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Lump waves and their dynamics in a generalized Kadomtsev–Petviashvili-like model

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This study explores dispersion-induced lump structures in a generalized (2+1)-dimensional Kadomtsev–Petviashvili-like framework. Starting from a generalized bilinear representation of the governing equation, we derive positive quadratic wave solutions through symbolic computation, which yield lump structures. The analysis reveals that the stationary points of these quadratic waves lie along a straight line in the spatial domain and move at constant velocities. Along this characteristic line, the lump wave amplitude becomes zero. The development of these lump waves is attributed to the combined effects of five distinct dispersion terms in the model.

Keywords: Generalized bilinear form; lump wave; symbolic computation; dispersion.

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1. Introduction

Exact and explicit closed-form solutions are of primary importance in mathematical physics and engineering sciences, as they provide deep insights and general frameworks for addressing complex nonlinear problems. Yet, obtaining such solutions is often a challenging task. Considerable research has therefore been devoted either to deriving closed-form expressions directly or to identifying conditions under which they can exist.

In the context of soliton theory and integrable models, wave structures, such as solitons, rogue waves and lump waves, are frequently constructed through symbolic analysis or computational techniques. These dispersive waves arise from the intricate interplay between nonlinearity and dispersion, making their construction, whether analytical or numerical, a key focus in the study of wave dynamics across diverse physical and engineering settings.

Two cornerstone techniques in soliton theory and the analysis of integrable models are the Inverse Scattering Technique (IST)[1] and the Hirota bilinear method.[2] The IST acts as a nonlinear generalization of the Fourier transform, tailored for integrable systems. It is widely used to solve initial data problems for nonlinear equations via their associated Lax pairs,[3] and to investigate the asymptotic dynamics of dispersive waves in the long-time limit, including those lacking solitons.[4] In contrast, the Hirota method provides an efficient framework for generating exact wave solutions, such as lump waves and solitons, especially for nonlinear dispersive equations in (2+1)- and (3+1)-dimensional settings.[5–10]

Let x and y denote spatial coordinates, and t represent time. Given a polynomial $P(x, y, t)$, we can formulate a Hirota bilinear differential equation in (2+1)-dimensions:

$$P(D_x, D_y, D_t) f \cdot f = 0, \tag{1.1}$$

where D_x, D_y and D_t are Hirotas bilinear operators,[2] defined by

$$\begin{aligned} D_x^m D_y^n D_t^k f \cdot f = & \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \\ & \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^k f(x, y, t) f(x', y', t')\bigg|_{x'=x, y'=y, t'=t}, \end{aligned}$$

with m, n, k being nonnegative integers. By using the Bell polynomial theory, one can often derive nonlinear PDEs for a scalar function u from Hirota bilinear forms via logarithmic derivative transformations (see, e.g. Ref. 11), for example,

$$u = \beta(\ln f)_{xx}, \quad \beta(\ln f)_{yy}, \quad \beta(\ln f)_{xy}, \quad \beta(\ln f)_x, \quad \beta(\ln f)_y, \tag{1.2}$$

where β is an appropriate nonzero constant. Hirotas bilinear method allows for the construction of N-soliton solutions in the exponential superposition form (see, e.g. Refs. 5 and 12):

$$f = \sum_{\lambda=0,1} \exp\left(\sum_{i=1}^N \lambda_i \eta_i + \sum_{i<j} \lambda_i \lambda_j c_{ij}\right), \tag{1.3}$$

where the sum $\sum_{\lambda=0,1}$ runs over all combinations $\lambda_1, \lambda_2, \ldots, \lambda_N \in \{0, 1\}$. Phase shifts c_{ij} and the wave variables η_i are given by

$$\exp(c_{ij}) = -\frac{P(\omega_j - \omega_i, k_i - k_j, l_i - l_j)}{P(\omega_j + \omega_i, k_i + k_j, l_i + l_j)}, \quad \text{where } 1 \le i < j \le N \tag{1.4}$$

and

$$\eta_i = k_i x + l_i y - \omega_i t + \eta_{i,0}, \quad \text{where } 1 \le i \le N. \tag{1.5}$$

The only requirement ensuring the existence of N-soliton solutions is the dispersion relation:

$$P(-\omega_i, k_i, l_i) = 0, \quad \text{where } 1 \le i \le N. \tag{1.6}$$

A central problem is to determine whether a function f of the form (1.3) indeed solves the bilinear equation (1.1) under the dispersion conditions (1.6). A systematic algorithm for this verification, with examples in both (1+1)-and (2+1)-dimensional cases, is presented in Refs. 12 and 13.

