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Lump-type and interaction solutions to an extended (3 + 1)-dimensional Jimbo–Miwa equation

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By using the Hirota bilinear method, we construct new lump-type solutions to an extended (3 + 1)-dimensional Jimbo–Miwa equation, which describes certain (3 + 1)-dimensional wave phenomena in physics. The presented solutions contain 10 arbitrary parameters and only need to satisfy four restrictive conditions to be analytic, thereby enriching the existing lump-type solutions. Moreover, we compute their interaction solutions with double exponential function waves, which include rogue wave solutions. Dynamical features of the obtained solutions are graphically exhibited.

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1. Introduction

As is known, nonlinear evolution equations describe various nonlinear phenomena in nature.[1–17] The $(3+1)$-dimensional Jimbo–Miwa equation,[18]

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

is the second member in the KP hierarchy and first introduced by Jimbo and Miwa, which models some fascinating $(3+1)$-dimensional waves in plasmas and optics.[18] Here, $u(x, y, z, t)$ is a function of the spatial variables x, y, z and the temporal variable t, and the subscripts denote the corresponding partial derivatives.[19] In recent years, Eq. (1) has received much attention, and different methods have been used to deal with it, which include the Hirota bilinear method, the tanh-coth method, the exp-function method and the extended homoclinic test approach.[20–31] Equation (1) possesses a variety of exact solutions exhibiting different structures, for example, lump-type solutions, kinky breather-soliton solutions, kinky periodic-soliton solutions, Wronskian determinant solutions and multi-soliton solutions.[20–31]

In 2017, Wazwaz[19] proposed two extended $(3+1)$-dimensional Jimbo–Miwa equations:

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0 \tag{2}$$

and

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} - 3u_{yt} + 2(u_{xt} + u_{yt} + u_{zt}) = 0. \tag{3}$$

Exact solitary wave solutions have been obtained by employing the Kudryashov method in Ref. 32, and multiple soliton solutions have been derived by the simplified Hirota's method.[19,33] Some meromorphic exact solutions have been achieved.[34] The resonant multi-soliton solutions have been established.[35] Periodic solitary wave solutions have been retrieved.[36] The lump solutions and the lump-kink solutions have been obtained[37] and the localized waves, solitons, breathers, lumps and rogue waves of Eq. (2) have been constructed in Ref. 38. Compared with Eq. (1), Eq. (2) and Eq. (3) have received much less attention.

In this paper, we will consider the extended $(3+1)$-dimensional Jimbo–Miwa equation (2). This paper will be structured as follows. In Sec. 2, we will present a new kind of lump-type solutions of Eq. (2) through the Hirota bilinear method. In Sec. 3, we will generate and analyze interaction solutions between the obtained lump-type solutions and double exponential function waves, and discuss dynamical properties of all the presented solutions, along with some graphical illustrations. In Sec. 4, we will provide our conclusions.

2. Lump-Type Solutions

Under the transformation $u = 2(\ln F)_x$, a Hirota bilinear form for Eq. (2) reads

$$(D_x^3 D_y + 2D_t D_y - 3D_x D_z - 3D_y D_z - 3D_z^2)F \cdot F = 0, \tag{4}$$

which equivalently gives rise to

$$\begin{aligned}&F_{xxxy}F - F_y F_{xxx} - 3F_x F_{xxy} + 3\,F_{xx}F_{xy} + 2F_{yt}F - 2F_y F_t - 3F_{xz}F + 3F_x F_z\\ &\quad - 3F_{yz}F + 3F_y F_z - 3F_{zz}F + 3F_z^2 = 0.\end{aligned} \tag{5}$$

To search for lump-type solutions of Eq. (2) (see, for example, Refs. 39–45), we begin by assuming

$$\begin{aligned}&F = G^2 + H^2 + L^2 + a_{16}, && G = a_1 x + a_2 y + a_3 z + a_4 t + a_5,\\ &H = a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, && L = a_{11}x + a_{12}y + a_{13}z + a_{14}t + a_{15},\end{aligned} \tag{6}$$

where $a_i (1 \le i \le 16)$ are real parameters to be determined. Substituting such a function F in Eqs. (6) into the bilinear equation (5), we can, with Maple symbolic computation, get

