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Painlevé 分析产生的 Tu 系统的精确解

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提 要 分析了非线性Tu系统的截断Painlevé 级数，得到了Tu系统与Riccati 型方程$\Phi_t - \Phi_x = \alpha\Phi^2 + \beta\Phi + \gamma$的关系，并揭示了Riccati 型方程本身的一些性质. 最后借助于Riccati 型方程给出了Tu 系统的三类显式的精确解.

关键词 可积系统；Painlevé 分析；截断 Painlevé 级数；Riccati 型方程

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Exact solutions to Tu system through Painlevé analysis*

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Abstract The truncated Painlevé series for nonlinear Tu system is carefully analysed. The relations of Tu system to the equation $\Phi_t - \Phi_x = \alpha\Phi^2 + \beta\Phi + \gamma$ of Riccati type are derived from the series and some properties on that equation of Riccati type itself are exposed. Further, three sorts of explicit exact solutions to Tu system are proposed by considering the above equation of Riccati type.

Keywords integrable system; Painlevé analysis; truncated Painlevé series; equation of Riccati type

Although the Painlevé property used as a test for integrability of nonlinear partial differential equations is only a Ablowitz-Ramani-Segur conjecture[1], the study of the Painlevé property may improve the understanding of algebraic, geometrical and analytic properties of integrability for PDEs. for example, Lax representation, Bäcklund transformation and bilinear form[2-5]. Moreover, by the truncated Painlevé series, we can often generate explicit exact solutions to nonlinear PDE's, including KdV equation, Sin-Gordon equation, KP equation and Burgers-KdV equation etc., no matter whether they possess integrability[5-8]. A point that we will explain by example is that the Painlevé series for a nonlinear PDE contains a lot of information about the solutions of the PDE.

We want to analyse, by the Painlevé expansion, a simple but interesting system of coupled equations

$$\begin{cases} u_t = u_x + 2v, \\ v_t = 2\varepsilon u v, \qquad \varepsilon = \pm 1. \end{cases} \tag{1}$$

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This system was first introduced by Tu[9] and then is called as Tu system. Tu system (1) has many nice properties, for instance, the existence of infinite many symmetries and conservation laws[9] and the nondegenerate Hamiltonian structure[10]. In fact, the second equation of Tu system may be easily written as

$$u_{tt}-u_{tx}=2\varepsilon u(u_t-u_x)$$

by using the first equation of (1). What is more, if setting $v=e^{\omega}$, the Tu system (1) can be reduced to the following nonlinear equation

$$\omega_{tt}-\omega_{tx}+4\varepsilon e^{\omega}=0\,.$$

A trivial solution for (1) is that $u=f(x+t)$, $v=0$ with an arbitrary function $f\in C^{\infty}(R)$.

Now let u be a solution of Tu sysem (1) and assume that

$$u=\Phi^{-\alpha_1}\sum_{i=0}^{\infty}u_i\Phi^i,\quad v=\Phi^{-\alpha_2}\sum_{i=0}^{\infty}v_i\Phi^i, \tag{2}$$

where $\Phi=\Phi(x,\ t)$ and $u_i=u_i(x,\ t)$ are analytic functions of x, t in a neigbborhood of the singularity manifold determined by $\Phi(x,\ t)=0$. Substituting (2) into Tu system (1), we can determine the possible values of $(\alpha_1,\ \alpha_2)$: $(\alpha_1,\ \alpha_2)=(1,\ 2)$. In §2, we further discuss the Painlevé series (2) and in §3, establish two relations of the Tu system to an equation of Riccati type. Eventually in §4, we exhibit three sorts of explicit exact solutions, derived form the trnucated Painlevé series, to Tu system (1).

