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Practical analytical approaches for finding novel optical solitons in the single-mode fibers

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ABSTRACT

In this work, under consideration is a $(2+1)$-dimensional generic model, known as the Hirota-Maccari system, which was studied in plasma physics, fluid dynamics and fiber optics communication systems. The article aims to extract novel soliton solutions of this model with the aid of symbolic computation. Three robust and versatile integration tools of the $\exp(-\Omega(\chi))$-expansion, the first integral, and the sine-Gordon expansion methods are used to extract dark, bright, and singular solitons solutions. Constraints conditions are explicitly presented for the resulting solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) have gained much attraction in the study of physical sciences due to important applications in many physical phenomena. Different exact solutions of NLEEs, which are obtained through numerous mathematical tools, are useful in many physical fields such as biology, organic chemistry, fluid mechanics, population dynamics, space technology, engineering methodology, hydrodynamics, theory of Bose–Einstein condensates, computer engineering, solid state physics and applied mathematics. Among the most important solutions of NLEEs are soliton solutions [1–3]. On one hand, those solutions can be constructed through the Hirota bilinear method, the multiple exp-function method, the nonlinear superposition principle, the inverse scattering transform, and the Riemann-Hilbert approach [4–14]. On the other hand, they possess elastic interactions and arises from a balance between nonlinear and dispersive effects.

Recently, the main goal for many researchers is to find exact and approximate solutions of NLEEs with the help of various tech-

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niques. Indeed, explicit solutions have a great impact in different engineering and scientific fields such as fluid mechanics, nonlinear optics, plasma physics and solid state physics [15–40]. Many strong techniques have been developed to procure approximated and exact solutions of NLEEs such as $\exp(-\phi(\xi))$-expansion method [41,42], the modified extended tanh-function method [43,44], new extended direct algebraic method [45,46], the (G'/G)-expansion method [47,48], the modified simple equation method [49], the trial equation method [50], the sine-Gordon expansion method [51,52], and the first integral method [53,54].

The (2+1)-dimensional Hirota-Maccari system is a special type of NLEEs given by [55,56]

$$\begin{aligned} is_\tau + s_{xy} + is_{xxx} + s\,r - i|s|^2 s_x = 0, \\ 3r_x + (|s|^2)_y = 0, \end{aligned}$$

which explains the dynamical behaviors of the femto-second soliton pulse in single-mode fibers, where $s = s(\tau, x, y)$ and $r = r(\tau, x, y)$ describe the complex and the real scalar fields, respectively. Both dependent variables s and r depend upon independent variables τ, x and y, where x, y and τ, respectively, denote spatial and temporal variables. Many authors have obtained exact solutions of the Hirota-Maccari system. For example, Malik et al. [57] obtained generalized traveling wave solutions of the system by using (G'/G)-expansion method, and Demiray et al. [58] demonstrated the solutions of above system, with the aid of Extended trial equation and generalized Kudryashov methods. Sulaiman et al. [59] used extended sinh-Gordon equation to explore several exact solutions of the nonlinear fractional Hirota-Maccari equation with a truncated M-fractional derivative. Yu et al. [60] by means of Hirota bilinear method and E. Fan [61] by a new unified algebraic method computed the solutions of Hirota-Maccari system. Complex hyperbolic-function method technique is used by C.L. Bai and H. Zhao [62] to elaborate the exact solutions for the same system.

The present work focuses on studying the integrating aspect of the Hirota-Maccari system. The process is based on three well-known integration tools: the $\exp(-\Omega(\chi))$-expansion, the first integral, and the sine-Gordon expansion methods. Those methods are extremely helpful in extracting a wide class of soliton solutions, which are applicable in different fields of natural sciences.

The principal advantage of the techniques implemented in this study over all the other methods is that they provide further new computable solutions, including additional free parameters. Most of the obtained solutions in literatures are taken via these applied approaches as a particular case, and more importantly, we receive some new solutions as well. The recommended computational methods here are uncomplicated, outspoken, consistent, and minimizing the computational work size, which give Their wide-range applicability. With all these properties, our studies are effectiveness and influence to deal with other nonlinear partial differential equations arising in different fields of science and deserve future research.

2. Overview of the applied integration tools

Herein, three integration gadgets namely the $\exp(-\Omega(\chi))$-expansion, the first integral, and the sine-Gordon expansion methods are introduced.

Let us consider a nonlinear partial differential equation (PDE),

$$\Xi\big(\upsilon, D_\tau\upsilon, D_x\upsilon, D_\tau^2\upsilon, D_{x\tau}\upsilon, D_x^2\upsilon, \ldots\big) = 0, \tag{1}$$

where Ξ is a polynomial in the arguments. By means of the variable transformation

$$\upsilon(x,\tau) = \Gamma(\chi), \qquad \chi = x - \kappa\tau, \tag{2}$$

where κ is a constant, Eq. (1) is converted to

$$\Xi_1(\Gamma, \Gamma', \Gamma'', \Gamma''', \ldots) = 0, \tag{3}$$

where Ξ_1 is in general a polynomial function of the arguments and $\Gamma' = \frac{d\Gamma}{d\chi}$.

