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Dynamics of analytical solutions and Soliton-like profiles for the nonlinear complex-coupled Higgs field equation

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ABSTRACT

In this work, the closed-form analytical solutions have been generated for the complex coupled Higgs field equation through newly two efficient techniques, namely the auxiliary equation method and the extended Sinh-Gordon expansion approach. The equation under consideration introduces a quantum field, often referred to as the Higgs field, to elucidate the mechanism responsible for generating mass in gauge bosons. The approaches used achieve an extensive variety of solutions, including rational functions, hyperbolic functions, exponential functions, trigonometric functions, and Jacobian elliptical functions. Moreover, to understand the properties of the attained solutions, combined 3D-graphics and contour plots are demonstrated for specified parametric values. In particular, it has been extensively discussed that wave position and category changes with respect to different parameters for some solutions. Various attractive soliton-like solutions have been extracted, such as bell-shaped, travelling waves, periodic solitary waves, singular kink-shaped solitons, and many others. All derived solutions are substituted into the original model to ensure their accuracy. The derived solitons can be employed to investigate numerous complex phenomena associated with this model. Soliton-like solutions and travelling waves are incredible phenomena seen in a variety of domains of physics, including nonlinear waves, nonlinear optics, nonlinear dynamics, quantum physics, dusty plasma physics, engineering physics, and other nonlinear sciences fields.

1. Introduction

In recent years, soliton-like solutions and travelling waves of the nonlinear partial differential equations (NPDEs) have become popular subjects in the disciplines of physical engineering and nonlinear sciences because of their potential applications. Exact closed-form solutions can take numerous forms, including travelling wave solutions, solitons, solitary waves, and many others.[1–7] In the study of many scientific and technological fields such as fluid dynamics, engineering physics, nonlinear optics, biology, plasma physics, nonlinear physical science, condensed matter physics, applied mathematics, etc., nonlinear partial differential equations (NPDEs) are of great significance. Therefore, a deep investigation of PDEs and the construction of new methods[8–14] to find closed analytical solutions are essential for many scientific and technological advancements. Expertise in solving PDEs helps practitioners develop the latest technologies, make predictions, and design new experiments. Consequently, in recent decades, numerous advanced methodologies have been developed, extended, and employed effectively by many researchers. In this direction few proposed approaches[15,16] and mathematical tools include, the direct algebraic approach,[17] the Jacobi elliptic function expansion approach,[18–20] the $\frac{G'}{G}$-expansion method,[21] the bifurcation method,[22] the generalized auxiliary equation approach,[23] the simplest equation method,[24] the generalized Kudryashov method,[25,26] the Bäcklund transformation method,[27] the generalized Riccati equation method,[28,29] Hirota's bilinear method,[30] the generalized exponential rational function method,[31] the inverse scattering method,[32] F-expansion technique,[33] Lie-symmetry reduction approach,[34–36] and many others.

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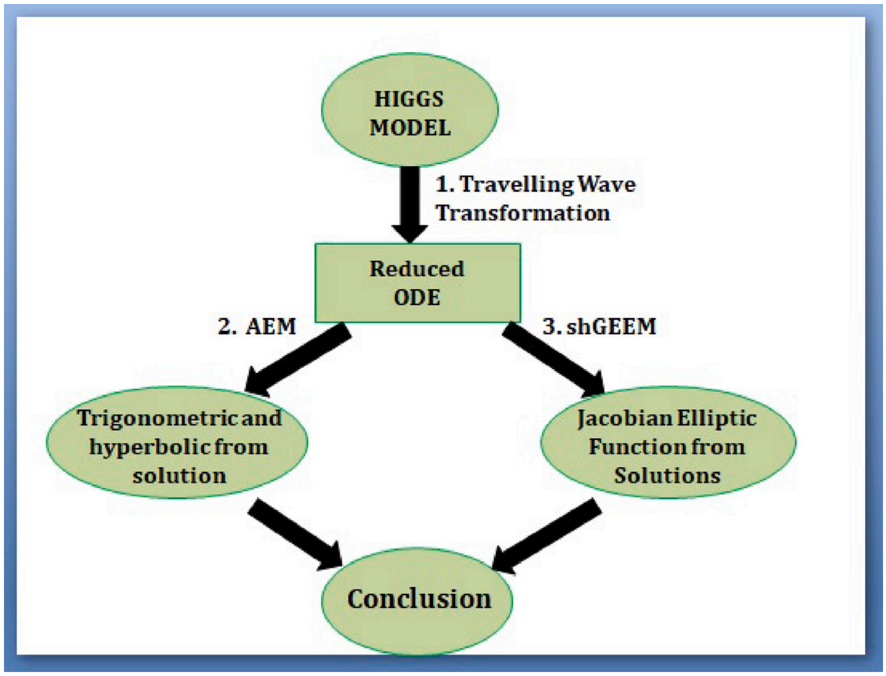


Fig. 1. Visual representation illustrating the structure of the paper.

In the article, we consider the nonlinear complex coupled Higgs field equation (NLccHF) which is associated with the classical Klein–Gordon equation.[37] The NLccHF equation incorporates a quantum field, often referred to as the Higgs field, to elucidate the mechanism by which mass is generated for gauge bosons. This equation can be expressed in the following form:

$$\phi_{tt} - \phi_{xx} - \alpha\phi + \beta|\phi|^2\phi - 2\phi\psi = 0$$
$$\psi_{tt} + \psi_{xx} - \beta(|\phi|^2)_{xx} = 0. \quad (1)$$

Understanding the Higgs mechanism is essential for grasping the theory that explains how gauge bosons acquire mass in the conventional framework of particle physics. In the mechanism, quantum field theory is used to construct physical models of subatomic particles. The masses of $W^{\pm}$ and Z weak gauge bosons are generated through electroweak symmetry breaking. Because of the significance and notable uses of the nonlinear complex coupled Higgs field Eq. (1), a lot of work has been done by researchers on the solutions of this model using various tactics. This equation was first investigated by Tajiri[38] in 1983. Salam Subhaschandra Singh derived soliton solutions of the Coupled Higgs Field Equation via the Trial Equation Method.[39] Kumar et al.[40] reported symmetry reductions and exact solutions for the Higgs equation and the Hamiltonian amplitude equation. Abdelkawy et al.[41] utilized the Tanh method to investigate several coupled nonlinear evolution equations in the complex domain, such as the generalized complex Higgs field equations. B. Talukdar et al.[42] converted the coupled Higgs equations into Hamiltonian form and further examined the resulting equation using dynamical system theory.

Considering the context provided earlier, we have applied the auxiliary equation method (AEM) and the extended sinh-Gordon equation expansion method (shGEEM) to the specified model, both of which have not been utilized on the stated model in previous literature. These methods are based on the transformation of the PDEs into ODEs using the travelling wave transformation. Afterwards, the trial solutions of the obtained ODE are considered as per the choice of method. We derive a set of algebraic equations that can be solved using a variety of computational resources currently available. As a result, we attain the precise travelling wave solution for the model under consideration. Employing the described methods, we have generated a considerable number of solitary wave and periodic wave solutions. Additionally, we have discovered solutions expressed in terms of Jacobian elliptic functions, exponential, trigonometric, hyperbolic, and rational functions, enhancing the effectiveness and novelty of our work. Furthermore, we have skillfully depicted the dynamics of these solutions using 3D plots and combined 3D and contour plots, varying the involved parameters and time values .

The paper is organized in the following pattern(see Fig. 1): An introduction for the nonlinear complex coupled Higgs field equation is given in Section “Introduction”. In Section “Description of algorithms”, a brief introduction of the method used is given. In Section “Implementation of methods”, we compute various analytical solutions to the considered model. In Section “Graphical illustrations of the solutions”, the obtained solutions are graphically analysed in details. Some concluding remarks are given in the Section “Conclusion”.

2. Overview of algorithms

With two independent variables, x and t, consider the following general nonlinear PDE:

$$\Sigma(\phi, \phi_x, \phi_t, \phi_{xx}, \phi_{xt}, \phi_{tt}, \dots) = 0, \quad (2)$$

where Σ is a polynomial of $\phi(x,t)$ and its partial derivatives containing the highest order derivatives and nonlinear terms. The space and time coordinates x and t are combined by the following wave transformation Υ,

$$\phi(x,t) = \Phi(\Upsilon),\ \Upsilon = px + \mu t, \quad (3)$$

in order to transform equation (2) into ordinary differential equation as follows:

$$\mathbb{M}(\Phi, \Phi', \Phi'', \Phi''', \ldots) = 0, \tag{4}$$

where differentiation with regard to Y is shown by the prime($'$).

2.1. Methodology of Auxiliary equation Method (AEM)

Step 1: In view of this method, the exact solutions of Eq. (4) is assumed in the following expansion form[43]:

$$\Phi(Y) = \sum_{i=0}^{p} B_i a^{ih(Y)}, \tag{5}$$

where B_i, $(i = 0, 1, 2, \ldots, p)$ are parameters to be acquired subsequently such that B_p is non-zero and the homogeneous balance approach determines the value of p between the highest order derivative and the nonlinear term. Here, $h(Y)$ is the solution of following equation

$$\ln(a)h'(Y) = \rho a^{-h(Y)} + \sigma + n a^{h(Y)}. \tag{6}$$

Step 2: Inserting Eqs. (5) in association with (6) in Eq. (4) including the value of p obtained above, we will get an algebraic expression in powers of $a^{h(Y)}$.

Step 3: Upon solving this family of algebraic equations we derive the values for $B_i, (i = 0, 1, 2, \ldots, p)$ and the values of other needed constraints. The some known solutions of Eq. (6) is publicized in Akbar et al..[43]

Step 4: Restoring the values of B_i, σ, ρ, n and $h(Y)$ in solution (5), one can construct the wide spectrum of closed-form travelling wave solutions of Eq. (2).

