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On global behavior for complex soliton solutions of the perturbed nonlinear Schrödinger equation in nonlinear optical fibers

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ABSTRACT

In this research article, the perturbed nonlinear Schrödinger equation (P-NLSE) is examined by utilizing two analytical methods, namely the extended modified auxiliary equation mapping and the generalized Riccati equation mapping methods. Consequently, we establish several sorts of new families of complex soliton wave solutions such as hyperbolic functions, trigonometric functions, dark and bright solitons, periodic solitons, singular solitons, and kink-type solitons wave solutions of the P-NLSE. Using the mentioned methods, the results are displayed in 3D and 2D contours for specific values of the open parameters. The obtained findings demonstrate that the implemented techniques are capable of identifying the exact solutions of the other complex nonlinear evolution equations (C-NLEEs) that arise in a range of applied disciplines.

1. Introduction

In the field of engineering, optics and mathematical physics, the nonlinear evolution equation has attracted more attention of several researchers for its wide-range of characteristics [1–6,6–16]. The nonlinear Schrödinger equation is a subpart of the nonlinear evolution equation and it is applied in many fields such as plasma physics, chemical kinematics, fluid mechanics, elastic media, solid-state, quantum mechanics, bio-genetics, nonlinear optics, and hydrodynamics [17–34]. A solitary wave or soliton is produced by eliminating the dispersive and nonlinear effects in the medium propagating at constant velocity without changing its shape [35–42]. Most recently, optical solitons have demonstrated significant effects in the telecommunications industry. The P-NLSE evaluates the propagation of optical solitons in nonlinear optical fibers and other telecommunication systems. For the past few years, to find the solitons of P-NLSE and exact solutions in the form of solitary waves, several mathematicians and scientists have been studied broadly and they established the effective and powerful approaches such as modified Kudryashov method [43], Hirota's method [44–53] and direct algebraic method [54,55] which discuss the interaction between lumps waves and also the mixed soliton solutions [56–59], residual power series method [60–62], sine-cosine method [63,64], Fan sub-equation method [65,66], inverse scattering method [67–69], homotopy Perturbation scheme [70,71], Bäcklund transform method [72–76], the unified method [77,78], sine-Gordon approach [79,80] and simple equation method [81].

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In this paper, the extended modified auxiliary equation mapping (AEM) method and the generalized Riccati equation mapping (REM) method [82,83] are employed to study the P-NLSE [84] which reads

$$i\mu_t + \alpha\mu_{xx} + \beta|\mu|^2\mu - i(\gamma\mu_x - \delta(|\mu|^2\mu)_x - \sigma(|\mu|^2)_x\mu) = 0, \tag{1}$$

where α, β, γ, δ, σ are constants. Here, $\mu = \mu(x,t)$ refers to the wave profile while the coefficient of group velocity dispersion term, the coefficient of nonlinearity, the coefficient of inter-modal dispersion, the self-steepening term for short pulses and the coefficient of nonlinear dispersion term are stated by the parameters α, β, γ, δ and σ, respectively. In optics, Eq. (1) is commonly used as a useful model for optical pulse propagation in nonlinear fibers. Furthermore, it is the simplest model for the propagation of the a laser beam in a medium with a Kerr nonlinearity.

The structure of this paper is designed as follows: The mathematical analysis to the Eq. (1) is developed in Section 2. The geometrical behavior of the solutions is demonstrated in Section 3. Finally, the conclusions are extracted in Section 4.

2. Mathematical analysis

Consider

$$\mu(x,t) = \Omega(\zeta)e^{i\theta} \quad \zeta = x - \lambda t, \quad \theta = -\omega_1 x + \omega_0 t, \tag{2}$$

where λ, ω_0 and ω_1 are real constants.

Plugging the transformation given by Eq. (9) into Eq. (1) and from the real and imaginary parts, respectively, we get,

$$\alpha\Omega'' - (\omega_0 + \alpha\omega_1^2 + \omega_1\gamma)\Omega + (\beta - \delta\omega_1)\Omega^3 = 0, \tag{3}$$

and

$$\lambda = -(2\alpha\omega_1 + \gamma), \quad \delta = -\frac{2}{3}\sigma. \tag{4}$$

2.1. The AEM method

By applying the AEM method on Eq. (3) and by balancing Ω'' with Ω^3, we obtain n=1. Therefore, the general solution will take the form Cheemaa et al. [82]

$$\Omega(\zeta) = a_0 + a_1\phi(\zeta) + \frac{b_1}{\phi(\zeta)} + d_1\frac{\phi'(\zeta)}{\phi(\zeta)}, \tag{5}$$

where $\phi(\zeta)$ satisfying the following auxiliary ordinary differential equation with its derivatives:

$$\phi'(\zeta)^2 = F_3\phi(\zeta)^4 + F_2\phi(\zeta)^3 + F_1\phi(\zeta)^2, \tag{6}$$

$$\phi''(\zeta) = 2F_3\phi(\zeta)^3 + \frac{3}{2}F_2\phi(\zeta)^2 + F_1\phi(\zeta), \tag{7}$$

$$\phi'''(\zeta) = (6F_3\phi(\zeta)^2 + 3F_2\phi(\zeta) + F_1)\phi'(\zeta), \tag{8}$$

where a_0, a_1, b_1, d_1, F_1, F_2 and F_3 are constants to be determined later.

Inserting Eq. (5) into Eq. (3) with the aid of Eq. (6) and collecting the coefficients of $\phi'^k\phi^j$, $(k = 0, 1, \ j = 0, 1, 2, 3, \ldots, n)$ yields an algebraic system of equations in the assumed parameters and the coefficients exists in Eq. (3). Solving this system of algebraic equations yields different sets of values represented by the following families.

Family 1:

$$\begin{aligned} a_0 &= \pm\frac{\sqrt{-\alpha\omega_1^2 - \gamma\omega_1 - \omega_0}}{\sqrt{\delta\omega_1 - \beta}}, \\ a_1 &= \frac{\alpha F_2}{2\sqrt{\delta\omega_1 - \beta}\sqrt{-\alpha\omega_1^2 - \gamma\omega_1 - \omega_0}}, \ b_1 = 0, \\ d_1 &= 0, \ F_1 = -\frac{2(\alpha\omega_1^2 + \gamma\omega_1 + \omega_1)}{\alpha}, F_3 = -\frac{\alpha F_2^2}{8(\alpha\omega_1^2 + \gamma\omega_1 + \omega_0)}. \end{aligned} \tag{9}$$

Putting all these values into Eq. (3) with the help of Eq. (6), the solutions of Eq. (1) are obtained in the following forms

$$\mu_{1,1}(\zeta) = \left[\frac{-2(\omega_0 + \omega_1(\gamma + \alpha\omega_1)) - \alpha F_1(1 + \epsilon\tanh[\frac{1}{2}\sqrt{F_1}(\zeta + \xi_0)])}{2\sqrt{\delta\omega_1 - \beta}\sqrt{-\omega_1(\gamma + \alpha\omega_1) - \omega_0}}\right]e^{i\theta}, \tag{10}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $\epsilon = \pm 1$ and $F_2 = 2\sqrt{F_1F_3}$.

$$\mu_{1,2}(\zeta) = \left[\frac{-4(\omega_0 + \omega_1(\gamma + \alpha\omega_1)) + (1 + \frac{\epsilon\sinh[\sqrt{F_1}(\zeta+\xi_0)]}{\kappa+\cosh[\sqrt{F_1}(\zeta+\xi_0)]}\alpha F_2\sqrt{\frac{F_1}{F_3}})}{4\sqrt{\delta\omega_1 - \beta}\sqrt{-\omega_1(\gamma + \alpha\omega_1) - \omega_0}}\right]e^{i\theta}, \tag{11}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $F_3 > 0$, $F_2 = -2\sqrt{F_1 F_3}$ and ϵ and κ are any combination of ± 1.

$$\mu_{1,3}(\zeta) = \left[\frac{-2(\omega_0 + \omega_1(\gamma + \alpha\omega_1)) - \alpha\left(1 + \frac{\epsilon\sqrt{1+p^2}\kappa + \cosh[\sqrt{F_1}(\zeta+\xi_0)]}{p + \sinh[\sqrt{F_1}(\zeta+\xi_0)]}\right)F_1}{2\sqrt{\delta\omega_1 - \beta}\sqrt{-\omega_1(\gamma + \alpha\omega_1) - \omega_0}}\right]e^{i\theta}, \tag{12}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, p is an arbitrary constant, $F_1 > 0$ and ϵ and κ are any combination of ± 1.