Another class of explicit wave solutions in nonlinear integrable models consists of lump waves and rogue waves, which are closely related to solitons and capture a broad range of nonlinear phenomena.[14] Lump waves are rationally localized in space, vanishing at infinity in all directions at fixed time.[14,15] For example, the Kadomtsev–Petviashvili I (KPI) equation admits a wide variety of lump solutions,[6] some of which emerge in the long-wave limit of multi-soliton configurations.[16] These localized structures are not confined to integrable systems: Lump-type waves also emerge in nonintegrable (2+1)D KP, BKP and KP–Boussinesq-type extensions,[17] and even in certain linear wave settings in higher dimensions through linear superposition.[18,19]

A common technique for constructing lump solutions is the sum-of-squares ansatz, in which a positive quadratic function is substituted into a bilinear equation.[6,14] Applying logarithmic derivative transformations to such quadratic forms yields lump solutions for various nonlinear model equations. In this paper, we employ this method to examine a (2+1)-D generalized Kadomtsev–Petviashvili(gKP)-like equation incorporating four nonlinear and five distinct dispersive terms. These nonlinear and dispersive effects act as the balancing mechanisms that sustain lump structures. We derive lump solutions symbolically using computer algebra systems and analyze the stationary points of the underlying quadratic function to gain insight into the wave dynamics. The paper concludes with remarks and prospective directions for related problems.

2. A Generalized Kadomtsev–Petviashvili-like Model

We consider a category of generalized bilinear differential operators, introduced in Ref. 20

$$\begin{aligned} D_{p,x}^m D_{p,y}^n D_{p,t}^k f \cdot f = &\left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} + \alpha_p \frac{\partial}{\partial y'}\right)^n \\ &\left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'}\right)^k f(x,y,t) f(x',y',t')\Big|_{x'=x, y'=y, t'=t}, \end{aligned} \tag{2.1}$$

where

$$\alpha_p^k = (-1)^{r(k)} \quad \text{where } k \equiv r(k) \bmod p, \ 0 \le r(k) < p. \tag{2.2}$$

For example, taking $p = 3$ yields the pattern

$$\alpha_3 = -1, \quad \alpha_3^2 = \alpha_3^3 = 1, \quad \alpha_3^4 = -1, \quad \alpha_3^5 = \alpha_3^6 = 1, \ldots \tag{2.3}$$

and taking $p = 5$ gives

$$\begin{gathered}\alpha_5 = -1, \quad \alpha_5^2 = 1, \quad \alpha_5^3 = -1, \quad \alpha_5^4 = \alpha_5^5 = 1, \quad \alpha_5^6 = -1, \\ \alpha_5^7 = 1, \quad \alpha_5^8 = -1, \quad \alpha_5^9 = \alpha_5^{10} = 1, \ldots.\end{gathered} \tag{2.4}$$

Setting $p = 3$, we consider a gKP-like bilinear equation:

$$\begin{aligned}F_{\text{gKP-like}}(f) :={}& (D_{3,x}^4 + \gamma_1 D_{3,t} D_{3,x} + \gamma_2 D_{3,t} D_{3,y} + \gamma_3 D_{3,x}^2 \\ &+ \gamma_4 D_{3,x} D_{3,y} + \gamma_5 D_{3,y}^2) f \cdot f \\ ={}& 2[3 f_{xx}^2 + \gamma_1 (f_{tx} f - f_t f_x) + \gamma_2 (f_{ty} f - f_t f_y) + \gamma_3 (f_{xx} f - f_x^2) \\ &+ \gamma_4 (f_{xy} f - f_x f_y) + \gamma_5 (f_{yy} f - f_y^2)] = 0,\end{aligned} \tag{2.5}$$

where $D_{3,x}, D_{3,y}$ and $D_{3,t}$ denote the generalized bilinear derivatives, and γ_i for $1 \le i \le 5$ are arbitrary constants. This represents a natural generalization of the standard KP equation. By redefining the dependent variable through

$$u = 2(\ln f)_x, \tag{2.6}$$

we obtain the corresponding generalized gKP-like model equation:

$$\begin{aligned}P_{\text{gKP-like}}(u) :={}& 3 u_x u_{xx} + 3 u u_x^2 + \frac{3}{2} u^3 u_x + \frac{3}{2} u^2 u_{xx} \\ &+ \gamma_1 u_{tx} + \gamma_2 u_{ty} + \gamma_3 u_{xx} + \gamma_4 u_{xy} + \gamma_5 u_{yy} = 0.\end{aligned} \tag{2.7}$$

This new model includes four nonlinear terms and five dispersion terms.

