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An explicit plethora of different classes of interactive lump solutions for an extension form of 3D-Jimbo–Miwa model

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Abstract A versatile integration gadget namely the Hirota bilinear (HB) technique is devoted to retrieving different categories of interactive lump solutions of an extension form to the 3D-Jimbo–Miwa (3D-JM) model with exponential and hyperbolic functions under some specific constraint conditions. The article studies the dynamics for these acquired solutions through 3D figures by selecting adequate measurable factors.

1 Introduction

The attainment of analytical solutions for different models described by NLPDEs has a major part in various fields of applied physics. The solutions of NLPDEs have attracted appreciably more attention of physicists as well as mathematicians [1–15]. Many effective methods have been presented to solve these models availing from the headway of symbolic computation [16–21]. Abundant of approaches are investigated to create, analyze, and study the analytical solutions (especially soliton wave solutions) of NLPDEs. These methods include, inter alia, the Hirota direct [22,23], exp-function [24], three-wave [25–27], and the generalized unified [28–32] approaches.

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Lump solutions, which are rational functional solutions, are important category of soliton solutions [33,34]. The studying of their interactions has been drawing the research community [35–48]. It is worth to be noticing that the lump waves preserved their behaviors before and after the encounter between them which can be indicated as an elastic collision.

In the literature, one typical approach to studying lump solutions is to utilize HB equations. In this paper, by employing symbolic computation [49–55] and the HB form, we address the following 3D-JM model [56]:

$$u_{xxxy} + 3\,u_x\,u_{xy} + 3\,u_y\,u_{xx} + 2\,(u_{yt} + u_{xt} + u_{zt}) - 3\,u_{xz} = 0, \tag{1}$$

which may be applied to illustrate the importance of 3D-waves in interdisciplinary research. Equation (1) is an extended form to the well-known (3+1)-dimensional Jimbo–Miwa equation which is the type (II) of the KP hierarchy [57] by adding the two linear terms u_{xt} and u_{zt}. Furthermore, it did not pass any of the conventional integrability tests. Deng and et al. [58] discussed the Bilinear form, Bäcklund transformation, Lax pair, and infinitely many conservation laws for the 3D-JM model through the binary Bell polynomials and symbolic computation. Wazwaz [56] used Hirota's technique to derive twofold soliton solutions for Eq. (1). Lump solutions and their dynamics are studied in [59–62].

The arrangement of this essay will be reported as; Sect. 2 constructed different classes of interactive lump solutions for the 3D-JM model established by the HB technique for Eq. (1). Section 3 discussed the dynamics property of the gained solutions. Sections 4, 5, and 6 reported the dynamics property between lump and exponential function, hyperbolic function, and hyperbolic tangent function solutions, respectively. Section 7 confined to the conclusion.

2 Implementation and discussion of the interactive lump solutions for the 3D-JM model via the HB form

Based on the processing $u = 2(ln\varpi)_x$, we achieve the following HB form of Eq. (1)

$$[2\,(\Gamma_t\Gamma_x + \Gamma_t\Gamma_y + \Gamma_t\Gamma_z) + \Gamma_x^3\Gamma_y - 3\,\Gamma_z\,\Gamma_x]\varpi \cdot \varpi = 0, \tag{2}$$

which is valent to:

$$\begin{aligned}&(\varpi_{xxxy} + 2\,\varpi_{tx} + 2\,\varpi_{ty} + 2\,\varpi_{tz} - 3\,\varpi_{xz})\,\varpi - 3\varpi_{xxy}\varpi_x + 3\varpi_{xy}\,\varpi_{xx}\\ &-\varpi_y\,\varpi_{xxx} - 2\,\varpi_t\varpi_x - 2\,\varpi_t\varpi_y - 2\,\varpi_t\varpi_z + 3\,\varpi_x\,\varpi_z = 0.\end{aligned} \tag{3}$$

