Wen-Xiu Ma*

Lump-Type Solutions to the (3+1)-Dimensional Jimbo-Miwa Equation

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Abstract: Taking advantage of the Hirota bilinear form, four classes of lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation are presented through symbolic computation with Maple. Special choices of the involved parameters guaranteeing analyticity of the fourth solution are given, together with two particular lump-type solutions.

Keywords: Hirota bilinear form, Jimbo-Miwa equation, Lump-type solution

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1 Introduction

Nonlinear integrable equations can be transformed into Hirota bilinear equations, and such examples of equations include the Korteweg-de Vries (KdV) equation, the Boussinesq equation, the Kadomtsev–Petviashvili (KP) equation and the Toda lattice equation [1]. All these integrable equations possess exponentially localized solutions – soliton solutions [2]. Hirota bilinear forms play a crucial role in generating soliton solutions, though some intelligent guesswork is often needed [3].

Recently, there has been a renewed and growing interest in rational solutions to nonlinear partial differential equations (see, e.g., [4, 5]). Particularly important are rationally localized solutions, called lump solutions, and examples of lump solutions are found for many integrable equations such as the KP equation I [6], the three-dimensional three-wave resonant interaction [7], the B-KP equation [8], the Davey–Stewartson equation II [9] and the Ishimori-I equation [10]. The KP equation I of the form

$$(u_t + 6uu_x + u_{xxx})_x - 3u_{yy} = 0 \tag{1}$$

possesses the following lump solution [6]:

$$u = 4\frac{-[x + ay + 3(a^2 - b^2)t]^2 + b^2(y + 6at)^2 + 1/b^2}{\{[x + ay + 3(a^2 - b^2)t]^2 + b^2(y + 6at)^2 + 1/b^2\}^2}, \tag{2}$$

where a and b are real free parameters. Rogue wave solutions, which draw a big attention of mathematicians and physicists worldwide, are a particularly interesting kind of lump or lump-type solutions, and such solutions, usually with rational function amplitudes, could be used to describe significant nonlinear wave phenomena in both oceanography [11] and nonlinear optics [12]. It is natural and interesting to search for lump or lump-type solutions to nonlinear partial differential equations, on the basis of Hirota bilinear forms.

General rational solutions to nonlinear integrable equations have been considered within the Wronskian formulation, the Casoratian formulation and the Pfaffian formulation (see [1, 2]). The KdV equation and the Boussinesq equation in (1+1)-dimensions, the KP equation in (2+1)-dimensions and the Toda lattice equation in (0+1)-dimensions are such typical examples (see, e.g., [13–16]). Several attempts have also been made to search for rational solutions to the non-integrable (3+1)-dimensional KP I [17, 18] and KP II [19] by direct approaches such as the tanh-function method and the $\frac{G'}{G}$-expansion method (see, e.g., [20, 21]). Rational solutions to the (3+1)-dimensional KP II are linked to the good Boussinesq equation by a transformation of dependent variables [19]. Moreover, bilinear Bäcklund transformations are used to construct rational solutions to (3+1)-dimensional generalized KP equations (see, e.g., [22]), and there is some direct search for rational solutions to generalized bilinear equations (see, e.g., [23]), formulated in terms of generalized bilinear derivatives [24].

In this paper, we would like to focus on the (3+1)-dimensional Jimbo-Miwa equation and present four classes of its lump-type solutions by symbolic computation with Maple. The (3+1)-dimensional Jimbo-Miwa equation has a Hirota bilinear form, and so, we will do a search for positive quadratic function solutions to the corresponding (3+1)-dimensional bilinear Jimbo-Miwa equation. The obtained quadratic function solutions contain a set

***Corresponding author: Wen-Xiu Ma,** Department of Mathematics, Zhejiang Normal University, Jinhua 321004, PR China; College of Mathematics and Physics, Shanghai University of Electric Power, Shanghai 200090, PR China; Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, USA, E-mail: mawx@cas.usf.edu

of free parameters, and taking special choices of parameters involved, particular lump-type solutions will be generated from the fourth quadratic function solution. A few concluding remarks are given finally at the end of the paper.

2 Lump-type solutions to the Jimbo-Miwa equation

2.1 The Jimbo-Miwa equation

The (3+1)-dimensional Jimbo-Miwa equation reads [25]

$$P_{JM}(u) := u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \quad (3)$$

called the Jimbo-Miwa equation in [26]. The equation is the second member in the entire KP hierarchy [25], originally defined by a Hirota bilinear equation

$$\begin{aligned} B_{JM}(f) := & \ (D_x^3 D_y + 2D_t D_y - 3D_x D_z) f \cdot f \\ = & \ 2(f_{xxxy} f - f_y f_{xxx} - 3f_x f_{xxy} + 3f_{xx} f_{xy} \\ & + 2f_{yt} f - 2f_y f_t - 3f_{xz} f + 3f_x f_z) = 0, \end{aligned} \quad (4)$$

under the link from f to u:

$$u = 2(\ln f)_x. \quad (5)$$

This link is also a characteristic transformation used in Bell polynomial theories of soliton equations [27, 28] and it actually presents

$$P_{JM}(u) = \frac{f(B_{JM}(f))_x - 2f_x B_{JM}(f)}{f^3}. \quad (6)$$

Therefore, if f solves the bilinear Jimbo-Miwa equation (4), then $u = 2(\ln f)_x$ will engender a solution to the Jimbo-Miwa equation (3).