In the case $\gamma_1 = \gamma_5 = 1$ and all other coefficients set to zero, the generalized equation reduces to the standard KP-like equation:

$$3 u_x u_{xx} + 3 u u_x^2 + \frac{3}{2} u^3 u_x + \frac{3}{2} u^2 u_{xx} + u_{tx} + u_{yy} = 0, \tag{2.8}$$

whose rational solutions have been studied in Ref. 21.

The model equation (2.7) is exactly related to the generalized bilinear equation (2.5) via

$$P_{\text{gKP-like}}(u) = \left[\frac{F_{\text{gKP-like}}(f)}{f^2}\right]_x. \tag{2.9}$$

Hence, u, defined by (2.6), solves the nonlinear generalized model equation (2.7) whenever f satisfies the generalized bilinear equation (2.5).

Many questions remain regarding the integrability of this new model, including whether it admits lump solutions, which are typical in integrable systems. In the following section, we analyze this problem, focusing particularly on a class of lump solutions influenced by the dispersion terms.

3. Lump Waves Managed by Dispersion

We now focus on constructing lump wave solutions for the gKP-like model equation (2.7) by performing symbolic computations on the corresponding generalized bilinear equation (2.5). In particular, we demonstrate that the five dispersion terms are essential for generating lump waves and examine the stationary points of the resulting quadratic function.

3.1. *Implementation of the sum-of-squares ansatz*

The sum-of-squares ansatz is a well-established framework for formulating lump solutions of nonlinear evolution equations in higher dimensions, as demonstrated in Ref. 6. The approach begins by assuming that the dependent variable can be represented in terms of a logarithmic derivative of a positive quadratic function. Typically, this quadratic form is written as a sum of squared linear terms plus a constant:

$$f = \xi_1^2 + \xi_2^2 + a_9, \quad \xi_1 = a_1x + a_2y + a_3t + a_4, \quad \xi_2 = a_5x + a_6y + a_7t + a_8, \tag{3.1}$$

which ensures the rationally localized solution in all spatial directions. Substituting this form into the generalized bilinear representation of the target equation reduces the problem to solving an algebraic system for the nine parameters a_i. This framework provides a foundation for generating general lump wave structures of lower order in (2+1)-dimensional settings,[14] with symbolic computation used to determine the coefficients.

Plugging the function f from (3.1) into the generalized bilinear equation (2.5) yields a system of algebraic equations. Solving this system via computer algebra gives explicit expressions for a_3, a_7 and a_9:

$$\begin{aligned} a_3 = -&\frac{1}{(a_1\gamma_1 + a_2\gamma_2)^2 + (a_5\gamma_1 + a_6\gamma_2)^2}[a_1(a_1^2 + a_5^2)\gamma_1\gamma_3 + a_2(a_1^2 + a_5^2)\gamma_1\gamma_4 \\ &+ (a_1a_2^2 - a_1a_6^2 + 2a_2a_5a_6)\gamma_1\gamma_5 + (a_1^2a_2 + 2a_1a_5a_6 - a_2a_5^2)\gamma_2\gamma_3 \\ &+ a_1(a_2^2 + a_6^2)\gamma_2\gamma_4 + a_2(a_2^2 + a_6^2)\gamma_2\gamma_5], \end{aligned} \tag{3.2}$$

$$\begin{aligned} a_7 = -&\frac{1}{(a_1\gamma_1 + a_2\gamma_2)^2 + (a_5\gamma_1 + a_6\gamma_2)^2}[a_5(a_1^2 + a_5^2)\gamma_1\gamma_3 + a_6(a_1^2 + a_5^2)\gamma_1\gamma_4 \\ &+ (2a_1a_2a_6 - a_2^2a_5 + a_5a_6^2)\gamma_1\gamma_5 + (2a_1a_2a_5 - a_1^2a_6 + a_5^2a_6)\gamma_2\gamma_3 \\ &+ a_5(a_2^2 + a_6^2)\gamma_2\gamma_4 + a_6(a_2^2 + a_6^2)\gamma_2\gamma_5] \end{aligned} \tag{3.3}$$

