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Rational solutions to an extended Kadomtsev-Petviashvili-like equation with symbolic computation

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ABSTRACT

Associated with the prime number $p = 3$, the generalized bilinear operators are adopted to yield an extended Kadomtsev-Petviashvili-like (eKP-like) equation. With symbolic computation, eighteen classes of rational solutions to the resulting eKP-like equation are generated from a search for polynomial solutions to the corresponding generalized bilinear equation.

1. Introduction

In recent years, rogue waves have become the subject of intensive investigations in oceanography [1] and nonlinear optics [2]. Actually, rogue waves belong to a kind of rational solutions [3,4], and it is of interest and importance for us to discuss about rational solutions to a new kind of nonlinear differential equations associated with generalized bilinear equations [5,6]. By involving different prime numbers, Hirota bilinear equations have been generalized to generate diverse nonlinear differential equations possessing potential applications [5,6].

For example, via the dependent variable transformation $u(x,t) = 2\big[\ln f(x,t)\big]_{xx}$, the Korteweg–de Vries (KdV) equation

$$u_t + 6\,u\,u_x + u_{xxx} = 0, \tag{1}$$

enjoys the bilinear representation in the sense of Hirota as

$$(D_xD_t + D_x^4)f\cdot f = 0, \tag{2}$$

where the Hirota derivatives D_x, D_t and D_x^4 [7] are defined by

$$D_x^\alpha D_y^\beta D_t^\gamma(f\cdot g) = \left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^\alpha\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^\beta\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^\gamma f(x,y,t)g(x',y',t')\bigg|_{x'=x,y'=y,t'=t}.$$

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Based on a prime number p, the generalized differential operators have been introduced in [5] as:

$$D_{p,x}^{a} D_{p,t}^{b}(f\cdot g)=\left(\frac{\partial}{\partial x}+\alpha_p\frac{\partial}{\partial x'}\right)^a\left(\frac{\partial}{\partial t}+\alpha_p\frac{\partial}{\partial t'}\right)^b f(x,t)g(x',t')\Big|_{x'=x,t'=t}$$
$$=\sum_{i=0}^{a}\sum_{j=0}^{b}\binom{a}{i}\binom{b}{j}\alpha_p^i\alpha_p^j\frac{\partial^{a-i}}{\partial x^{a-i}}\frac{\partial^{i}}{\partial x'^{(i)}}\frac{\partial^{b-j}}{\partial t^{b-j}}\frac{\partial^{j}}{\partial t'^{(j)}}f(x,t)g(x',t')\Big|_{x'=x,t'=t},$$

where

$$\alpha_p^s=(-1)^{r_p(s)},\quad s=r_p(s)\mod p. \tag{3}$$

Under the rule given by Eq. (3), we find that if $p=2k$ ($k\in\mathbb{N}$), all bilinear differential operators defined above turn out to be the Hirota bilinear operators, since $D_{2k,x}=D_x$ [7]. If $p=3$, we particularly have

$$\alpha_3=-1,\qquad \alpha_3^2=\alpha_3^3=1,\qquad \alpha_3^4=-1,\qquad \alpha_3^5=\alpha_3^6=1,\ldots.$$

We can correspondingly extend the Hirota bilinear equation (2), with $p=3$, into

$$(D_{3,x}D_{3,t}+D_{3,x}^4)f\cdot f=2f_{xt}f-2f_xf_t+6f_{xx}^2=0, \tag{4}$$

which is a generalized bilinear KdV-like equation [5].

In this paper, we would like to introduce an extended Kadomtsev-Petviashvili-like (eKP-like) equation by using a generalized bilinear differential equation of KP type. Based on polynomial solutions to the associated generalized bilinear equation, we will construct eighteen classes of rational solutions to the presented eKP-like equation with symbolic computation [8], and some of the solutions will be described graphically. Finally, a few concluding remarks will be given at the end of the paper.

2. An extended Kadomtsev-Petviashvili-like equation

As a (2 + 1)-dimensional generalization of KdV Eq. (1), the well-known Kadomtsev-Petviashvili equation reads [9,7]

$$\left(u_t+6\,u\,u_x+u_{xxx}\right)_x+u_{yy}=0, \tag{5}$$

which enjoys the bilinear representation in the sense of Hirota as

$$(D_xD_t+D_x^4+D_y^2)f\cdot f=0 \tag{6}$$

through the transformation $u=2\left[\ln f(x,y,t)\right]_{xx}$.