$$\begin{aligned}
a_3 &= \frac{\begin{array}{l}a_2 a_{12} a_1^2 + (-a_{11}a_2^2 + a_{11}a_{12}^2 + a_6 a_7 a_{12} + a_7^2 a_{13} + a_{12}^2 a_{13})a_1\\ \quad - a_2(a_6 a_7 + a_{11}a_{12})(a_{11} + a_{13})\end{array}}{a_{11}a_2^2 - a_1 a_{12} a_2 + a_7^2 a_{11} - a_6 a_7 a_{12}},\\
a_4 &= -\frac{3\beta_2 + 3[\beta_4 - a_7(a_7 a_{11} - a_6 a_{12})\beta_5]a_1 + 3a_2(a_{11} + a_{13})(\beta_6 + \beta_7)}{2(a_{11}a_2^2 - a_1 a_{12} a_2 + a_7^2 a_{11} - a_6 a_7 a_{12})^2},\\
a_8 &= \frac{\begin{array}{l}-a_7 a_{12} a_6^2 + [a_{11}a_7^2 - a_{11}a_{12}^2 - (a_2^2 + a_{12}^2)a_{13}]a_6 + a_7 a_{11} a_{12}(a_{11} + a_{13})\\ \quad + a_1 a_2[a_7(a_{11} + a_{13}) - a_6 a_{12}]\end{array}}{-a_{11}a_2^2 + a_1 a_{12} a_2 + a_7(a_6 a_{12} - a_7 a_{11})},\\
a_9 &= \frac{\begin{array}{l}3\{\beta_9 + \beta_{12} + a_7(a_7 a_{11} - a_6 a_{12})^2[a_6 a_7 + (a_{11} + a_{13})(a_{12} + a_{13})]\\ \quad + \beta_{13} - a_2^2 a_7(a_{11} + a_{12})(a_{11} + a_{13})\}\end{array}}{-2(a_{11}a_2^2 - a_1 a_{12} a_2 + a_7^2 a_{11} - a_6 a_7 a_{12})^2},\\
a_{14} &= -\frac{\begin{array}{l}3\{\beta_{15} + a_1[a_{11}^3 a_2^2 + \beta_{16} - 2a_6 a_7 a_{12}(2a_{11}^2 + 3a_{13}a_{11}\\ \quad + a_{13}a_{12} + a_{13}^2)]a_2 + \beta_{17} + \beta_{18}\}\end{array}}{2(a_{11}a_2^2 - a_1 a_{12} a_2 + a_7^2 a_{11} - a_6 a_7 a_{12})^2},\\
a_{16} &= \frac{\begin{array}{l}a_1[a_2(2a_6\beta_{43} + a_7^2 a_{12}^2 a_6^4 + \beta_{41} + \beta_{45}) + a_{11}^2(a_6^2 + a_{11}^2)a_2^5\\ \quad + \beta_{47} + \beta_{37} + \beta_{38}] + \beta_{50} + \beta_{48}\end{array}}{(a_{11} + a_{13})(a_{13}a_2^2 + a_1 a_{12} a_2 + a_{11}a_{12}^2 + a_6 a_7 a_{12} + a_7^2 a_{13} + a_{12}^2 a_{13})\beta_{19}},
\end{aligned} \tag{7}$$

where

$$\beta_1 = [a_{11}^2 - (a_{12} - 2a_{13})a_{11} + a_{13}(a_{12} + a_{13})]a_7^2 - a_6 a_{12}(a_{11} - 2a_{12} + a_{13})a_7,$$

$$\beta_2 = a_2^2 a_{12}^2 a_1^3 + a_2[\beta_1 + a_{12}^2(a_{11} + a_{13})(a_{12} + a_{13}) - 2a_2^2 a_{11} a_{12}]a_1^2,$$

$$\beta_3 = a_{12}(a_{11}+a_{13})a_6^2 - a_7[a_{11}^2 + 3(a_{12}+a_{13})a_{11} + a_{13}(a_{12}+2a_{13})]a_6,$$

$$\beta_4 = a_{11}^2 a_2^4 + [\beta_3 + a_{11}a_7^2(a_{11}-a_{13}) - 2a_{11}a_{12}(a_{11}+a_{13})(a_{12}+a_{13})]a_2^2,$$

$$\beta_5 = -a_{12}a_{11}^2 + a_{12}(a_{12}-a_{13})a_{11} + (a_7^2+a_{12}^2)a_{13} - a_6a_7(a_{11}-a_{12}+a_{13}),$$

$$\beta_6 = [a_{13}a_6^2 + a_7a_{11}a_6 + a_{11}^2(a_{12}+a_{13})]a_2^2,$$

$$\beta_7 = (a_7a_{11}-a_6a_{12})\{-a_7a_6^2 + [a_7^2 - a_{12}(a_{11}+a_{13})]a_6 + a_7a_{11}(a_{12}+a_{13})\},$$