and

$$a_9 = -\frac{3(a_1^2 + a_5^2)^2[(a_1\gamma_1 + a_2\gamma_2)^2 + (a_5\gamma_1 + a_6\gamma_2)^2]}{(a_1a_6 - a_2a_5)^2(\gamma_1^2\gamma_5 - \gamma_1\gamma_2\gamma_4 + \gamma_2^2\gamma_3)}, \tag{3.4}$$

while all other parameters can be chosen arbitrarily. Frequency parameters a_3 and a_7 encode dispersion relations in (2+1)-dimensional nonlinear dispersive wave systems, whereas the constant a_9 depends intricately on the wave numbers, playing a key role in the formulation of lump waves. Some dispersion relations of higher order have appeared in studies of lump waves associated with the second flow of the integrable KP hierarchy,[22] and related dynamical behaviors have been explored in various generalized KP-type models (see, e.g. Refs. 23 and 24).

All expressions, (3.2), (3.3) and (3.4), are simplified using symbolic computation. To ensure the parameters are well defined, the following two conditions on the dispersion coefficients and wave numbers must hold:

$$\gamma_1^2\gamma_5 - \gamma_1\gamma_2\gamma_4 + \gamma_2^2\gamma_3 \neq 0, \tag{3.5}$$

which implies

$$\gamma_1^2 + \gamma_2^2 \neq 0 \tag{3.6}$$

and the determinant condition

$$a_1a_6 - a_2a_5 \neq 0, \tag{3.7}$$

which ensures

$$a_1^2 + a_5^2 \neq 0, \quad a_2^2 + a_6^2 \neq 0 \tag{3.8}$$

and also guarantees that the solution u, defined via the logarithmic derivative transformation (2.6), decays to zero as $x^2 + y^2 \to \infty$, thereby confirming its spatial localization.

Regarding positivity of the solution f defined by (3.1), a necessary and sufficient condition on the dispersion coefficients is

$$\gamma_1^2\gamma_5 - \gamma_1\gamma_2\gamma_4 + \gamma_2^2\gamma_3 < 0, \tag{3.9}$$

which guarantees $a_9 > 0$, and therefore $f > 0$. This condition can be satisfied in three specific cases:

$$\gamma_1 \neq 0, \quad \gamma_5 < 0, \tag{3.10}$$

$$\gamma_2 \neq 0, \quad \gamma_3 < 0 \tag{3.11}$$

and

$$\gamma_1\gamma_2\gamma_4 > 0, \tag{3.12}$$

while the other (unmentioned) dispersion coefficients are all assumed to be zero. Therefore, the existence of lump waves is fundamentally ensured by the effect of dispersion (see, also Ref. 25).

In summary, the construction of lump wave solutions via the logarithmic derivative transformation requires the essential conditions (3.7) and (3.9). The former, a determinate condition, ensures spatial localization in all directions, while the latter, a dispersion condition, is both necessary and sufficient to ensure the positivity of the solution f, thereby guaranteeing the well posedness of u throughout the spatial-temporal domain. Under these conditions, the resulting u indeed represents a localized lump wave solution.

3.2. *Characteristic trajectory of stationary points*

Let us now compute the stationary points of the quadratic function f, defined by (3.1). To achieve this, we solve the system given by

$$f_x(x(t), y(t), t) = 0, \quad f_y(x(t), y(t), t) = 0.$$

Since f is a quadratic function in x and y, this leads to the following linear system:

$$a_1\xi_1 + a_5\xi_2 = 0, \quad a_2\xi_1 + a_6\xi_2 = 0,$$

where ξ_1 and ξ_2 are defined as in (3.1). Assuming the nondegeneracy condition (3.7), we obtain

$$\xi_1 = a_1x + a_2y + a_3t + a_4 = 0, \quad \xi_2 = a_5x + a_6y + a_7t + a_8 = 0. \tag{3.13}$$