Lump and lump-kink solutions are an example of the interactive lump solutions for the 3D-JM model which can be explored by the aid of Eq. (3) when shape takes

$$\begin{aligned} g &= x\chi_1 + y\chi_2 + z\chi_3 + t\chi_4 + \chi_5,\\ h &= x\chi_6 + y\chi_7 + z\chi_8 + t\chi_9 + \chi_{10},\\ l &= ke^{xk_1+yk_2+zk_3+tk_4},\\ m &= j\cosh\left(xj_1 + yj_2 + zj_3 + tj_4\right),\\ \varpi &= g^2 + h^2 + \chi_{11} + l + m, \end{aligned} \tag{4}$$

where $\chi_\sigma(1 \leq \sigma \leq 11)$, $k_\sigma(1 \leq \sigma \leq 4)$, $j_\sigma(1 \leq \sigma \leq 4)$, k and j are real parameters. A principle system of these parameters can be obtained by inserting Eq. (4) into Eq. (3) which derives different states of solutions for $\chi_\sigma(1 \leq \sigma \leq 11)$, $k_\sigma(1 \leq \sigma \leq 4)$, $j_\sigma(1 \leq \sigma \leq 4)$, k and j.

State(1)

$$\chi_4=\frac{3\left(\chi_1\chi_3^2+\chi_6^2\chi_3+\chi_1\chi_2\chi_3\right)}{2\left(\chi_2^2+2\chi_3\chi_2+\chi_3^2+\chi_6^2\right)},\ \chi_8=-\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=-\frac{\chi_1\chi_2}{\chi_6},$$
$$\chi_9=-\frac{3\left(\chi_1-\chi_2-\chi_3\right)\chi_3\chi_6}{2\left(\left(\chi_2+\chi_3\right)^2+\chi_6^2\right)},\ \chi_{11}=j=k=0, \tag{5}$$

where $(\chi_2+\chi_3)^2+\chi_6^2\neq 0$, $\chi_6\neq 0$.

State(2)

$$\chi_4=\frac{3\left(\chi_1\chi_3^2+\chi_6^2\chi_3+\chi_1\chi_2\chi_3\right)}{2\left(\chi_2^2+2\chi_3\chi_2+\chi_3^2+\chi_6^2\right)},\ \chi_8=-\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=-\frac{\chi_1\chi_2}{\chi_6},$$
$$\chi_9=-\frac{3\left(\chi_1-\chi_2-\chi_3\right)\chi_3\chi_6}{2\left(\left(\chi_2+\chi_3\right)^2+\chi_6^2\right)},\ \chi_{11}=k=0,\ j_4=\frac{3j_1j_3-j_1^3j_2}{2\left(j_1+j_2+j_3\right)},$$
$$\chi_6=-\frac{\sqrt{-j_1\left(\chi_2+\chi_3\right)^2\left(\left(j_1^3+\left(j_1^2+3\right)j_3\right)\chi_2-\left(3j_1+\left(j_1^2+3\right)j_2\right)\chi_3\right)}}{\sqrt{j_1\left(j_1^3+\left(j_1^2+3\right)j_3\right)\chi_2-\left(j_2j_1^3-3\left(j_2+2j_3\right)j_1-3\left(j_2+j_3\right)^2\right)\chi_3}},$$
$$\begin{aligned}j_3=&[\chi_2j_1^3+3j_2\left(\chi_2+\chi_3\right)j_1^2+3\left(\chi_2+2\chi_3\right)j_1-3j_2\left(\chi_2-\chi_3\right)\\&-[[[j_1\left(j_1^2+3\right)+3\left(j_1^2-1\right)j_2]\chi_2+3[2j_1+\left(j_1^2+1\right)j_2]\chi_3]^2+12\chi_2[(j_1^2\\&+2j_2j_1+3j_2^2)\chi_2j_1^2+j_2\left(j_1\left(j_1^2+3\right)+3\left(j_1^2+1\right)j_2\right)\chi_3]]^{1/2}]/(6\chi_2),\end{aligned} \tag{6}$$

where $j_1\left(j_1^3+\left(j_1^2+3\right)j_3\right)\chi_2-\left(j_2j_1^3-3\left(j_2+2j_3\right)j_1-3\left(j_2+j_3\right)^2\right)\chi_3>0$, $\chi_2\neq 0$, $-j_1\left(\chi_2+\chi_3\right)^2\left(\left(j_1^3+\left(j_1^2+3\right)j_3\right)\chi_2-\left(3j_1+\left(j_1^2+3\right)j_2\right)\chi_3\right)\geq 0$, $j_1+j_2+j_3\neq 0$.