The $\exp(-\Omega(\chi))$-expansion method

Step 1. Assume that

$$\Gamma(\chi) = \sum_{l=0}^{S} \mu_l(\exp(-\Omega(\chi)))^l, \tag{4}$$

where μ_l are unknowns and $\Omega(\chi)$ is a solution of the auxiliary linear ordinary differential equation

$$\Omega'(\chi) = \exp(-\Omega(\chi)) + c_2\exp(\Omega(\chi)) + c_1, \tag{5}$$

where c_1 and c_2 are arbitrary constants.

Step 2. Eq. (5) exerts different solutions classified into the following situations (see, for example, [37]):

Situation 1. When $c_1^2 - 4c_2 > 0$, $c_2 \neq 0$.

$$\Omega_1(\chi)=\ln\left(\frac{-\sqrt{c_1^2-4c_2}\tanh\left(\frac{\sqrt{c_1^2-4c_2}}{2}(\chi+A)\right)-c_1}{2c_2}\right).$$

Situation 2. When $c_1^2-4c_2<0$, $c_2\neq 0$.

$$\Omega_2(\chi)=\ln\left(\frac{\sqrt{4c_2-c_1^2}\tan\left(\frac{\sqrt{4c_2-c_1^2}}{2}(\chi+A)\right)-c_1}{2c_2}\right).$$

Situation 3. When $c_1^2-4c_2>0$, $c_2=0$.

$$\Omega_3(\chi)=-\ln\left(\frac{c_1}{\cosh(c_1(\chi+A))+\sinh(c_1(\chi+A))-1}\right).$$

Situation 4. When $c_1^2-4c_2=0$, $c_2\neq 0$ and $c_1\neq 0$.

$$\Omega_4(\chi)=\ln\left(-\frac{2(c_1(\chi+A))+2}{c_1^2(\chi+A)}\right).$$

Situation 5. When $c_1^2-4c_2=0$, $c_1=c_2=0$.

$$\Omega_5(\chi)=\ln(\chi+A),$$

where A is the constant of integration.

Step 3. The value of S in Eq. (4) is determined by the balancing principle for Eq. (3). Substituting Eq. (4) into Eq. (3) yields a set of algebraic equations in μ_l by equating each coefficient of every power of $\exp(-\Omega(\chi))$ to zero. Substituting the values of these constants in Eq. (4), we obtain exact traveling wave solutions of Eq. (1).

The first integral method

Step 1. Consider Eqs. (1)-(3).
Step 2. Let $\Lambda=\Lambda(\chi)$ be a new independent variable which satisfies

$$\Gamma(\chi)=\Theta(\chi),\qquad \Lambda(\chi)=\Theta^{'}(\chi).$$

Step 3. By using the previous relation, Eq. (1) can be transformed to

$$\Lambda(\chi)=\Theta^{'}(\chi),\qquad \Lambda^{'}(\chi)=\Upsilon(\Theta(\chi),\Lambda(\chi)).$$

where Υ is a polynomial. Applying the division theorem to the last system of ODEs, Eq. (3) is reduced to a first integral ODE which can be solved to find solutions of Eq. (1).

The sine-Gordon expansion method

Consider the sine-Gordon equation

$$u_{xx}-u_{\tau\tau}=a^2\sin u, \tag{6}$$

where $u=u(x,\tau)$ and $a\neq 0$.

Step 1. Eq. (2) transformed Eq. (6) to

$$\Gamma''=\frac{a^2}{1-\kappa^2}\sin\Gamma. \tag{7}$$

Eq. (7) can be integrated to

$$\left[\left(\frac{\Gamma}{2}\right)^{'}\right]^2=\frac{a^2}{1-\kappa^2}\sin^2\left(\frac{\Gamma}{2}\right)+A_0, \tag{8}$$

where A_0 is the integration constant.

Setting $A_0=0$, $\frac{\Gamma}{2}=\wp(\chi)$ and $\omega^2=\frac{a^2}{1-\kappa^2}$, Eq. (8) can be written in the form

$$\wp' = \omega \sin(\wp). \tag{9}$$

For $\omega = 1$, we have

$$\wp' = \sin(\wp), \tag{10}$$

which leads to

$$\sin\wp = \sin(\wp(\chi)) = \frac{2\rho \exp(\chi)}{\rho^2 \exp(2\chi) + 1}\Big|_{\rho=1} = \mathrm{sech}(\chi), \tag{11}$$

$$\cos\wp = \cos(\wp(\chi)) = \frac{\rho^2 \exp(2\chi) - 1}{\rho^2 \exp(2\chi) + 1}\Big|_{\rho=1} = \tanh(\chi), \tag{12}$$

where $\rho \neq 0$ is the integration constant.