The stationary points of the quadratic function f are determined by solving (3.13) for x and y as functions of t:

$$\begin{aligned} x(t) = {} & \frac{[(a_1^2 + a_5^2)\gamma_3 - (a_2^2 + a_6^2)\gamma_5]\gamma_1 + [2(a_1a_2 + a_5a_6)\gamma_3 + (a_2^2 + a_6^2)\gamma_4]\gamma_2}{(a_1\gamma_1 + a_2\gamma_2)^2 + (a_5\gamma_1 + a_6\gamma_2)^2} t \\ & + \frac{a_2a_8 - a_4a_6}{a_1a_6 - a_2a_5}, \end{aligned} \tag{3.14}$$

$$\begin{aligned} y(t) = {} & \frac{[(a_1^2 + a_5^2)\gamma_4 + 2(a_1a_2 + a_5a_6)\gamma_5]\gamma_1 - [(a_1^2 + a_5^2)\gamma_3 - (a_2^2 + a_6^2)\gamma_5]\gamma_2}{(a_1\gamma_1 + a_2\gamma_2)^2 + (a_5\gamma_1 + a_6\gamma_2)^2} t \\ & - \frac{a_1a_8 - a_4a_5}{a_1a_6 - a_2a_5}. \end{aligned} \tag{3.15}$$

These expressions describe the trajectory of stationary points at any fixed time t.

Evidently, they lie along a straight line, referred to as a characteristic trajectory, along which both spatial coordinates propagate at constant speeds. Along this trajectory, the lump wave u attains a zero value.

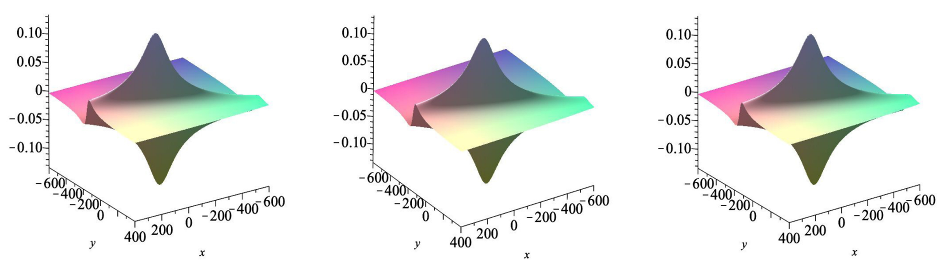

Fig. 1. 3d-plots of u with $t = 0$ (left), $t = 5$ (middle) and $t = 10$ (right).

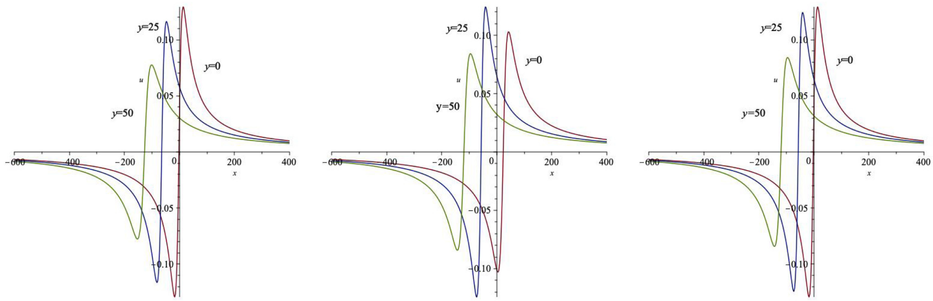

Fig. 2. x curves of u with $t = 0$ (left), $t = 10$ (middle) and $t = 20$ (right).

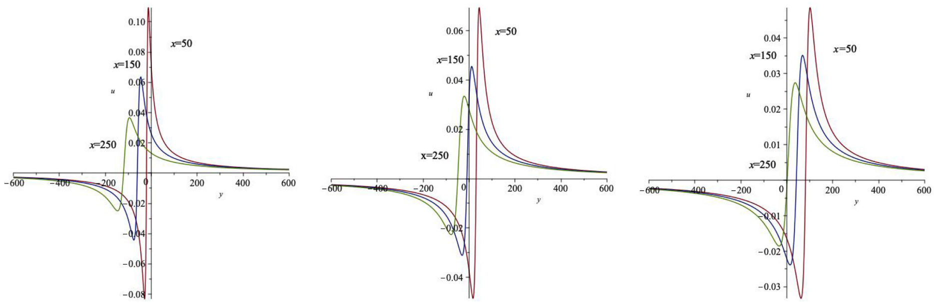

Fig. 3. y curves of u with $t = 0$ (left), $t = 50$ (middle) and $t = 100$ (right).

