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Rational solutions of a (2 + 1)-dimensional Sharma-Tasso-Olver equation

Bo Ren[*,a,b], Wen-Xiu Ma[b,c,d,e]

[a] Institute of Nonlinear Science, Shaoxing University, Shaoxing 312000, China
[b] Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620-5700, USA
[c] Department of Mathematics, Zhejiang Normal University, Jinhua 321004, China
[d] College of Mathematics and Physics, Shanghai University of Electric Power, Shanghai 200090, China
[e] College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China

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ABSTRACT

We extend the (1 + 1)-dimensioanl Sharma-Tasso-Olver (STO) equation to a (2 + 1)-dimensional one by adding one additional term u_{yy}. A tri-linear form of the (2 + 1)-dimensional STO equation is obtained by the Painlevé analysis. A family of rational solutions for the (2 + 1)-dimensional STO equation is constructed by using the resulting tri-linear form. Associated 3-dimensional plot and density plot with particular choices of the involved parameters are given to show the charateristics of the rational solutions.

1. Introduction

Exact solutions of nonlinear partial differetial equations are very important to predict and understand possible behaviors of physics phenomena [1–3]. As a kind of special localized wave solutions, special lump solutions which are rationally decaying in all directions in the space, were obtained for the Kadomtsev-Petviashvili by Manakov et. al. [4]. Subsequently, a class of rational solutions was investigated with many methods [5–10]. These wave solutions have potential applications in the open ocean and coastal areas [11]. Therefore, direct methods for construction of these kinds of solutions have attracted particular attention in mathematical physics [8–26]. Several traditional methods are developed to construct these kinds of waves, such as the inverse scattering transformation [12], the Bäcklund transformation [13], the Darboux transformation [14] and the Hirota bilinear method [15–31].

In this paper, we shall focus on rational solutions of a new (2 + 1)-dimensional STO equation by solving a tri-linear STO form that we are going to present. The paper is organized as follows. In Section 2, a (2 + 1)-dimensional STO equation is constructed by adding one additional term u_{yy}. A dipole-mode soliton solution and lump-like solutions of the (2 + 1)-dimensional STO equation can be obtained by solving the tri-linear form. The last section is devoted to summary and discussions.

2. Rational solutions of a (2 + 1)-dimensional STO equation

The (1 + 1)-dimensional STO equation reads

* Corresponding author at: Institute of Nonlinear Science, Shaoxing University, Shaoxing 312000, China.
E-mail address: renbo0012@163.com (B. Ren).

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$$u_t + \alpha u_x^3 + \frac{3}{2}\alpha u_{xx}^2 + \alpha u_{xxx} = 0, \tag{1}$$

where α is an arbitary constant. The $(1+1)$-dimensional STO equation can describe the propagation of nonlinear dispersive waves in inhomogeneous media. This equation has gained much attentions due to its appearance in scientific applications. Many integrability properties for the STO Eq. (1) have been obtained [32–35]. The fission and fusion of the solitary wave were found by means of the Hirota direct method and the Bäcklund transformation [32]. The symmetry reduction procedure was used to obtain infinitely many symmetries and exact solutions [33,34].

The $(2+1)$-dimensional KP equation can be taken a generalized two-spatial dimensional version of the $(1+1)$-dimensional Korteweg-de Vries (KdV) equation. According to the form of the KP equation, we take a $(2+1)$-dimensional STO equation as the following form as a generalization of the $(1+1)$-dimensional STO equation

$$\left(u_t + \alpha u_x^3 + \frac{3}{2}\alpha u_{xx}^2 + \alpha u_{xxx}\right)_x + \beta u_{yy} = 0, \tag{2}$$

where β is an arbitary constant. By the truncated Painlevé analysis [36], the solution for the STO Eq. (2) reads

$$u = \frac{u_0}{\phi} + u_1, \tag{3}$$

where u and u_1 are the solution of the STO Eq. (2). By substituting of the expansion (3) into (2) and vanishing the coefficient of ϕ^{-5}, we get

$$u_0 = \phi_x. \tag{4}$$

By substituting (3) and (4) into (2), a tri-linear form for the $(2+1)$-dimensional STO equation is obtained

$$\phi^2\phi_{xxt} + 2\phi_x^2\phi_t - 2\phi\phi_x\phi_{xt} - \phi\phi_{xx}\phi_t + \alpha\phi^2\phi_{xxxxx} - 2\alpha\phi\phi_x\phi_{xxxx} - \alpha\phi\phi_{xx}\phi_{xxx} + 2\alpha\phi_x^2\phi_{xxx} + \beta\phi^2\phi_{xyy} - 2\beta\phi\phi_y\phi_{xy} + 2\beta\phi_x\phi_y^2 - \beta\phi\phi_x\phi_{yy} = 0. \tag{5}$$

We want to use a tri-linear Eq. (5) to calculate rational solutions. The quadratic function solutions to a $(2+1)$-dimensional tri-linear Eq. (5) assume

$$\phi = \xi_1^2 + \xi_2^2 + a_9, \xi_1 = a_1x + a_2y + a_3t + a_4, \xi_2 = a_5x + a_6y + a_7t + a_8, \tag{6}$$

where a_i, $1 \le i \le 9$ are constant parameters to be determined. Substituting the expression (6) into (5) and vanishing the coefficients of different powers of x, y and t, we can get the relationship among parameters for the Eq. (5). Two sets for the parameters a_i are expressed as the following forms.