It is recognized that the Jimbo-Miwa equation (3) passes the Painlevé test only for a subclass of solutions [26] and does not possess a Kac–Moody–Virasoro symmetry algebra [29]. Nevertheless, different types of exact solutions to the Jimbo-Miwa equation (3) are found (see, e.g., [30]–[32]). The Hirota perturbation technique generates one- and two-soliton solutions [26] and dromion-type solutions [32], and the transformed rational function algorithm yields various traveling wave solutions [33]. It is obvious that the Jimbo-Miwa equation (3) has the following y- or x-independent solutions:

$$u = F(x, t) + H(z, t), \ u = G(y, z) + +H(z, t), \quad (7)$$

where F, G and H are arbitrary functions in the indicated variables. These solutions contain more special solutions: $u = F(x, t)$, $u = G(y, z)$ and $u = H(z, t)$, which are independent of two variables of x, y, z, t. Beginning with such solutions, various variable separated solutions are presented by performing Painlevé analysis and abundant nonlinear coherent structures are explored [34]. Moreover, among polynomial functions with individual degrees of the independent variables less than 2, the Jimbo-Miwa equation (3) has the following polynomial solutions:

$$\begin{aligned} u = & \ a_0 + a_1 x + a_2 y + a_3 z + a_4 t + a_5 xz \\ & + a_6 xt + a_7 yz + \frac{3}{2} a_5 yt + a_8 zt, \end{aligned} \quad (8)$$

where $a_i, \ 0 \le i \le 8$, are arbitrary parameters.

In what follows, we concentrate on presenting lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation (3) by formulating a problem of searching for positive quadratic function solutions to the bilinear Jimbo-Miwa equation (4).

2.2 Lump-type solutions

We apply the computer algebra system Maple to search for quadratic function solutions to the (3+1)-dimensional bilinear Jimbo-Miwa equation (4). A direct Maple symbolic computation with

$$\begin{aligned} f & = g^2 + h^2 + a_{11}, \ g = a_1 x + a_2 y + a_3 z + a_4 t + a_5, \\ h & = a_6 x + a_7 y + a_8 z + a_9 t + a_{10}, \end{aligned} \quad (9)$$

generates the following four sets of solutions for the parameters $a_i, \ 1 \le i \le 11$:

$$\Big\{ a_1 = a_1, \ a_2 = -\frac{a_6 a_7}{a_1}, \ a_3 = -\frac{2 a_4 a_6 a_7}{3 a_1^2}, \ a_4 = a_4, \ a_5 = a_5, \ a_6 = a_6, \ a_7 = a_7, \ a_8 = \frac{2 a_4 a_7}{3 a_1}, \ a_9 = \frac{a_4 a_6}{a_1}, \ a_{10} = a_{10}, \ a_{11} = a_{11} \Big\},$$

$$\Big\{ a_1 = a_1, \ a_2 = a_7, \ a_3 = a_8, \ a_4 = \frac{3 a_1 a_8}{2 a_7}, \ a_5 = a_5, \ a_6 = -a_1, \ a_7 = a_7, \ a_8 = a_8, \ a_9 = -\frac{3 a_1 a_8}{2 a_7}, \ a_{10} = a_{10}, a_{11} = a_{11} \Big\},$$

$$\Big\{ a_1 = 0, \ a_2 = a_2, \ a_3 = -\frac{2 a_2(-3 a_6^3 a_9 + 2 a_4^2 a_{11})}{9 a_6^4}, \ a_4 = a_4, \ a_5 = a_5, \ a_6 = a_6,$$

$$a_7 = -\frac{2\,a_2a_4a_{11}}{3\,a_6^3},\ a_8 = -\frac{2\,a_2a_4(a_6^3+2\,a_9a_{11})}{9\,a_6^4},$$
$$\left. a_9 = a_9,\ a_{10} = a_{10},\ a_{11} = a_{11} \right\},$$

$$\left\{ a_1 = a_1, a_2 = a_2, a_3 = \frac{2}{3}\frac{a_1a_2a_4 - a_1a_7a_9 + a_2a_6a_9 + a_4a_6a_7}{a_1^2+a_6^2}, \right.$$
$$a_4 = a_4,\ a_5 = a_5, a_6 = a_6,\ a_7 = a_7,$$
$$a_8 = \frac{2}{3}\frac{a_1a_2a_9 + a_1a_4a_7 - a_2a_4a_6 + a_6a_7a_9}{a_1^2+a_6^2},$$
$$a_9 = a_9, a_{10} = a_{10},$$
$$\left. a_{11} = -\frac{3}{2}\frac{(a_1^5a_2 + a_1^4a_6a_7 + 2\,a_1^3a_2a_6^2 + 2\,a_1^2a_6^3a_7 + a_1a_2a_6^4 + a_6^5a_7)}{(a_1a_9 - a_4a_6)(a_1a_7 - a_2a_6)} \right\}.$$