Figures 1–3 illustrate the spatial profiles and amplification characteristics of the lump wave described by $u = 2(\ln f)_x$, based on the following parameter configurations:

$$\gamma_1 = 1, \quad \gamma_2 = 3, \quad \gamma_3 = -2, \quad \gamma_4 = -3, \quad \gamma_5 = 5$$

and

$$a_1 = 1, \quad a_2 = 3, \quad a_4 = -2, \quad a_5 = 1, \quad a_6 = 2, \quad a_8 = 6.$$

4. Concluding Remarks

A (2+1)-dimensional generalized Kadomtsev–Petviashvili-like model was analyzed, and its lump wave solutions were obtained via symbolic computation using computer algebra systems. The resulting lump wave vanishes along a characteristic curve specified by the stationary points of the associated quadratic function.

Lump waves appear in a wide range of physical and mathematical contexts, highlighting their versatility and the complexity involved in modeling nonlinear dispersive wave phenomena. Prior studies have examined lump solutions in linear wave models,[18,19] as well as in nonlinear, nonintegrable models in both (2+1)-dimensions[26–32] and (3+1)-dimensions.[23,33] The construction of lump waves often leverages Hirota bilinear forms and their generalizations, which provide efficient tools for analyzing these localized structures.[14]

Furthermore, lump waves exhibit diverse interactions with other coherent structures in (2+1)-dimensional integrable models, including homoclinic and heteroclinic waves.[34–36] Meanwhile, N-soliton solutions and integrability have been extensively studied in both local and nonlocal integrable systems using Riemann–Hilbert problems, group reductions and bi-Hamiltonian structures (see, e.g. Refs. 37–47). In particular, a class of coupled and mixed nonlinear Schrödinger equations has been investigated via the Riemann–Hilbert approach in Refs. 48 and 49, and general dynamical behaviors associated with the propagation and interaction of solitons were conjectured for multi-component nonlinear Schrödinger equations in Ref. 49. The existence, structure, and dynamics of lump waves in (2+1)-dimensional generalizations of integrable systems, whether standard or generalized bilinear, scalar or multi-component, remain open and compelling topics for further investigation (see, e.g. Refs. 50–52).

In conclusion, investigating lump waves provides deeper insight into nonlinear dispersive dynamics and may inform applications in physical and engineering systems where localized, coherent, and energy-concentrated structures are significant.

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References

1. M. J. Ablowitz and H. Segur, *Solitons and the Inverse Scattering Transform* (SIAM, Philadelphia, 1981).
2. R. Hirota, *The Direct Method in Soliton Theory* (Cambridge Univ. Press, 2004).
3. V. E. Zakharov and A. B. Shabat, *Funct. Anal. Appl.* **8**, 226 (1974).
4. M. J. Ablowitz and A. C. Newell, *J. Math. Phys.* **14**, 1277 (1973).
5. R. Hirota, *Phys. Rev. Lett.* **27**, 1192 (1971).
6. W. X. Ma, *Phys. Lett. A* **379**, 1975 (2015).
7. J. G. Liu, L. Zhou and Y. He, *Appl. Math. Lett.* **80**, 71 (2018).
8. J. Manafian, *Comput. Math. Appl.* **76**, 1246 (2018).
9. R. F. Zhang and S. D. Bilige, *Nonlinear Dyn.* **95**, 3041 (2019).
10. J. Y. Chu, Y. Q. Liu and W. X. Ma, *Mod. Phys. Lett. B* **39**, 2550060 (2025).
11. W. X. Ma, *Rep. Math. Phys.* **72**, 41 (2013).
12. W. X. Ma, *Opt. Quantum Electron.* **52**, 511 (2020).
13. W. X. Ma, *Int. J. Nonlinear Sci. Numer. Simul.* **23**, 123 (2022).
14. W. X. Ma and Y. Zhou, *J. Differ. Equ.* **264**, 2633 (2018).