State(3)

$$\chi_4=\frac{3\chi_1\chi_3\left(\chi_1-\chi_2+\chi_3\right)\chi_6^2}{2\chi_1^2\chi_2^2+2\left(\chi_1+\chi_3\right)^2\chi_6^2},\ \chi_8=\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=-\frac{\chi_1\chi_2}{\chi_6},$$
$$\chi_9=\frac{3\chi_3\chi_6\left(\chi_2\chi_1^2+\left(\chi_1+\chi_3\right)\chi_6^2\right)}{2\chi_1^2\chi_2^2+2\left(\chi_1+\chi_3\right)^2\chi_6^2},\ \chi_{11}=j=k=0, \tag{7}$$

where $2\chi_1^2\chi_2^2+2\left(\chi_1+\chi_3\right)^2\chi_6^2\neq 0$, $\chi_6\neq 0$.

State(4)

$$\chi_4=\frac{3\chi_1\chi_3\left(\chi_1-\chi_2+\chi_3\right)\chi_6^2}{2\chi_1^2\chi_2^2+2\left(\chi_1+\chi_3\right)^2\chi_6^2},\ \chi_8=\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=-\frac{\chi_1\chi_2}{\chi_6},$$
$$\chi_9=\frac{3\chi_3\chi_6\left(\chi_2\chi_1^2+\left(\chi_1+\chi_3\right)\chi_6^2\right)}{2\chi_1^2\chi_2^2+2\left(\chi_1+\chi_3\right)^2\chi_6^2},\ \chi_{11}=k=0,\ j_4=\frac{3j_1j_3-j_1^3j_2}{2\left(j_1+j_2+j_3\right)},$$
$$\begin{aligned}j_2=&\frac{2\chi_1\chi_2^2\chi_3}{j_1\left(\chi_1^2\chi_2^2+\left(\chi_1+\chi_3\right)^2\chi_6^2\right)},\ j_3=[3j_1\chi_1\chi_3[\left(j_1^2-1\right)\chi_1^5\chi_2^6\\&+\chi_1^3\left(\chi_1+\chi_3\right)[2\chi_3j_1^2+\left(3j_1^2-2\right)\chi_1]\chi_6^2\chi_2^4+\chi_1\left(\chi_1+\chi_3\right)^3[\left(3j_1^2-1\right)\chi_1\\&+\left(j_1^2+1\right)\chi_3]\chi_6^4\chi_2^2+j_1^2\left(\chi_1+\chi_3\right)^5\chi_6^6]-\sqrt{3}[j_1^2\chi_1^3\chi_2^2\chi_3[\chi_1^2\chi_2^2\\&+\left(\chi_1+\chi_3\right)^2\chi_6^2]^4[\left(\chi_1+\chi_3\right)^2\chi_6^2j_1^4+\chi_1\chi_2^2[\left(\chi_1+\chi_3\right)j_1^4+3\chi_3]]]^{1/2}]\\&/[3j_1^2\chi_1^2\left(\chi_1\chi_2^2+\left(\chi_1+\chi_3\right)\chi_6^2\right)\left(\chi_1^2\chi_2^2+\left(\chi_1+\chi_3\right)^2\chi_6^2\right)^2],\end{aligned} \tag{8}$$

where $2\chi_1^2\chi_2^2 + 2(\chi_1+\chi_3)^2\chi_6^2 \neq 0$, $\chi_6 \neq 0$, $j_1 + j_2 + j_3 \neq 0$ and