1 Truncated Painlevé series

Make the Painlevé series for Tu system (1)

$$u=\Phi^{-1}\sum_{i=0}^{\infty}u_i\Phi^i,\quad v=\Phi^{-2}\sum_{i=0}^{\infty}v_i\Phi^i.$$

Substitution into Tu system (1) defines the following recurrence relations for $(u_i,\ v_i)$, $i\geqslant 0$,

$$\begin{cases}u_{i-1,t}+(i-1)\Phi_t u_i=u_{i-1,x}+(i-1)\Phi_x u_i+2v_i, i\geqslant 0\,,\\ v_{i-1,t}+(i-2)\Phi_t v_i=2\varepsilon\sum_{\kappa+l=i}u_\kappa v_l,\ \ i\geqslant 0\,,\end{cases} \tag{3}$$

where we accept $u_{-1}=v_{-1}=0$. When $i=0$, (3) engenders

$$\begin{cases}-\Phi_t u_0=-\Phi_x u_0+2v_0,\\ -2\Phi_t v_0=2\varepsilon u_0 v_0\end{cases}$$

This system has a solution

$$\begin{cases}u_0=-\varepsilon\Phi_t,\\ v_0=\dfrac{\varepsilon}{2}\,\Phi_t(\Phi_t-\Phi_x).\end{cases} \tag{4}$$

Therefore the coefficient determinant of the recurrence relation (3) at the $i-$th step reads as

$$r(i)=\det\begin{vmatrix}(i-1)(\Phi_t-\Phi_x) & -2\\ -\Phi_t\ (\Phi_t-\Phi_x) & i\Phi_t\end{vmatrix}=\Phi_t(\Phi_t-\Phi_x)(i+1)(i-2).$$

Resonances occur at $i=-1$, 2 where the u_i, v_i are arbitrary. The Resonance at $i=-1$ corresponds to the arbitrary function Φ. It is easy to see that the recurrence relation is consistently satisfied at the resonance $i=2$ (for more information, see Ref. [11]). Thus Tu system (1) possesses the WTC−Painlevé property[2].

In the following, we would like to analyse the truncated Painlevé series.

Proposition 1 Let $n\geqslant 0$. If $u_i=0$, $i\geqslant n$, then $v_i=0$, $i\geqslant n+1$.

Proof It follows from the first recurrence relation in (3).

Proposition 2 Let $n\geqslant 3$. If $u_i=0$, $i\geqslant n$, but $u_{n-1}\neq 0$, then $v_i=0$, $i\geqslant 0$.

Proof By Proposition 1, we know that $v_i=0$, $i\geqslant n+1$. Then we can prove that $v_i=0$, $0\leqslant i\leqslant n$ by the mathematical induction for i.

First, from the second recurrence relation with $i=2n-1$ in (3), we obtain

$$u_{n-1}u_n=\sum_{\kappa+l=2n-1}u_\kappa v_l=0,$$

and thus $v_n=0$ because $u_{n-1}\neq 0$. Here we have used $(2n-1)-1\geqslant n+1$, i. e. $n\geqslant 3$. Now we assume that $v_i=0$, $i\geqslant m$ for some $0\leqslant m\leqslant n$. Set $i=n+m-2$ and note $(n+m-2)-1\geqslant m$. This moment the second recurrence relation in (3) reduces

$$u_{n-1}v_{m-1}=\sum_{\kappa+l=n+m-2}u_\kappa v_l=0,$$

which means $v_{m-1}=0$, Therefore we have $v_i=0$, $i\geqslant 0$, by the mathematical induction.

From Propositions 1 and 2, the trivial truncated Painlevé series of Tu system (1) must be of the following form

$$u=\frac{u_0}{\Phi}+u_1,\quad v=\frac{u_0}{\Phi^2}+\frac{u_1}{\Phi}+v_2. \tag{5}$$

In the case of the above Painlevé series, the recurrence relations (3) become some identities for $i\geqslant 4$, and give

$$\begin{cases}u_{0t}=u_{0x}+2v_1,\\ v_{0t}-\Phi_t v_1=2\varepsilon(u_0v_1+u_1v_0);\end{cases} \tag{6}$$

$$\begin{cases}u_{1t}=u_{1x}+2v_2,\\ v_{1t}=2\varepsilon(u_0v_2+u_1v_1);\end{cases} \tag{7}$$

$$v_{2t}=2\varepsilon u_1v_2; \tag{8}$$

respectively for $i=1$, 2, 3.