Based on the theory of generalized differential operators introduced in [5], it is natural to extend Eq. (6) into

$$(D_{3,x}D_{3,t}+D_{3,x}^4+D_{3,y}^2+D_{3,z}^2)f\cdot f=2f_{xt}f-2f_xf_t+6f_{xx}^2+2f_{yy}f-2f_y^2+2f_{zz}f-2f_z^2=0, \tag{7}$$

which is a generalized bilinear equation. Bell polynomial theories (see, e.g., [5,10]) motivate us to consider a dependent variable transformation

$$u=2\left[\ln f(x,y,z,t)\right]_x=2\frac{f_x(x,y,z,t)}{f(x,y,z,t)}, \tag{8}$$

and find that

$$\left[\frac{(D_{3,x}D_{3,t}+D_{3,x}^4+D_{3,y}^2+D_{3,z}^2)f\cdot f}{f^2}\right]_x=\left(u_t+\frac{3}{2}\,u_x^2+\frac{3}{8}\,u^4+\frac{3}{2}\,u^2u_x\right)_x+u_{yy}+u_{zz}. \tag{9}$$

Then Equality (9) shows that we can treat Eq. (7) as a generalized bilinear representation of the following eKP-like equation

$$\left(u_t+\frac{3}{2}\,u_x^2+\frac{3}{8}\,u^4+\frac{3}{2}\,u^2u_x\right)_x+u_{yy}+u_{zz}=0, \tag{10}$$

which is a (3 + 1)-dimensional model.

Attention should be emphasized on ($\mathcal{A}$) the eKP-like equation (10) possesses more terms and higher nonlinearity than the standard KP Eq. (5), ($\mathcal{B}$) if f solves Eq. (7), then $u=2\left[\ln f(x,y,z,t)\right]_x$ will present a solution to the eKP-like Eq. (10), and ($\mathcal{C}$) the Transformation (8) provides us with a formula of rational solutions.

3. Rational solutions by Maple

Within the framework of investigation on resonant solutions to generalized bilinear equations, we find that Eq. (10) does not satisfy the conditions given in Refs. [5,11] for resonant solutions. *In this paper, we will discuss rational solutions to the eKP-like Eq. (10) based on polynomial solutions to the generalized bilinear equation (7).*

We will use the computer algebra system Maple to search for polynomial solutions to the generalized bilinear equation (7). With symbolic computation, a direct substitution of

$$f = \sum_{i=0}^{4}\sum_{j=0}^{3}\sum_{k=0}^{3}\sum_{l=0}^{5} c_{i,j,k,l} x^i y^j z^k t^l$$

into Eq. (7) solves the polynomial solutions. In turn, the polynomial solutions lead to eighteen classes of rational solutions to the presented eKP-like Eq. (10) through the Transformation (8).

The *first* class of rational solutions to Eq. (10) reads

$$u_1 = \frac{2\, c_{1,1,0,2}}{x\, c_{1,1,0,2} + c_{0,1,0,2}}. \tag{11}$$

The *second* class of rational solutions to Eq. (10) reads

$$u_2 = \frac{2\, c_{1,0,0,0}^2}{x\, c_{1,0,0,0}^2 - t\, c_{0,1,0,0}^2 + \varepsilon y\, c_{1,0,0,0} c_{0,1,0,0} + \varepsilon c_{1,0,0,0} c_{0,0,0,0}} \tag{12}$$

with $\varepsilon = \pm 1$.

The *third* class of rational solutions to Eq. (10) reads

$$u_3 = \frac{2\, c_{0,1,1,1}^2}{x\, c_{0,1,1,1}^2 - t\, c_{0,1,0,2}^2 + \varepsilon z\, c_{0,1,1,1} c_{0,1,0,2} + \varepsilon c_{0,1,0,1} c_{0,1,0,2}} \tag{13}$$

with $\varepsilon = \pm 1$.

The *fourth* class of rational solutions to Eq. (10) reads

$$u_4 = \frac{2\, c_{1,0,0,0}^2}{x\, c_{1,0,0,0}^2 - t\,(c_{0,0,1,0}^2 + c_{0,1,0,0}^2) + y\, c_{0,1,0,0} c_{1,0,0,0} + z\, c_{0,0,1,0} c_{1,0,0,0} + c_{0,0,0,0} c_{1,0,0,0}}. \tag{14}$$

