

Department of Mathematics, University of South Florida
MAT5932-004 Methods of Mathematical Physics

Test 3, Fall 2004

Instruction: *There are five questions and each is worth 20 points. Please show all work necessary to produce your solutions.*

1. (20 points) Note that $y = x$ would be a solution of

$$(1 - x)y'' + xy' - y = 2(x - 1)^2$$

if the right side were zero. Use this fact to find the general solution of the equation as given.

2. (20 points) Solve the integral equation for $f(x)$:

$$f(x) = e^{-|x|} + 2 \int_0^{\infty} f(y) \cos xy \, dy.$$

3. (20 points) Use the separation of variables to solve the vibrating beam problem:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} + \frac{\partial^4 u}{\partial x^4} = 0, & 0 < x < 1, \quad t > 0, \\ u(x = 0, t) = u(x = 1, t) = \frac{\partial^2 u}{\partial x^2}(x = 0, t) = \frac{\partial^2 u}{\partial x^2}(x = 1, t) = 0, & t > 0, \\ u(x, t = 0) = f(x), \quad \frac{\partial u}{\partial t}(x, t = 0) = g(x), & 0 \leq x \leq 1. \end{cases}$$

4. (20 points) Show that for the potential

$$u(x) = \begin{cases} U_0, & -1 < x < 0, \\ 0, & x \leq -1, \quad x \geq 0, \end{cases}$$

where U_0 is a positive constant, the Sturm-Liouville problem

$$\psi'' + \{\lambda - u(x)\}\psi = 0$$

has no discrete eigenvalue.

5. (20 points) Solve the Cauchy problem of the Korteweg-de Vries equation on the whole x -axis by the inverse scattering transform:

$$\begin{cases} \frac{\partial u}{\partial t} - 6u \frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0, & t > 0, \\ u(x, t = 0) = -\frac{3}{2} \operatorname{sech}^2 \frac{x}{2}. \end{cases}$$

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