$$\begin{aligned}
&\chi_1(\chi_1+\chi_3)\left(\chi_1\chi_2^2+(\chi_1+\chi_3)\chi_6^2\right)\left(\chi_1^2\chi_2^2+(\chi_1+\chi_3)^2\chi_6^2\right)^3 j_1^5\\
&\quad-3\chi_1^2\chi_2^2\chi_3\left(\chi_1^2\chi_2^2+(\chi_1+\chi_3)^2\chi_6^2\right)^3 j_1+\sqrt{3}[j_1^2\chi_1^3\chi_2^2\chi_3[\chi_1^2\chi_2^2\\
&\quad+(\chi_1+\chi_3)^2\chi_6^2]^4[(\chi_1+\chi_3)^2\chi_6^2 j_1^4+\chi_1\chi_2^2[(\chi_1+\chi_3)j_1^4\\
&\quad+3\chi_3]]]^{1/2}\left(\chi_3^2\chi_6^2-\chi_1^2\left(\chi_2^2+\chi_6^2\right)\right)=0.
\end{aligned}$$

State(5)

$$\begin{aligned}
\chi_4 &= \frac{3\chi_1\chi_3(\chi_1-\chi_2+\chi_3)\chi_6^2}{2\chi_1^2\chi_2^2+2(\chi_1+\chi_3)^2\chi_6^2},\ \chi_8=\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=-\frac{\chi_1\chi_2}{\chi_6},\ k_3=k_2k_1^2+k_1,\\
\chi_9 &= \frac{3\chi_3\chi_6\left(\chi_2\chi_1^2+(\chi_1+\chi_3)\chi_6^2\right)}{2\chi_1^2\chi_2^2+2(\chi_1+\chi_3)^2\chi_6^2},\ \chi_{11}=j=0,\ \chi_6=\frac{1}{2}k_1^2\chi_2,\ \chi_3=-\chi_1,\\
k_2 &= \frac{4\sqrt{k_1^8\chi_2^2}-2k_1^2\left(3k_1^4+4k_1^2+3\right)\chi_2}{3k_1^3\left(k_1^2+1\right)^2\chi_2},\ k_4=\frac{3k_1k_3-k_1^3k_2}{2(k_1+k_2+k_3)},
\end{aligned} \tag{9}$$

where $2\chi_1^2\chi_2^2 + 2(\chi_1+\chi_3)^2\chi_6^2 \neq 0$, $\chi_6 \neq 0$, $\chi_2 < 0$, $k_1 \neq 0$.

State(6)

$$\begin{aligned}
\chi_4 &= \frac{3\chi_1\chi_3\left(\chi_3\chi_1^2+(\chi_1+\chi_2)\chi_6^2\right)}{2\chi_1^2\chi_3^2+2(\chi_1+\chi_2)^2\chi_6^2},\ \chi_8=-\frac{\chi_1\chi_3}{\chi_6},\ \chi_7=\frac{\chi_2\chi_6}{\chi_1},\\
\chi_9 &= -\frac{3\chi_1^2(\chi_1+\chi_2-\chi_3)\chi_3\chi_6}{2\chi_1^2\chi_3^2+2(\chi_1+\chi_2)^2\chi_6^2},\ j=k=0,\\
\chi_{11} &= -\frac{\chi_2\left(\chi_1^2+\chi_6^2\right)\left(\chi_1^2\chi_3^2+(\chi_1+\chi_2)^2\chi_6^2\right)}{\chi_1^2(\chi_1+\chi_2)\chi_3^2},
\end{aligned} \tag{10}$$

where $\chi_1^2(\chi_1+\chi_2)\chi_3^2 \neq 0$, $\chi_1\chi_6 \neq 0$, $2\chi_1^2\chi_3^2 + 2(\chi_1+\chi_2)^2\chi_6^2 \neq 0$.

