartic–linear forms of the mDWW equation. Some novel lumps and their interaction solutions are obtained with the quartic–linear forms of the mDWW equation. We can also select the seed solution

$$
u_1 = 0, \qquad w_2 = \frac{\phi_{yt}}{\phi_x} - \frac{\phi_{xxy}}{\phi_x} - \frac{\phi_t\phi_{xy}}{\phi_x^2} - \frac{\phi_{xx}\phi_{xy}}{\phi_x^2}, \tag{42}
$$

to yield different multi-linear forms of the mDWW equation. We can study lumps and their interaction solutions by solving different multi-linear forms. In addition, we find a new idea to get lump solutions by solving the quartic–linear forms of the mDWW equation. We can explore lumps and their interaction solutions by multi-linear forms of the nonlinear differential equations, which are not usually given by their bilinear forms.

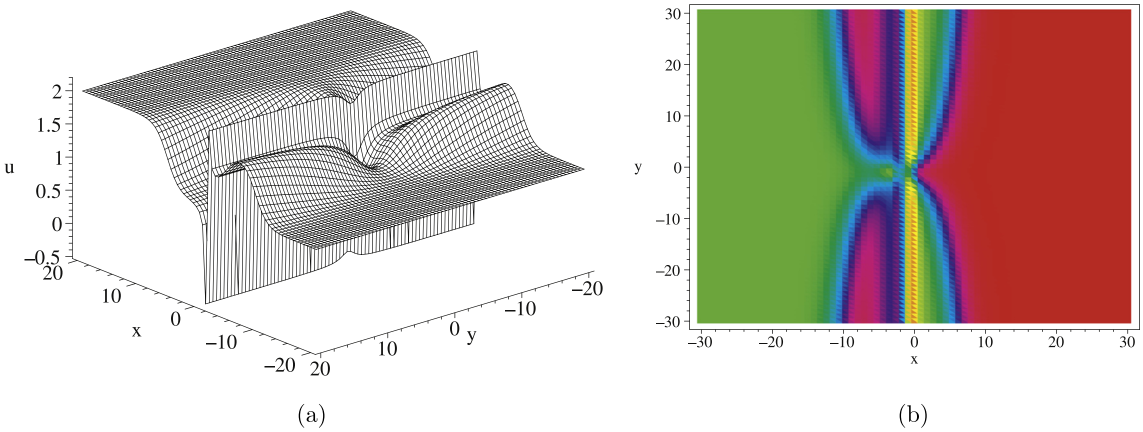


Fig. 7. Profile of the solution (40). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

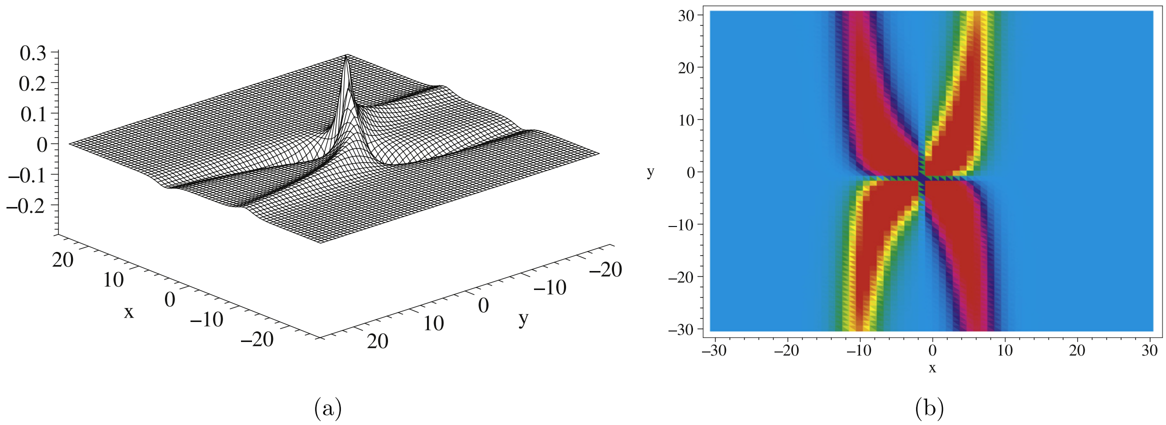


Fig. 8. Profile of the solution (40). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

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