Case I.

$$a_2 = \frac{a_1a_6}{a_5}, \qquad a_3 = -\beta\frac{a_1a_6^2}{a_5^2}, \qquad a_7 = -\frac{\beta a_6^2}{a_5}. \tag{7}$$

The corresponding quadratic function solution of (5) yields

$$\phi = \left(a_1x + \frac{a_1a_6}{a_5}y - \beta\frac{a_1a_6^2}{a_5^2}t + a_4\right)^2 + \left(a_5x + a_6y - \frac{\beta a_6^2}{a_5}t + a_8\right)^2 + a_9. \tag{8}$$

It is easily seen that the coefficients of x and y are direct ratio for the square terms in (8). It can not be given lump solution in case I. By substituting (4), (6) and (7) into (3), the solution of (2) writes

$$u = \frac{2a_1\xi_1 + 2a_5\xi_2}{\phi}, \tag{9}$$

where $\xi_1 = a_1x + \frac{a_1a_6}{a_5}y - \beta\frac{a_1a_6^2}{a_5^2}t + a_4$, $\xi_2 = a_5x + a_6y - \frac{\beta a_6^2}{a_5}t + a_8$ and ϕ satifies (8). By selecting the parameters $\beta = 1$, $a_1 = -3$, $a_4 = 2$, $a_5 = 2$, $a_6 = 2$, $a_8 = 1$, $a_9 = 3$, we get the solution

$$u = \frac{2(13x + 13y - 13t - 4)}{(13t^2 - 26xt - 26yt + 13x^2 + 26xy + 13y^2 + 8t - 8x - 8y + 8)^2}. \tag{10}$$

The 3-dimensional plot and density plot for a dipole-mode soliton (10) are depicted in Fig. 1. The dipole-mode soliton solution can be viewed a slit laser beam propagates in Kerr-type nonlinear, nonlocal media with exponential response function [37].

Case II.

$$a_3 = -\beta\frac{a_1a_2^2 - a_1a_6^2 + 2a_2a_5a_6}{a_1^2 + a_5^2}, \qquad a_7 = -\beta\frac{a_5a_6^2 - a_5a_2^2 + 2a_1a_2a_6}{a_1^2 + a_5^2}, \qquad a_9 = 0, \tag{11}$$

which need to satisfy the condition $a_1a_5 \neq 0$ to guarantee the well-posedness. It will yield the lump-like solution due to $a_9 = 0$. The corresponding quadratic function solution of (5) yields

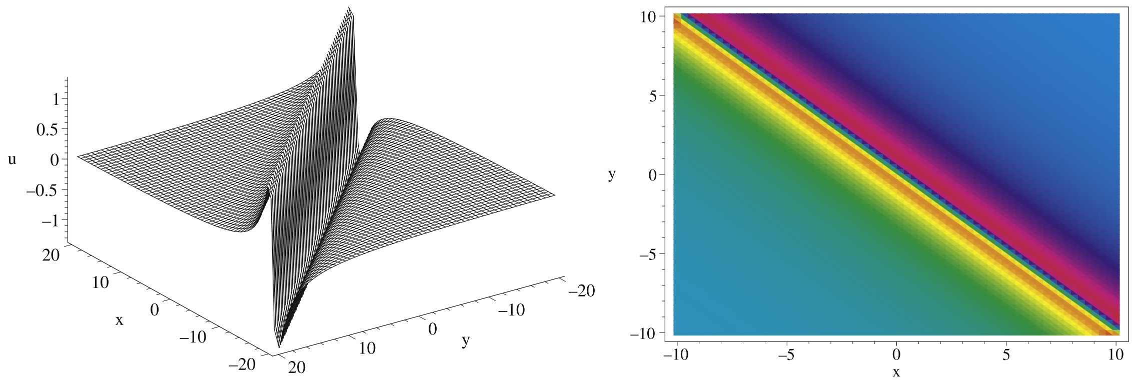

Fig. 1. Profile of a dipole-mode soliton (10). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

$$\phi = \left(a_1x + a_2y - \beta\frac{a_1a_2^2 - a_1a_6^2 + 2a_2a_5a_6}{a_1^2 + a_5^2}t + a_4\right)^2 + \left(a_5x + a_6y - \beta\frac{a_5a_6^2 - a_5a_2^2 + 2a_1a_2a_6}{a_1^2 + a_5^2}t + a_8\right)^2. \tag{12}$$