State(7)

$$\begin{aligned}
\chi_4 &= \frac{3\chi_6(\chi_3(\chi_6+\chi_7)-\chi_2\chi_8)}{2\left((\chi_2+\chi_3)^2+(\chi_6+\chi_7+\chi_8)^2\right)},\ \chi_1=0,\\
\chi_9 &= \frac{3\chi_6(\chi_3(\chi_2+\chi_3)+\chi_8(\chi_6+\chi_7+\chi_8))}{2\left((\chi_2+\chi_3)^2+(\chi_6+\chi_7+\chi_8)^2\right)},\ j=k=0,\\
\chi_{11} &= \frac{\chi_6^2\chi_7\left((\chi_2+\chi_3)^2+(\chi_6+\chi_7+\chi_8)^2\right)}{(\chi_2+\chi_3)(\chi_2\chi_8-\chi_3(\chi_6+\chi_7))},
\end{aligned} \tag{11}$$

where $(\chi_2+\chi_3)^2 + (\chi_6+\chi_7+\chi_8)^2 \neq 0$, $(\chi_2+\chi_3)(\chi_2\chi_8-\chi_3(\chi_6+\chi_7)) \neq 0$.

State(8)

$$\begin{aligned}
\chi_4 &= \frac{3\chi_1(\chi_3(\chi_1+\chi_2+\chi_3)+\chi_8(\chi_7+\chi_8))}{2\left((\chi_1+\chi_2+\chi_3)^2+(\chi_7+\chi_8)^2\right)},\ \chi_6=0,\\
\chi_9 &= \frac{3\chi_1((\chi_1+\chi_2)\chi_8-\chi_3\chi_7)}{2\left((\chi_1+\chi_2+\chi_3)^2+(\chi_7+\chi_8)^2\right)},\ j=k=0,\\
\chi_{11} &= \frac{\chi_1^2\chi_2\left((\chi_1+\chi_2+\chi_3)^2+(\chi_7+\chi_8)^2\right)}{(\chi_7+\chi_8)(\chi_3\chi_7-(\chi_1+\chi_2)\chi_8)},
\end{aligned} \tag{12}$$

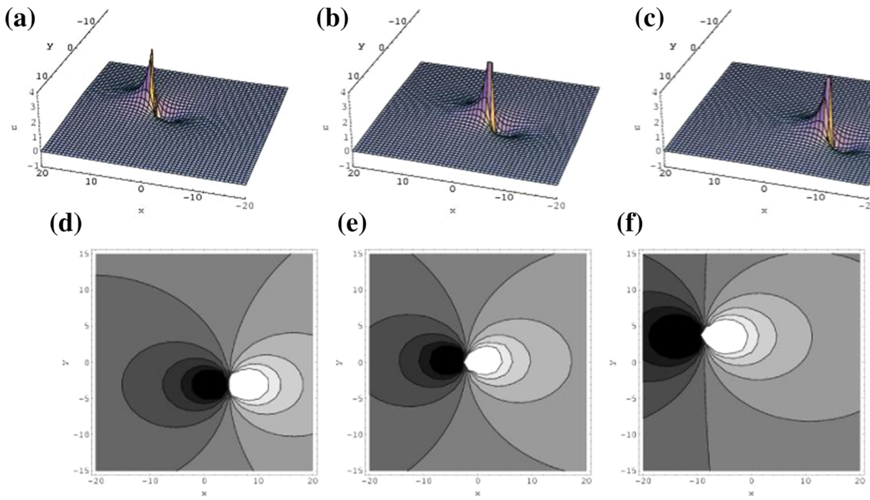

Fig. 1 The lump solution of Eq. (1) for $\chi_1 = \chi_3 = z = 2$, $\chi_2 = 5$, $\chi_6 = 3$, $\chi_{10} = 1$, $\chi_5 = -1$. **a**, **d** $t = -15$. **b**, **e** $t = 0$. **c**, **f** $t = 15$

where $(\chi_1 + \chi_2 + \chi_3)^2 + (\chi_7 + \chi_8)^2 \neq 0$, $(\chi_7 + \chi_8)(\chi_3\chi_7 - (\chi_1 + \chi_2)\chi_8) \neq 0$.
State(9)