$$\begin{aligned}\beta_8 &= a_7[a_{11}^2 - (a_{12}-2a_{13})a_{11} + a_{13}(a_{12}+a_{13})]a_6^2 + a_{11}[(a_{11}-a_{13})a_7^2\\ &\quad + (a_{11}-a_{12})a_{12}(a_{11}+a_{13})]a_6,\end{aligned}$$

$$\beta_9 = -a_6a_{11}a_{13}a_2^4 + [\beta_8 + a_7a_{11}^2(a_{11}+a_{13})(a_{12}+a_{13})]a_2^2 + a_1^3a_7a_{12}(a_{11}+a_{13})a_2,$$

$$\begin{aligned}\beta_{10} &= \{a_7a_{11}(a_{11}+a_{13}) + a_6[a_{11}^2 + (a_{13}-a_{12})a_{11} + a_{12}a_{13}]\}a_2^2\\ &\quad - a_6^2a_7a_{12}(a_{11}-2a_{12}+a_{13}),\end{aligned}$$

$$\beta_{11} = \beta_{10} - a_7a_{11}(a_{11}+a_{13})[a_{12}(a_{11}+a_{12}+2a_{13}) - a_7^2],$$

$$\begin{aligned}\beta_{12} &= a_1a_2\beta_{11} - a_1a_2a_6\{[a_{11}^2 + 3(a_{12}+a_{13})a_{11} + a_{13}(a_{12}+2a_{13})]a_7^2\\ &\quad + (a_{11}-a_{12})a_{12}^2(a_{11}+a_{13})\},\end{aligned}$$

$$\begin{aligned}\beta_{13} &= a_1^2a_7(a_{11}+a_{13})[a_{11}a_{12}^2 + a_6a_7a_{12} + (a_7^2+a_{12}^2)a_{13}]\\ &\quad - a_1^2a_2^2a_6a_{12}(a_{11}-a_{12}+a_{13}),\end{aligned}$$

$$\begin{aligned}\beta_{14} &= a_7a_{11}(a_{11}^2 + a_{13}a_{11} + 2a_{12}a_{13})a_6 + a_{11}^2[a_{12}a_{11}^2 + 2a_{12}a_{13}a_{11}\\ &\quad + a_{13}(a_{12}a_{13} - 2a_7^2)],\end{aligned}$$

$$\beta_{15} = -a_{11}^2a_{13}a_2^4 + [a_{12}a_{13}(a_{11}+a_{13})a_6^2 + \beta_{14}]a_2^2 + a_1^3a_{12}^2(a_{11}+a_{13})a_2,$$

$$\begin{aligned}\beta_{16} &= a_{11}(a_{13}a_{11} + 2a_{12}a_{13})a_2^2 + a_7^2a_{11}(a_{11}^2 + a_{13}a_{11} + 2a_{12}a_{13})\\ &\quad + a_{12}^2(a_{11}+a_{13})[a_6^2 - 2a_{11}(a_{11}+a_{13})],\end{aligned}$$

$$\beta_{17} = (a_7a_{11}-a_6a_{12})^2[a_{12}a_{11}^2 + 2a_{12}a_{13}a_{11} + a_6a_7(a_{11}+a_{13}) + a_{13}(a_{12}a_{13}-a_7^2)],$$

$$\begin{aligned}\beta_{18} &= a_1^2a_{12}(a_{11}+a_{13})[a_{11}a_{12}^2 + a_6a_7a_{12} + (a_7^2+a_{12}^2)a_{13}]\\ &\quad - a_1^2a_{12}a_2^2(2a_{11}^2 + 2a_{13}a_{11} + a_{12}a_{13}),\end{aligned}$$

$$\beta_{19} = (a_7^2+a_{12}^2)a_1^2 - 2a_2(a_6a_7 + a_{11}a_{12})a_1 + (a_7a_{11}-a_6a_{12})^2 + a_2^2(a_6^2+a_{11}^2),$$

$$\begin{aligned}\beta_{20} &= a_6^2a_{11}a_{12}a_7^2 + (a_{12}a_{11}^3 - a_5^2a_{13}a_{11}\\ &\quad - a_5^2a_{13}^2)a_7^2 + a_6[a_7^2a_{11}^2 - a_5^2a_{12}(a_{11}+a_{13})]a_7 - a_5^2a_{12}^2(a_{11}+a_{13})^2,\end{aligned}$$