Step 2. Let the general solution of Eq. (1) be in the form

$$\Gamma(\chi) = \sum_{i=1}^{N} \tanh^{i-1}(\chi)[B_i \mathrm{sech}(\chi) + A_i \tanh(\chi)] + A_0. \tag{13}$$

With the aid of Eqs. (11) and (12), Eq. (13) is transformed to

$$\Gamma(\wp) = \sum_{i=1}^{N} \cos^{i-1}(\wp)[B_i \sin(\wp) + A_i \cos(\wp)] + A_0. \tag{14}$$

The homogeneous balance rule gives the value of N.

Step 3. Inserting Eq. (14) into Eq. (3), and equating the coefficients of every power of $\sin(\wp)\cos(\wp)$ to zero gives a bunch of nonlinear algebraic equations in A_0, A_i, B_i. By solving the resulting algebraic system of A_0, A_i, B_i and substituting their values in Eq. (14), a variety of exact solutions for Eq. (1) are established.

3. Applications and discussions

The Hirota-Maccari system can be written as

$$is_\tau + s_{xy} + is_{xxx} + s\,r - i|s|^2 s_x = 0, \tag{15}$$

$$3r_x + \left(|s|^2\right)_y = 0, \tag{16}$$

where, $s(\tau, x, y)$ and $r(\tau, x, y)$ represent respective scalar complex and real fields. x, y and τ, respectively, are spatial and temporal coordinates.

To solve Eqs. (15) and (16), we suppose

$$s(\tau, x, y) = \Gamma_1(\chi)\exp(i\psi(\tau, x, y)), \tag{17}$$

$$r(\tau, x, y) = \Gamma_2(\chi), \tag{18}$$

where χ, has the form

$$\chi = x + y + \omega\tau. \tag{19}$$

The amplitude portion of the soliton wave is given by $\Gamma_l(\chi)$ ($l = 1, 2$), while ω is the velocity profile of the soliton. Moreover, the phase component $\psi(\tau, x, y)$ bears the subsequent definition

$$\psi(\tau, x, y) = px + qy + \kappa\tau, \tag{20}$$

where p, q, and κ are arbitrary constants. Substituting (17) and (18) into (15) and (16) respectively, the real and imaginary components yield

$$3(1 - 3p)\Gamma_1'' + 3\left(p^3 - pq - \kappa\right)\Gamma_1 + (3p - 1)\Gamma_1^3 = 0, \tag{21}$$

$$\Gamma_2 = -\frac{1}{3}\Gamma_1^2, \quad p \neq \frac{1}{3}. \tag{22}$$

3.1. The $\exp(-\Omega(\chi))$-expansion method

Here, Eq. (21) is solved via the $\exp(-\Omega(\chi))$-expansion method.
Balancing Γ_1'' with Γ_1^3 in Eq. (21) yields $S=1$. So, the supposed solution becomes

$$\Gamma_1(\chi)=A_0+A_1\exp(-\Omega(\chi)). \tag{23}$$

By applying step 3 introduced by this method in Section 2, we get a system of polynomial equations which can be solved to present the following sets

SET 1:

$$A_0=\pm\sqrt{\frac{3}{2}}c_1,\quad A_1=\pm\sqrt{6},\quad c_2=\frac{1}{4}\left(\frac{2(p^3-pq-\kappa)}{-1+3p}+c_1^2\right)$$

SET 2:

$$A_0=0,\quad A_1=\pm\sqrt{6},\quad c_1=0,\quad c_2=\frac{-p^3+pq+\kappa}{2-6p}$$

From **SET 1:**(1)When $c_1^2-4c_2>0$, $c_2\neq 0$.

$$\begin{aligned} s(\tau,x,y) &= \sqrt{\frac{3}{2}}\left[\pm c_1\pm\frac{-2p^3+2pq+2\kappa+c_1^2-3pc_1^2}{(-1+3p)c_1+\sqrt{(-2+6p)(-p^3+pq+\kappa)}\tanh\left[\sqrt{\frac{-p^3+pq+\kappa}{-2+6p}}(E+\chi)\right]}\right] \\ &\quad\times\exp(i\psi(\tau,x,y)), \end{aligned}$$

$$r(\tau,x,y)=-\frac{1}{2}\left[\pm c_1\pm\frac{-2p^3+2pq+2\kappa+c_1^2-3pc_1^2}{(-1+3p)c_1+\sqrt{(-2+6p)(-p^3+pq+\kappa)}\tanh\left[\sqrt{\frac{-p^3+pq+\kappa}{-2+6p}}(E+\chi)\right]}\right]^2. \tag{24}$$