Family 2:

$$d_1 = \frac{i\sqrt{\alpha}}{\sqrt{2\beta - 2\delta\omega_1}},\ a_0 = 0,\ b_1 = 0,\ a_1 = -\frac{i\sqrt{\alpha}\sqrt{F_3}}{\sqrt{2\beta - 2\delta\omega_1}},\ F_1 = -\frac{2(\alpha\omega_1^2 + \gamma\omega_1 + \omega_0)}{\alpha}. \tag{13}$$

Putting the values of Family 2 into Eq. (3) with the help of Eq. (6), the solutions of Eq. (1) are obtained in the following forms

$$\mu_{1,4}(\zeta) = \left[\frac{i\sqrt{\alpha}(\epsilon\sec[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)]^2\sqrt{F_1}F_2 + 2F_1\sqrt{F_3}(1 + \epsilon\tanh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])^2)}{2\sqrt{2}\sqrt{-\delta\omega_1 + \beta}F_2(1 + \epsilon\tanh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])}\right]e^{i\theta}, \tag{14}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $\epsilon = \pm 1$, $F_2 = 2\sqrt{F_1 F_3}$.

$$\mu_{1,5}(\zeta) = \left[\frac{i\sqrt{\alpha}(2\epsilon(1 + \kappa\cosh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])\pi + ((\kappa + \cosh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)] + \epsilon\sinh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])^2\pi)}{2\sqrt{2}\sqrt{-\delta\omega_1 + \beta}(\kappa + \epsilon\cosh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])((\kappa + \cosh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)] + \epsilon\sinh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])}\right] \times e^{i\theta}, \tag{15}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $\epsilon = \pm 1$, $F_2 = -2\sqrt{F_1 F_3}$, and ϵ and κ are any combination of ± 1.

$$\mu_{1,6}(\zeta) = \left[\left(\frac{i\sqrt{\alpha}\sqrt{F_1}}{2\beta - 2\sqrt{\delta\omega_1}}\right) \times \left(\frac{\epsilon(-1 - \sqrt{1+p^2}\kappa\cosh[\sqrt{F_1}(\zeta+\xi_0)] + p\sinh[\sqrt{F_1}(\zeta+\xi_0)])}{(p + \sinh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])^2} + \frac{\left(1 + \frac{\epsilon\sqrt{1+p^2}\kappa + \cosh[\sqrt{F_1}(\zeta+\xi_0)]}{p+\sinh[\sqrt{F_1}(\zeta+\xi_0)]}\right)^2\sqrt{F_3}\sqrt{F_1}}{F_2}\right)\right]e^{i\theta}, \tag{16}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, p is an arbitrary constant, $F_1 > 0$ and ϵ and κ are any combination of ± 1.

Family 3:

$$d_1 = \frac{\sqrt{-\omega_0 - \omega_1(\omega_1\alpha + \gamma)}}{2\sqrt{(-\beta + \delta\omega_1)}},\ a_1 = 0,\ b_1 = 0,$$
$$F_2 = 0,\ F_3 = 0,\ a_0 = -\frac{\sqrt{-\omega_0 - \omega_1(\alpha\omega_1 + \gamma)}}{2\sqrt{F_1}\sqrt{(-\beta + \delta\omega_1)}}, \tag{17}$$

Using the obtained values in (17) in Eq. (3) with the help of Eq. (6), we get the solutions of Eq. (1) in the following forms

$$\mu_{1,7}(\zeta) = \left[\frac{\left(-2 - \epsilon\sec[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)]^2(-1 + \sinh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])\right)\sqrt{-\omega_0 - \omega_1(\alpha\omega_1 + \gamma)}}{4\sqrt{-\beta + \delta\omega_1}(1 + \epsilon\tanh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])}\right]e^{i\theta}, \tag{18}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $\epsilon = \pm 1$ and $F_2 = 2\sqrt{F_1 F_3}$.

$$\mu_{1,8}(\zeta) = \left[\frac{-\left(1 - \frac{\epsilon(1+\kappa\cosh[\sqrt{F_1}(\zeta+\xi_0)])}{(\kappa+\cosh[\sqrt{F_1}(\zeta+\xi_0)])(\Delta_2)}\right)\sqrt{-\omega_0 - \omega_1(\alpha\omega_1 + \gamma)}}{2\sqrt{-\beta + \delta\omega_1}}\right]e^{i\theta}, \tag{19}$$

where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, $F_2 = -2\sqrt{F_1 F_3}$ and ϵ and κ are any combination of ± 1.

$$\mu_{1,9}(\zeta) = \left[\frac{-\pi\left(1 + \frac{\epsilon(1+\sqrt{1+p^2}\kappa\cosh[\sqrt{F_1}(\zeta+\xi_0)] - p\sinh[\sqrt{F_1}(\zeta+\xi_0)])}{(p+\sinh[\sqrt{F_1}(\zeta+\xi_0)])(p\sqrt{1+p^2}\kappa\epsilon + \epsilon\cosh[\sqrt{F_1}(\zeta+\xi_0)] + \sinh[\sqrt{F_1}(\zeta+\xi_0)])}\right)}{2\sqrt{-\beta + \delta\omega_1}}\right]e^{i\theta}, \tag{20}$$

Where $\zeta = x + (2\alpha\omega_1 + \gamma)t$, ξ_0 is an integration constant, $\theta = -\omega_1 x + \omega_0 t$, $F_1 > 0$, ξ_0, p are arbitrary constants and ϵ and κ are any combination of ± 1.