2.2. A method of generalized sinh-Gordon equation expansion approach

Step 1: The method explains how to solve Eq. (4) in the following form as Ref. 44

$$\Phi(Y) = A_0 + \sum_{i=1}^{n} [A_i \sinh w(Y) + B_i \cosh w(Y)]^i, \tag{7}$$

where the following equation is satisfied by $w(Y)$

$$w' = \sqrt{p + q \sinh^2(w)}. \tag{8}$$

Deduction of Eq. (8) can be obtained from Ref. 44. In addition, the following multiple solutions of Eq. (8) are obtained for different unique values of parameters p and q:

Case(i): Assuming $p = 0$ and $q = 1$, Eq. (8) transforms into the following first order ODE:

$$w'(Y) = \sinh(w(Y)), \tag{9}$$

which has the solutions

$$\sinh(w(Y)) = \pm i sech(Y) \text{ or } \cosh(w(Y)) = \pm \tanh(Y), \tag{10}$$

and

$$\sinh(w(Y)) = \pm csch(Y) \text{ or } \cosh(w(Y)) = \pm \coth(Y). \tag{11}$$

Case(ii): Assuming $p = 1$ and $q = 1$, Eq. (8) get transformed to

$$w'(Y) = \cosh(w(Y)), \tag{12}$$

which provides

$$\sinh(w(Y)) = tan(Y) \text{ or } \cosh(w(Y)) = \pm \sec(Y) \tag{13}$$

and

$$\sinh(w(Y)) = -cot(Y) \text{ or } \cosh(w(Y)) = \pm \csc(Y) \tag{14}$$

Case(iii): Assuming $p = 1 - m^2$ and $q = 1$, Eq. (8) transforms to

$$w'(Y) = \sqrt{\sinh^2(w(Y)) + 1 - m^2}, \tag{15}$$

which gives

$$\sinh(w(Y)) = cs(Y, m) \text{ or } \cosh(w(Y)) = ns(Y, m). \tag{16}$$

Case(iv): By assuming $p = q = 1 - m^2$, Eq. (8) converted into

$$w'(Y) = \sqrt{(1 - m^2)\sinh^2(w(Y)) + 1}, \tag{17}$$

which gives

$$\sinh(w(Y)) = sc(Y, m) \text{ or } \cosh(w(Y)) = nc(Y, m). \tag{18}$$

Step 2: The parameter n is determined by balancing the dominant power of the nonlinear terms and the highest order derivatives in Eq. (4).
Step 3: Using Eq. (7) into Eq. (4) yields a nonlinear algebraic expression in $w'^r(Y)\sinh^s(w(Y))\cosh^t(w(Y))(r=0,1;s=0,1;t=0,1)$. On comparing the coefficients of $w'^r(Y)\sinh^s(w(Y))\cosh^t(w(Y))$ to zero separately, we shall acquire systems of equations.
Step 4: Then, using several packages of symbolic computing tools on the resulting set of algebraic equations, we determine the parameters values $A_0, A_i, B_i, \alpha_1, \mu_1$.
Step 5: Upon reinstating the parameter values obtained from cases (i) to (iv) and reapplying the derived solutions, one can obtain the solution for Eq. (2).

3. Implementation of methods

In this section, we investigate the novel and further generic travelling wave solutions of the governing equation (1) through the implementation of the Auxiliary equation method. In order to accomplish this, we consider the transformation as

$$\phi(x,t)=\exp(i\theta)U(\xi) \text{ and } \psi(x,t)=V(\xi), \text{ where } \xi=x+vt, \text{ and } \theta=vrx+rt. \tag{19}$$

Then, Eq. (1) yields,

$$\begin{aligned}&(v^2r^2-\alpha-r^2)U(\xi)+(v^2-1)U''(\xi)+\beta U(\xi)^3-2U(\xi)V(\xi)=0,\\&2\beta(U'(\xi))^2+2\beta U(\xi)U''(\xi)-(1+v^2)V''(\xi)=0.\end{aligned} \tag{20}$$

Integrating second part of Eq. (20) two times yields,

$$V(\xi)=\frac{\beta}{v^2+1}U(\xi)^2. \tag{21}$$

Substituting (21) in first equation of (20) yields

$$\left(v^2+1\right)\left(-\alpha+v^2r^2-r^2\right)U(\xi)+\beta\left(v^2-1\right)U(\xi)^3+\left(v^2-1\right)\left(v^2+1\right)U''(\xi)=0. \tag{22}$$

3.1. Application of new Auxiliary equation method(AEM) to the aforementioned equation

The homogeneous balance approach between the highest order derivative $U''(\xi)$ and the nonlinear term $U^3(\xi)$ determines the value of $p=1$. Following the AEM approach, and using the value of p together with Eq. (5), the solution of (22) is of the form:

$$U(\xi)=B_0+B_1a^{h(\xi)}. \tag{23}$$

We attain the following algebraic expression in the power of $a^{h(\xi)}$ by substituting Eq. (23) along with (6) into Eq. (22):

$$\begin{aligned}&-\alpha B_1v^2a^{h(\xi)}-\alpha B_1a^{h(\xi)}+3\beta B_0^2B_1v^2a^{h(\xi)}+3\beta B_0B_1^2v^2a^{2h(\xi)}+\beta B_1^3v^2a^{3h(\xi)}-3\beta B_0^2B_1a^{h(\xi)}\\&-3\beta B_0B_1^2a^{2h(\xi)}-\beta B_1^3a^{3h(\xi)}+B_1v^4\sigma^2a^{h(\xi)}+2B_1v^4n^2a^{3h(\xi)}-2B_1n^2a^{3h(\xi)}+2B_1v^4n\rho a^{h(\xi)}\\&+3B_1v^4n\sigma a^{2h(\xi)}-2B_1n\rho a^{h(\xi)}-3B_1n\sigma a^{2h(\xi)}-B_1\sigma^2a^{h(\xi)}+B_1v^4r^2a^{h(\xi)}-B_1r^2a^{h(\xi)}\\&-\alpha B_0v^2-\alpha B_0+\beta B_0^3v^2-\beta B_0^3+B_1v^4\rho\sigma-B_1\rho\sigma+B_0v^4r^2-B_0r^2.\end{aligned} \tag{24}$$

Equalizing the coefficient of like powers of $a^{h(\xi)}$ of Eq. (24) provides following system of algebraic equations:

$$\begin{aligned}&-\alpha B_0v^2-\alpha B_0+\beta B_0^3v^2-\beta B_0^3+B_1v^4\rho\sigma-B_1\rho\sigma+B_0v^4r^2-B_0r^2=0,\\&-\alpha B_1v^2-\alpha B_1+3\beta B_0^2B_1v^2-3\beta B_0^2B_1+B_1v^4\sigma^2+2B_1v^4n\rho-2B_1n\rho+B_1v^4r^2-B_1r^2-B_1\sigma^2=0,\\&3\beta B_0B_1^2v^2-3\beta B_0B_1^2+3B_1v^4n\sigma-3B_1n\sigma=0,\\&\beta B_1^3v^2-\beta B_1^3+2B_1v^4n^2-2B_1n^2=0.\end{aligned}$$

Solving the above algebraic equation via MATHEMATICA, we attain

$$B_0=-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}},\quad B_1=-\frac{i\sqrt{2}n\sqrt{v^2+1}}{\sqrt{\beta}},\quad r=\frac{\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}. \tag{25}$$

We accomplish a number of exact closed-form solutions to the nonlinear complex coupled Higgs field equation (1) by using (25) as follows:

- When $\sigma^2-4\rho n<0$ and $n\neq 0$;

Using (25) into solution (23), we attain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\begin{aligned}\phi_{1,1}(x,t)=&\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\\&\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{\left(i\sqrt{2}\sqrt{v^2+1}n\right)\left(\frac{\sqrt{4n\rho-\sigma^2}\tan\left(\frac{1}{2}\sqrt{4n\rho-\sigma^2}(vt+x)\right)}{2n}-\frac{\sigma}{2n}\right)}{\sqrt{\beta}}\right),\end{aligned}$$

$$\psi_{1,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\frac{\sqrt{4n\rho-\sigma^2}\tan\left(\frac{1}{2}\sqrt{4n\rho-\sigma^2}(vt+x)\right)}{2n}-\frac{\sigma}{2n}\right)}{\sqrt{\beta}}\right)^2,$$

or

$$\phi_{2,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)$$
$$\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(-\frac{\sigma}{2n}-\frac{\sqrt{4n\rho-\sigma^2}\cot\left(\frac{1}{2}\sqrt{4n\rho-\sigma^2}(vt+x)\right)}{2n}\right)}{\sqrt{\beta}}\right),$$
$$\psi_{2,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(-\frac{\sigma}{2n}-\frac{\sqrt{4n\rho-\sigma^2}\cot\left(\frac{1}{2}\sqrt{4n\rho-\sigma^2}(vt+x)\right)}{2n}\right)}{\sqrt{\beta}}\right)^2.$$

- When $\sigma^2-4\rho n>0$ and $n\neq 0$;

Using (25) into solution (23), we achieve the closed-form solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{3,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)$$
$$\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(-\frac{\sigma}{2n}-\frac{\sqrt{\sigma^2-4n\rho}\tanh\left(\frac{1}{2}\sqrt{\sigma^2-4n\rho}(vt+x)\right)}{2n}\right)}{\sqrt{\beta}}\right),$$
$$\psi_{3,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(-\frac{\sigma}{2n}-\frac{\sqrt{\sigma^2-4n\rho}\tanh\left(\frac{1}{2}\sqrt{\sigma^2-4n\rho}(vt+x)\right)}{2n}\right)}{\sqrt{\beta}}\right)^2,$$

or

$$\phi_{4,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)$$
$$\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(-\frac{\sigma}{2n}-\frac{\sqrt{\sigma^2-4n\rho}\coth\left(\frac{1}{2}\sqrt{\sigma^2-4n\rho}(vt+x)\right)}{2n}\right)}{\sqrt{\beta}}\right),$$
$$\psi_{4,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\frac{\sqrt{\sigma^2-4n\rho}\coth\left(\frac{1}{2}\sqrt{\sigma^2-4n\rho}(vt+x)\right)}{2n}-\frac{\sigma}{2n}\right)}{\sqrt{\beta}}\right)^2.$$

Similarly, we can establish more solution by taking particular values in above two cases:

(i) For $r=-p$ and $r=p$ in solution $(\phi_{1,1}(x,t),\psi_{1,1}(x,t))$ and $(\phi_{2,1}(x,t),\psi_{2,1}(x,t))$, four more solutions can be obtained.
(ii) For $r=-p$ and $r=p$ in solution $(\phi_{3,1}(x,t),\psi_{3,1}(x,t))$ and $(\phi_{4,1}(x,t),\psi_{4,1}(x,t))$ another four more solutions can be obtained.