The *fifth* class of rational solutions to Eq. (10) reads

$$u_5 = \frac{2p}{q} \tag{15}$$

with

$$\begin{aligned} p &= t\, c_{0,0,0,2}^3 c_{2,0,0,1} c_{0,1,0,1} - 2x\, c_{0,1,0,1}^3 c_{0,0,0,2} c_{2,0,0,1} + y\, c_{0,0,0,2}^2 c_{0,1,0,1}^2 c_{2,0,0,1} + c_{0,0,0,2}^2 c_{0,0,0,1} c_{2,0,0,1} c_{0,1,0,1} \\ &\quad - c_{0,0,0,2}^3 c_{0,1,0,0} c_{2,0,0,1} + c_{0,1,0,1}^5 - 12 c_{0,1,0,1}^3 c_{2,0,0,1}^2, \\ q &= t\, x\, c_{0,0,0,2}^3 c_{2,0,0,1} c_{0,1,0,1} - x^2\, c_{0,1,0,1}^3 c_{0,0,0,2} c_{2,0,0,1} + x\, y\, c_{0,0,0,2}^2 c_{0,1,0,1}^2 c_{2,0,0,1} + x\,(c_{0,0,0,2}^2 c_{0,0,0,1} c_{2,0,0,1} c_{0,1,0,1} \\ &\quad - c_{0,0,0,2}^3 c_{0,1,0,0} c_{2,0,0,1} + c_{0,1,0,1}^5 - 12 c_{0,1,0,1}^3 c_{2,0,0,1}^2) - t\, c_{0,1,0,1}^3 c_{0,0,0,2}^2 - y\, c_{0,1,0,1}^4 c_{0,0,0,2} \\ &\quad - c_{0,1,0,1}^3 c_{0,0,0,2} c_{0,0,0,1} + c_{0,0,0,2}^2 c_{0,1,0,1}^2 c_{0,1,0,0}. \end{aligned}$$

The *sixth* class of rational solutions to Eq. (10) reads

$$u_6 = \frac{2p}{q} \tag{16}$$

with

$$\begin{aligned} p &= c_{1,0,0,3}^2 (t\, c_{1,0,0,3}^2 c_{1,1,0,2} - 2x\, c_{1,1,0,2}^3 + y\, c_{1,1,0,2}^2 c_{1,0,0,3} + c_{1,0,0,2} c_{1,0,0,3} c_{1,1,0,2} - c_{1,0,0,3}^2 c_{1,1,0,1}), \\ q &= t\, x\, c_{1,0,0,3}^4 c_{1,1,0,2} - x^2\, c_{1,1,0,2}^3 c_{1,0,0,3}^2 + x\, y\, c_{1,0,0,3}^3 c_{1,1,0,2}^2 + x\, c_{1,0,0,3}^2 (c_{1,0,0,2} c_{1,0,0,3} c_{1,1,0,2} - c_{1,0,0,3}^2 c_{1,1,0,1}) \\ &\quad + t\, c_{1,0,0,3}^3 c_{0,0,0,3} c_{1,1,0,2} + y\, c_{1,0,0,3}^2 c_{1,1,0,2}^2 c_{0,0,0,3} + c_{1,1,0,2}^3 c_{0,0,0,3}^2 + c_{1,0,0,3}^2 c_{0,0,0,3} c_{1,0,0,2} c_{1,1,0,2} \\ &\quad - c_{1,0,0,3}^3 c_{0,0,0,3} c_{1,1,0,1} - 12 c_{1,1,0,2}^5. \end{aligned}$$

The *seventh* class of rational solutions to Eq. (10) reads

$$u_7 = \frac{2p}{q} \tag{17}$$

with

$$p = c_{0,0,1,3}(108\,x^2\,c_{3,0,0,2}^2 - c_{0,0,1,2}^2),$$
$$q = 36\,x^3\,c_{3,0,0,2}^2 c_{0,0,1,3} - x\,c_{0,0,1,2}^2 c_{0,0,1,3} + 1296\,t\,c_{3,0,0,2}^2 c_{0,0,1,3} + 36\,z\,c_{0,0,1,2}c_{0,0,1,3}c_{3,0,0,2} + 36\,c_{3,0,0,2}(c_{0,0,0,3}c_{0,0,1,2} - 36\,c_{0,0,1,2}c_{3,0,0,2}).$$

The *eighth* class of rational solutions to Eq. (10) reads

$$u_8 = \frac{6p}{q} \tag{18}$$

with

$$p = c_{0,0,1,2}(x^2\,c_{0,0,0,3}^2 + 24x\,c_{0,0,0,3}c_{2,0,0,2} - 12c_{0,0,1,2}^2 - 12c_{0,1,0,2}^2 + 144c_{2,0,0,2}^2),$$
$$q = x^3\,c_{0,0,0,3}^2 c_{0,0,1,2} + 36x^2\,c_{0,0,0,3}c_{0,0,1,2}c_{2,0,0,2} - 36x\,c_{0,0,1,2}(c_{0,0,1,2}^2 + c_{0,1,0,2}^2 - 12c_{2,0,0,2}^2) + 36\,t\,c_{0,0,0,3}^2 c_{0,0,1,2} + 36\,c_{0,0,0,3}c_{0,0,1,2}(y\,c_{0,1,0,2} + z\,c_{0,0,1,2}) + 36\,c_{0,0,0,3}(c_{0,0,0,2}c_{0,0,1,2} - c_{0,0,0,3}c_{0,0,1,1}).$$