$$\begin{aligned}\beta_{21} &= 2a_2a_5(a_7a_{11}-a_6a_{12})(a_{11}+a_{13})[a_{11}a_{12}^2 + a_6a_7a_{12}\\ &\quad + (a_7^2+a_{12}^2)a_{13}](a_{10}a_{11}-a_6a_{15}),\end{aligned}$$

$$\beta_{22} = a_{11}^2 a_2^4 + [2a_7^2 a_{11}^2 - 4a_6 a_7 a_{12} a_{11} + (a_6^2 - a_{11}^2) a_{12}^2] a_2^2 + a_7^4 a_{11}^2 - 4a_6 a_7^3 a_{11} a_{12} - a_{10}^2 a_{12}^3 (a_{11} + a_{13}),$$

$$\beta_{23} = a_{12}(3a_{12} a_6^2 - a_{11} a_{15}^2 - a_{13} a_{15}^2 - 2a_{11}^2 a_{12}) a_7^2 + 2a_{12}^2 [a_6 a_{11} a_{12} + a_{10}(a_{11} + a_{13}) a_{15}] a_7 + \beta_{22},$$

$$\beta_{24} = (a_{12} a_{11}^3 - a_{10}^2 a_{13} a_{11} - a_{10}^2 a_{13}^2) a_{11}^2 + a_6 [a_7 a_{11}^3 + 2a_{10} a_{13} (a_{11} + a_{13}) a_{15}] a_{11},$$

$$\beta_{25} = a_{11}^2 \{[-2a_{12} a_{11}^3 + (a_5^2 + a_{10}^2) a_{13} a_{11} + (a_5^2 + a_{10}^2) a_{13}^2] a_7^2 + a_{10}^2 a_{12}^2 (a_{11} + a_{13})^2\},$$

$$\beta_{26} = 2a_7^3 a_{11}^3 - a_7 a_{12} [a_{10}^2 a_{11} (a_{11} + a_{13}) - 2(-a_{12} a_{11}^3 + a_5^2 a_{13} a_{11} + a_5^2 a_{13}^2)],$$

$$\beta_{27} = (a_7 a_{11} - a_6 a_{12})^2 \beta_{20} + a_2^3 a_{12}^2 a_1^5 + a_2^2 a_{12} (-2a_{11} a_2^2 + a_{11} a_{12}^2 - 2a_7^2 a_{11} + 3a_6 a_7 a_{12}) a_1^4,$$

$$\beta_{28} = \beta_{25} - a_6 a_{11} [2a_{10} a_{13} (a_{11} + a_{13}) a_{15} a_7^2 + 2a_{10} a_{12}^2 (a_{11} + a_{13})^2 a_{15} + \beta_{26}],$$

$$\beta_{29} = a_6^2 (a_{11} + a_{13}) \{a_5^2 a_{13} a_{12}^2 + a_{15} [a_{13} a_{15} a_7^2 - 2a_{10} a_{11} a_{12} a_7 + a_{12}^2 (a_{11} + a_{13}) a_{15}]\},$$

$$\beta_{30} = a_{11}(2a_{12} a_6^2 - a_7 a_{11} a_6 + a_{11}^2 a_{12}) a_2^4 + a_{10}^2 a_{11}^2 a_{12}^4 - a_6^3 a_7^3 a_{12}^2 - a_7^4 a_{11}^3 a_{12} + a_6^2 a_7^2 a_{11} a_{12} (2a_7^2 - a_{12}^2),$$

$$\beta_{31} = a_{10}^2 a_{13}^2 a_{12}^4 + 2a_{10}^2 a_{11} a_{13} a_{12}^4 - 2a_7 a_{10} a_{11}^2 a_{15} a_{12}^3 + a_7^2 a_{10}^2 a_{13}^2 a_{12}^2 + a_7^2 a_{10}^2 a_{11} a_{13} a_{12}^2,$$

$$\beta_{32} = -2a_{10} a_{12} a_{13}^2 a_{15} a_7^3 - 2a_{10} a_{11} a_{12} a_{13} a_{15} a_7^3 - 2a_{10} a_{12}^3 a_{13}^2 a_{15} a_7 - 4a_{10} a_{11} a_{12}^3 a_{13} a_{15} a_7,$$

$$\beta_{33} = a_{13}^2 a_{15}^2 a_7^4 + a_{11} a_{13} a_{15}^2 a_7^4 + a_{11}^2 a_{12}^2 a_{15}^2 a_7^2 + a_{12}^2 a_{13}^2 a_{15}^2 a_7^2 + 2a_{11} a_{12}^2 a_{13} a_{15}^2 a_7^2 + \beta_{30} + \beta_{31} + \beta_{32},$$