- When $\sigma^2=4\rho n$;

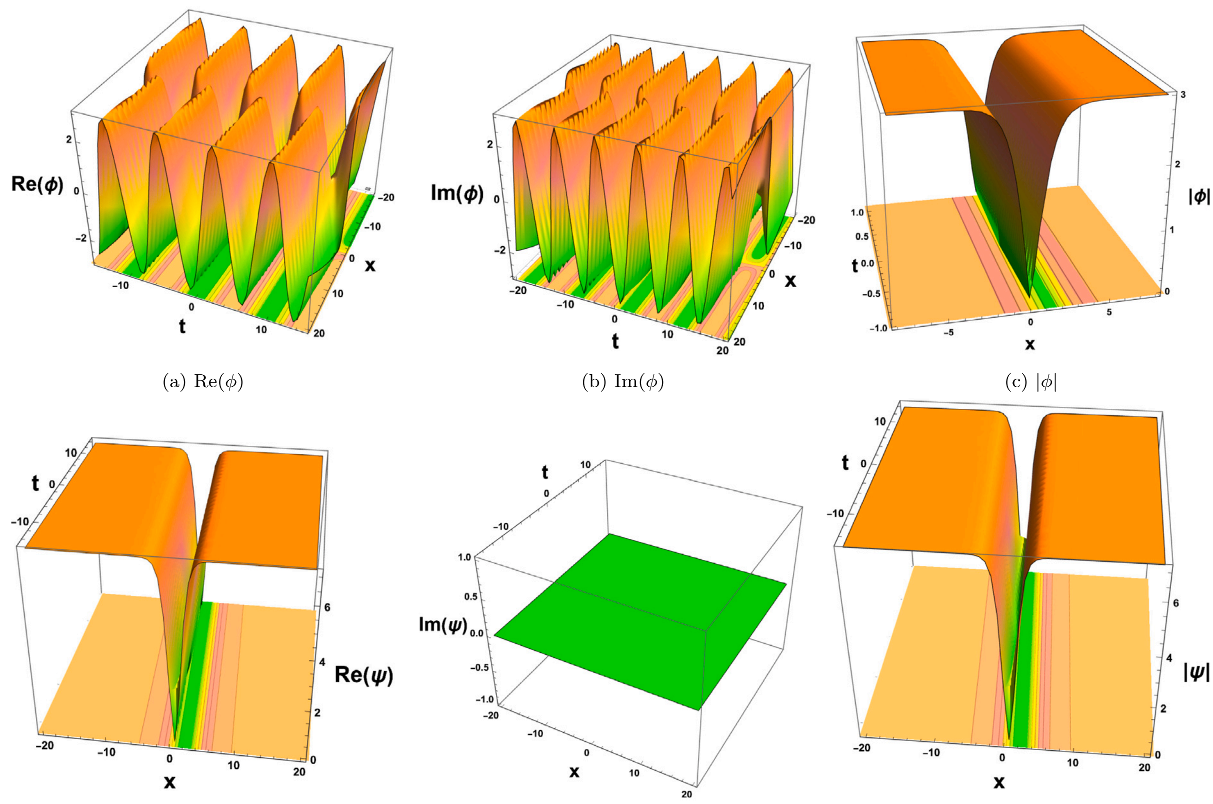
Using (25) into solution (23), we attain the closed-form solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{5,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n\rho+4n\rho-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)$$
$$\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{v^2+1}(\sigma(vt+x)-2)}{\sqrt{2}\sqrt{\beta}(vt+x)}\right),$$
$$\psi_{5,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{v^2+1}(\sigma(vt+x)-2)}{\sqrt{2}\sqrt{\beta}(vt+x)}\right)^2.$$

- When $\rho n<0$, $\sigma=0$ and $r\neq 0$;

Using (25) into solution (23), we achieve the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{6,1}(x,t)=\sqrt{-\frac{\rho}{n}}\tanh\left(\sqrt{-n\rho}(vt+x)\right)\left(-\exp\left(i\left(\frac{t\sqrt{2\alpha-4v^2n\rho+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha-4v^2n\rho+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\right),$$



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 2. Graphical formation for solution $\phi_{6,1}(x,t)$ and $\psi_{6,1}(x,t)$ with specified parameters value set at $\alpha = 0.65$, $n = 0.28$, $\rho = -2.5$, $\nu = 0.05$ and $\beta = 0.8$..

$$\psi_{6,1}(x,t) = -\frac{\beta\rho\tanh^2\left(\sqrt{-n\rho}(\nu t + x)\right)}{\left(\nu^2+1\right)n},$$

and

$$\phi_{7,1}(x,t) = \sqrt{-\frac{\rho}{n}}\coth\left(\sqrt{-n\rho}(\nu t + x)\right)\left(-\exp\left(i\left(\frac{t\sqrt{2\alpha - 4\nu^2 n\rho + 4n\rho}}{\sqrt{2}\sqrt{\nu^2-1}} + \frac{\nu x\sqrt{2\alpha - 4\nu^2 n\rho + 4n\rho}}{\sqrt{2}\sqrt{\nu^2-1}}\right)\right)\right),$$

$$\psi_{7,1}(x,t) = -\frac{\beta\rho\coth^2\left(\sqrt{-n\rho}(\nu t + x)\right)}{\left(\nu^2+1\right)n},$$

- When $\rho = -n$ and $\sigma = 0$;

Using (25) into solution (23), we attain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{8,1}(x,t) = -\frac{i\sqrt{2}\sqrt{\nu^2+1}n\left(e^{-2n(\nu t + x)}+1\right)\exp\left(i\left(\frac{t\sqrt{2\alpha+4\nu^2n^2-4n^2}}{\sqrt{2}\sqrt{\nu^2-1}} + \frac{\nu x\sqrt{2\alpha+4\nu^2n^2-4n^2}}{\sqrt{2}\sqrt{\nu^2-1}}\right)\right)}{\sqrt{\beta}\left(e^{-2n(\nu t+x)}-1\right)},$$

$$\psi_{8,1}(x,t) = -\frac{2n^2\left(e^{-2n(\nu t+x)}+1\right)^2}{\left(e^{-2n(\nu t+x)}-1\right)^2}.$$

- When $\rho = n = 0$;

Using (25) into solution (23), we obtain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{9,1}(x,t) = \exp\left(i\left(\frac{t\sqrt{2\alpha+\nu^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{\nu^2-1}} + \frac{\nu x\sqrt{2\alpha+\nu^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{\nu^2-1}}\right)\right)(\sinh(\sigma(\nu t+x)) + \cosh(\sigma(\nu t+x))),$$

$$\psi_{9,1}(x,t) = \frac{\beta(\sinh(\sigma(\nu t+x)) + \cosh(\sigma(\nu t+x)))^2}{\nu^2+1}.$$

- When $\rho = \sigma = K$ and $n = 0$;

Using (25) into solution (23), we accomplish the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{10,1}(x,t) = \left(e^{K(\nu t+x)}-1\right)\exp\left(i\left(\frac{t\sqrt{2\alpha+K^2\nu^2-K^2}}{\sqrt{2}\sqrt{\nu^2-1}} + \frac{\nu x\sqrt{2\alpha+K^2\nu^2-K^2}}{\sqrt{2}\sqrt{\nu^2-1}}\right)\right),$$

$$\psi_{10,1}(x,t)=\frac{\beta\left(e^{K(vt+x)}-1\right)^2}{v^2+1}.$$

- When $\sigma=n=K$ and $\rho=0$;

Using (25) into solution (23), we obtain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{11,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+K^2v^2-K^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+K^2v^2-K^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\left(-\frac{iK\sqrt{v^2+1}}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}K\sqrt{v^2+1}e^{K(vt+x)}}{\sqrt{\beta}\left(1-e^{K(vt+x)}\right)}\right),$$

$$\psi_{11,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{iK\sqrt{v^2+1}}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}K\sqrt{v^2+1}e^{K(vt+x)}}{\sqrt{\beta}\left(1-e^{K(vt+x)}\right)}\right)^2.$$

- When $\sigma=\rho+n$;

Using (25) into solution (23), we achieve the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{12,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2(n+\rho)^2-4v^2n\rho-(n+\rho)^2+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2(n+\rho)^2-4v^2n\rho-(n+\rho)^2+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\left(-\frac{i\sqrt{v^2+1}(n+\rho)}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\rho e^{(\rho-n)(vt+x)}-1\right)}{\sqrt{\beta}\left(1-ne^{(\rho-n)(vt+x)}\right)}\right),$$

$$\psi_{12,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}(n+\rho)}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\rho e^{(\rho-n)(vt+x)}-1\right)}{\sqrt{\beta}\left(1-ne^{(\rho-n)(vt+x)}\right)}\right)^2.$$

- When $\sigma=-(\rho+n)$;

Using (25) into solution (23), we attain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{13,1}(x,t)=\left(-\frac{i\sqrt{v^2+1}(-n-\rho)}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\rho-e^{(\rho-n)(vt+x)}\right)}{\sqrt{\beta}\left(n-e^{(\rho-n)(vt+x)}\right)}\right)\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2(-n-\rho)^2-4v^2n\rho-(-n-\rho)^2+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2(-n-\rho)^2-4v^2n\rho-(-n-\rho)^2+4n\rho}}{\sqrt{2}\sqrt{v^2-1}}\right)\right),$$

$$\psi_{13,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}(-n-\rho)}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\left(\rho-e^{(\rho-n)(vt+x)}\right)}{\sqrt{\beta}\left(n-e^{(\rho-n)(vt+x)}\right)}\right)^2.$$

- When $\rho=0$;

Using (25) into solution (23), we observe the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{14,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\sigma e^{\sigma(vt+x)}}{\sqrt{\beta}\left(1-ne^{\sigma(vt+x)}\right)}\right),$$