$$\begin{aligned}
\chi_4 &= \frac{3\chi_8(\chi_1(\chi_7+\chi_8) - \chi_2\chi_6)}{2\left((\chi_1+\chi_2)^2 + (\chi_6+\chi_7+\chi_8)^2\right)}, \chi_3 = 0,\\
\chi_9 &= \frac{3\chi_8(\chi_1(\chi_1+\chi_2) + \chi_6(\chi_6+\chi_7+\chi_8))}{2\left((\chi_1+\chi_2)^2 + (\chi_6+\chi_7+\chi_8)^2\right)}, j = k = 0,\\
\chi_{11} &= -\frac{\left(\chi_1^2+\chi_6^2\right)(\chi_1\chi_2+\chi_6\chi_7)\left((\chi_1+\chi_2)^2 + (\chi_6+\chi_7+\chi_8)^2\right)}{(\chi_1+\chi_2)\chi_8(\chi_1(\chi_7+\chi_8) - \chi_2\chi_6)},
\end{aligned} \tag{13}$$

where $(\chi_1 + \chi_2)^2 + (\chi_6 + \chi_7 + \chi_8)^2 \neq 0$, $(\chi_1 + \chi_2)\chi_8(\chi_1(\chi_7 + \chi_8) - \chi_2\chi_6) \neq 0$.

Above these solutions for the parameters engender nine classes of linear combination solutions to the Hirota's bilinear Eq. (3), defined by Eq. (4), under the transformation $u = 2(ln\varpi)_x$).

3 The dynamics property of lump solution

Herein, we analyze the dynamical characteristic of the acquired lump solutions of the 3D-JM model. The acquired solutions stated in Eqs. (5)–(13) own the same style but vary in their coefficients. In observance of this, we confine ourself to state (3) to shed light on their dynamical behaviors through 3D- and contour plots.

Figure 1a–c shows the 3D-plots of the lump solutions in the xy-plane, while Fig. 1d–f shows the corresponding contour plot at the time instance $t = -15, 0, 15$, respectively.

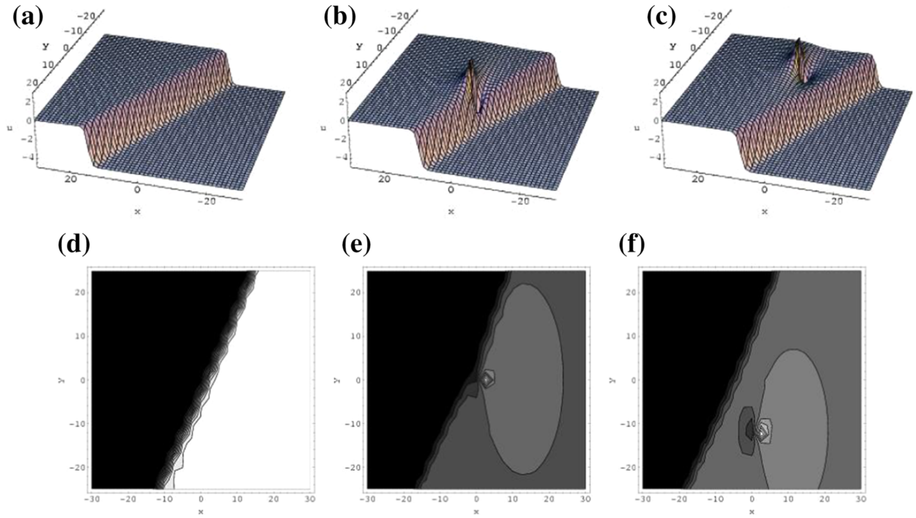

Fig. 2 Plots of the lump solution with exponential function of Eq. (1) for $\chi_1 = 8$, $\chi_5 = \chi_{10} = 1$, $\chi_2 = -5$, $k_1 = -2$, $k = 0.8$, $z = 2$. **a**, **d** $t = -5$. **b**, **e** $t = 0$. **c**, **f** $t = 2$

4 The dynamics property between lump and exponential function solutions

In this part, we focus on studying the interaction between lump and exponential function solutions. To explain this case, we investigate 3D- and contour plots for State (5). Figure 2a–c shows the 3D-plots of this kind of solutions in xy-plane at $t = -5, 0, 2$, respectively. Figure 2d–f represents their corresponding contour plot.

5 The dynamics property between lump and hyperbolic function solutions

Now, the dynamics analysis between lump and hyperbolic function solutions will be discussed in this part. State (4) will cover graphically this discussion through some 3D- and contour plots when $t = -15, 0, 15$. Figure 3a–c gives the 3D-plots of this type of solutions xy-plane, while the contour plots for the solutions are introduced in Fig. 3d–f.