They lead to four classes of quadratic function solutions to the bilinear Jimbo-Miwa equation (4), and the resulting quadratic function solutions, in turn, yield four classes of lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation (3) through the transformation (5). We list these classes of lump-type solutions as follows.

The first class of lump-type solutions to the Jimbo-Miwa equation (3) reads

$$u_1 = \frac{4}{f_1}\frac{a_4a_1^2t + a_4a_6^2t + a_1^3x + a_1a_6^2x + a_1^2a_5 + a_1a_6a_{10}}{a_1} \tag{10}$$

with

$$f_1 = \left(a_4t + a_1x - \frac{a_6a_7y}{a_1} - \frac{2\,a_4a_6a_7z}{3\,a_1^2} + a_5\right)^2 + \left(\frac{a_4a_6t}{a_1} + a_6x + a_7y + \frac{2\,a_4a_7z}{3\,a_1} + a_{10}\right)^2 + a_{11}. \tag{11}$$

The second class of lump-type solutions to the Jimbo-Miwa equation (3) reads

$$u_2 = \frac{4}{f_2}\frac{a_1(3\,a_1a_8t + 2\,a_1a_7x + a_5a_7 - a_7a_{10})}{a_7} \tag{12}$$

with

$$f_2 = \left(\frac{3\,a_1a_8t}{2\,a_7} + a_1x + a_7y + a_8z + a_5\right)^2 + \left(-\frac{3\,a_1a_8t}{2\,a_7} - a_1x + a_7y + a_8z + a_{10}\right)^2 + a_{11}. \tag{13}$$

The third class of lump-type solutions to the Jimbo-Miwa equation (3) reads

$$u_3 = \frac{4}{9\,f_3}\frac{(9\,a_6^4a_9t + 9\,a_6^5x - 6\,a_2a_4a_6a_{11}y - 6\,a_2a_4a_6^3z - 4\,a_2a_4a_9a_{11}z + 9\,a_6^4a_{10}\ 9\,a_6^4a_9t)}{a_6^3} \tag{14}$$

with

$$f_3 = \left[a_4t + a_2y - \frac{2\,a_2(-3\,a_6^3a_9 + 2\,a_4^2a_{11})}{9\,a_6^4}z + a_5\right]^2 + \left[a_9t + a_6x - \frac{2\,a_2a_4a_{11}y}{3\,a_6^3} - \frac{2\,a_2a_4(3\,a_6^3 + 2\,a_9a_{11})}{9\,a_6^4}z + a_{10}\right]^2 + a_{11}. \tag{15}$$

The fourth class of lump-type solutions to the Jimbo-Miwa equation (3) reads

$$\begin{aligned} u_4 = \frac{4}{3f_4(a_1^2+a_6^2)}\big(&3\,a_1^3a_4t + 3\,a_1^2a_6a_9t + 3\,a_1a_4a_6^2t \\ &+ 3\,a_6^3a_9t + 3\,a_1^4x + 6\,a_1^2a_6^2x + 3\,a_6^4x + 3\,a_1^3a_2y \\ &+ 3\,a_1^2a_6a_7y + 3\,a_1a_2a_6^2y + 3\,a_6^3a_7y + 2\,a_1^2a_2a_4z \\ &- 2\,a_1^2a_7a_9z + 4\,a_1a_2a_6a_9z + 4\,a_1a_4a_6a_7z \\ &- 2\,a_2a_4a_6^2z + 2\,a_6^2a_7a_9z + 3\,a_1^3a_5 + 3\,a_1^2a_6a_{10} \\ &+ 3\,a_1a_5a_6^2 + 3\,a_6^3a_{10}\big) \end{aligned} \tag{16}$$

with

$$\begin{aligned} f_4 = &\Big(a_4t + a_1x + a_2y + \frac{2}{3}\frac{a_1a_2a_4 - a_1a_7a_9 + a_2a_6a_9 + a_4a_6a_7}{a_1^2+a_6^2}z + a_5\Big)^2 \\ &+\Big(a_9t + a_6x + a_7y + \frac{2}{3}\frac{a_1a_2a_9 + a_1a_4a_7 - a_2a_4a_6 + a_6a_7a_9}{a_1^2+a_6^2}z + a_{10}\Big)^2 \\ &-\frac{3}{2}\frac{(a_1^5a_2 + a_1^4a_6a_7 + 2\,a_1^3a_2a_6^2 + 2\,a_1^2a_6^3a_7 + a_1a_2a_6^4 + a_6^5a_7)}{(a_1a_7 - a_2a_6)(a_1a_9 - a_4a_6)}. \end{aligned} \tag{17}$$