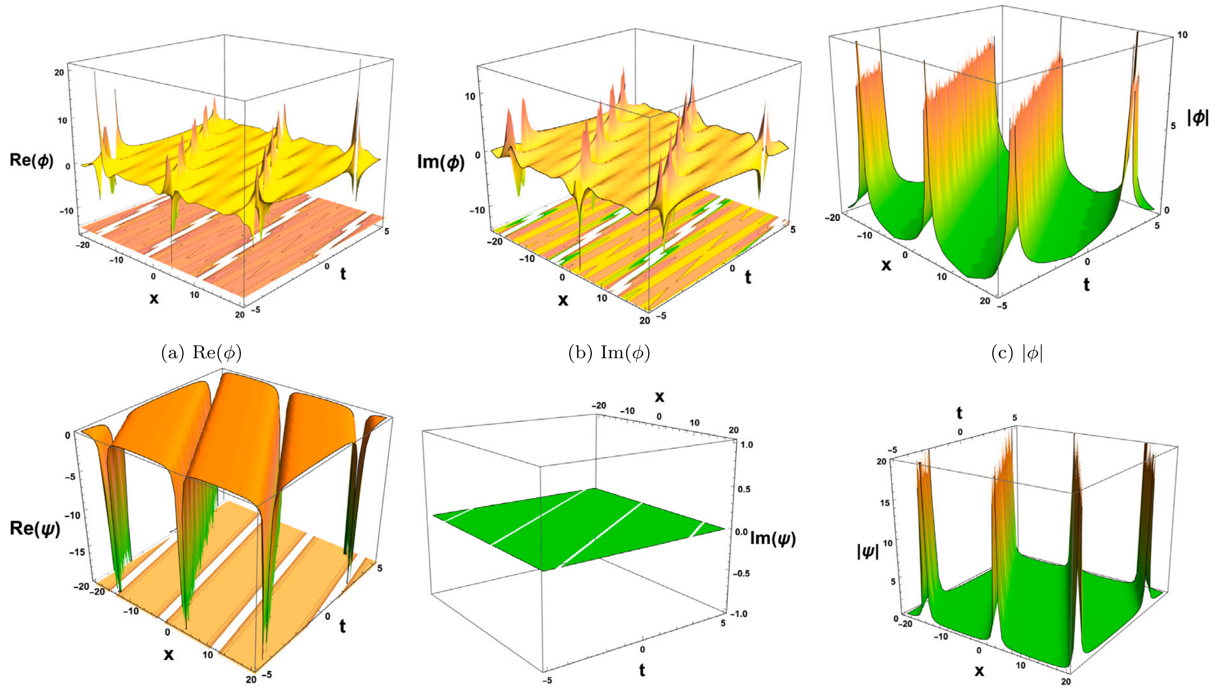
$$\psi_{14,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{2}\sqrt{v^2+1}n\sigma e^{\sigma(vt+x)}}{\sqrt{\beta}\left(1-ne^{\sigma(vt+x)}\right)}\right)^2.$$

- When $n=\sigma=\rho\neq 0$;

Using (25) into solution (23), we attain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{15,1}(x,t)=\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-4v^2n^2+4n^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-4v^2n^2+4n^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{v^2+1}n\left(\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}n(vt+x)\right)-1\right)}{\sqrt{2}\sqrt{\beta}}\right),$$

$$\psi_{15,1}(x,t)=\frac{\beta}{v^2+1}\left(-\frac{i\sqrt{v^2+1}\sigma}{\sqrt{2}\sqrt{\beta}}-\frac{i\sqrt{v^2+1}n\left(\sqrt{3}\tan\left(\frac{1}{2}\sqrt{3}n(vt+x)\right)-1\right)}{\sqrt{2}\sqrt{\beta}}\right)^2.$$



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 3. Graphical formation for solution $\phi_{15,1}(x,t)$ and $\psi_{15,1}(x,t)$ with specified parameters value set at $\alpha = 1.65$, $n = 0.2$, $\rho = 0.2$, $\sigma = 0.2$, $v = 2.55$ and $\beta = 2.2$.

- When $\rho = \sigma = 0$;

Using (25) into solution (23), we achieve the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{16,1}(x,t) = \frac{i\sqrt{2}\sqrt{v^2+1}e^{i\left(\frac{\sqrt{\alpha}t}{\sqrt{v^2-1}}+\frac{\sqrt{\alpha}vx}{\sqrt{v^2-1}}\right)}}{\sqrt{\beta}(vt+x)},$$

$$\psi_{16,1}(x,t) = -\frac{2}{(vt+x)^2}.$$

- When $n = \rho$ and $\sigma = 0$;

Using (25) into solution (23), we obtain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{17,1}(x,t) = -\frac{i\sqrt{2}n\sqrt{v^2+1}\tan(n(vt+x))\exp\left(i\left(\frac{t\sqrt{2\alpha-4v^2n^2+4n^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha-4v^2n^2+4n^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)}{\sqrt{\beta}},$$

$$\psi_{17,1}(x,t) = -2n^2\tan^2(n(vt+x)).$$

- When $n = 0$;

Using (25) into solution (23), we attain the solution of considered Eq. (1) through (23) and (21) as follows:

$$\phi_{18,1}(x,t) = -\frac{i\sqrt{v^2+1}\sigma\exp\left(i\left(\frac{t\sqrt{2\alpha+v^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}+\frac{vx\sqrt{2\alpha+v^2\sigma^2-\sigma^2}}{\sqrt{2}\sqrt{v^2-1}}\right)\right)}{\sqrt{2}\sqrt{\beta}},$$

$$\psi_{18,1}(x,t) = -\frac{\sigma^2}{2}.$$

3.2. Application of extended sinh*-Gordon equation expansion(shGEE) algorithm to the aforementioned equation*

Applying the homogeneous balancing concept in Eq. (22), we obtain $n = 1$. By substituting $n = 1$ in (7), we begin solving as

Case 1:

$$U(\xi) = R_1\cosh(\omega(\xi)) + R_0 + S_1\sinh(\omega(\xi)). \tag{26}$$

Substituting Eq. (26) into (22), the expanded shGEE approach and symbolic computing with MATHEMATICA can be applied to obtain non-trivial solutions to the governing equation.

Solution set 1.1:

$$R_0 = 0,\quad R_1 = \frac{\sqrt{2}\sqrt{-\alpha-2r^2+4}}{\sqrt{\beta r^2-2\beta}},\quad S_1 = 0,\quad v = \frac{\sqrt{\alpha+r^2-2}}{\sqrt{r^2-2}}. \tag{27}$$

From Eq. (27), (26) and (10), we get

$$\phi_{1,2}(x,t) = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 + 4}e^{i\left(\frac{rx\sqrt{\alpha+r^2-2}}{\sqrt{r^2-2}}+rt\right)} \coth\left(\frac{t\sqrt{\alpha+r^2-2}}{\sqrt{r^2-2}} + x\right)}{\sqrt{\beta r^2 - 2\beta}},$$

$$\psi_{1,2}(x,t) = \frac{2\beta\left(-\alpha - 2r^2 + 4\right)\coth^2\left(\frac{t\sqrt{\alpha+r^2-2}}{\sqrt{r^2-2}} + x\right)}{\left(\frac{\alpha+r^2-2}{r^2-2} + 1\right)\left(\beta r^2 - 2\beta\right)}. \tag{28}$$

Solution set 1.2:

$$R_0 = 0, \quad R_1 = 0, \quad S_1 = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 - 2}}{\sqrt{\beta + \beta r^2}}, \quad v = -\frac{\sqrt{\alpha + r^2 + 1}}{\sqrt{r^2 + 1}}. \tag{29}$$

From Eq. (27), (26) and (10), we get

$$\phi_{2,2}(x,t) = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 - 2}e^{i\left(rt-\frac{rx\sqrt{\alpha+r^2+1}}{\sqrt{r^2+1}}\right)} \operatorname{csch}\left(x - \frac{t\sqrt{\alpha+r^2+1}}{\sqrt{r^2+1}}\right)}{\sqrt{\beta + \beta r^2}},$$

$$\psi_{2,2}(x,t) = \frac{2\beta\left(-\alpha - 2r^2 - 2\right)\operatorname{csch}^2\left(x - \frac{t\sqrt{\alpha+r^2+1}}{\sqrt{r^2+1}}\right)}{\left(\frac{\alpha+r^2+1}{r^2+1} + 1\right)\left(\beta + \beta r^2\right)}. \tag{30}$$

Solution set 1.3:

$$R_0 = 0, \quad R_1 = \frac{\sqrt{-\alpha - 2r^2 + 1}}{\sqrt{2\beta r^2 - \beta}}, \quad S_1 = -\frac{\sqrt{-\alpha - 2r^2 + 1}}{\sqrt{2\beta r^2 - \beta}}, \quad v = \frac{\sqrt{2\alpha + 2r^2 - 1}}{\sqrt{2r^2 - 1}}. \tag{31}$$

From Eq. (27), (26) and (10), we get

$$\phi_{3,2}(x,t) = \exp\left(i\left(\frac{rx\sqrt{2\alpha + 2r^2 - 1}}{\sqrt{2r^2 - 1}} + rt\right)\right)\left(\frac{\sqrt{-\alpha - 2r^2 + 1}\coth\left(\frac{t\sqrt{2\alpha+2r^2-1}}{\sqrt{2r^2-1}} + x\right)}{\sqrt{2\beta r^2 - \beta}} - \frac{\sqrt{-\alpha - 2r^2 + 1}\operatorname{csch}\left(\frac{t\sqrt{2\alpha+2r^2-1}}{\sqrt{2r^2-1}} + x\right)}{\sqrt{2\beta r^2 - \beta}}\right),$$

$$\psi_{3,2}(x,t) = \frac{\beta\left(\frac{\sqrt{-\alpha-2r^2+1}\coth\left(\frac{t\sqrt{2\alpha+2r^2-1}}{\sqrt{2r^2-1}}+x\right)}{\sqrt{2\beta r^2-\beta}} - \frac{\sqrt{-\alpha-2r^2+1}\operatorname{csch}\left(\frac{t\sqrt{2\alpha+2r^2-1}}{\sqrt{2r^2-1}}+x\right)}{\sqrt{2\beta r^2-\beta}}\right)^2}{\frac{2\alpha+2r^2-1}{2r^2-1} + 1}. \tag{32}$$

Case 2:

$$U(\xi) = R_1\cosh(\omega(\xi)) + R_0 + S_1\sinh(\omega(\xi)). \tag{33}$$

Substituting Eq. (33) into (22), the implementing expanded shGEE approach and symbolic computing with MATHEMATICA can be applied to obtain non-trivial solutions to the governing equation.