$$\beta_{34} = a_6 a_7 [-a_{11}^2 a_7^4 + a_{12}(2a_{12} a_{11}^2 + a_{15}^2 a_{11} + a_{13} a_{15}^2) a_7^2 - 2a_{10} a_{12}^2 (a_{11} + a_{13}) a_{15} a_7 + a_{10}^2 a_{12}^3 (a_{11} + a_{13})],$$

$$\beta_{35} = 2a_7 a_{12}^2 a_6^3 + a_{11} a_{12} (a_{12}^2 - 4a_7^2) a_6^2 + a_{11}^3 a_{12}^3 + a_{10}^2 a_{12}^2 (2a_{11}^2 + a_{13} a_{11} - a_{13}^2),$$

$$\beta_{36} = a_6 [2a_{11}^2 a_7^3 + a_{12}(a_{12} a_{11}^2 + 2a_{15}^2 a_{11} + 2a_{13} a_{15}^2) a_7 - 2a_{10} a_{12}^2 (a_{11} + a_{13}) a_{15}],$$

$$\beta_{37} = 2a_5 (a_7 a_{11} - a_6 a_{12})(a_{11} + a_{13}) [a_{11} a_{12}^2 + a_6 a_7 a_{12} + (a_7^2 + a_{12}^2) a_{13}] (a_7 a_{15} - a_{10} a_{12}),$$

$$\beta_{38} = 2a_2^2 a_5 (a_7 a_{11} - a_6 a_{12})(a_{11} + a_{13}) [a_{10} a_{12} (a_{11} - a_{13}) + (a_7 a_{13} - a_6 a_{12}) a_{15}],$$

$$\beta_{39} = a_{11} \{-2a_7^2 a_{11}^3 + a_{12} [2a_{12} a_{11}^3 + a_{10}^2 (a_{11}^2 - a_{13} a_{11} - 2a_{13}^2)] + 2a_7 a_{10} a_{13} (a_{11} + a_{13}) a_{15}\},$$

$$\beta_{40} = a_{11}^3 a_7^4 - a_{12}[a_5^2 a_{11}(a_{11} + a_{13}) - 2(-a_{12}a_{11}^3 + a_{10}^2 a_{13}a_{11} + a_{10}^2 a_{13}^2)]a_7^2$$
$$+ 2a_{10}^2 a_{12}^3 (a_{11} + a_{13})^2,$$
$$\beta_{41} = a_6^2[a_{11}^2 a_7^4 + a_{12}(a_{12}a_{11}^2 + 2a_{15}^2 a_{11} + 2a_{13}a_{15}^2)a_7^2 - 2a_{10}a_{12}^2(a_{11} + a_{13})a_{15}a_7$$
$$- a_5^2 a_{12}^3(a_{11} + a_{13})],$$
$$\beta_{42} = (-2a_{12}a_{11}^3 + a_{13}a_{15}^2 a_{11} + a_{13}^2 a_{15}^2)a_7^3 - a_{10}a_{12}(a_{11} + a_{13})^2 a_{15}a_7^2$$
$$- a_{10}a_{12}^3(a_{11} + a_{13})^2 a_{15},$$
$$\beta_{43} = a_7[a_{12}a_{11}^3 + a_{15}^2 a_{11}^2 + 2a_{13}a_{15}^2 a_{11} + a_5^2(a_{11} + a_{13})a_{11}$$
$$+ a_{10}^2(a_{11} + a_{13})a_{11} + a_{13}^2 a_{15}^2]a_{12}^2 + \beta_{42},$$
$$\beta_{44} = 2a_7 a_{11}a_{12}a_6^3 + [a_{12}(2a_{12}a_{11}^2 + a_{15}^2 a_{11} + a_{13}a_{15}^2) - 2a_7^2 a_{11}^2]a_6^2 + \beta_{39},$$
$$\beta_{45} = a_{11}[-2a_{10}a_{13}(a_{11} + a_{13})a_{15}a_7^3 - 2a_{10}a_{12}^2(a_{11} + a_{13})^2 a_{15}a_7 + \beta_{40}]$$
$$+ 2a_7 a_{11}a_{12}(a_{12}^2 - a_7^2)a_6^3,$$
$$\beta_{46} = a_2^4[(a_{12}a_{11}^3 - a_{13}a_{15}^2 a_{11} - a_{13}^2 a_{15}^2)a_6^2 + \beta_{24}] - a_2^2[a_7 a_{12}(2a_{12}a_{11}^2 + a_{15}^2 a_{11}$$
$$+ a_{13}a_{15}^2)a_6^3 + \beta_{28} + \beta_{29}],$$
$$\beta_{47} = -a_2^3\{2a_6[a_{10}a_{12}(a_{13}^2 - a_{11}^2)a_{15} + a_7(2a_{12}a_{11}^3 - a_{13}a_{15}^2 a_{11} - a_{13}^2 a_{15}^2)] + \beta_{44}\},$$
$$\beta_{48} = a_2 a_1^3 \beta_{23} + 2a_2^3 a_5(a_7 a_{11} - a_6 a_{12})a_{13}(a_{11} + a_{13})(a_{10}a_{11} - a_6 a_{15})$$
$$+ \beta_{21} + \beta_{27} + \beta_{46},$$
$$\beta_{49} = -2a_2 a_5 a_{12}(a_6 a_{12} - a_7 a_{11})(a_{11} + a_{13})(a_{10}a_{12} - a_7 a_{15}) + \beta_{33} + \beta_{34},$$
$$\beta_{50} = -a_1^2 \beta_{49} + a_1^2 a_2^2[-a_7^2 a_{13}^2 a_{15}^2 - a_7^2 a_{11}a_{13}a_{15}^2 + 2a_7 a_{10}a_{12}(a_{13}^2 - a_{11}^2)a_{15}$$
$$+ \beta_{35} + \beta_{36}].$$