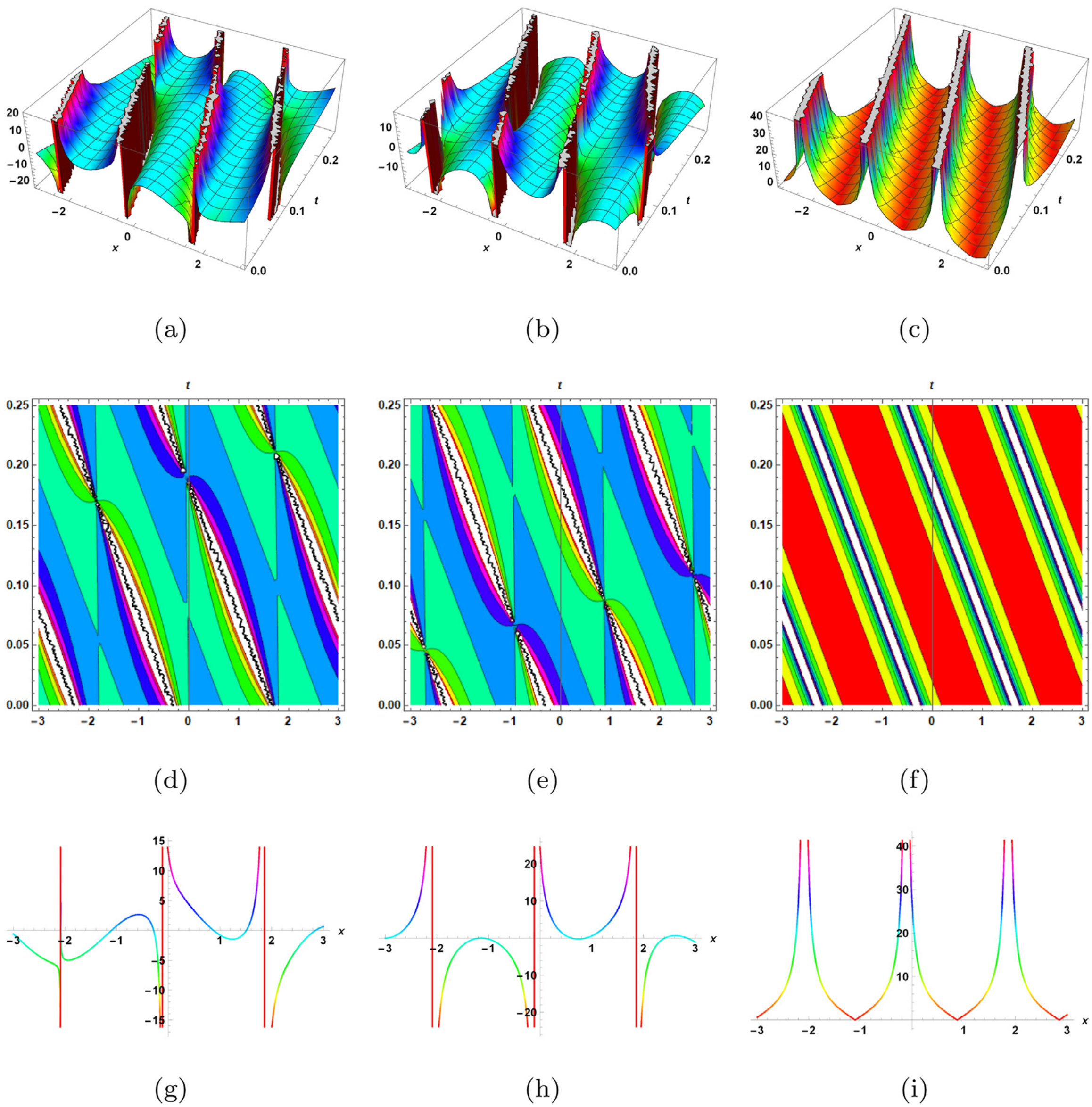

Fig. 1. $\mu_{1,1}(x,t): \alpha=2,\ \beta=0.25,\ \gamma=2,\ \omega_1=1.75,\ F_2=-1,\ \sigma=0.75,\ \omega_0=0.5,\ \xi_0=1.3,\ \epsilon=1.5$. In (g)-(i), $t=1$.

Family 4:

$$a_1=\pm\frac{\sqrt{2}\sqrt{\alpha}\sqrt{F_3}}{\sqrt{\delta\omega_1-\beta}},\ b_1=0,\ d_1=0,$$
$$a_0=0,\ F_2=0,\ F_1=\frac{\alpha\omega_1^2+\gamma\omega_1+\omega_0}{\alpha}. \tag{21}$$

Similarly, by using the obtained values in Eq. (21) and inserting them in Eq. (3) with the help of Eq. (6), we get the solutions of Eq. (1) in the following forms

$$\mu_{1,10}(\zeta)=\left[-\frac{\sqrt{2}\sqrt{\alpha}(1+\epsilon\tanh[\frac{1}{2}\sqrt{F_1}(\zeta+\xi_0)])F_1\sqrt{F_3}}{\sqrt{\delta\omega_1-\beta}F_2}\right]e^{i\theta}, \tag{22}$$

where $\zeta=x+(2\alpha\omega_1+\gamma)t$, ξ_0 is an integration constant, $\theta=-\omega_1x+\omega_0t$, $F_1>0,\ \epsilon=\pm1$ and $F_2=2\sqrt{F_1F_3}$.

$$\mu_{1,11}(\zeta)=\left[\frac{\sqrt{2}(1+\frac{\epsilon\sinh[\sqrt{F_1}(\zeta+\xi_0)]}{\kappa+\cosh[\sqrt{F_1}(\zeta+\xi_0)]})\sqrt{\frac{F_1}{F_3}}\sqrt{F_3}}{\sqrt{\delta\omega_1-\beta}}\right]\times e^{i\theta}, \tag{23}$$

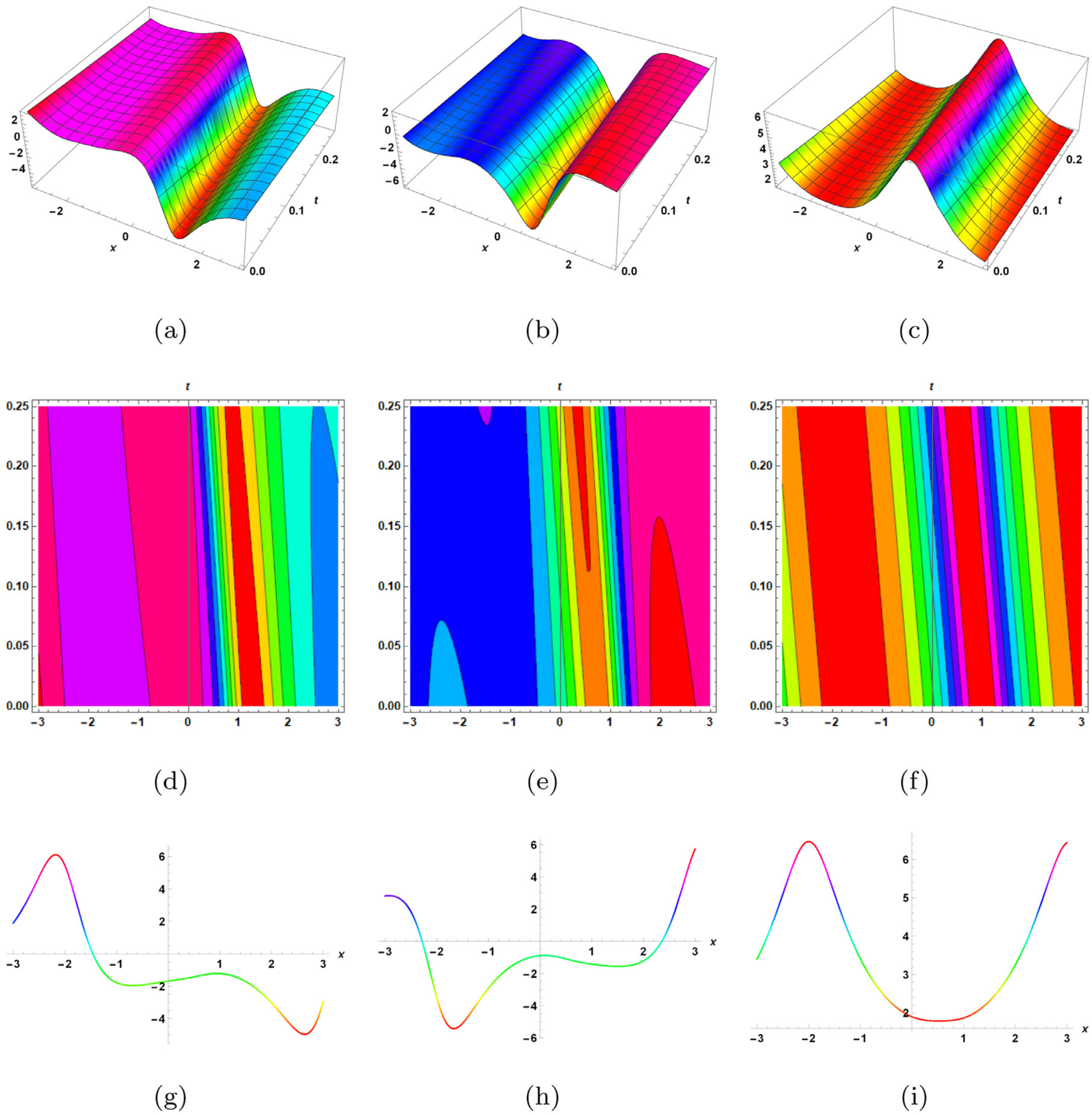

Fig. 2. $\mu_{1,3}(x,t)$: $\alpha=1.5,\ \beta=-1.25,\ \gamma=-0.25,\ \omega_1=0.75,\ F_2=-1,\ \sigma=2,\ \omega_0=0.5,\ \xi_0=-1.3,\ \epsilon=1.3, \kappa=1,\ p=0.92$. In (g)-(i), $t=1$.

where $\zeta=x+(2\alpha\omega_1+\gamma)t$, ξ_0 is an integration constant, $\theta=-\omega_1 x+\omega_0 t$, $F_1>0$, $F_3>0$, $F_2=-2\sqrt{F_1F_3}$ and ϵ and κ are any combination of ± 1.

$$\mu_{1,12}(\zeta)=\left[-\frac{\sqrt{2}\sqrt{\alpha}\left(1+\frac{\epsilon\sqrt{1+p^2}\kappa+\cosh[\sqrt{F_1}(\zeta+\xi_0)]}{p+\sinh[\sqrt{F_1}(\zeta+\xi_0)]}\right)\sqrt{F_3}F_1}{\sqrt{\delta\omega_1-\beta F_2}}\right]e^{i\theta}, \tag{24}$$

where $\zeta=x+(2\alpha\omega_1+\gamma)t$, ξ_0 is an integration constant, $\theta=-\omega_1 x+\omega_0 t$, p, ξ_0 are arbitrary constants and ϵ and κ are any combination of ± 1.