(2) When $c_1^2-4c_2<0$, $c_2\neq 0$.

$$\begin{aligned} s(\tau,x,y) &= \sqrt{\frac{3}{2}}\left[\pm c_1\mp\frac{-2p^3+2pq+2\kappa+c_1^2-3pc_1^2}{(1-3p)c_1+\sqrt{(-2+6p)(p^3-pq-\kappa)}\tan\left[\sqrt{\frac{p^3-pq-\kappa}{-2+6p}}(E+\chi)\right]}\right] \\ &\quad\times\exp(i\psi(\tau,x,y)), \end{aligned}$$

$$r(\tau,x,y)=-\frac{1}{2}\left[\pm c_1\mp\frac{-2p^3+2pq+2\kappa+c_1^2-3pc_1^2}{(1-3p)c_1+\sqrt{(-2+6p)(p^3-pq-\kappa)}\tan\left[\sqrt{\frac{p^3-pq-\kappa}{-2+6p}}(E+\chi)\right]}\right]^2. \tag{25}$$

(3) When $c_1^2-4c_2>0$, $c_2=0$.

$$s(\tau,x,y)=\pm\sqrt{\frac{3}{2}}c_1\coth\left[\frac{1}{2}c_1(E+\chi)\right]\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=-\frac{1}{2}c_1^2\coth^2\left[\frac{1}{2}c_1(E+\chi)\right]. \tag{26}$$

(4) When $c_1^2-4c_2=0$, $c_2\neq 0$ and $c_1\neq 0$.

$$s(\tau,x,y)=\mp\frac{\sqrt{\frac{3}{2}}c_1}{-1+c_1(E+\chi)}\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=\frac{-\frac{1}{2}c_1^2}{(-1+c_1(E+\chi))^2}. \tag{27}$$

(5) When $c_1^2-4c_2=0$, $c_2=c_1=0$.

$$s(\tau,x,y)=\pm\frac{\sqrt{6}}{E+\chi}\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=\frac{-2}{(E+\chi)^2}, \tag{28}$$

where E denotes the integration constant.

From **SET 2:**

(1) When $c_1^2-4c_2>0$, $c_2\neq 0$.

$$s(\tau,x,y)=\pm\left[\frac{\sqrt{-p^3+pq+\kappa}\coth\left[\sqrt{\frac{-p^3+pq+\kappa}{-2+6p}}(E+\chi)\right]}{\sqrt{-\frac{1}{3}+p}}\right]\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=-\frac{1}{3}\left[\frac{(-p^3+pq+\kappa)\coth^2\left[\sqrt{\frac{-p^3+pq+\kappa}{-2+6p}}(E+\chi)\right]}{\left(-\frac{1}{3}+p\right)}\right]. \tag{29}$$

(2) When $c_1^2-4c_2<0$, $c_2\neq 0$.