All that remains is to study the analyticity of the solutions determined by $u = 2(\ln F)_x$. It is direct to see that it is sufficient for u to be analytic if the involved constants satisfy

$$\begin{cases} a_{11}a_2^2 - a_1 a_{12}a_2 + a_7^2 a_{11} - a_6 a_7 a_{12} \neq 0, \\ a_{13}a_2^2 + a_1 a_{12}a_2 + a_{11}a_{12}^2 + a_6 a_7 a_{12} + a_7^2 a_{13} + a_{12}^2 a_{13} \neq 0, \\ a_{11} + a_{13} \neq 0, \quad \beta_{19} \neq 0, \quad a_{16} > 0. \end{cases}$$

Based on the transformation $u = 2(\ln F)_x$, a kind of lump-type solutions of Eq. (2) can be presented as

$$u = [4a_1(a_4 t + a_1 x + a_2 y + a_3 z + a_5) + 4a_6(a_9 t + a_6 x + a_7 y + a_8 z + a_{10})$$
$$+ 4a_{11}(a_{14}t + a_{11}x + a_{12}y + a_{13}z + a_{15})][(a_4 t + a_1 x + a_2 y + a_3 z + a_5)^2$$

$$+ (a_9 t + a_6 x + a_7 y + a_8 z + a_{10})^2$$
$$+ (a_{14} t + a_{11} x + a_{12} y + a_{13} z + a_{15})^2 + a_{16}]^{-1}. \tag{8}$$

The 10 involved parameters, $a_1, a_2, a_5, a_6, a_7, a_{10}, a_{11}, a_{12}, a_{13}, a_{15}$, are all arbitrary, and thus, with such diverse parametric choices, we can expect to use those solutions to model more complicated wave phenomena in reality. As an example, we specially take the parameters to be

$$a_1 = 1, \quad a_2 = 1, \quad a_5 = 2, \quad a_6 = 1, \quad a_7 = 1, \quad a_{10} = 1,$$
$$a_{11} = 2, \quad a_{12} = 1, \quad a_{13} = 1 \quad \text{and} \quad a_{15} = 1,$$

and then obtain the following lump-type solution:

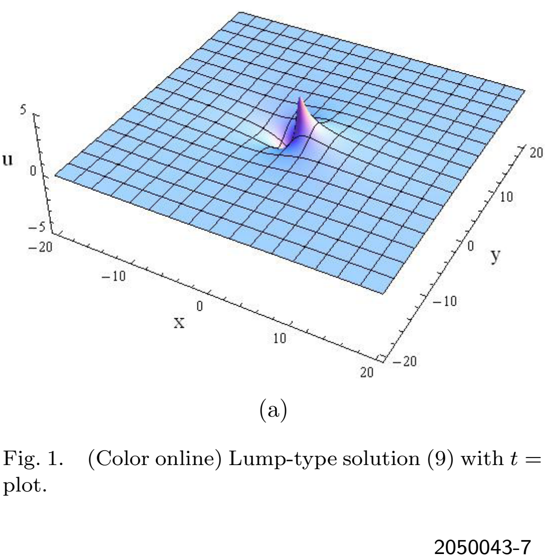
$$u = \frac{24x + 16y - 12z - 126t + 20}{(1 - 6t + x + y - \frac{5z}{2})^2 + (2 - 6t + x + y - \frac{5z}{2})^2 + (1 - \frac{39t}{4} + 2x + y + z)^2 + \frac{71}{42}}. \tag{9}$$