2.2. The REM method

According to the REM method, Eq. (3) has the formal solution [83]

$$\mu(x,t)=g_1\phi(\zeta)+g_0+\frac{g_{-1}}{\phi(\zeta)}. \tag{25}$$

where g_0, g_1, g_{-1} are constants and $\phi(\zeta)$ satisfies the following ODE

$$\phi'(\zeta)=\eta_1+\eta_2\phi(\zeta)+\eta_3\phi^2(\zeta), \tag{26}$$

where η_1, η_2 and η_3 are arbitrary constants.

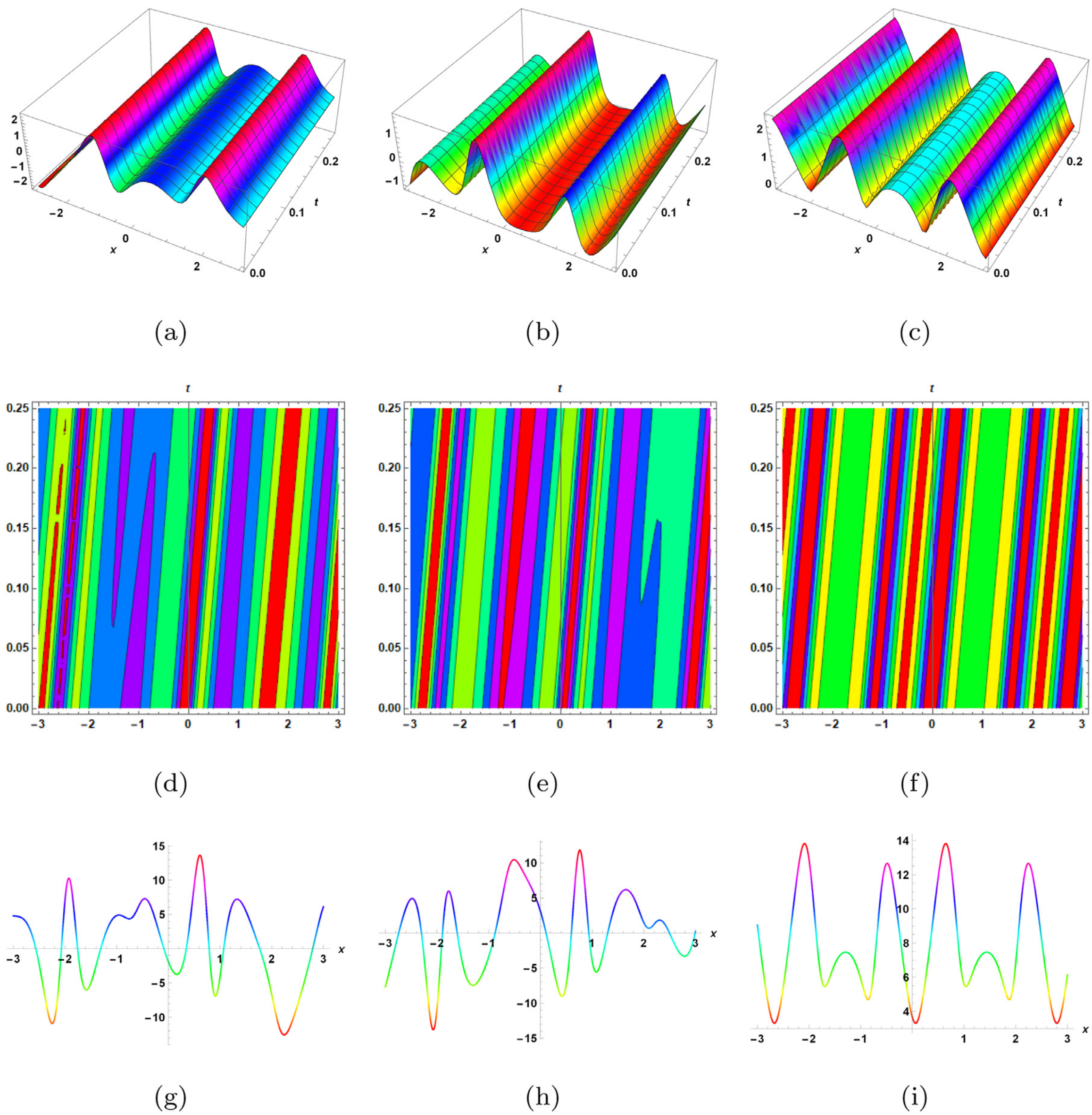

Fig. 3. $\mu_{1,6}(x,t)$: $\alpha = 0.65,\ \beta = 1.35,\ \gamma = -1.15,\ \omega_1 = 0.75,\ F_2 = -1.25,\ \sigma = 2.5,\ \omega_0 = 0.49,\ \xi_0 = 1.3,\ \epsilon = -1,\ \kappa = 0.73,\ p = 1$. In (g)-(i), $t = 1$.

Substituting Eq. (25) into Eq. (3) with the aid of Eq. (26) and collecting the coefficients of ϕ^i, $(i = 0, 1, 2, \cdots, n)$ to zero, yields a system of algebraic equations in the assumed parameters and the coefficients of Eq. (3). Solving this system via Mathematica Software, the values of the constants g_0, g_1, g_{-1} and ω_0 are obtained as mentioned below, $\zeta = x + (2\alpha\omega_1 + \gamma)t$ and $\theta = -\omega_1 x + \omega_0 t$.

Case I

$$g_0 = \frac{\sqrt{\alpha}\eta_2}{\sqrt{2}\sqrt{\delta\omega_1 - \beta}}, \quad g_1 = \frac{\sqrt{2}\sqrt{\alpha}\eta_3}{\sqrt{\delta\omega_1 - \beta}}, \quad g_{-1} = 0$$

$$\omega_0 = \frac{1}{2}(-\alpha\eta_2^2 + 4\alpha\eta_1\eta_3 - 2\gamma\omega_1 - 2\alpha\omega_1^2). \tag{27}$$

Case II

$$g_0 = -\frac{\eta_2\sqrt{\alpha}}{\sqrt{2}\sqrt{\delta\omega_1 - \beta}}, \quad g_{-1} = -\frac{\sqrt{2}\sqrt{\alpha}\eta_1}{\sqrt{\delta\omega_1 - \beta}}, \quad g_1 = 0, \tag{28}$$

$$\omega_0 = \frac{1}{2}(-\alpha\eta_2^2 + 4\alpha\eta_1\eta_3 - 2\gamma\omega_1 - 2\alpha\omega_1^2). \tag{29}$$

From Case I, the solutions of Eq. (1) are obtained in the following sorts:

Family 1: when ${\eta_2}^2 - 4\eta_1\eta_3 > 0$ and $\eta_2\eta_3 \neq 0$ $(\eta_1\eta_3 \neq 0)$.