The *ninth* class of rational solutions to Eq. (10) reads

$$u_9 = \frac{2p}{q} \tag{19}$$

with

$$p = c_{1,0,0,1}^2\left[t\,c_{1,0,0,1}^2 - 2x\,(c_{1,0,1,0}^2 + c_{1,1,0,0}^2) + c_{1,0,0,1}(y\,c_{1,1,0,0} + z\,c_{1,0,1,0}) + c_{1,0,0,0}c_{1,0,0,1}\right],$$
$$q = t\,x\,c_{1,0,0,1}^4 - x^2\,c_{1,0,0,1}^2(c_{1,0,1,0}^2 + c_{1,1,0,0}^2) + c_{1,0,0,1}^3(x\,y\,c_{1,1,0,0} + x\,z\,c_{1,0,1,0}) + x\,c_{1,0,0,1}^3 c_{1,0,0,0} - 12(c_{1,0,1,0}^2 + c_{1,1,0,0}^2)^2.$$

The *tenth* class of rational solutions to Eq. (10) reads

$$u_{10} = \frac{2p}{q} \tag{20}$$

with

$$p = (c_{1,0,1,1}^2 + c_{1,1,0,1}^2)\left[t\,c_{1,0,1,1}(c_{1,0,1,1}^2 + c_{1,1,0,1}^2) - c_{1,0,1,1}c_{2,0,0,1}(2x\,c_{2,0,0,1} + y\,c_{1,1,0,1} + z\,c_{1,0,1,1} + c_{1,0,0,1}) - c_{1,0,1,0}(c_{1,0,1,1}^2 + c_{1,1,0,1}^2)\right],$$
$$q = t\,x\,c_{1,0,1,1}(c_{1,0,1,1}^2 + c_{1,1,0,1}^2)^2 - c_{2,0,0,1}c_{1,0,1,1}(c_{1,0,1,1}^2 + c_{1,1,0,1}^2)(x^2\,c_{2,0,0,1} + xy\,c_{1,1,0,1} + xz\,c_{1,0,1,1}) - x\,(c_{1,0,1,1}^2 + c_{1,1,0,1}^2)(c_{1,0,0,1}c_{1,0,1,1}c_{2,0,0,1} + c_{1,0,1,1}^2 c_{1,0,1,0} + c_{1,1,0,1}^2 c_{1,0,1,0}) - 12c_{2,0,0,1}^4 c_{1,0,1,1}.$$

The *eleventh* class of rational solutions to Eq. (10) reads

$$u_{11} = \frac{2p}{q} \tag{21}$$

with

$$p = c_{2,0,0,1}c_{0,0,1,1}(t\,c_{0,0,0,2}^3 - 2x\,c_{0,0,1,1}^2 c_{0,0,0,2} + z\,c_{0,0,0,2}^2 c_{0,0,1,1} + c_{0,0,0,2}^2 c_{0,0,0,1} - 12c_{0,0,1,1}^2 c_{2,0,0,1}) - c_{0,0,0,2}^3 c_{0,0,1,0}c_{2,0,0,1} + c_{0,0,1,1}^5,$$
$$q = c_{0,0,0,2}c_{2,0,0,1}c_{0,0,1,1}(t\,x\,c_{0,0,0,2}^2 - x^2\,c_{0,0,1,1}^2 + x\,z\,c_{0,0,0,2}c_{0,0,1,1}) - c_{0,0,0,2}c_{0,0,1,1}^3(t\,c_{0,0,0,2} + z\,c_{0,0,1,1}) + x\,(c_{0,0,0,2}^2 c_{0,0,0,1}c_{2,0,0,1}c_{0,0,1,1} - c_{0,0,0,2}^3 c_{2,0,0,1}c_{0,0,1,0} + c_{0,0,1,1}^5 - 12c_{0,0,1,1}^3 c_{2,0,0,1}^2) + c_{0,0,1,1}^2 c_{0,0,0,2}(c_{0,0,0,2}c_{0,0,1,0} - c_{0,0,0,1}c_{0,0,1,1}).$$

The *twelfth* class of rational solutions to Eq. (10) reads

$$u_{12} = \frac{2p}{q} \tag{22}$$

with

$$p = c_{0,0,0,3}^3 c_{1,0,0,3}(t\,c_{0,0,0,3}^2 - 2x\,c_{0,0,1,2}^2 + z\,c_{0,0,0,3}c_{0,0,1,2}) - c_{0,0,1,2}^2(c_{0,0,0,3}^4 - 12c_{1,0,0,3}^2 c_{0,0,1,2}^2),$$
$$q = c_{0,0,0,3}^3 c_{1,0,0,3}(t\,x\,c_{0,0,0,3}^2 - x^2\,c_{0,0,1,2}^2 + x\,z\,c_{0,0,0,3}c_{0,0,1,2}) - x\,c_{0,0,1,2}^2(c_{0,0,0,3}^4 - 12c_{1,0,0,3}^2 c_{0,0,1,2}^2) + c_{0,0,0,3}^5(t\,c_{0,0,0,3} + z\,c_{0,0,1,2}).$$