Solution set 2.1:

$$R_0 = 0, \quad R_1 = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 + 2}}{\sqrt{\beta r^2 - \beta}}, \quad S_1 = 0, \quad v = \frac{\sqrt{\alpha + r^2 - 1}}{\sqrt{r^2 - 1}}. \tag{34}$$

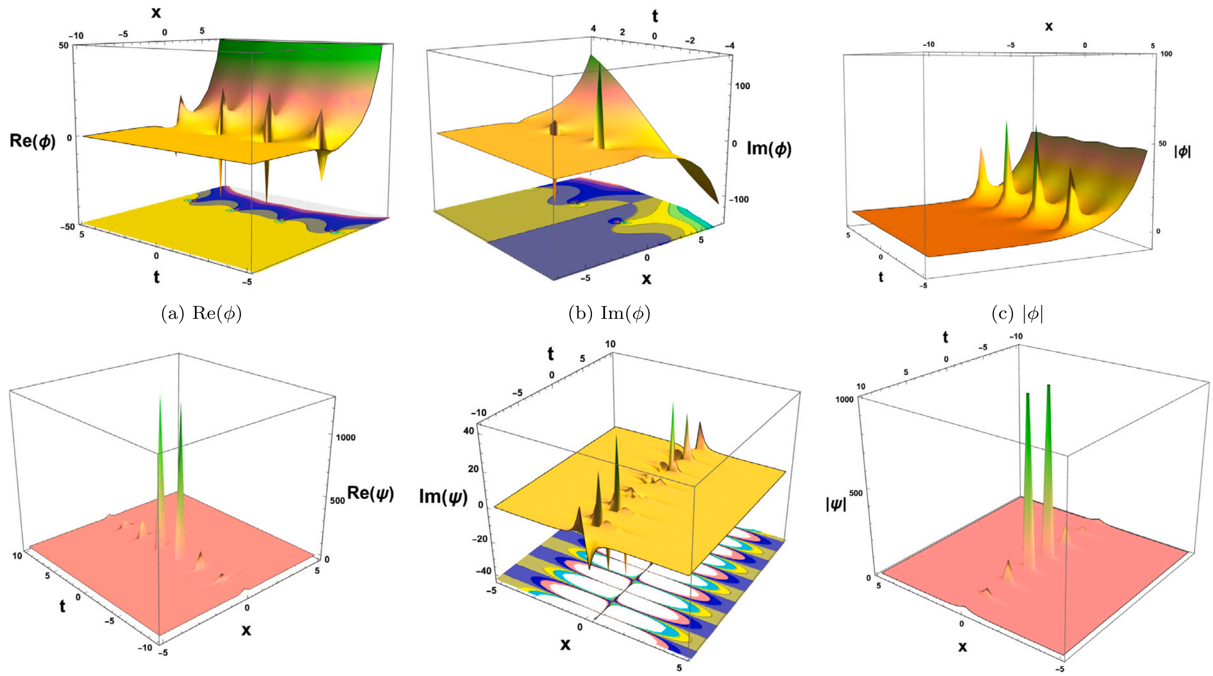
From Eq. (38), (26) and (13), we get

$$\phi_{4,2}(x,t) = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 + 2}e^{i\left(\frac{rx\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}}+rt\right)} \sec\left(\frac{t\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}} + x\right)}{\sqrt{\beta r^2 - \beta}},$$

$$\psi_{4,2}(x,t) = \frac{2\beta\left(-\alpha - 2r^2 + 2\right)\sec^2\left(\frac{t\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}} + x\right)}{\left(\frac{\alpha+r^2-1}{r^2-1} + 1\right)\left(\beta r^2 - \beta\right)} \tag{35}$$

Solution set 2.2:

$$R_0 = 0, \quad R_1 = 0, \quad S_1 = \frac{\sqrt{2}\sqrt{-\alpha - 2r^2 - 4}}{\sqrt{2\beta + \beta r^2}}, \quad v = \frac{\sqrt{\alpha + r^2 + 2}}{\sqrt{r^2 + 2}}. \tag{36}$$



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 4. Graphical formation for the solution $\phi_{3,2}(x,t)$ and $\psi_{3,2}(x,t)$ with specified parameters value set at $\alpha = 2.5$, $\beta = 0.2$ and $r = 0.25$.

From Eq. (38), (26) and (13), we get

$$\phi_{5,2}(x,t) = \frac{\sqrt{2}\sqrt{-\alpha-2r^2-4}\,e^{i\left(\frac{rx\sqrt{\alpha+r^2+2}}{\sqrt{r^2+2}}+rt\right)}\tan\left(\frac{t\sqrt{\alpha+r^2+2}}{\sqrt{r^2+2}}+x\right)}{\sqrt{2\beta+\beta r^2}},$$

$$\psi_{5,2}(x,t) = \frac{2\beta\left(-\alpha-2r^2-4\right)\tan^2\left(\frac{t\sqrt{\alpha+r^2+2}}{\sqrt{r^2+2}}+x\right)}{\left(\frac{\alpha+r^2+2}{r^2+2}+1\right)\left(2\beta+\beta r^2\right)} \tag{37}$$

Solution set 2.3:

$$R_0 = 0,\quad R_1 = \frac{\sqrt{-\alpha-2r^2-1}}{\sqrt{\beta+2\beta r^2}},\quad S_1 = \frac{\sqrt{-\alpha-2r^2-1}}{\sqrt{\beta+2\beta r^2}},\quad v = \frac{\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}. \tag{38}$$

From Eq. (38), (26) and (13), we get

$$\phi_{6,2}(x,t) = \exp\left(i\left(\frac{rx\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+rt\right)\right)\left(\frac{\sqrt{-\alpha-2r^2-1}\tan\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta+2\beta r^2}}\right.$$
$$\left.+\frac{\sqrt{-\alpha-2r^2-1}\sec\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta+2\beta r^2}}\right),$$

$$\psi_{6,2}(x,t) = \frac{\beta\left(\frac{\sqrt{-\alpha-2r^2-1}\tan\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta+2\beta r^2}}+\frac{\sqrt{-\alpha-2r^2-1}\sec\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta+2\beta r^2}}\right)^2}{\frac{2\alpha+2r^2+1}{2r^2+1}+1}. \tag{39}$$

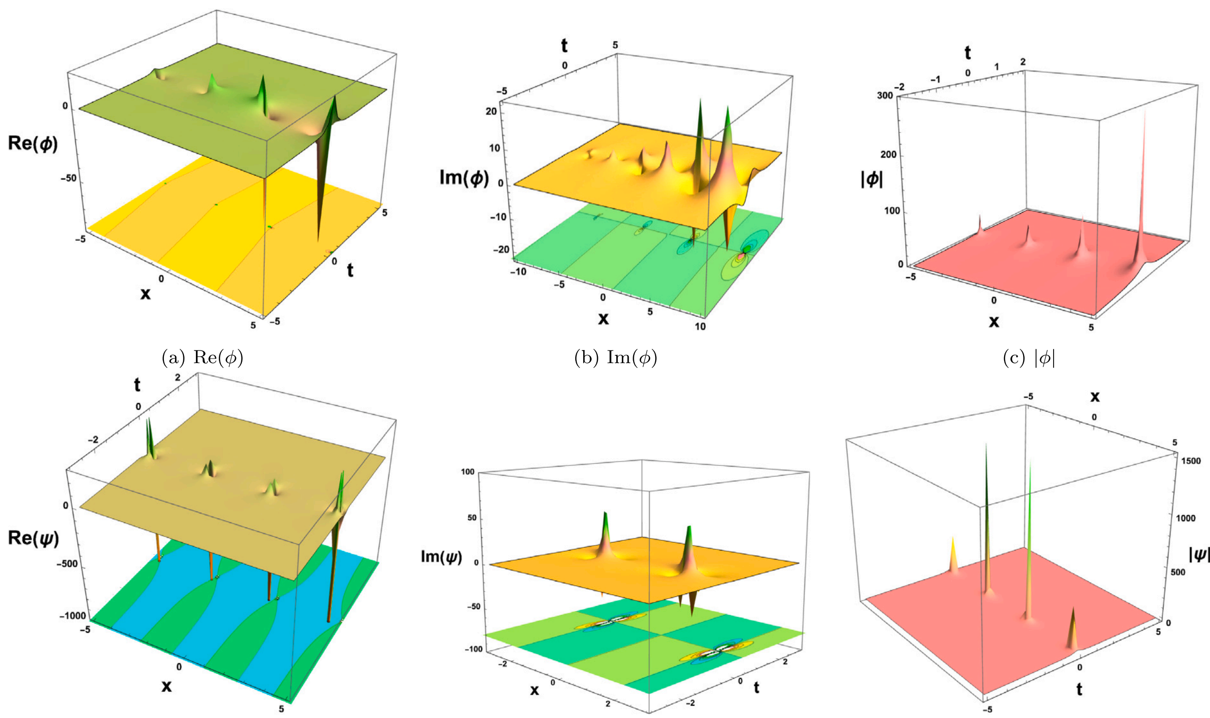
Case 3:

$$U(\xi) = R_1\sinh(\omega(\xi)) + R_0 + S_1\cosh(\omega(\xi)). \tag{40}$$

Substituting Eq. (53) into (22) and the implementing expanded shGEE approach and symbolic computing with MATHEMATICA can be applied to obtain non-trivial solutions to the governing equation.