$$\mu_{2,1}(\zeta) = \left[g_0 - \frac{g_1}{2\eta_3}\left(\sqrt{\eta_2^2 - 4\eta_1\eta_3}\tanh\left[\frac{\sqrt{\eta_2^2 - 4\eta_1\eta_3}}{2}\zeta\right] + \eta_2\right)\right]e^{i\theta}. \tag{30}$$

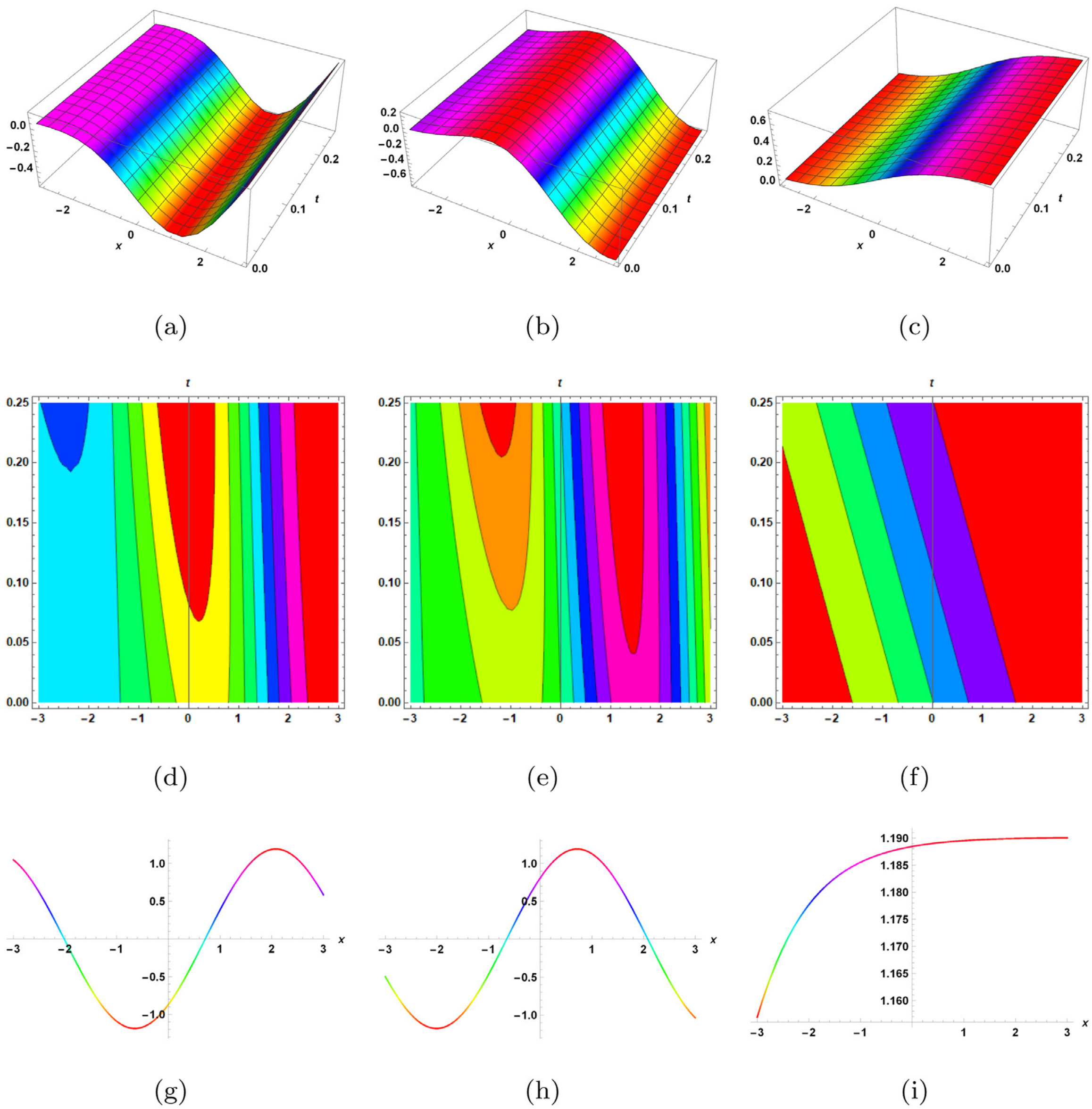

Fig. 4. $\mu_{1,8}(x,t): \alpha = 2,\ \beta = 0.65,\ \gamma = 1.95,\ \omega_1 = 1.15,\ F_1 = 1,\ \sigma = 0.45,\ \omega_0 = 0.75,\ \xi_0 = 2.25,\ \epsilon = 1,\ \kappa = 1$. In (g)-(i), $t = 1$.

$$\mu_{2,2}(\zeta) = \left[g_0 - \frac{g_1}{2\eta_3}\left(\sqrt{\eta_2^2 - 4\eta_1\eta_3}\coth\left[\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{2}\zeta\right]) + \eta_2\right)\right]e^{i\theta}. \tag{31}$$

$$\mu_{2,3}(\zeta) = \left[g_0 - \frac{g_1}{2\eta_3}\left(\eta_2 + \sqrt{\eta_2^2 - 4\eta_1\eta_3}\operatorname{sech}\left[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta\right]\right.\right.$$
$$\left.\left.\times(\sinh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta] \pm i)\right)\right]e^{i\theta}. \tag{32}$$

$$\mu_{2,4}(\zeta) = \left[g_0 - \frac{g_1}{2\eta_3}\left(\eta_2 + \sqrt{\eta_2^2 - 4\eta_1\eta_3}\operatorname{csch}[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta](\pm 1 + \cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta])\right)\right]e^{i\theta}$$
$$. \tag{33}$$

$$\mu_{2,5}(\zeta) = \left[g_0 - \frac{g_1}{4\eta_3}\left(\sqrt{\eta_2^2 - 4\eta_1\eta_3}\left(\tanh\left[\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{4}\zeta\right] \pm \coth\left[\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{4}\zeta\right]\right) + 2\eta_2\right)\right]e^{i\theta}. \tag{34}$$