3. Interaction Solutions with Double Exponential Function Waves

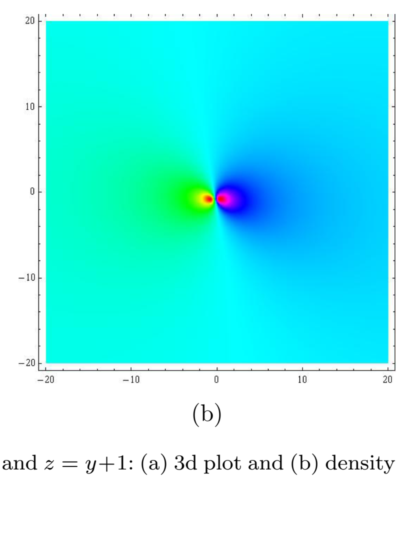
In this section, we will construct and analyze interaction solutions between the presented lump-type solutions and double exponential function waves (see, for example, Refs. 43, 46, 47 and Refs. 48–50 for other examples of nonlinear PDEs and linear PDEs, respectively). To the end, we begin by assuming that

$$\begin{aligned} F &= G^2 + H^2 + L + b_{11}, & G &= b_1 x + b_2 y + b_3 z + b_4 t + b_5, \\ H &= b_6 x + b_7 y + b_8 z + b_9 t + b_{10}, & L &= v_1 e^{k_1 x + k_2 y + k_3 z + k_4 t + k_5} \\ & & &\quad + v_2 e^{-k_1 x - k_2 y - k_3 z - k_4 t - k_5}, \end{aligned} \tag{10}$$

where $b_i (1 \le i \le 11)$, $k_j (1 \le j \le 5)$, v_1 and v_2 are all the real parameters to be determined. Substituting such a function F by Eqs. (10) into the bilinear form (5),



(a)



(b)

Fig. 1. (Color online) Lump-type solution (9) with $t = 0$ and $z = y+1$: (a) 3d plot and (b) density plot.

we can, through Maple, obtain

$$\begin{cases} b_1 = -\dfrac{2b_3b_8^2}{b_3^2+b_8^2}, \quad b_2 = \dfrac{b_3(b_3^2-b_8^2)k_2}{(b_3^2+b_8^2)k_3}, \quad b_4 = \dfrac{3b_3(k_2+k_3)}{2k_2}, \quad b_6 = \dfrac{b_8(b_3^2-b_8^2)}{b_3^2+b_8^2}, \\ b_7 = \dfrac{2b_3^2b_8k_2}{(b_3^2+b_8^2)k_3}, \quad b_9 = \dfrac{3b_8(k_2+k_3)}{2k_2}, \quad k_1 = 0, \quad k_4 = \dfrac{3k_3^2+3k_2k_3}{2k_2}, \end{cases} \tag{11}$$