The *thirteenth* class of rational solutions to Eq. (10) reads

$$u_{13} = \frac{2p}{q} \tag{23}$$

with

$$\begin{aligned} p &= c_{0,1,0,1}c_{1,0,1,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)\big[t\, c_{1,0,1,0}^2(c_{0,0,1,0}^2 + c_{0,1,0,0}^2) - c_{2,0,0,0}c_{0,0,1,0}(2x\, c_{0,0,1,0}c_{2,0,0,0} \\ &\quad + y\, c_{0,1,0,0}c_{1,0,1,0} + z\, c_{0,0,1,0}c_{1,0,1,0})\big] - c_{1,0,1,0}^2 c_{0,0,0,1}c_{2,0,0,0}c_{0,0,1,0}c_{0,1,0,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2) \\ &\quad - c_{0,0,1,0}^3 c_{2,0,0,0}^2 c_{0,1,0,1}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2 + 12c_{2,0,0,0}^2) - c_{1,0,1,0}^3 c_{0,1,0,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)^2, \\ q &= c_{0,1,0,1}c_{1,0,1,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)\big[t\, x\, c_{1,0,1,0}^2(c_{0,0,1,0}^2 + c_{0,1,0,0}^2) - c_{2,0,0,0}c_{0,0,1,0}(x^2\, c_{0,0,1,0}c_{2,0,0,0} + x\, y\, c_{0,1,0,0}c_{1,0,1,0} \\ &\quad + x\, z\, c_{0,0,1,0}c_{1,0,1,0} + y\, c_{0,0,1,0}c_{0,1,0,0} + z\, c_{0,0,1,0}^2)\big] - x\big[c_{1,0,1,0}^2 c_{0,0,0,1}c_{2,0,0,0}c_{0,0,1,0}c_{0,1,0,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2) \\ &\quad + c_{0,0,1,0}^3 c_{2,0,0,0}^2 c_{0,1,0,1}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2 + 12c_{2,0,0,0}^2) + c_{1,0,1,0}^3 c_{0,1,0,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)^2\big] \\ &\quad - c_{0,0,1,0}c_{0,1,0,0}c_{1,0,1,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)\big[c_{0,0,0,1}c_{2,0,0,0}c_{0,0,1,0} + c_{1,0,1,0}(c_{0,0,1,0}^2 + c_{0,1,0,0}^2)\big]. \end{aligned}$$

The *fourteenth* class of rational solutions to Eq. (10) reads

$$u_{14} = \frac{2p}{q} \tag{24}$$

with

$$\begin{aligned} p &= c_{0,0,0,2}^2 c_{0,1,0,1}c_{1,1,0,1}c_{0,0,1,1}\big[t\, c_{0,0,0,2}^2 - 2x\,(c_{0,0,1,1}^2 + c_{0,1,0,1}^2) + y\, c_{0,0,0,2}c_{0,1,0,1} + z\, c_{0,0,1,1}c_{0,0,0,2} \\ &\quad + c_{0,0,0,2}c_{0,0,0,1}\big] - c_{0,0,0,2}^2 c_{0,1,0,1}(c_{0,0,0,2}^2 c_{0,0,1,0}c_{1,1,0,1} + c_{0,0,1,1}^3 c_{0,1,0,1} + c_{0,1,0,1}^3 c_{0,0,1,1}) \\ &\quad + 12c_{1,1,0,1}^2 c_{0,0,1,1}(c_{0,0,1,1}^2 + c_{0,1,0,1}^2)^2, \\ q &= c_{0,0,0,2}^2 c_{0,1,0,1}c_{1,1,0,1}c_{0,0,1,1}\big[x\, t\, c_{0,0,0,2}^2 - x^2\,(c_{0,0,1,1}^2 + c_{0,1,0,1}^2) + x\, y\, c_{0,0,0,2}c_{0,1,0,1} + x\, z\, c_{0,0,1,1}c_{0,0,0,2}\big] \\ &\quad + x\, c_{0,0,0,2}^2 c_{0,1,0,1}(c_{0,0,0,2}c_{0,0,0,1}c_{0,0,1,1}c_{1,1,0,1} - c_{0,0,0,2}^2 c_{0,0,1,0}c_{1,1,0,1} - c_{0,0,1,1}^3 c_{0,1,0,1} - c_{0,1,0,1}^3 c_{0,0,1,1}) \\ &\quad + 12x\, c_{1,1,0,1}^2 c_{0,0,1,1}(c_{0,0,1,1}^2 + c_{0,1,0,1}^2)^2 + c_{0,0,0,2}^3 c_{0,1,0,1}^2 c_{0,0,1,1}(t\, c_{0,0,0,2} + y\, c_{0,1,0,1} + z\, c_{0,0,1,1}) \\ &\quad + c_{0,0,0,2}^3 c_{0,1,0,1}^2(c_{0,0,1,1}c_{0,0,0,1} - c_{0,0,0,2}c_{0,0,1,0}). \end{aligned}$$