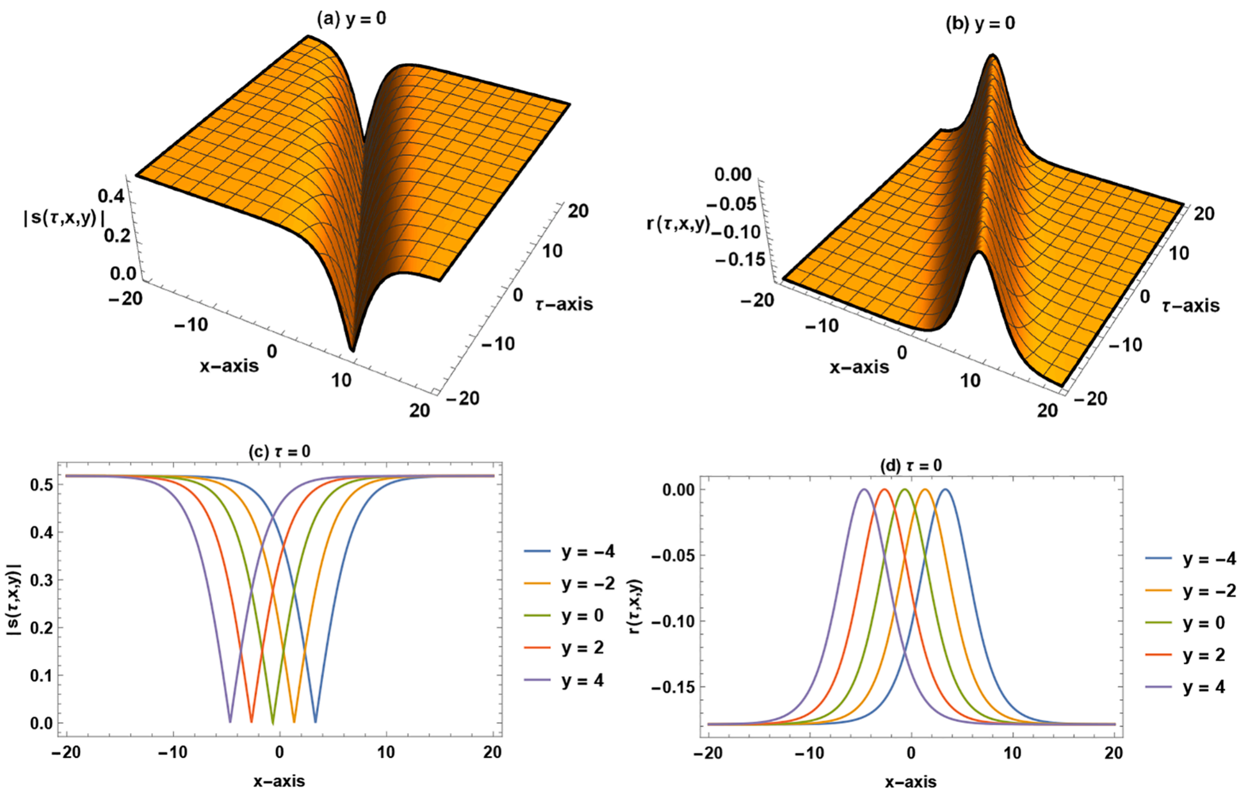

Fig. 1. The soliton wave solution given by (24).

$$s(\tau,x,y)=\pm\left[\frac{\sqrt{-p^3+pq+\kappa}\cot\left[\sqrt{\frac{-p^3+pq+\kappa}{2-6p}}(E+\chi)\right]}{\sqrt{\frac{1}{3}-p}}\right]\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=-\frac{1}{3}\left[\frac{(-p^3+pq+\kappa)\cot^2\left[\sqrt{\frac{-p^3+pq+\kappa}{2-6p}}(E+\chi)\right]}{\left(\frac{1}{3}-p\right)}\right], \tag{30}$$

where E denotes the integration constant.

In this section, we confine ourselves to discuss the obtained nonsingular solutions and illustrate their physical meaning.

The dynamical property of the wave structures via the solutions given by Eq. (24) for different choices of the free parameters, is presented in Fig. 1. The Figs. 1(a) and (c), depict the 3D and 2D charts of the absolute value of $s(\tau,x,y)$. Moreover, $r(\tau,x,y)$ is plotted in Figs. 1(b) and (d).

Fig. 1 represents the complex soliton wave solution given by Eq. (24) with the parameters $p=1.5$, $q=2$, $\kappa=1$, $\omega=0.5$, $E=0$, and $c_1=3$. We observe that in Figs. 1(a), (c) the absolute value of $s(\tau,x,y)$ is a dark soliton wave, while in Figs. 1(b), (c) $r(\tau,x,x)$ is a bright soliton wave.

3.2. The first integral method

First, we consider the following transformation

$$\Gamma_1(\chi)=\Theta(\chi),\qquad \Lambda(\chi)=\Theta^{'}(\chi). \tag{31}$$

Eq. (21) takes the following form

$$(3(1-3p))\Lambda^{'}+3\left(p^3-pq-\kappa\right)\Theta+(3p-1)\Theta^3=0. \tag{32}$$

Or

$$\Lambda^{'}(\chi)=\frac{(p^3-pq-\kappa)}{3p-1}\Theta+\frac{1}{3}\Theta^3. \tag{33}$$

For non-trivial solutions $\Theta(\chi)$ and $\Lambda(\chi)$, we assume that $\Pi(\Theta,\Lambda)=\sum_{j=0}^{m}a_j(\Theta)\Lambda^j(\chi)$ is an irreducible polynomial in $C[\Theta,\Lambda]$, where C is complex.

$$\Pi(\Theta(\chi),\Lambda(\chi))=\sum_{j=0}^{m}a_j(\Theta)\Lambda^j=0, \tag{34}$$

where $a_j(\Theta)(j=0,1,2,3,\ldots,m)$ denote polynomials in the arguments and $a_m(\Theta)\neq0$. From the division theorem, we have a polynomial $[g(\Theta)+h(\Theta)\Lambda]$ existing in $C[\Theta,\Lambda]$ such that

$$\frac{d\Pi}{d\chi}=\frac{\partial\Pi}{\partial\Theta}\frac{d\Theta}{d\chi}+\frac{\partial\Pi}{\partial\Lambda}\frac{d\Lambda}{d\chi}=[g(\Theta)+h(\Theta)\Lambda(\chi)]\sum_{j=0}^{m}a_j(\Theta)\Lambda^j. \tag{35}$$