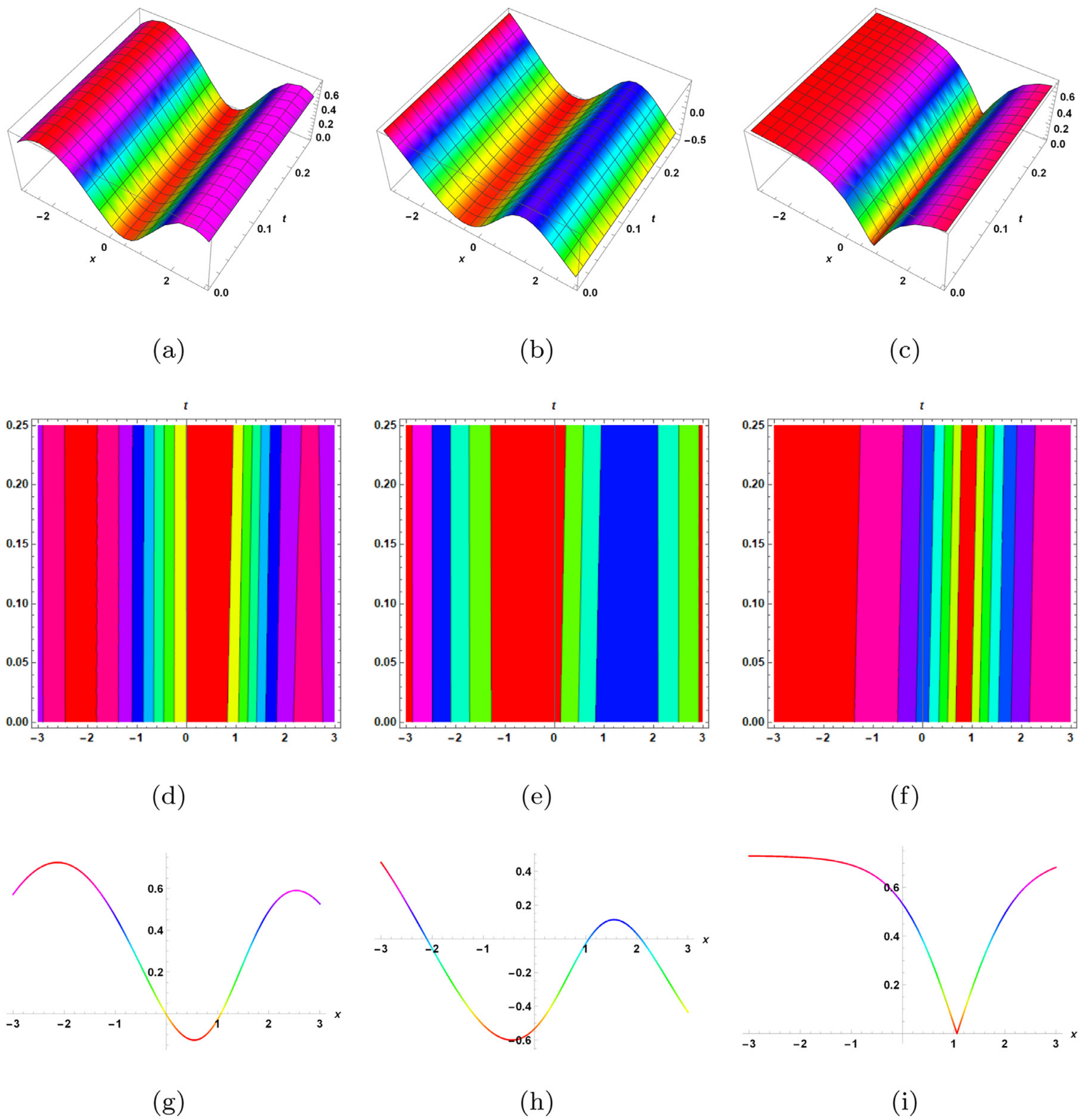

Fig. 5. $\mu_{2,6}(x,t):$ α=0.35, β=0.5, γ=-1, σ=1, η_1=0.2, η_2=1.5, η_3=-1, ω_1=0.75, A=1, B=2. In (g)-(i), $t=0.5$.

$$\mu_{2,6}(\zeta)=\left[\frac{g_1\left(\frac{\sqrt{(A^2+B^2)(\eta_2^2-4\eta_1\eta_3)}-A(\sqrt{\eta_2^2-4\eta_1\eta_3}(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta]))}{A\sinh(\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta)+B}-\eta_2\right)}{2\eta_3}+g_0\right]e^{i\theta}, \tag{35}$$

$$\mu_{2,7}(\zeta)=\left[\frac{g_1\left(-\frac{\sqrt{(B^2-A^2)(\eta_2^2-4\eta_1\eta_3)}+A(\sqrt{\eta_2^2-4\eta_1\eta_3}(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta]))}{A\sinh(\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta)+B}-\eta_2\right)}{2\eta_3}+g_0\right]e^{i\theta}, \tag{36}$$

where A and B are arbitrary constants such that $B^2-A^2>0$.

$$\mu_{2,8}(\zeta)=\left[g_0+\frac{2g_1\eta_1(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta])}{\sqrt{\eta_2^2-4\eta_1\eta_3}\sinh(\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{2}\zeta)-\eta_2(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta])}\right]e^{i\theta}. \tag{37}$$

$$\mu_{2,9}(\zeta)=\left[g_0+\frac{2g_1\eta_1\sinh(\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{2}\zeta)}{\sqrt{\eta_2^2-4\eta_1\eta_3}(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta])-\eta_2\sinh(\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{2}\zeta)}\right]e^{i\theta}. \tag{38}$$

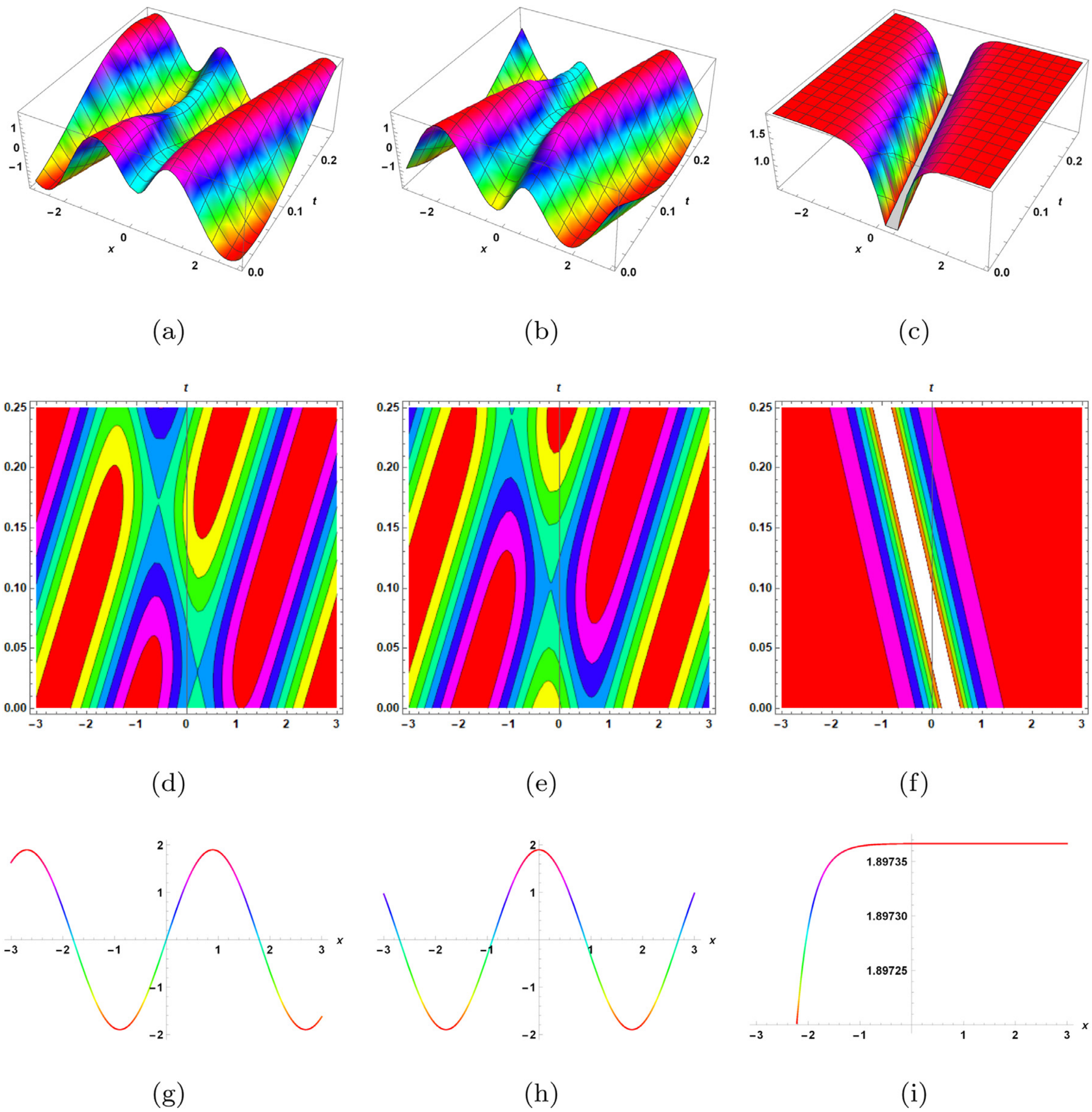

Fig. 6. $\mu_{2,9}(x,t)$: α=1, β=0.25, γ=2, σ=1, η_1=1, η_2=2, η_3=-2, ω_1=1.75. In (g)-(i), $t=1$.