The *fifteenth* class of rational solutions to Eq. (10) reads

$$u_{15} = \frac{2p}{q} \tag{25}$$

with

$$\begin{aligned} p &= (c_{0,0,1,1}^2 + c_{0,1,0,1}^2)\big[t\, c_{1,0,1,1}^3(c_{0,0,1,1}^2 + c_{0,1,0,1}^2) - 2x\, c_{0,0,1,1}^2 c_{2,0,0,1}^2 c_{1,0,1,1} - y\, c_{1,0,1,1}^2 c_{0,0,1,1}c_{0,1,0,1}c_{2,0,0,1} \\ &\quad - z\, c_{0,0,1,1}^2 c_{1,0,1,1}^2 c_{2,0,0,1}\big] - c_{0,0,1,1}^3 c_{2,0,0,1}^2(c_{0,0,1,1}^2 + c_{0,1,0,1}^2 - 12c_{2,0,0,1}^2), \\ q &= (c_{0,0,1,1}^2 + c_{0,1,0,1}^2)\big[t\, x\, c_{1,0,1,1}^3(c_{0,0,1,1}^2 + c_{0,1,0,1}^2) - x^2\, c_{0,0,1,1}^2 c_{2,0,0,1}^2 c_{1,0,1,1} - x\, y\, c_{1,0,1,1}^2 c_{0,0,1,1}c_{0,1,0,1}c_{2,0,0,1} \\ &\quad - x\, z\, c_{0,0,1,1}^2 c_{1,0,1,1}^2 c_{2,0,0,1} + t\, c_{1,0,1,1}^2 c_{0,0,1,1}(c_{0,0,1,1}^2 + c_{0,1,0,1}^2) - y\, c_{0,0,1,1}^2 c_{0,1,0,1}c_{1,0,1,1}c_{2,0,0,1} \\ &\quad - z\, c_{0,0,1,1}^3 c_{1,0,1,1}c_{2,0,0,1}\big] - x\, c_{0,0,1,1}^3 c_{2,0,0,1}^2(c_{0,0,1,1}^2 + c_{0,1,0,1}^2 - 12c_{2,0,0,1}^2). \end{aligned}$$

The *sixteenth* class of rational solutions to Eq. (10) reads

$$u_{16} = -\frac{2p}{q} \tag{26}$$

with

$$\begin{aligned} p &= (c_{0,0,1,2}^4 c_{2,0,0,1}^2 + c_{0,1,1,1}^6)\big[t\,(c_{0,0,1,2}^4 c_{2,0,0,1}^2 + c_{0,1,1,1}^6) - c_{2,0,0,1}c_{0,1,1,1}c_{0,0,1,2}(2x\, c_{2,0,0,1}c_{0,1,1,1}c_{0,0,1,2} \\ &\quad - y\, c_{0,0,1,2}^2 c_{2,0,0,1} - z\, c_{0,1,1,1}^3 + c_{1,0,0,1}c_{0,1,1,1}c_{0,0,1,2})\big], \\ q &= (c_{0,0,1,2}^4 c_{2,0,0,1}^2 + c_{0,1,1,1}^6)\big[c_{0,1,1,1}^2 c_{0,0,1,2}(t^2\, c_{0,1,1,1}^2 c_{0,0,1,2} + t\, y\, c_{0,1,1,1}^3 + t\, z\, c_{0,0,1,2}^2 c_{2,0,0,1}) \\ &\quad + c_{0,0,1,2}c_{0,1,1,1}c_{2,0,0,1}(x^2\, c_{0,0,1,2}c_{0,1,1,1}c_{2,0,0,1} - x\, y\, c_{0,0,1,2}^2 c_{2,0,0,1} - x\, z\, c_{0,1,1,1}^3 + y\, z\, c_{0,1,1,1}^2 c_{0,0,1,2} \\ &\quad + x\, c_{0,0,1,2}c_{0,1,1,1}c_{1,0,0,1} - y\, c_{0,0,1,2}^2 c_{1,0,0,1}) - t\, c_{0,0,1,2}^4 c_{1,0,0,1}c_{2,0,0,1} - t\, x\,(c_{0,0,1,2}^4 c_{2,0,0,1}^2 + c_{0,1,1,1}^6)\big] \\ &\quad + 12c_{0,0,1,2}^4 c_{0,1,1,1}^4 c_{2,0,0,1}^4. \end{aligned}$$