6 The dynamics property between the rational and the hyperbolic tangent function solutions

Finally in this section, we investigate the dynamics property among the rational and hyperbolic tangent function solutions. To reach our aim, we assume Eq. (3) in the form

$$\begin{aligned} g &= x\aleph_1 + y\aleph_2 + z\aleph_3 + t\aleph_4 + \aleph_5, \\ h &= x\aleph_6 + y\aleph_7 + z\aleph_8 + t\aleph_9 + \aleph_{10}, \\ \varpi &= g^2 + h^2 + k \tanh\left(\varpi_1 x + \varpi_2 y + \varpi_3 z + \varpi_4 t\right) + \aleph_{11}, \end{aligned} \tag{14}$$

where $\aleph_i (1 \leq i \leq 11)$, $\varpi_i (1 \leq i \leq 4)$, and k are undetermined constants. Let us take Eq. (14) into Eq. (3), the values of the undetermined parameters $\aleph_i (1 \leq i \leq 11)$, $\varpi_i (1 \leq i \leq 4)$, and k are calculated by using the Mathematical software as

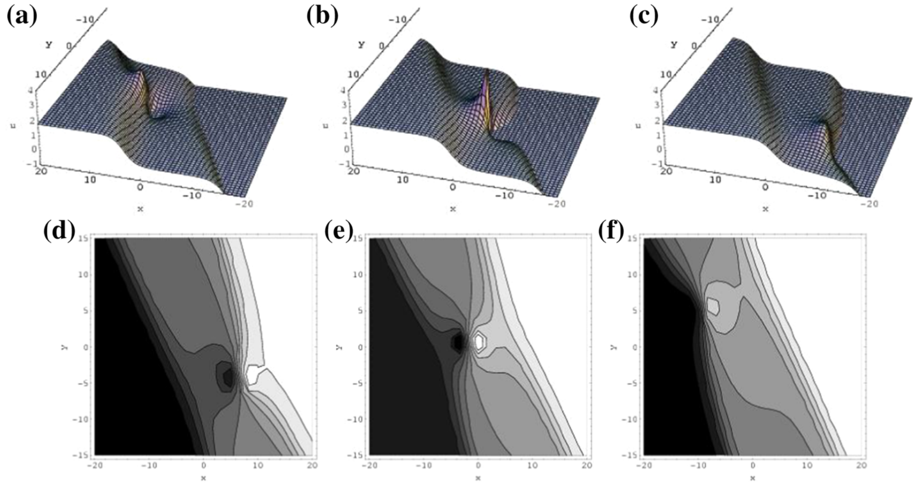

Fig. 3 Plots of the lump solution with hyperbolic function of Eq. (1) $\chi_1 = \chi_3 = z = 2$, $\chi_2 = \chi_6 = \chi_{10} = j = 1$, $\chi_5 = -1$. **a**, **d** $t = -15$. **b**, **e** $t = 0$. **c**, **f** $t = 15$

$$\begin{aligned}
&\varpi_1 = \aleph_2 = \aleph_7 = 0, \varpi_3 = \varpi_2 = -\frac{2\left(\aleph_1 + \aleph_3\right)\varpi_4}{3\aleph_1}, \aleph_8 = \frac{\aleph_3\aleph_6}{\aleph_1},\\
&\aleph_4 = \frac{3\aleph_1\aleph_3}{2\left(\aleph_1 + \aleph_3\right)}, \aleph_9 = \frac{3\aleph_3\aleph_6}{2\left(\aleph_1 + \aleph_3\right)},
\end{aligned} \tag{15}$$

where $\aleph_1 \neq 0$, $\aleph_1 + \aleph_3 \neq 0$. Substituting Eqs. (14) and (15) into the formula $u = 2(ln\varpi)_x$, we gain the solution of Eq. (1) as