$$\mu_{2,10}(\zeta) = \left[g_0 + \frac{2g_1\eta_1(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta])}{\sqrt{\eta_2^2-4\eta_1\eta_3}\sinh(\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta) - \eta_2(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta]) \pm i\sqrt{\eta_2^2-4\eta_1\eta_3}} \right] e^{i\theta}. \tag{39}$$

$$\mu_{2,11}(\zeta) = \left[g_0 + \frac{2g_1\eta_1 \sinh[\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{2}\zeta]}{\sqrt{\eta_2^2-4\eta_1\eta_3}(\cosh[\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta]) - \eta_2 \sinh(\sqrt{\eta_2^2-4\eta_1\eta_3}\zeta \pm \sqrt{\eta_2^2-4\eta_1\eta_3}} \right] e^{i\theta}.$$

$$\mu_{2,12}(\zeta) = \left[\frac{4g_1\eta_1 \sinh[\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{4}\zeta]\Phi)}{-2\eta_1 \sinh(\frac{\sqrt{\eta_2^2-4\eta_1\eta_3}}{4}\zeta)(\Phi) + 2\sqrt{\eta_2^2-4\eta_1\eta_3}(\Phi^2) - \sqrt{\eta_2^2-4\eta_1\eta_3}} + g_0 \right] e^{i\theta}.$$

Family 2: when $\eta_2^2 - 4\eta_1\eta_3 < 0$ and $\eta_2\eta_3 \neq 0(\eta_1\eta_3 \neq 0)$.

$$\mu_{2,13}(\zeta) = \left[g_0 - \frac{g_1}{2\eta_3}\left(-\sqrt{-\eta_2^2+4\eta_1\eta_3}\tan[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta] + \eta_2 \right) \right] e^{i\theta}. \tag{40}$$

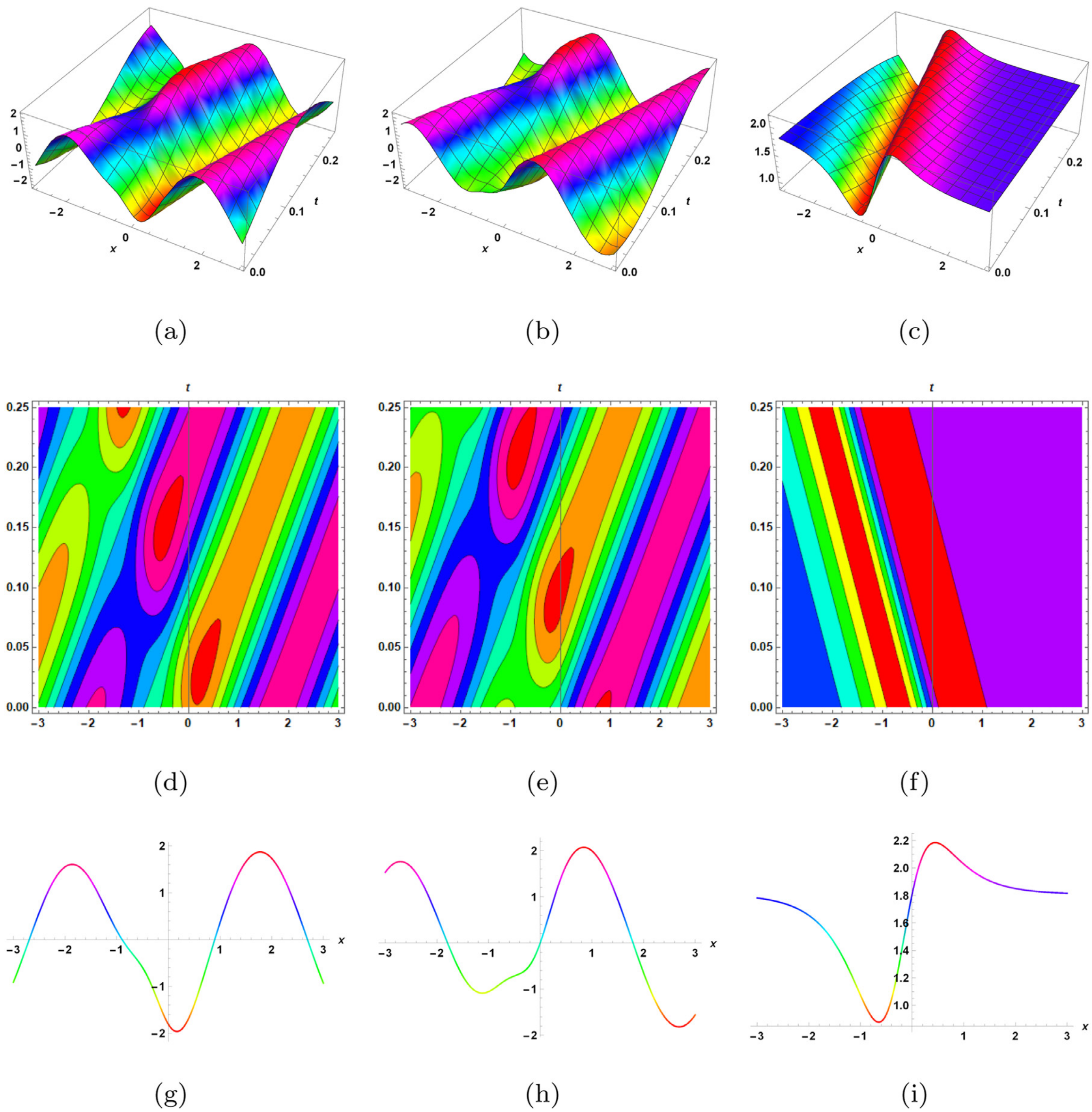

Fig. 7. $\mu_{2,11}(x,t)$: α=1.50, β=0.25, γ=1, σ=-1, η_1=1, η_2=-2, η_3=-2, ω_1=1.75. In (g)-(i), $t=0$.

$$\mu_{2,14}(\zeta)=\left[g_0-\frac{g_1}{2\eta_3}\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\cot[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta]+\eta_2\right)\right]e^{i\theta}. \tag{41}$$

$$\mu_{2,15}(\zeta)=\left[g_0+\frac{g_1}{2\eta_3}\left(-\eta_2+\sqrt{-\eta_2^2+4\eta_1\eta_3}\sec[\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta]\times(\pm1+\sin[\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta])\right)\right]e^{i\theta}. \tag{42}$$

$$\mu_{2,16}(\zeta)=\left[g_0-\frac{g_1}{2\eta_3}\left(\eta_2+\sqrt{-\eta_2^2+4\eta_1\eta_3}\csc[\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta](\pm1+\cos(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta))\right)\right]e^{i\theta}.$$

$$\mu_{2,17}(\zeta)=\left[g_0-\frac{g_1}{4\eta_3}\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\left(\tanh\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{4}\zeta\right]-\coth\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{4}\zeta\right]\right)-2\eta_2\right)\right]e^{i\theta}. \tag{43}$$

$$\mu_{2,18}(\zeta)=\left[\frac{g_1\left(\frac{\pm\sqrt{(B^2-A^2)(-\eta_2^2+4\eta_1\eta_3)}-A(\sqrt{-\eta_2^2+4\eta_1\eta_3}(\cos(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta)))}{A\sinh(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta)+B}-\eta_2\right)}{2\eta_3}+g_0\right]e^{i\theta},$$