2 Relations of Tu system to the equation of Riccati type

Let us carefully consider the truncated Painlevé series (5). Noticing our selection (4), we obtain from (6)

$$v_1 = \frac{1}{2}(u_{0t} - u_{0x}) = -\frac{\varepsilon}{2}(\Phi_{tt} - \Phi_{tx}), \tag{9}$$

$$u_0 u_1 = \frac{1}{2\varepsilon}(v_{0t} - \Phi_t v_1) - u_0 v_1 = -\frac{1}{4}\Phi_{tt}(\Phi_t - \Phi_x),$$

and thus we may choose

$$u_1 = \frac{\varepsilon}{2}\frac{\Phi_{tt}}{\Phi_t}. \tag{10}$$

Further from the first equality of (7), we obtain

$$v_2 = \frac{1}{2}(u_{1t} - u_{1x}) = \frac{\varepsilon}{4}\frac{(\Phi_t - \Phi_x)_{tt} + \Phi_{tt}(\Phi_x - \Phi_t)_t}{\Phi_t^2}, \tag{11}$$

and now the second equality of (7) automatically holds. Let us now give some manipulations with the equality (8). We first have

$$v_{2t} = 2\varepsilon u_1 v_2 = \frac{\Phi_{tt}}{\Phi_t} v_2,$$

which means that

$$\partial_t \frac{v_2}{\Phi_t} = 0 \text{ or } \partial_t \left\{ \frac{1}{\Phi_t}\left[(\partial_t - \partial_x)\frac{\Phi_{tt}}{\Phi_t}\right]\right\} = 0.$$

Note that $\partial_x \frac{\Phi_{tt}}{\Phi_t} = \partial_t \frac{\Phi_{tx}}{\Phi_t}$. The last equality may be rewritten as

$$\partial_t \left\{ \frac{1}{\Phi_t}\left[\partial_t\left(\frac{1}{\Phi_t}\partial_t(\Phi_t - \Phi_x)\right)\right]\right\} = 0. \tag{12}$$

Integrating the above equation three times with respect to t, we obtain

$$\Phi_t - \Phi_x = \alpha\Phi^2 + \beta\Phi + \gamma, \quad \Phi_t \neq 0, \tag{13}$$

where $\alpha_t = \beta_t = \gamma_t = 0$, but α, β, γ can be chosen as arbitrary functions of χ. The partial differential equation (13) is of Riccati type and the coefficients α, β, γ are three changeable functions of χ.

Summing up, we obtain

Theorem 1 If $\Phi = \Phi(\chi, t)$ satisfies the equation (13) of Riccati type, then $(u, v) = (u, v)(\Phi)$ of

$$\begin{cases} u = \dfrac{u_0}{\Phi} + u_1 = -\dfrac{\varepsilon\Phi_i}{\Phi} + \dfrac{\varepsilon}{2}\dfrac{\Phi_{ii}}{\Phi_i}, \\ v = \dfrac{v_0}{\Phi^2} + \dfrac{v_1}{\Phi} + v_2 = \dfrac{\varepsilon}{2}(\partial_i - \partial_x)(-\dfrac{\Phi_i}{\Phi} + \dfrac{\Phi_{ii}}{2\Phi_i}), \end{cases} \tag{14}$$

and $(\bar{u}, \bar{v}) = (\bar{u}, \bar{v})(\Phi)$ of

$$\bar{u} := u_1 = \frac{\varepsilon}{2}\frac{\Phi_{ii}}{\Phi_i}, \quad \bar{v} := v_2 = \frac{\varepsilon}{4}(\partial_i - \partial_x)\frac{\Phi_{ii}}{\Phi_i}, \tag{15}$$

are two solutions to Tu system (1).

This theorem shows two relations between Tu system (1) and the equation (13) of Riccati type. We also have the following result about the equation (13).