For $m=1$, Eq. (34) becomes,

$$\Pi(\Theta,\Lambda)=a_0(\Theta)+a_1(\Theta)\Lambda=0. \tag{36}$$

Eq. (35) becomes

$$\begin{aligned}&\tfrac{da_0(\Theta)}{d\Theta}\Lambda+\tfrac{da_1(\Theta)}{d\Theta}\Lambda^2+a_1(\Theta)\left[\tfrac{(p^3-pq-\kappa)}{3p-1}\Theta+\tfrac{1}{3}\Theta^3\right]\\&=a_0(\Theta)g(\Theta)+[a_1(\Theta)g(\Theta)+a_0(\Theta)h(\Theta)]\Lambda+a_1(\Theta)h(\Theta)\Lambda^2.\end{aligned} \tag{37}$$

Setting the coefficients of $\Lambda^j(j=0,1,2)$ in Eq. (37) gives

$$\Lambda^0:\qquad a_0(\Theta)g(\Theta)=a_1(\Theta)\left[\frac{(p^3-pq-\kappa)}{3p-1}\Theta+\frac{1}{3}\Theta^3\right], \tag{38}$$

$$\Lambda^1: \qquad \frac{da_0(\Theta)}{d\Theta} = a_1(\Theta)g(\Theta) + a_0(\Theta)h(\Theta), \tag{39}$$

$$\Lambda^2: \qquad \frac{da_1(\Theta)}{d\Theta} = a_1(\Theta)h(\Theta). \tag{40}$$

$a_j(\Theta)(j=0,1)$ are polynomials. So, if $h(\Theta)$=0, Eq. (40) yields $a_1(\Theta)=$ constant (we put $a_1(\Theta)\ =1$). The balancing rule between the degrees of $a_0(\Theta)$ and $g(\Theta)$ gives the degree of $g(\Theta)$ equals 1. Now, we suppose that

$$g(\Theta) = A\Theta + B, \tag{41}$$

where $A \neq 0$. From Eq. (39), we obtain

$$a_0(\Theta) = \frac{A\Theta^2}{2} + B\Theta + C, \tag{42}$$

where C is integration constant. Substituting the values of $a_0(\Theta), a_1(\Theta)$, and $g(\Theta)$ in Eq. (38) and equating the coefficients of $\Theta^j(j=0,1,2,3)$, we obtain a system of algebraic equations in A, B, and C which solves to

$$A = \pm\sqrt{\frac{2}{3}}, \qquad B=0, \qquad C = \pm\frac{\sqrt{6}(p^3-pq-\kappa)}{-2+6p}. \tag{43}$$

Plugging Eq. (43) and Eq. (42) into Eq. (37), we get

$$\Lambda(\chi) = \frac{\pm 3\kappa \pm \Theta^2 \mp 3p(p^2-q+\Theta^2)}{\sqrt{6}(-1+3p)}. \tag{44}$$

From Eq. (38), we get

$$\Theta = \Gamma_1(\chi) = \pm\frac{\sqrt{-p^3+pq+\kappa}\tanh\left[\frac{\sqrt{-p^3+pq+\kappa}(A_0+\chi)}{\sqrt{-2+6p}}\right]}{\sqrt{p-\frac{1}{3}}}. \tag{45}$$