The *seventeenth* class of rational solutions to Eq. (10) reads

$$u_{17}=-\frac{2p}{q} \tag{27}$$

with

$$\begin{aligned}
p={}&c_{2,0,0,0}(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)\big[t\,(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)-c_{2,0,0,0}c_{0,0,1,1}c_{0,1,1,0}(2x\,c_{2,0,0,0}c_{0,1,1,0}c_{0,0,1,1}\\
&-\varepsilon\,y\,c_{0,0,1,1}^2c_{2,0,0,0}-\varepsilon\,z\,c_{0,1,1,0}^4+c_{1,0,0,0}c_{0,0,1,1}c_{0,1,1,0})\big],\\
q={}&(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)\big[c_{2,0,0,0}c_{0,0,1,1}c_{0,1,1,0}(t^2\,c_{0,1,1,0}^3c_{0,0,1,1}+\varepsilon\,t\,y\,c_{0,1,1,0}^4+\varepsilon\,t\,z\,c_{0,0,1,1}^2c_{0,1,1,0}c_{2,0,0,0}\\
&+x^2\,c_{2,0,0,0}^2c_{0,0,1,1}c_{0,1,1,0}-\varepsilon\,x\,y\,c_{0,0,1,1}^2c_{2,0,0,0}^2-\varepsilon\,x\,z\,c_{0,1,1,0}^3c_{2,0,0,0}+y\,z\,c_{0,1,1,0}^2c_{0,0,1,1}c_{2,0,0,0}\\
&-\varepsilon\,z\,c_{0,1,1,0}^3c_{1,0,0,0})-t\,x\,c_{2,0,0,0}(c_{0,0,1,2}^4c_{2,0,0,1}^2+c_{0,1,1,1}^6)-t\,c_{0,1,1,0}^6c_{1,0,0,0}+x\,c_{0,0,1,1}^2c_{0,1,1,0}^2c_{2,0,0,0}^2c_{1,0,0,0}\big]\\
&+12c_{2,0,0,0}^5c_{0,0,1,1}^4c_{0,1,1,0}^4,\\
\varepsilon={}&\pm1.
\end{aligned}$$

The *eighteenth* class of rational solutions to Eq. (10) reads

$$u_{18}=-\frac{2p}{q} \tag{28}$$

with

$$\begin{aligned}
p={}&(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)\big[t\,c_{0,1,0,0}(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)-2x\,c_{2,0,0,0}^2c_{0,0,1,1}^2c_{0,1,0,0}+\varepsilon\,y\,c_{2,0,0,0}^2c_{0,1,1,0}c_{0,1,0,0}\\
&+\varepsilon\,z\,c_{0,1,1,0}^4c_{0,0,1,1}c_{0,1,0,0}c_{2,0,0,0}+\varepsilon(c_{0,0,1,1}^3c_{2,0,0,0}^2c_{0,1,1,0}c_{0,0,0,0}+c_{0,1,1,0}^3c_{0,1,0,0}^2c_{0,0,1,1}c_{2,0,0,0})\big]\\
&-12c_{2,0,0,0}^5c_{0,0,1,1}^5c_{0,1,1,0}^3,\\
q={}&(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)\big[t^2\,c_{0,1,1,0}^4c_{0,0,1,1}^2c_{0,1,0,0}-t\,x\,c_{0,1,0,0}(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)+\varepsilon\,t\,y\,c_{0,1,1,0}^5c_{0,0,1,1}c_{0,1,0,0}\\
&+\varepsilon\,t\,z\,c_{0,0,1,1}^3c_{0,1,1,0}^2c_{0,1,0,0}c_{2,0,0,0}+x^2\,c_{2,0,0,0}^2c_{0,0,1,1}^2c_{0,1,1,0}^2c_{0,1,0,0}-\varepsilon\,x\,y\,c_{0,0,1,1}^3c_{2,0,0,0}^2c_{0,1,0,0}c_{0,1,1,0}\\
&-\varepsilon\,x\,z\,c_{0,1,1,0}^4c_{2,0,0,0}c_{0,1,0,0}c_{0,0,1,1}+y\,z\,c_{0,1,1,0}^3c_{0,0,1,1}^2c_{2,0,0,0}c_{0,1,0,0}+y\,c_{0,0,1,1}^2c_{0,1,0,0}^2c_{0,1,1,0}^2c_{2,0,0,0}\\
&+c_{0,0,1,1}^2c_{0,1,1,0}^2c_{2,0,0,0}c_{0,1,0,0}c_{0,0,0,0}\big]+\varepsilon\,t\,\big[(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)(c_{0,1,1,0}^5c_{0,0,1,1}c_{0,0,0,0}\\
&+c_{0,0,1,1}^3c_{0,1,0,0}^2c_{0,1,1,0}c_{2,0,0,0})-12c_{0,1,1,0}^7c_{0,0,1,1}^3c_{2,0,0,0}^3\big]-\varepsilon\,x\,\big[(c_{0,0,1,1}^4c_{2,0,0,0}^2+c_{0,1,1,0}^6)\\
&\times(c_{0,0,1,1}^3c_{2,0,0,0}^2c_{0,1,1,0}c_{0,0,0,0}+c_{0,1,1,0}^3c_{0,1,0,0}^2c_{0,0,1,1}c_{2,0,0,0})-12c_{2,0,0,0}^5c_{0,0,1,1}^5c_{0,1,1,0}^3\big],\\
\varepsilon={}&\pm1.
\end{aligned}$$