Theorem 2 Let $f \in C^\infty(R)$ be an arbitrary function of χ. if $\Phi = \Phi(\chi, t)$ is a solution to the equation (13) of Riccati type, then $\dfrac{1}{\Phi}$ and $\Phi + f$ are still solutions to the equation (13) of Riccati type.

Proof From the equation (13), after some calculation we may obtain

$$(\frac{1}{\Phi})_i - (\frac{1}{\Phi})_x = -\gamma(\frac{1}{\Phi})^2 - \beta(\frac{1}{\Phi}) - \alpha,$$

$$(\Phi + f)_i - (\Phi + f)_x = \bar{\alpha}(\Phi + f)^2 + \bar{\beta}(\Phi + f) + \bar{\gamma},$$

where

$$\bar{\alpha} = \alpha, \ \bar{\beta} = \bar{\beta} - 2f\bar{\alpha}, \ \bar{\gamma} = \bar{\gamma} - f_x - \bar{\alpha} f^2 - \bar{\beta} f.$$

In addition $(\dfrac{1}{\Phi})i = -\dfrac{\Phi_i}{\Phi^2} \neq 0$, $(\Phi + f)_i = \Phi_i \neq 0$. Therefore $\dfrac{1}{\Phi}$ and $\Phi + f$ are two solutions of (13). The proof is completed.

By Theorems land 2, a direct calculation may show

Corollary 1 Let $\Phi = \Phi(\chi, t)$ is a solution of (13). Then (i) for an arbitrary χ-function $f \in C^\infty(R)$,

$$u = -\frac{\varepsilon\Phi_i}{\Phi + f} + \frac{\varepsilon}{2}\frac{\Phi_{ii}}{\Phi_i}, \quad v = \frac{\varepsilon}{2}(\partial_i - \partial_x)(-\frac{\Phi_i}{\Phi + f} + \frac{\Phi_{ii}}{2\Phi_i})$$

is still a solution of Tu system (1), and (ii) we have the relation $(u, v)(\dfrac{1}{\Phi}) = (\bar{u}, \bar{v})(\Phi)$.

Also through Theorem 2, for the equation (13) we can generate a sort of new solutions from one known solution $\Phi=\Phi(\chi, t)$:

$$\bar{\Phi}=f_1+\cfrac{1}{f_2+\cfrac{1}{f_3+\ddots+\cfrac{1}{f_{n-1}+\cfrac{1}{f_n+\Phi}}}}$$

provided that the functions f_i, $1\leqslant i\leqslant n$, only depend on the variable χ.

3 Exact solutions to Tu system

In this secition, we will exhibit some exact solutions to the Tu system (1) by analysing another equivalent form of the equation (12). Directly calculating the derivative of ∂_t in the equation (12), we may obtain that equivalent form

$$\Phi_{tttt}\Phi_t^2-4\Phi_{ttt}\Phi_{tt}\Phi_t+3\Phi_{tt}^3-\Phi_{xttt}\Phi_t^2+3\Phi_{xtt}\Phi_{tt}\Phi_t+\Phi_{xt}\Phi_{ttt}\Phi_t-3\Phi_{xt}\Phi_{tt}^2=0,\quad \Phi_t\neq 0, \qquad (16)$$

which is homogeneous with respect to Φ.

Case 1 Let us choose $\Phi_{tt}=0$, Hence the solutions of the equation (16) possess the form $\Phi=f(\chi)t+g(\chi), f\neq 0$, with two changeable functions $f, g\in C^\infty(R)$. In this case, we have $u_1=v_2=0$. Further we obtain exact solutions of Tu system (1)

$$u=\frac{u_0}{\Phi}=-\frac{\varepsilon f}{ft+g},\quad v=\frac{v_0}{\Phi^2}+\frac{v_1}{\Phi}=\frac{\varepsilon}{2}\,\frac{f^2-fg_x+f_xg}{(ft+g)^2} \qquad (17)$$

with two arbitrary χ-functions $f, g\in C^\infty(R)$ satisfying $f^2+g^2\neq 0$.