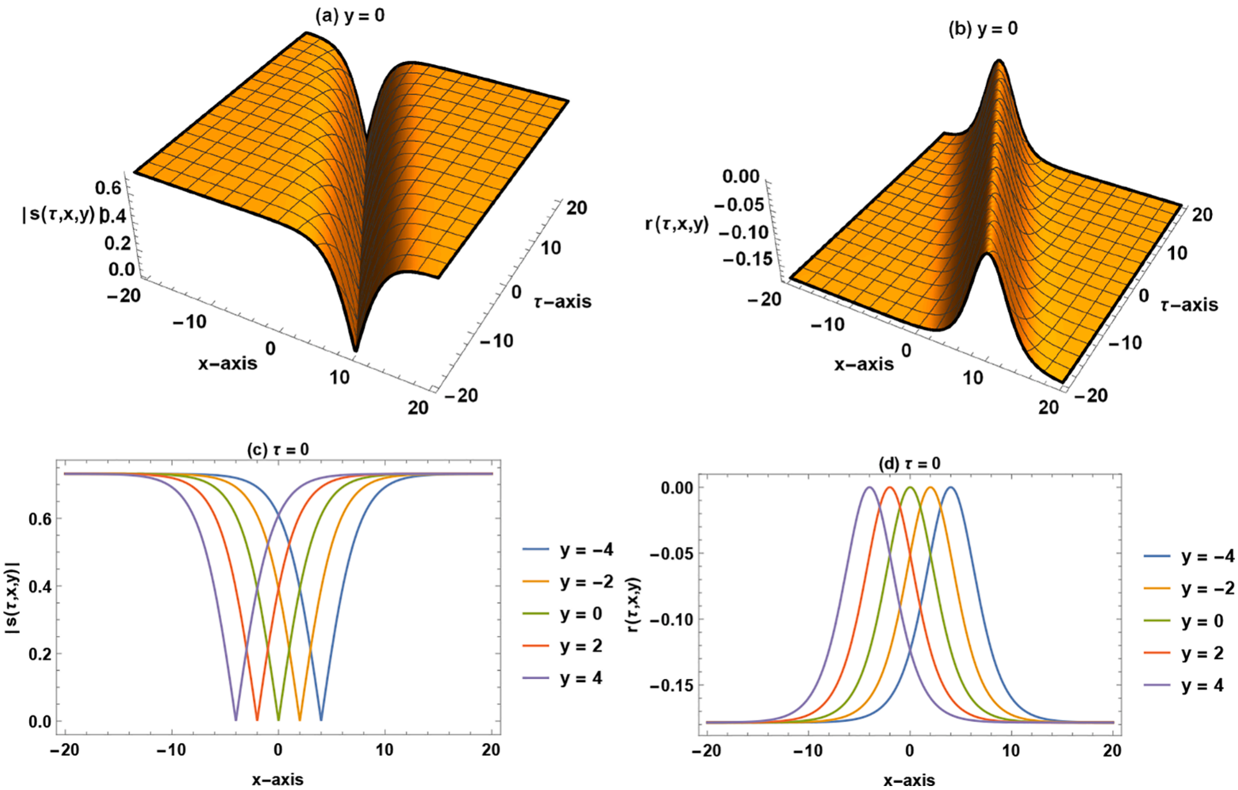

Fig. 2. The soliton wave solution given by (46).

where A_0 is integration constant. Hence, we obtain dark soliton solutions given by

$$s(\tau,x,y)=\pm\frac{\sqrt{-p^3+pq+\kappa}\tanh\left[\frac{\sqrt{-p^3+pq+\kappa}(A_0+\chi)}{\sqrt{6\left(p-\frac{1}{3}\right)}}\right]}{\sqrt{p-\frac{1}{3}}}\times\exp(i\psi(\tau,x,y)),$$

$$r(\tau,x,y)=\frac{(p^3-pq-\kappa)\tanh^2\left[\frac{\sqrt{-p^3+pq+\kappa}(A_0+\chi)}{\sqrt{6\left(p-\frac{1}{3}\right)}}\right]}{3\left(p-\frac{1}{3}\right)},\tag{46}$$

where $(-p^3+pq+\kappa)\left(p-\frac{1}{3}\right)>0$.

The dynamical property of the wave structures via the solutions given by Eq. (47) for different choices of the free parameters, are presented in Fig. 2. The Figs. 2(a) and (c), depict the 3D and 2D charts of the absolute value of $s(\tau,x,y)$. Moreover, $r(\tau,x,y)$ is plotted in Figs. 2(b) and (d).

Fig. 2 represents the complex soliton wave solution given by Eq. (47) with the same parameters used in Fig. 1 and with the same assumption (24), in which $A_0=0$.

3.3. The sine-Gordon expansion method

This subsection is devoted to the sine-Gordon expansion method and its application on (21).

Using the balancing rule between Γ_1'' and Γ_1^3 in Eq. (21) gives $N=1$. Therefore, we have