Solution set 3.1:

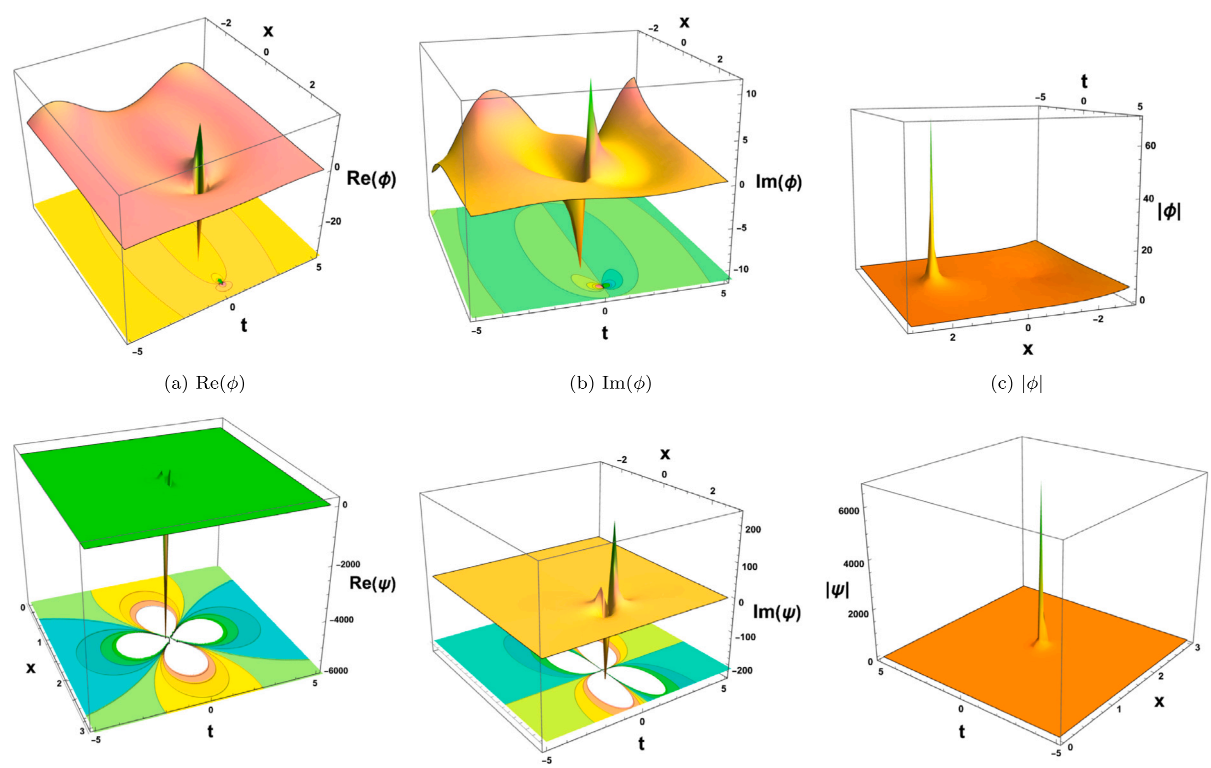
$$R_0 = 0,\quad R_1 = 0,\quad S_1 = \frac{\sqrt{2}\sqrt{\alpha-2m^2+2r^2-2}}{\sqrt{\beta+\beta m^2-\beta r^2}},\quad v = \frac{\sqrt{-\alpha+m^2-r^2+1}}{\sqrt{m^2-r^2+1}}. \tag{41}$$



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 5. Graphical formation for solution $\phi_{4,2}(x,t)$ and $\psi_{4,2}(x,t)$ with specified parameters value set at $\alpha = 2.58$, $\beta = 0.25$ and $r = 0.152$.



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 6. Graphical formation for solution $\phi_{6,2}(x,t)$ and $\psi_{6,2}(x,t)$ with specified parameter set at $\alpha = -2.22$, $\beta = -0.25$ and $r = 1.18$.

From Eq. (60), (53) and (16), we get

$$\phi_{7,2}(x,t) = \frac{\sqrt{2}\sqrt{\alpha - 2m^2 + 2r^2 - 2}\exp\left(i\left(\frac{rx\sqrt{-\alpha+m^2-r^2+1}}{\sqrt{m^2-r^2+1}} + rt\right)\right)\operatorname{ns}\left(x + \frac{t\sqrt{m^2-r^2-\alpha+1}}{\sqrt{m^2-r^2+1}}\middle| m\right)}{\sqrt{\beta + \beta m^2 - \beta r^2}},$$

$$\psi_{7,2}(x,t)=\frac{2\beta\left(\alpha-2m^2+2r^2-2\right)\mathrm{ns}\left(x+\frac{t\sqrt{m^2-r^2-\alpha+1}}{\sqrt{m^2-r^2+1}}\middle|m\right)^2}{\left(\frac{-\alpha+m^2-r^2+1}{m^2-r^2+1}+1\right)\left(\beta+\beta m^2-\beta r^2\right)}. \tag{42}$$

For $m=0$, we get

$$\phi_{8,2}(x,t)=\frac{\sqrt{2}\sqrt{\alpha+2r^2-2}e^{i\left(\frac{rx\sqrt{-\alpha-r^2+1}}{\sqrt{1-r^2}}+rt\right)}\csc\left(\frac{t\sqrt{-\alpha-r^2+1}}{\sqrt{1-r^2}}+x\right)}{\sqrt{\beta-\beta r^2}},$$

$$\psi_{8,2}(x,t)=\frac{2\beta\left(\alpha+2r^2-2\right)\csc^2\left(\frac{t\sqrt{-\alpha-r^2+1}}{\sqrt{1-r^2}}+x\right)}{\left(\frac{-\alpha-r^2+1}{1-r^2}+1\right)\left(\beta-\beta r^2\right)}. \tag{43}$$

For $m=1$, we get

$$\phi_{9,2}(x,t)=\frac{\sqrt{2}\sqrt{\alpha+2r^2-4}e^{i\left(\frac{rx\sqrt{-\alpha-r^2+2}}{\sqrt{2-r^2}}+rt\right)}\coth\left(\frac{t\sqrt{-\alpha-r^2+2}}{\sqrt{2-r^2}}+x\right)}{\sqrt{2\beta-\beta r^2}},$$

$$\psi_{9,2}(x,t)=\frac{2\beta\left(\alpha+2r^2-4\right)\coth^2\left(\frac{t\sqrt{-\alpha-r^2+2}}{\sqrt{2-r^2}}+x\right)}{\left(\frac{-\alpha-r^2+2}{2-r^2}+1\right)\left(2\beta-\beta r^2\right)}. \tag{44}$$

Solution set 3.2:

$$R_0=0,\quad R_1=\frac{\sqrt{2}\sqrt{\alpha-2m^2+2r^2+4}}{\sqrt{-2\beta+\beta m^2-\beta r^2}},\quad S_1=0,\quad \nu=-\frac{\sqrt{-\alpha+m^2-r^2-2}}{\sqrt{m^2-r^2-2}}. \tag{45}$$

From Eq. (60), (53) and (16), we get

$$\phi_{10,2}(x,t)=\frac{\sqrt{2}\sqrt{\alpha-2m^2+2r^2+4}\exp\left(i\left(rt-\frac{rx\sqrt{-\alpha+m^2-r^2-2}}{\sqrt{m^2-r^2-2}}\right)\right)\mathrm{cs}\left(x-\frac{t\sqrt{m^2-r^2-\alpha-2}}{\sqrt{m^2-r^2-2}}\middle|m\right)}{\sqrt{-2\beta+\beta m^2-\beta r^2}},$$

$$\psi_{10,2}(x,t)=\frac{2\beta\left(\alpha-2m^2+2r^2+4\right)\mathrm{cs}\left(x-\frac{t\sqrt{m^2-r^2-\alpha-2}}{\sqrt{m^2-r^2-2}}\middle|m\right)^2}{\left(\frac{-\alpha+m^2-r^2-2}{m^2-r^2-2}+1\right)\left(-2\beta+\beta m^2-\beta r^2\right)}. \tag{46}$$

For $m=0$, we get

$$\phi_{11,2}(x,t)=\frac{\sqrt{2}\sqrt{\alpha+2r^2+4}\exp\left(i\left(rt-\frac{rx\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)\right)\cot\left(x-\frac{t\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)}{\sqrt{-2\beta-\beta r^2}},$$

$$\psi_{11,2}(x,t)=\frac{2\beta\left(\alpha+2r^2+4\right)\cot^2\left(x-\frac{t\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)}{\left(\frac{-\alpha-r^2-2}{-r^2-2}+1\right)\left(-2\beta-\beta r^2\right)}. \tag{47}$$

For $m=1$, we get

$$\phi_{12,2}(x,t)=\frac{\sqrt{2}\sqrt{\alpha+2r^2+2}\exp\left(i\left(rt-\frac{rx\sqrt{-\alpha-r^2-1}}{\sqrt{-r^2-1}}\right)\right)\mathrm{csch}\left(x-\frac{t\sqrt{-\alpha-r^2-1}}{\sqrt{-r^2-1}}\right)}{\sqrt{-\beta-\beta r^2}},$$

$$\psi_{12,2}(x,t)=\frac{2\beta\left(\alpha+2r^2+2\right)\mathrm{csch}^2\left(x-\frac{t\sqrt{-\alpha-r^2-1}}{\sqrt{-r^2-1}}\right)}{\left(\frac{-\alpha-r^2-1}{-r^2-1}+1\right)\left(-\beta-\beta r^2\right)}. \tag{48}$$

Solution set 3.3:

$$R_0=0,\quad R_1=-\frac{\sqrt{\alpha-2m^2+2r^2+1}}{\sqrt{-\beta+2\beta m^2-2\beta r^2}},\quad S_1=\frac{\sqrt{\alpha-2m^2+2r^2+1}}{\sqrt{-\beta+2\beta m^2-2\beta r^2}},\quad \nu=\frac{\sqrt{-2\alpha+2m^2-2r^2-1}}{\sqrt{2m^2-2r^2-1}}. \tag{49}$$