Case 2 We choose the second simple case: $\Phi_{ttt}=0$; the equation (16) becomes

$$\Phi_{tt}^3+\Phi_{xtt}\Phi_{tt}\Phi_t-\Phi_{xt}\Phi_{tt}^2=0,\quad \Phi_t\neq 0. \qquad (18)$$

Let $\Phi=f(\chi)t^2+g(\chi)t+h(\chi)$, $f, g, h\in C^\infty(R), f^2+g^2\neq 0$. Then Φ satisfies (18) if and only if

$$2f^2+f_xg-fg_x=0. \qquad (19)$$

This equation has many trivial solutions. For example,

$$f=a_0\chi+a_0a_1,\ g=2a_0x^2+a_2x+(a_1a_2-2a_0a_1^2),\ a_i\in R,\ 0\leqslant i\leqslant 2;$$

$$f=ax^{n+1}+bx^n,\ g=2ax^{n+2}+2bx^{n+1},\ a, b\in R,\ n\geqslant 0.$$

Now we have

$$u_0=-\varepsilon(2ft+g),\ u_1=\frac{\varepsilon f}{2ft+g},$$

$$v_0=-\frac{\varepsilon}{2}(2ft+g)[f_xt^2+(g_x-2f)t+(h_x-g)],$$

$$v_1=\frac{\varepsilon}{2}(2f_xt+g_x-2f),\ v_2=0.$$

Therefore we obtain exact solutions of Tu system (1)

$$u=\frac{u_0}{\Phi}+u_1=-\frac{\varepsilon(2ft+g)}{ft^2+gt+h}+\frac{\varepsilon f}{2ft+g},\tag{20a}$$

$$v=\frac{v_0}{\Phi^2}+\frac{v_1}{\Phi}=-\frac{\varepsilon}{2}\frac{(2ft+g)[f_xt^2+(g_x-2f)t+(h_x-g)]}{(ft^2+gt+h)^2}+\frac{\varepsilon}{2}\frac{2f_xt+g_x-2f}{ft^2+gt+h},\tag{20b}$$

where the x-functions $f, g, h \in C^\infty(R)$ may be all changeable, but need to satisfy (19) and $f^2+g^2+h^2\neq 0$. In addition, we see by Theorem 1 that $(\partial_t-\partial_x)\frac{f}{2ft+g}=0$ iff (19) holds.

Case 3 Finally we choose

$$\Phi=f(x)e^{at}+g(x),\ f,\ g\in C^\infty(R),\ a\in R,\ af\neq 0.\tag{21}$$

Then Φ always satisfies (16). This moment $v_2=0$. Therefore we obtain exact solutions of Tu system (1)

$$u=\frac{u_0}{\Phi}+u_1=-\frac{a\varepsilon}{2}\frac{fe^{at}-g}{fe^{at}+g},\tag{22a}$$

$$v=\frac{v_0}{\Phi^2}+\frac{v_1}{\Phi}=-\frac{a\varepsilon}{2}\frac{(afg+fg_x-f_xg)e^{at}}{(fe^{at}+g)^2},\tag{22b}$$

with an arbitrary coefficient $a\in R$ and two arbitrary x-functions $f, g\in C^\infty(R)$ satisfying $f^2+g^2\neq 0$. In particular, if we choose that $f=e^{h(x)}$, $h\in C^\infty(R)$, $g=1$, we obtain a special analytic solution

$$u=-\frac{a\varepsilon}{2}\,\mathrm{th}\frac{1}{2}[at+h(x)],$$

$$v=-\frac{a\varepsilon}{8}[a-h_x(x)]\mathrm{sech}^2\frac{1}{2}[at+h(x)],$$

which includes a solution given by Li and Tian [12] for the case of $\varepsilon=-1$. When $h=bx+c$, $b, c\in R$, $b\neq 0$ or a, the above solution reduces a soliton solution with the speed $-a/b$.

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