Hereby, we give some figures to describe the rational solutions graphically. Two special cases of rational solutions (16) and (17) with

$$c_{i,j,k,l}=1+i^2+j^2+k^2+l^2$$

turn out, respectively, to be

$$u=\frac{204974t-166012x+130438y}{195657t-41503x^2+65219xy-2662x+59290y-169804}, \tag{29}$$

and

$$u=\frac{12936x^2-22}{77616t+2156x^3-11x+924z+41496}. \tag{30}$$

4. Concluding remarks

With the generalized bilinear operators based on a prime number $p=3$, an extended Kadomtsev-Petviashvili-like equation was proposed (see Eq. (10)), which possesses more terms and higher nonlinearity than the standard KP Eq. (5). Eighteen classes of rational solutions were constructed based on polynomial solutions to the generalized bilinear equation (7) by using symbolic computation software Maple. Note that the rational solutions (11), (12), (15) and (16) are independent of the variable z, while the others depend on z. Finally, we give some figures to describe the shape and surface for the rational solutions (29) and (30) as seen in Figs. 1 and 2, respectively. We hope our work in this paper contributes to the study of multi-dimensional and higher order rogue waves.

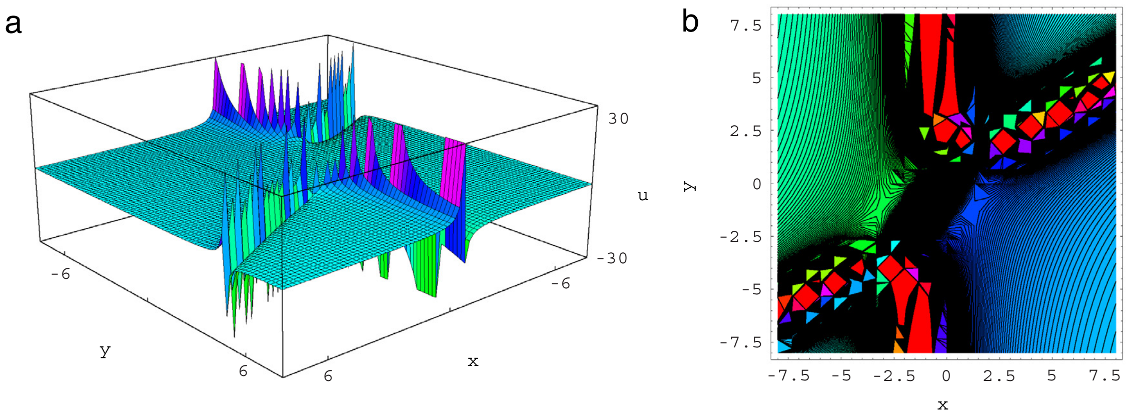

Fig. 1. Shape and surface for rational solution (29) with $t = 0$: (a) 3d plot and (b) density plot.

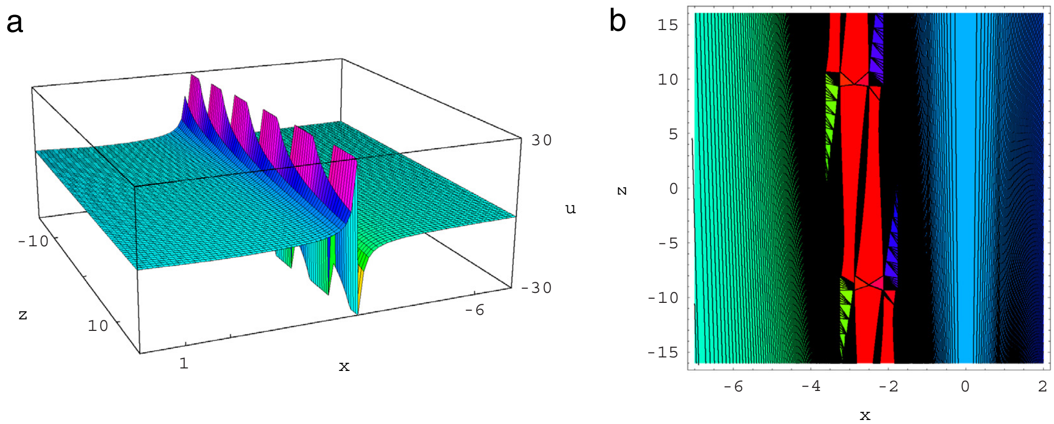

Fig. 2. Shape and surface for rational solution (30) with $t = 0$: (a) 3d plot and (b) density plot.

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