15. W. Tan, H. P. Dai, Z. D. Dai and W. Y. Zhong, *Pramana–J. Phys.* **89**, 77 (2017).
16. S. V. Manakov, V. E. Zakharov, L. A. Bordag and V. B. Matveev, *Phys. Lett. A* **63**, 205 (1977).
17. W. X. Ma, Z. Y. Qin and X. Lü, *Nonlinear Dyn.* **84**, 923 (2016).
18. W. X. Ma, *Mod. Phys. Lett. B* **33**, 1950457 (2019).
19. W. X. Ma, *East Asian J. Appl. Math.* **9**, 185 (2019).
20. W. X. Ma, *Stud. Nonlinear Sci.* **2**, 140 (2011).
21. Y. F. Zhang and W. X. Ma, *Z. Naturforsch. A* **70**, 263 (2015).
22. W. X. Ma and L. Q. Zhang, *Pramana–J. Phys.* **94**, 43 (2020).
23. Y. Sun, B. Tian, X. Y. Xie, J. Chai and H. M. Yin, *Wave Random Complex* **28**, 544 (2018).
24. M. M. Liu, J. P. Yu, W. X. Ma, C. M. Khalique and Y. L. Sun, *Mod. Phys. Lett. B* **37**, 2250203 (2023).
25. W. X. Ma, S. Batwa and S. Manukure, *East Asian J. Appl. Math.* **13**, 246 (2023).
26. S. Manukure, Y. Zhou and W. X. Ma, *Comput. Math. Appl.* **75**, 2414 (2018).
27. B. Ren, W. X. Ma and J. Yu, *Nonlinear Dyn.* **96**, 717 (2019).
28. J. P. Yu and Y. L. Sun, *Nonlinear Dyn.* **87**, 2755 (2017).
29. W. X. Ma, Y. Zhang and Y. N. Tang, *East Asian J. Appl. Math.* **10**, 732 (2020).
30. W. X. Ma, Y. S. Bai and A. Adjiri, *Eur. Phys. J. Plus* **136**, 240 (2021).
31. H. C. Ma, S. P. Yue, Y. D. Gao and A. P. Deng, *Qual. Theory Dyn. Syst.* **22**, 72 (2023).
32. W. X. Ma, *Rom. Rep. Phys.* **76**, 108 (2024).
33. W. X. Ma, *Int. J. Nonlinear Sci. Numer. Simul.* **17**, 355 (2016).
34. R. X. Yao, Y. Li and S. Y. Lou, *Commun. Nonlinear Sci. Numer. Simul.* **99**, 105820 (2021).
35. T. C. Kofane, M. Fokou, A. Mohamadou and E. Yomba, *Eur. Phys. J. Plus* **132**, 465 (2017).
36. H. Yasmin, A. S. Alshehry, A. H. Ganie, A. M. Mahnashi and R. Shah, *Optik* **298**, 171576 (2024).
37. W. X. Ma, *Int. J. Geom. Methods Mod. Phys.* **20**, 2350098 (2023).
38. M. J. Ablowitz and Z. H. Musslimani, *Stud. Appl. Math.* **139**, 7 (2017).
39. W. X. Ma, *Rep. Math. Phys.* **93**, 313 (2024).
40. W. X. Ma, *Rom. J. Phys.* **70**, 110 (2025).
41. W. X. Ma, *Indian J. Phys.* **99**, 1063 (2025).
42. J. L. Ji and Z. N. Zhu, *Commun. Nonlinear Sci. Numer. Simul.* **42**, 699 (2017).
43. W. X. Ma, *Mod. Phys. Lett. B* **39**, 2550233 (2025).
44. M. Gürses and A. Pekcan, *J. Math. Phys.* **59**, 051501 (2018).
45. W. X. Ma, *Commun. Theor. Phys.* **78**, 015001 (2026).
46. W. K. Xun and S. F. Tian, *Chin. Ann. Math. Ser. A* **43**, 313 (2022).
47. X. F. Zhang, S. F. Tian and J. J. Yang, *Adv. Appl. Math. Mech.* **14**, 893 (2022).
48. W. X. Ma, *Nonlinear Anal.: Real World Appl.* **47**, 1 (2019).
49. Y. Li, S. F. Tian and J. J. Yang, *Stud. Appl. Math.* **148**, 577 (2021).
50. W. X. Ma, *Chaos Solitons Fractals* **195**, 116309 (2025).
51. W. X. Ma and Y. D. Zhong, *Axioms* **14**, 594 (2025).
52. W. X. Ma, *Pramana–J. Phys.* **99**, 109 (2025).