$$\begin{aligned}
u = &\left[2\left[2\aleph_1\left[\aleph_5 + \frac{3\aleph_3\aleph_1 t}{2\left(\aleph_1 + \aleph_3\right)} + \aleph_1 x + \aleph_3 z\right] + 2\aleph_6\left[\aleph_{10} + \frac{3\aleph_3\aleph_6 t}{2\left(\aleph_1 + \aleph_3\right)}\right.\right.\right.\\
&\left.\left.\left. + \aleph_6 x + \frac{\aleph_3\aleph_6 z}{\aleph_1}\right]\right]\right] \Big/ \left[\aleph_{11} + k\tanh\left[\varpi_4 t - \frac{2\left(\aleph_1 + \aleph_3\right)\varpi_4 y}{3\aleph_1} + \frac{2\left(\aleph_1 + \aleph_3\right)\varpi_4 z}{3\aleph_1}\right]\right.\\
&+ \left[\aleph_5 + \frac{3\aleph_3\aleph_1 t}{2\left(\aleph_1 + \aleph_3\right)} + \aleph_1 x + \aleph_3 z\right]^2\\
&\left.+ \left[\aleph_{10} + \frac{3\aleph_3\aleph_6 t}{2\left(\aleph_1 + \aleph_3\right)} + \aleph_6 x + \frac{\aleph_3\aleph_6 z}{\aleph_1}\right]^2\right].
\end{aligned} \tag{16}$$

The dynamical behaviors for the solution in Eq. (16) that are clarified in interaction solution between the rational function and the hyperbolic tangent function are shown in Fig. 4.

Figure 4a–c gives the 3D-plots in tz-plane at $x = -10, 0, 10$, respectively. Figure 4d–f describes the corresponding contour maps at the same values of x.

7 Conclusion

This research is successfully applied the HB form and symbolic computations to generate different classes of interactive lump solutions for the 3D-JM model with some nonzero determinant conditions. Some discussions are graphically introduced about the dynamic

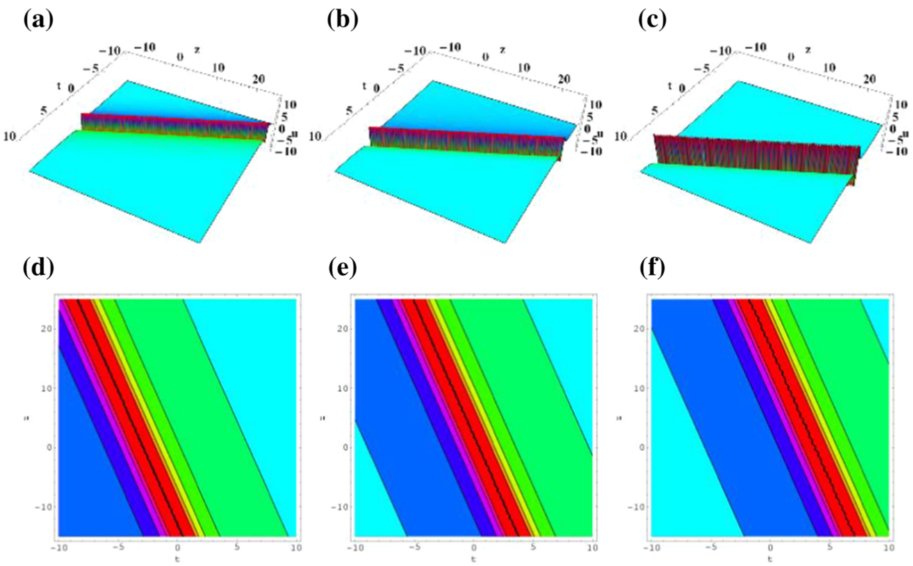

Fig. 4 The solution (6) for $\aleph_1 = \aleph_6 = 3$, $\aleph_{11} = -5$, $k = y = \aleph_{10} = 2$, $\aleph_5 = 6$, $\varpi_4 = -1$, $\aleph_3 = -2$. **a**, **d** $x = -10$. **b**, **e** $x = 0$. **c**, **f** $x = 10$

properties of the acquired solutions in Figs. 1, 2, 3, and 4 via suitable choices of the adequate parameters.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest concerning the publication of this manuscript.

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