In the above solutions, all parameters a_i involved are arbitrary provided that the solutions are well defined. The third lump-type solution (14) is similar to the lump solution (2) to the KP equation I, but the other three solutions are more general since the two linear waves involved contain all independent variables x, y, z, t.

The analyticity of the first three solutions is guaranteed if $a_{11} > 0$. In the fourth solution in (16), assume that all parameters a_i involved are positive, i.e., $a_i > 0$ for $1 \le i \le 4$ and $6 \le i \le 9$. Then if $\frac{a_2}{a_7} \neq \frac{a_4}{a_9}$, there are lump-type solutions when taking

$$\frac{a_1}{a_6} \in \left(\frac{a_2}{a_7}, \frac{a_4}{a_9}\right) \text{ or } \frac{a_1}{a_6} \in \left(\frac{a_4}{a_9}, \frac{a_2}{a_7}\right), \tag{18}$$

which leads to $a_{11} > 0$; and if $\frac{a_2}{a_7} = \frac{a_4}{a_9}$, which leads to $a_{11} < 0$, there are singularities in the resulting solution (noting that in the case of $\frac{a_2}{a_7} = \frac{a_4}{a_9}$, a_{11} is well defined iff $\frac{a_1}{a_6} \neq \frac{a_2}{a_7}$). All the above rational function solutions $u_i \to 0, 1 \le i \le 4$, when the corresponding sum of squares $g^2 + h^2 \to \infty$. But they do not approach zero in all directions in $\mathbb{R}^4$ due to the character of (3+1)-dimensions in the resulting solutions, and so we call them lump-type solutions.

For the fourth lump-type solution, let us now fix

$$a_5 = 1, \ a_{10} = -1,$$

and choose two particular sets of parameters:

$$a_1 = 3, \ a_2 = 5, \ a_4 = 4, \ a_6 = 3, \ a_7 = 6, \ a_9 = 3,$$

and

$$a_1 = 4, \ a_2 = 3, \ a_4 = 5, \ a_6 = 3, \ a_7 = 2, \ a_9 = 6,$$

which satisfy the two conditions in eq. (18), respectively. The corresponding two particular analytical lump-type solutions of (16) are given by

$$u_{4,1} = \frac{324\,(21\,t + 18\,x + 33\,y + 26\,z)}{g_1} \tag{19}$$

with

$$\begin{aligned} g_1 = {} & 2025\,t^2 + 3402\,tx + 6156\,ty + 4950\,tz + 1458\,x^2 \\ & + 5346\,xy + 4212\,xz + 4941\,y^2 + 7686\,yz + 3050\,z^2 \\ & + 162\,t - 162\,y + 72\,z + 144504, \end{aligned} \tag{20}$$

and

$$u_{4,2} = \frac{36\,(950\,t + 652\,x + 450\,y + 462\,z + 25)}{g_2} \tag{21}$$

with

$$\begin{aligned} g_2 = {} & 13725\,t^2 + 17100\,tx + 12150\,ty + 13176\,tz \\ & + 5625\,x^2 + 8100\,xy + 8316\,xz + 2925\,y^2 \\ & + 5928\,yz + 3172\,z^2 - 450\,t + 450\,x \\ & + 450\,y - 84\,z + 422325, \end{aligned} \tag{22}$$

respectively.

3 Concluding remarks

Based on the Hirota formulation and by a Maple symbolic computation, we presented four classes of lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation (3), and analyzed the positivity of the fourth quadratic function solution leading to analytical lump-type solutions, of which two particular analytical lump-type solutions were computed under special choices of parameters involved.

We remark that it would be very interesting to determine conditions under which there exist positive polynomial solutions to a kind of generalized bilinear and trilinear differential equations, as did for resonant solutions in terms of exponential functions [35, 36]. This kind of polynomial solutions will generate analytical lump or lump-type solutions, particularly rogue wave solutions, to the corresponding nonlinear equations through $u = 2(\ln f)_x$ or $u = 2(\ln f)_{xx}$. Higher-order rogue wave solutions will be linked to a wide variety of mathematical topics including generalized Wronskian solutions [37] and generalized Darboux transformations [38]. Multicomponent or higher-order generalizations of lump solutions, especially in (3+1)-dimensional cases and fully discrete cases, would be a good topic for future research, exhibiting more diverse soliton phenomena.

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