The solution for (2) gives

$$u = \frac{2a_1\xi_1 + 2a_5\xi_2}{\phi}, \tag{13}$$

where $\xi_1 = a_1x + a_2y - \beta\frac{a_1a_2^2 - a_1a_6^2 + 2a_2a_5a_6}{a_1^2 + a_5^2}t + a_4$, $\xi_2 = a_5x + a_6y - \beta\frac{a_5a_6^2 - a_5a_2^2 + 2a_1a_2a_6}{a_1^2 + a_5^2}t + a_8$ and ϕ satisfies (12). The critical point of the lump-like wave (13) is calculated by taking the partial derivatives ϕ_x and ϕ_y be zero. The moving path of the lump-like wave is read [38]

$$x = x(t) = \frac{\beta(a_2^2 + a_6^2)}{a_1^2 + a_5^2}t + \frac{a_2a_8 - a_4a_6}{a_1a_6 - a_2a_5}, y = y(t) = \frac{2\beta(a_1a_2 + a_5a_6)}{a_1^2 + a_5^2}t + \frac{a_4a_5 - a_1a_8}{a_1a_6 - a_2a_5}. \tag{14}$$

The lump-like wave move along the route line

$$y = -\frac{2(a_1a_2 + a_5a_6)}{a_2^2 + a_6^2}x + \frac{a_8(a_1a_2^2 - a_1a_6^2 + 2a_2a_5a_6) - a_4(a_5a_6^2 - a_2^2a_5 + 2a_1a_2a_6)}{(a_2^2 + a_6^2)(a_1a_6 - a_2a_5)}, \tag{15}$$

with the velocities

$$V_x = \frac{\beta(a_2^2 + a_6^2)}{a_1^2 + a_5^2}, \qquad V_y = \frac{2\beta(a_1a_2 + a_5a_6)}{a_1^2 + a_5^2}. \tag{16}$$

To describe this kind of lump-like solution (13), we take the parameters $\beta = 1$, $a_1 = -1$, $a_2 = -5$, $a_4 = 2$, $a_5 = -2$, $a_6 = 2$, $a_8 = 1$. The lump-like solution reads

$$u = \frac{2(25x + 5y + 143t - 20)}{(841t^2 + 286xt - 58yt + 25x^2 + 10xy + 145y^2 - 200t - 40x - 80y + 25)^2}. \tag{17}$$

The 3-dimensional plot and density plot for this lump-like solution (17) are presented in Fig. 2.

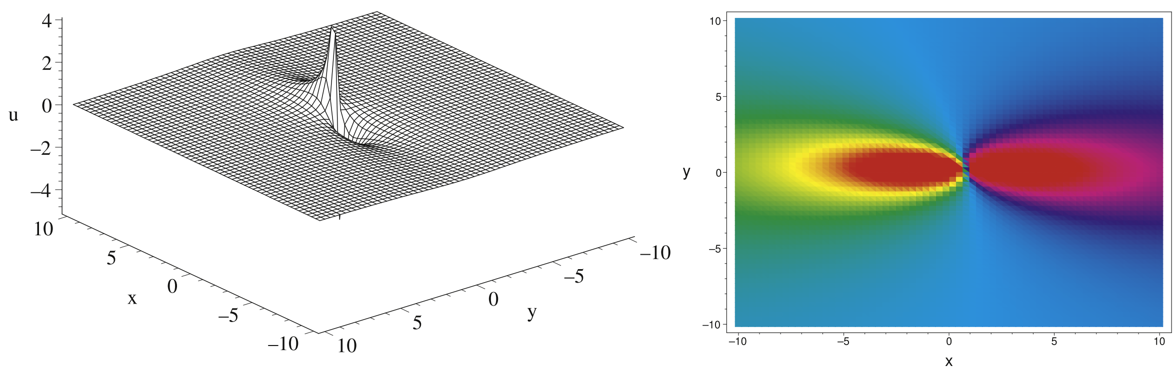

Fig. 2. Profile of the lump-like solution (17). (a) 3-dimensional plot with the time $t = 0$, (b) the corresponding density plot.

3. Summary and discussions

In summary, a (2 + 1)-dimensional STO equation is derived by adding one term u_{yy}. The tri-linear form of the (2 + 1)-dimensional STO equation is obtained by the Painlevé analysis. By solving the tri-linear form, we get two cases solutions of (8) and (12) by introducing the quadratic function solution. A dipole-mode soliton and the lump-like solution of the (2 + 1)-dimensional STO equation are generated by using the tri-linear form. The phenomena of the dipole-mode soliton and the lump-like solution are given by figures. Like the Hirota bilinear form, the lump-like solution can be also obtained with the tri-linear form. In this paper, we construct a new (2 + 1)-dimensional STO equation. We can study other integrability properties of the new (2 + 1)-dimensional STO equation in the future, such as lump-kink interaction solutions [39,40], lump-soliton interaction solutions [41–43], diversity of wave solutions [44] and interaction solutions among solitons and other complicated waves [45–49].

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