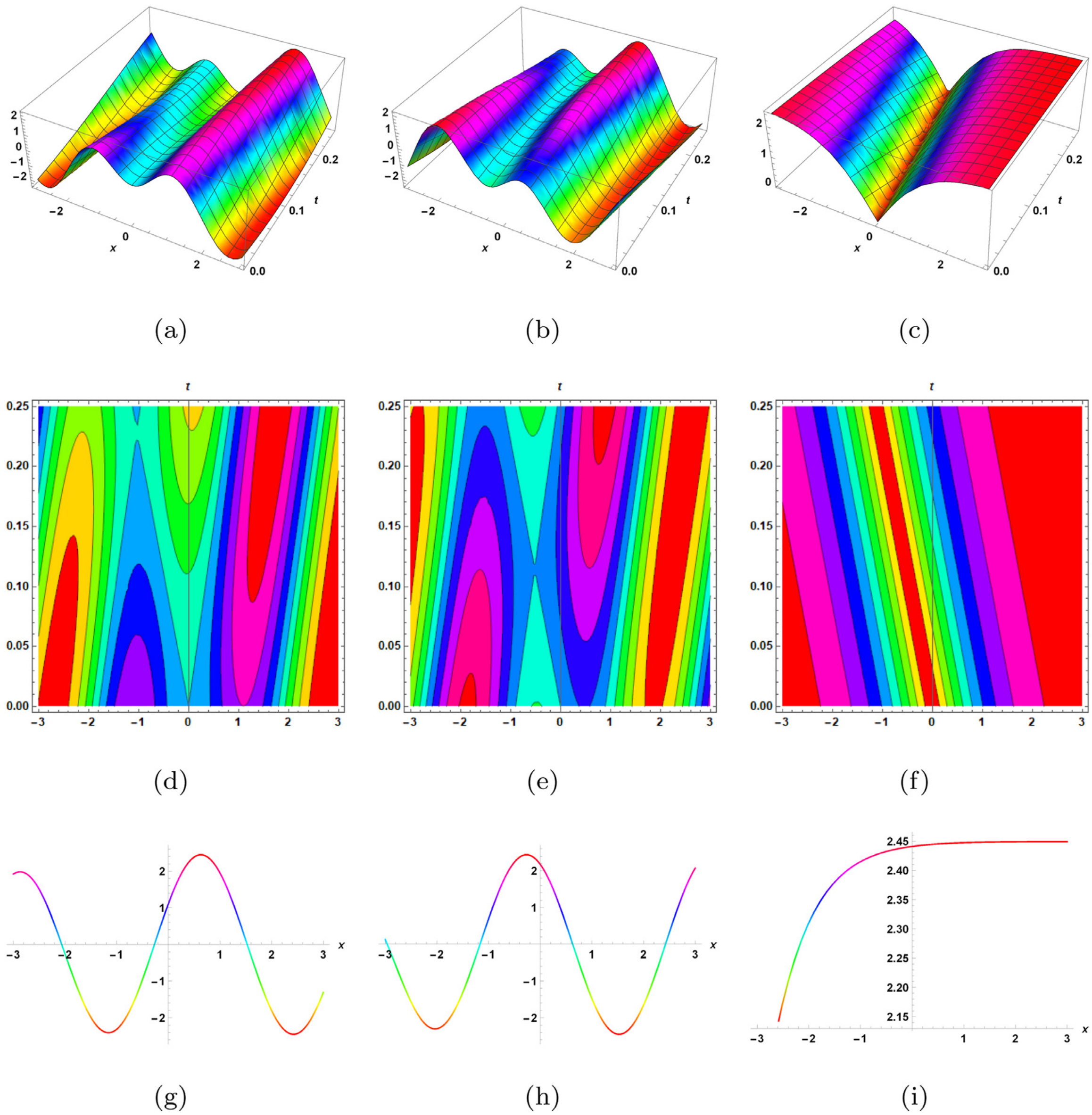

Fig. 8. $\mu_{2,13}(x,t):\ \alpha{=}1,\ \beta{=}2.5,\ \gamma{=}1,\ \sigma{=}{-}2,\ \eta_1{=}0.5,\ \eta_2{=}{-}2,\ \eta_3{=}1,\ \omega_1{=}1.75$. In (g)-(i), $t=1$.

$$\mu_{2,19}(\zeta)=\left[\frac{g_1\left(-\frac{\pm\sqrt{(B^2-A^2)(-\eta_2^2+4\eta_1\eta_3)}+A\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)\right)}{A\sinh\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)+B}-\eta_2\right)}{2\eta_3}+g_0\right]e^{i\theta},$$

where A and B are arbitrary constants such that $B^2-A^2>0$.

$$\mu_{2,20}(\zeta)=\left[g_0-\frac{2g_1\eta_1\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)}{\sqrt{-\eta_2^2+4\eta_1\eta_3}\sin\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta\right]+\eta_2\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)}\right]e^{i\theta}. \tag{44}$$

$$\mu_{2,21(\zeta)}=\left[g_0+\frac{2g_1\eta_1\sin\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta\right]}{\sqrt{-\eta_2^2+4\eta_1\eta_3}\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)-\eta_1\sin\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta\right]}\right]e^{i\theta}. \tag{45}$$

$$\mu_{2,22}(\zeta)=\left[g_0-\frac{2g_1\eta_1\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)}{\sqrt{-\eta_2^2+4\eta_1\eta_3}\left(\pm1+\sin\left[\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right]\right)+\eta_2\left(\cos\left(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta\right)\right)}\right]e^{i\theta}. \tag{46}$$

$$\mu_{2,23}(\zeta) = \left[g_0 + \frac{2g_1\eta_1 \sin\left[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{2}\zeta\right]}{\sqrt{-\eta_2^2+4\eta_1\eta_3}(\cos(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta)) - \eta_1 \sin[\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta] \pm \sqrt{-\eta_2^2+4\eta_1\eta_3}} \right] e^{i\theta}.$$

$$\mu_{2,24}(\zeta) = \left[\frac{4g_1\eta_1 \sin[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{4}\zeta](\cos(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta))}{-2\eta_1 \sin[\frac{\sqrt{-\eta_2^2+4\eta_1\eta_3}}{4}\zeta](\cos(\sqrt{-\eta_2^2+4\eta_1\eta_3}\zeta)) + 2\sqrt{-\eta_2^2+4\eta_1\eta_3}(\Phi^2) - \sqrt{-\eta_2^2+4\eta_1\eta_3}} + g_0 \right] \times e^{i\theta}. \tag{47}$$

Family 3: when $\eta_1 = 0$ and $\eta_2\eta_3 \neq 0$.

$$\mu_{2,25}(\zeta) = \left[g_0 - \frac{g_1 c_1 \eta_2}{\eta_3(c_1 - \sinh(\eta_2\zeta) + \cos[\eta_2\zeta])} \right] e^{i\theta}, \tag{48}$$

$$\mu_{2,26}(\zeta) = \left[g_0 - \frac{g_1\eta_2(\sinh(\eta_2\zeta) + \cos(\eta_2\zeta))}{\eta_3(c_1 + \sinh(\eta_2\zeta) + \cos(\eta_2\zeta))} \right] e^{i\theta}, \tag{49}$$

where c_1 is an arbitrary constant.

Family 4: when $\eta_3 \neq 0$ and $\eta_2 = \eta_1 = 0$

$$\mu_{2,27}(\zeta) = \left[g_0 - \frac{g_1}{c_2 + \eta_3\zeta} \right] e^{i\theta}, \tag{50}$$

. where c_2 is an arbitrary constant.

We mention that the general solutions of Eq. (1) can be obtained by the same techniques that was used in Case II but it will not be stated here.

3. Discussion and results

To demonstrate a set of new traveling wave solutions for Eq. (1), Mathematica 11.0 is employed to visualize the behavior of solitons with the aid of the 3D, 2D and contour plots for the various sets of parameters. We mention that in all Figs. 1–8: (a) and (d) represent the real part of the solution, (b) and (e) represent the imaginary part of the solution, and (c) and (f) represent the absolute value of the solution.

4. Conclusions

In this article, we have been employed the AEM and the REM methods to acquire exact and solitary wave solutions for the P-NLSE which is crucial for analyzing pulse propagation in optical fiber-based communications systems. A collection of different structures for these solutions were retrieved in the form of bright, dark, singular soliton, hyperbolic functions and trigonometric functions solutions that were investigated in Figs. 1–8. Our results demonstrated that the proposed methods are concise, effective and can be employed on other nonlinear evolution equation. Moreover, we visualize the physical behavior for some of the obtained results in 3D, 2D and counter plots by choosing appropriate values of the free parameters existed in these solutions. This research could be useful for future research in terms of solution technique and precision, especially in solving the C-NLEEs, which have a high level of efficacy in the nonlinear arena.

Declaration of Competing Interest

The authors have declared no conflict of interest.

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