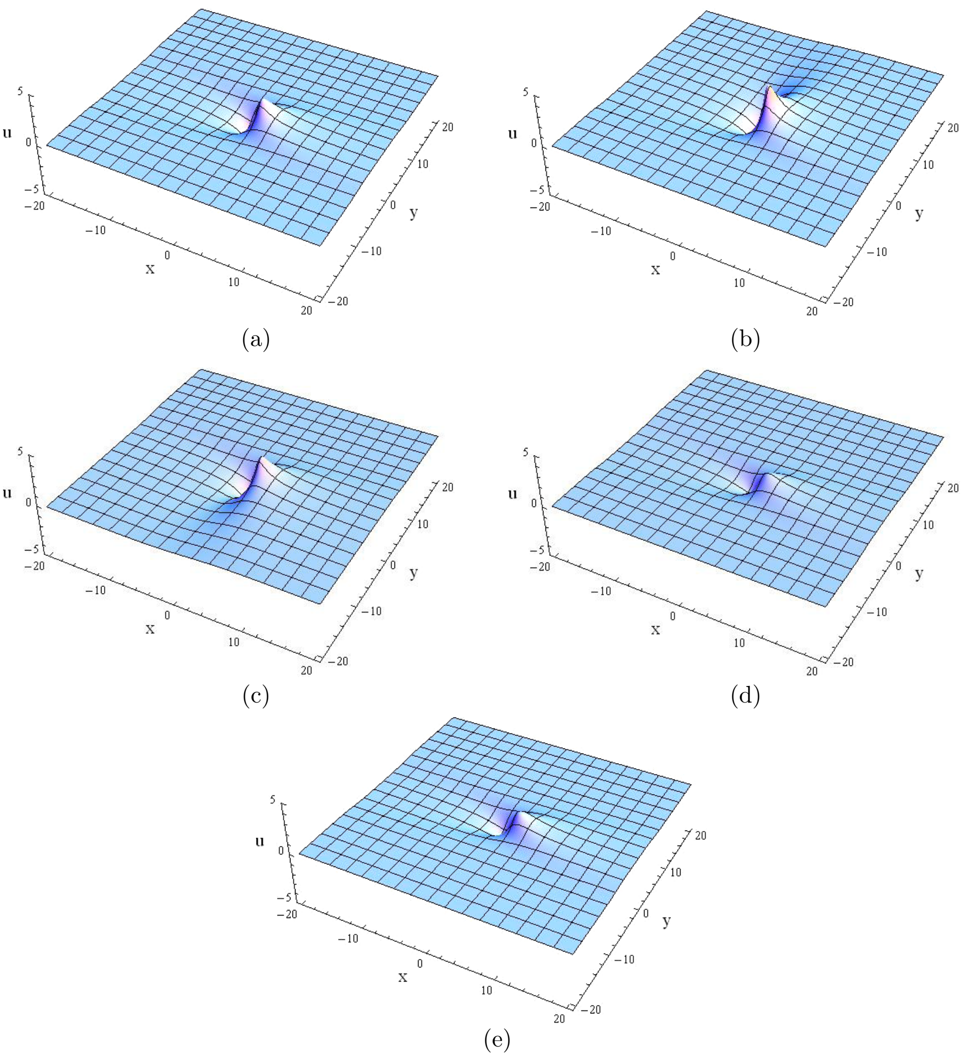


Fig. 2. (Color online) Interaction solution (12) with $b_3 = 1$, $b_5 = 1$, $b_8 = 2$, $b_{10} = 1$, $b_{11} = 1$, $k_2 = 1$, $k_3 = 1$, $k_5 = 1$: (a) $v_1 = 1$, $v_2 = 1$; (b) $v_1 = 0$, $v_2 = 1$; (c) $v_1 = 1$, $v_2 = 0$; (d) $v_1 = 8$, $v_2 = 1$ and (e) $v_1 = 1$, $v_2 = 8$.

which need to satisfy

$$(b_3^2 + b_8^2)k_3k_2 \neq 0,$$

to make F to be analytic.

With $u = 2(\ln F)_x$, a kind of interaction solutions of Eq. (2) can be presented as

$$\begin{aligned} u = {} & [4b_1(b_4t + b_1x + b_2y + b_3z + b_5) + 4b_6(b_9t + b_6x + b_7y + b_8z + b_{10})] \\ & \times [(b_4t + b_1x + b_2y + b_3z + b_5)^2 + (b_9t + b_6x + b_7y + b_8z + b_{10})^2 + b_{11} \\ & + v_1 e^{k_4t + k_2y + k_3z + k_5} + v_2 e^{-k_4t - k_2y - k_3z - k_5}]^{-1}. \end{aligned} \tag{12}$$

In the following, Figs. 1 and 2 depict dynamic features and energy distributions of the lump-type solutions Eq. (9) and the interaction solution Eq. (12) with $b_3 = 1, b_5 = 1, b_8 = 2, b_{10} = 1, b_{11} = 1, k_2 = 1, k_3 = 1$ and $k_5 = 1$.

Particularly, Fig. 2 sheds light on geometrical features of the interaction solution (12) under $b_3 = 1, b_5 = 1, b_8 = 2, b_{10} = 1, b_{11} = 1, k_2 = 1, k_3 = 1$ and $k_5 = 1$, but with the different values of v_1 and v_2 as (a) $v_1 = 1, v_2 = 1$; or (b) $v_1 = 0, v_2 = 1$; or (c) $v_1 = 1, v_2 = 0$; or (d) $v_1 = 8, v_2 = 1$; or (e) $v_1 = 1, v_2 = 8$.

4. Conclusions

In this paper, we have considered an extended $(3 + 1)$-dimensional Jimbo–Miwa equation, which has potential applications in physics. A kind of lump-type solutions and their interaction solutions with double exponential function waves were presented. Dynamical features and energy distributions of the presented solutions were graphically depicted. It is expected that our results could be helpful in exploring mathematical characteristics and practical applications of the discussed extended $(3+1)$-dimensional Jimbo–Miwa equation.

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