From Eq. (60), (53) and (16), we get

$$\phi_{13,2}(x,t)=\exp\left(i\left(\frac{rx\sqrt{-2\alpha+2m^2-2r^2-1}}{\sqrt{2m^2-2r^2-1}}+rt\right)\right)\left(\frac{\sqrt{\alpha-2m^2+2r^2+1}\,\mathrm{ns}\left(x+\frac{t\sqrt{2m^2-2r^2-2\alpha-1}}{\sqrt{2m^2-2r^2-1}}\middle|m\right)}{\sqrt{-\beta+2\beta m^2-2\beta r^2}}\right.$$
$$\left.-\frac{\sqrt{\alpha-2m^2+2r^2+1}\,\mathrm{cs}\left(x+\frac{t\sqrt{2m^2-2r^2-2\alpha-1}}{\sqrt{2m^2-2r^2-1}}\middle|m\right)}{\sqrt{-\beta+2\beta m^2-2\beta r^2}}\right),$$

$$\psi_{13,2}(x,t)=\frac{\beta\left(\frac{\sqrt{\alpha-2m^2+2r^2+1}\,\mathrm{ns}\left(x+\frac{t\sqrt{2m^2-2r^2-2\alpha-1}}{\sqrt{2m^2-2r^2-1}}\middle|m\right)}{\sqrt{-\beta+2\beta m^2-2\beta r^2}}-\frac{\sqrt{\alpha-2m^2+2r^2+1}\,\mathrm{cs}\left(x+\frac{t\sqrt{2m^2-2r^2-2\alpha-1}}{\sqrt{2m^2-2r^2-1}}\middle|m\right)}{\sqrt{-\beta+2\beta m^2-2\beta r^2}}\right)^2}{\frac{-2\alpha+2m^2-2r^2-1}{2m^2-2r^2-1}+1}. \tag{50}$$

For $m=0$, we get

$$\phi_{14,2}(x,t)=\exp\left(i\left(\frac{rx\sqrt{-2\alpha-2r^2-1}}{\sqrt{-2r^2-1}}+rt\right)\right)\left(\frac{\sqrt{\alpha+2r^2+1}\csc\left(\frac{t\sqrt{-2\alpha-2r^2-1}}{\sqrt{-2r^2-1}}+x\right)}{\sqrt{-\beta-2\beta r^2}}-\frac{\sqrt{\alpha+2r^2+1}\cot\left(\frac{t\sqrt{-2\alpha-2r^2-1}}{\sqrt{-2r^2-1}}+x\right)}{\sqrt{-\beta-2\beta r^2}}\right),$$

$$\psi_{14,2}(x,t)=\frac{\beta\left(\frac{\sqrt{\alpha+2r^2+1}\csc\left(\frac{t\sqrt{-2\alpha-2r^2-1}}{\sqrt{-2r^2-1}}+x\right)}{\sqrt{-\beta-2\beta r^2}}-\frac{\sqrt{\alpha+2r^2+1}\cot\left(\frac{t\sqrt{-2\alpha-2r^2-1}}{\sqrt{-2r^2-1}}+x\right)}{\sqrt{-\beta-2\beta r^2}}\right)^2}{\frac{-2\alpha-2r^2-1}{-2r^2-1}+1}. \tag{51}$$

For $m=1$, we get

$$\phi_{15,2}(x,t)=\exp\left(i\left(\frac{rx\sqrt{-2\alpha-2r^2+1}}{\sqrt{1-2r^2}}+rt\right)\right)\left(\frac{\sqrt{\alpha+2r^2-1}\coth\left(\frac{t\sqrt{-2\alpha-2r^2+1}}{\sqrt{1-2r^2}}+x\right)}{\sqrt{\beta-2\beta r^2}}-\frac{\sqrt{\alpha+2r^2-1}\,\mathrm{csch}\left(\frac{t\sqrt{-2\alpha-2r^2+1}}{\sqrt{1-2r^2}}+x\right)}{\sqrt{\beta-2\beta r^2}}\right),$$

$$\psi_{15,2}(x,t)=\frac{\beta\left(\frac{\sqrt{\alpha+2r^2-1}\coth\left(\frac{t\sqrt{-2\alpha-2r^2+1}}{\sqrt{1-2r^2}}+x\right)}{\sqrt{\beta-2\beta r^2}}-\frac{\sqrt{\alpha+2r^2-1}\,\mathrm{csch}\left(\frac{t\sqrt{-2\alpha-2r^2+1}}{\sqrt{1-2r^2}}+x\right)}{\sqrt{\beta-2\beta r^2}}\right)^2}{\frac{-2\alpha-2r^2+1}{1-2r^2}+1}. \tag{52}$$

Case 4:

$$U(\xi)=R_1\sinh(\omega(\xi))+R_0+S_1\cosh(\omega(\xi)). \tag{53}$$

Substituting Eq. (53) into (22) and the implementing expanded shGEE approach and symbolic computing with MATHEMATICA can be applied to obtain non-trivial solutions to the governing equation.

Solution set 4.1:

$$R_0=0,\quad R_1=0,\quad S_1=\frac{\sqrt{2}\sqrt{m^2-1}\sqrt{\alpha+4m^2+2r^2-2}}{\sqrt{\beta}\sqrt{2m^2+r^2-1}},\quad v=\frac{\sqrt{\alpha+2m^2+r^2-1}}{\sqrt{2m^2+r^2-1}}. \tag{54}$$

From Eq. (60), (53) and (16), we get

$$\phi_{16,2}(x,t)=\frac{\sqrt{2}\sqrt{m^2-1}\sqrt{\alpha+4m^2+2r^2-2}\exp\left(i\left(\frac{rx\sqrt{\alpha+2m^2+r^2-1}}{\sqrt{2m^2+r^2-1}}+rt\right)\right)\mathrm{nc}\left(x+\frac{t\sqrt{2m^2+r^2+\alpha-1}}{\sqrt{2m^2+r^2-1}}\middle|m\right)}{\sqrt{\beta}\sqrt{2m^2+r^2-1}},$$

$$\psi_{16,2}(x,t)=\frac{2\left(m^2-1\right)\left(\alpha+4m^2+2r^2-2\right)\mathrm{nc}\left(x+\frac{t\sqrt{2m^2+r^2+\alpha-1}}{\sqrt{2m^2+r^2-1}}\middle|m\right)^2}{\left(2m^2+r^2-1\right)\left(\frac{\alpha+2m^2+r^2-1}{2m^2+r^2-1}+1\right)}. \tag{55}$$

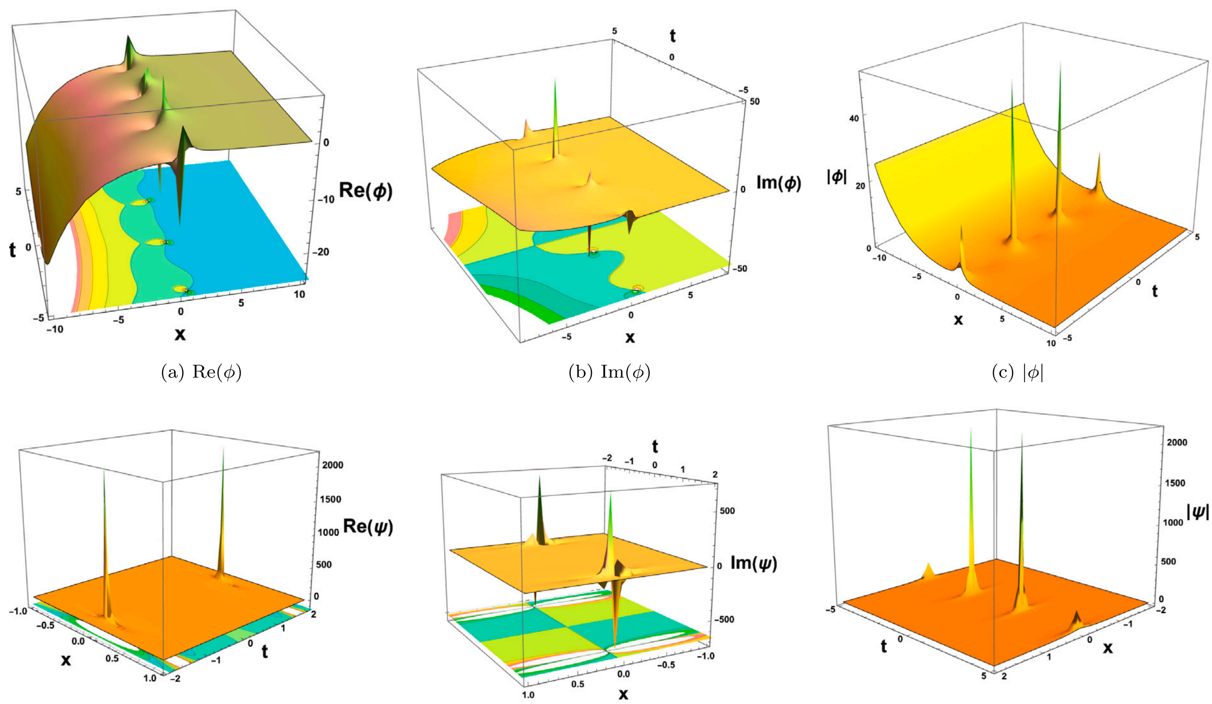
For $m=0$, we get

$$\phi_{17,2}(x,t)=\frac{i\sqrt{2}\sqrt{\alpha+2r^2-2}e^{i\left(\frac{rx\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}}+rt\right)}\sec\left(\frac{t\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}}+x\right)}{\sqrt{\beta}\sqrt{r^2-1}},$$

$$\psi_{17,2}(x,t)=-\frac{2\left(\alpha+2r^2-2\right)\sec^2\left(\frac{t\sqrt{\alpha+r^2-1}}{\sqrt{r^2-1}}+x\right)}{\left(r^2-1\right)\left(\frac{\alpha+r^2-1}{r^2-1}+1\right)}. \tag{56}$$

Solution set 4.2:

$$R_0=0,\quad R_1=\frac{\sqrt{2}\sqrt{m^2-1}\sqrt{-\alpha+2m^2-2r^2-4}}{\sqrt{\beta}\sqrt{m^2-r^2-2}},\quad S_1=0,\quad v=-\frac{\sqrt{-\alpha+m^2-r^2-2}}{\sqrt{m^2-r^2-2}}. \tag{57}$$



(a) Re(ϕ) (b) Im(ϕ) (c) $|\phi|$

(d) Re(ψ) (e) Im(ψ) (f) $|\psi|$

Fig. 7. Graphical formation for solution $u_{15,2}(x,t)$ and $v_{15,2}(x,t)$ with specified parameters value set at $\alpha = 2.22$, $\beta = 2.25$ and $r = 0.18$.