$$\Gamma_1(\chi)=B_1\mathrm{sech}(\chi)+A_1\tanh(\chi)+A_0.\tag{47}$$

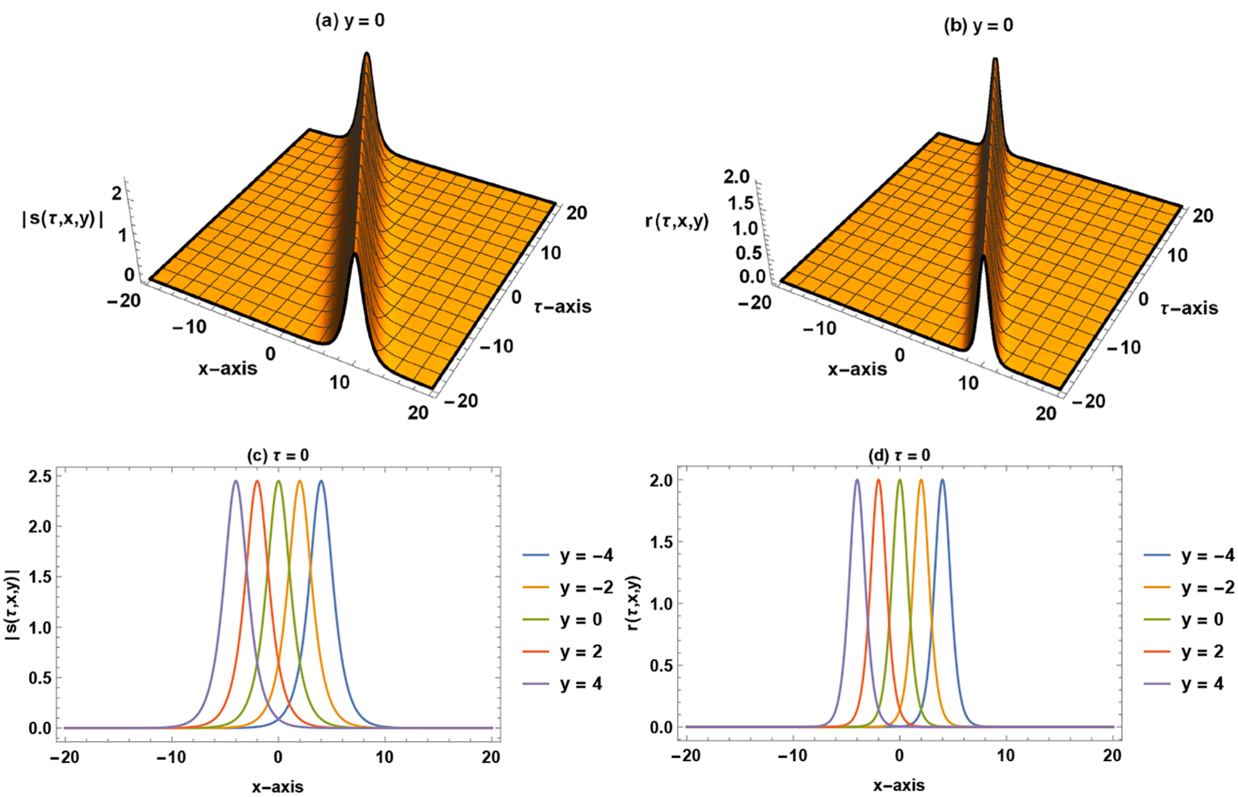

Fig. 3. The soliton wave solution given by (50).

With the aid of Eqs. (11) and (12), Eq. (14) can be stated as

$$\Gamma_1(\chi) = B_1\sin(\chi) + A_1\cos(\chi) + A_0. \tag{48}$$

By applying step 3 introduced by this method in Section 2, we get a system of polynomial equations in A_0, A_1, and B_1 which can be solved to present the following sets.

SET 1:

$$A_0 = 0,\quad A_1 = \pm\sqrt{6},\quad B_1 = 0,\quad \kappa = -2 + p^3 - p(-6 + q).$$

SET 2

$$A_0 = 0,\quad A_1 = 0,\quad B_1 = \pm i\sqrt{6},\quad \kappa = 1 + p^3 - p(3 + q).$$

From **SET 1**, we get the dark soliton solutions in the form

$$s(\tau, x, y) = \pm\sqrt{6}\tanh(\chi) \times \exp(i\psi(\tau, x, y)),$$

$$r(\tau, x, y) = -2\tanh^2(\chi). \tag{49}$$

Finally, from **SET 2**, we get the bright soliton solutions as

$$s(\tau, x, y) = \pm i\sqrt{6}\operatorname{sech}(\chi) \times \exp(i\psi(\tau, x, y)),$$

$$r(\tau, x, y) = 2\operatorname{sech}^2(\chi). \tag{50}$$

The dynamical property of the wave structures via the solutions given by Eq. (50) for different choices of the free parameters, are presented in Fig. 3. The Figs. 3(a) and (c), depict the 3D and 2D charts of the absolute value of $s(\tau,x,y)$. Moreover, $r(\tau,x,y)$ is plotted in Figs. 3(b) and (d).

Fig. 3 represents the complex soliton wave solution given by Eq. (50) with $\omega = 0.5$. Figs. 3(a), (c) and Figs. 3(b), (d) represents bright soliton waves of the absolute value of $s(\tau,x,y)$ and $r(\tau,x,y)$ respectively.

4. Conclusion

The spotlight of this work is to retrieve dark, bright, and singular solutions of the Hirota-Maccari system. The integration of the system was carried out with the help of the three different approaches: the $\exp(-\Omega(\chi))$-expansion, the first integral, and the Sine-Gordon expansion methods. The existence of these solutions is guaranteed by constraint conditions. The presented solutions would be helpful in observing optical solitons in nature. Our results show that the structures of the obtained wave solutions are multifarious in nonlinear dynamic system. In the near future, we will modify the algorithms presented here to deal with different NLEEs when their coefficients are variables, for exhaling nonautonomous wave solutions.

Declaration of Competing Interest

The authors have declared no conflict of interest.

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