From Eq. (60), (53) and (16), we get

$$\phi_{18,2}(x,t) = \frac{\sqrt{2}\sqrt{m^2-1}\sqrt{-\alpha+2m^2-2r^2-4}\exp\left(i\left(rt-\frac{rx\sqrt{-\alpha+m^2-r^2-2}}{\sqrt{m^2-r^2-2}}\right)\right)\mathrm{sc}\left(x-\frac{t\sqrt{m^2-r^2-\alpha-2}}{\sqrt{m^2-r^2-2}}\middle| m\right)}{\sqrt{\beta}\sqrt{m^2-r^2-2}},$$

$$\psi_{18,2}(x,t) = \frac{2\left(m^2-1\right)\left(-\alpha+2m^2-2r^2-4\right)\mathrm{sc}\left(x-\frac{t\sqrt{m^2-r^2-\alpha-2}}{\sqrt{m^2-r^2-2}}\middle| m\right)^2}{\left(m^2-r^2-2\right)\left(\frac{-\alpha+m^2-r^2-2}{m^2-r^2-2}+1\right)}. \tag{58}$$

For $m = 0$, we get

$$\phi_{19,2}(x,t) = \frac{i\sqrt{2}\sqrt{-\alpha-2r^2-4}\exp\left(i\left(rt-\frac{rx\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)\right)\tan\left(x-\frac{t\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)}{\sqrt{\beta}\sqrt{-r^2-2}},$$

$$\psi_{19,2}(x,t) = -\frac{2\left(-\alpha-2r^2-4\right)\tan^2\left(x-\frac{t\sqrt{-\alpha-r^2-2}}{\sqrt{-r^2-2}}\right)}{\left(-r^2-2\right)\left(\frac{-\alpha-r^2-2}{-r^2-2}+1\right)}. \tag{59}$$

Solution set 4.3:

$$R_0 = 0,\quad R_1 = \frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}}{\sqrt{\beta}\sqrt{m^2+2r^2+1}},\quad S_1 = \frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}}{\sqrt{\beta}\sqrt{m^2+2r^2+1}}, \tag{60}$$

$$v = \frac{\sqrt{2\alpha+m^2+2r^2+1}}{\sqrt{m^2+2r^2+1}}. \tag{61}$$

From Eq. (60), (53) and (16), we get

$$\phi_{20,2}(x,t) = \exp\left(i\left(\frac{rx\sqrt{2\alpha+m^2+2r^2+1}}{\sqrt{m^2+2r^2+1}}+rt\right)\right)\left(\frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}\mathrm{nc}\left(x+\frac{t\sqrt{m^2+2r^2+2\alpha+1}}{\sqrt{m^2+2r^2+1}}\middle| m\right)}{\sqrt{\beta}\sqrt{m^2+2r^2+1}}\right.$$
$$\left.+\frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}\mathrm{sc}\left(x+\frac{t\sqrt{m^2+2r^2+2\alpha+1}}{\sqrt{m^2+2r^2+1}}\middle| m\right)}{\sqrt{\beta}\sqrt{m^2+2r^2+1}}\right),$$

$$\psi_{20,2}(x,t)=\frac{\beta}{\frac{2\alpha+m^2+2r^2+1}{m^2+2r^2+1}+1}\left(\frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}\,\mathrm{nc}\left(x+\frac{t\sqrt{m^2+2r^2+2\alpha+1}}{\sqrt{m^2+2r^2+1}}\middle|m\right)}{\sqrt{\beta}\sqrt{m^2+2r^2+1}}\right. \tag{62}$$

$$\left.+\frac{\sqrt{m^2-1}\sqrt{\alpha+m^2+2r^2+1}\,\mathrm{sc}\left(x+\frac{t\sqrt{m^2+2r^2+2\alpha+1}}{\sqrt{m^2+2r^2+1}}\middle|m\right)}{\sqrt{\beta}\sqrt{m^2+2r^2+1}}\right)^2. \tag{63}$$

For $m=0$, we get

$$\phi_{21,2}(x,t)=\exp\left(i\left(\frac{rx\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+rt\right)\right)\left(\frac{i\sqrt{\alpha+2r^2+1}\tan\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta}\sqrt{2r^2+1}}\right.$$

$$\left.+\frac{i\sqrt{\alpha+2r^2+1}\sec\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta}\sqrt{2r^2+1}}\right),$$

$$\psi_{21,2}(x,t)=\frac{\beta\left(\frac{i\sqrt{\alpha+2r^2+1}\tan\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta}\sqrt{2r^2+1}}+\frac{i\sqrt{\alpha+2r^2+1}\sec\left(\frac{t\sqrt{2\alpha+2r^2+1}}{\sqrt{2r^2+1}}+x\right)}{\sqrt{\beta}\sqrt{2r^2+1}}\right)^2}{\frac{2\alpha+2r^2+1}{2r^2+1}+1}. \tag{64}$$

4. Graphical illustration of the solutions

In this part of the manuscript, we examine the obtained travelling wave solutions by presenting 3D and combined contour graphics and also inferred their dynamical behaviour using different parametric values with the aid of Mathematica software.

We receive the anti-bell shaped soliton structure for the absolute value of solution $\phi_{6,1}(x,t)$ and $\psi_{6,1}(x,t)$ with specified parameter values as $\alpha=0.65$, $n=0.28$, $\rho=-2.5$, $\nu=0.05$ and $\beta=0.8$ with $-8<x<8$ and $-1<t<1$ as shown in Fig. 2(c) and (f). Fig. 2(a) and (b) demonstrates the double-periodic wave like structure for real and imaginary value of solution $\phi_{6,1}(x,t)$, wherein in (d) and (e) we demonstrate the anti-bell shaped soliton and plane structure for the real and imaginary value of solution $\psi_{6,1}(x,t)$ respectively.

We obtain the singular periodic solitary wave structure for the absolute value of solution $\phi_{15,1}(x,t)$ and $\psi_{15,1}(x,t)$ with specified parameters value as $\alpha=1.65$, $n=0.2$, $\rho=0.2$, $\sigma=0.2$, $\nu=2.55$ and $\beta=2.2$ as shown in Fig. 3(c) and (f). Fig. 3(a) and (b) demonstrates the combined 3D and contour graphics of real and imaginary values of solution $\phi_{15,1}(x,t)$, whereas 3(d) and (e) demonstrates real and imaginary values of solution $\psi_{15,1}(x,t)$ showing periodic wave structure and constant plane structures.

We obtain the kink type solitons for the absolute value of solution $\phi_{3,2}(x,t)$ and $\psi_{3,2}(x,t)$ with the specified parameter values as $\alpha=2.5$, $\beta=0.2$ and $r=0.25$ with $-10<x<10$ and $-5<t<5$ as shown in Fig. 4(e) and (f). Fig. 4(a) and (b) demonstrates the corresponding combined 3D and contour graphics of real and imaginary values of solution $\phi_{3,2}(x,t)$, whereas 4(d) and (e) demonstrates real and imaginary values of solution $\psi_{15,1}(x,t)$ showing solitary wave structures.

We receive interaction of kink and soliton wave profile for the solution $phi_{4,2}(x,t)$ and $\psi_{4,2}(x,t)$ with specified parameter values as $\alpha=2.58$, $\beta=0.25$ and $r=0.152$ with $-10<x<10$ and $-5<t<5$ as shown in Fig. 5(c) and (f). Fig. 5(a) and (b) demonstrates the combined 3D and contour graphics of real and imaginary values of the solution $phi_{4,2}(x,t)$, whereas 5(d) and (e) represents graphics of real and imaginary values of solution $\psi_{4,2}(x,t)$

We obtain the lump like structure for the absolute for solution $\phi_{6,2}(x,t)$ and $\psi_{6,2}(x,t)$ with specified parameters value as $\alpha=-2.22$, $\beta=-0.25$ and $r=1.18$ as shown in Fig. 6(c) and (f). Fig. 6(a) and (b) demonstrates the corresponding 3D and contour combined graphics for real and imaginary values of solution $\phi_{6,2}(x,t)$, whereas 6(d) and (e) represents graphics for real and imaginary values of solution $\psi_{6,2}(x,t)$.

We receive interaction of plane-kink and soliton wave profile for the absolute value of solution $\phi_{15,6}(x,t)$ and $\psi_{15,6}(x,t)$ with specified parameter values as $\alpha=2.22$, $\beta=2.25$ and $r=0.18$ with $-10<x<10$ and $-5<t<5$ as shown in Fig. 7(c) and (f). Fig. 7(a) and (b) demonstrates the corresponding combined 3D and contour plots for real and imaginary values of solution $\phi_{15,6}(x,t)$, whereas 7(d) and (e) demonstrates the corresponding combined 3D and contour plots for real and imaginary values of solution $\psi_{15,6}(x,t)$.

5. Conclusion

In summary, the new auxiliary equation method and shGEEM are implemented for the first time in the (2+1)-dimensional NLccHF equation to achieve effective, competent, and further novel soliton solutions. We have obtained numerous solutions in terms of Jacobian elliptic functions, hyperbolic, trigonometric, exponential, and rational functions to the above-stated model with more parameters. The solutions obtained are visually represented through graphs, highlighting their outcomes. Additionally, the study explores the impact of time evolution through combined two-dimensional graphs. These graphical representations showcase diverse wave patterns, including irregular periodic solitons, singular bell-shaped solitons, and anti-bell-shaped solitons, depending on different parameter values within the system. The findings emphasize the substantial influence of free parameters on waveform behaviour, providing a versatile method to depict a plethora of unique and complex features observed across scientific domains. Furthermore, the solutions were verified for accuracy using symbolic computational software applications. They were incorporated into the main equations and confirmed to be correct, ensuring their reliability. This study affirms the trustworthiness and success of the two applied techniques in evaluating the optimal approach for well-established mathematical models.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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