Nonlinear Dyn (2018) 94:2643–2654
https://doi.org/10.1007/s11071-018-4514-5

ORIGINAL PAPER

Observation of interaction phenomena for two dimensionally reduced nonlinear models

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Received: 10 May 2018 / Accepted: 26 July 2018 / Published online: 20 August 2018
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Abstract To study the lump–soliton interaction phenomenon for the (3 + 1)-dimensional nonlinear model with dimensional reduction, interaction solutions have been formulated by combining positive quadratic functions with hyperbolic function in bilinear equations. The collision between lump and soliton has been analyzed and simulated. When the lump is induced by a bounded twin soliton, the rogue wave turns up, which can only be visible at an instant time. Based on the solutions, it is easy to find the amplitude, the place and the arrival time of the rogue waves. The mechanism investigated in this paper may shed some light on the study of rogue waves in oceanography, fluid dynamics and nonlinear optics.

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Keywords Lump · Soliton · Rogue wave · Oceanography

1 Introduction

Nonlinear evolution equations (NLEEs) arising in physically relevant situations possess potential applications, especially, the (2 + 1)- and (3 + 1)-dimensional ones [1–3]. For instance, the Kadomtsev–Petviashvili (KP) equation, as a two-dimensional generalization of the one-dimensional Korteweg–de Vries (KdV) equation, can be applied to model water waves of long wavelengths with weakly nonlinear restoring forces and frequency dispersion [4,5]. The (2 + 1)-dimensional breaking soliton equation can be used to describe the (2 + 1)-dimensional interaction of Riemann wave propagation with the long-wave propagation [6,7]. Moreover, some (3 + 1)-dimensional NLEEs have also been proposed and solved to investigate the spatiotemporal features of the associated nonlinear phenomena (see, e.g., Refs. [1,8–12]).

The exact solutions to NLEEs are helpful in studying the physical mechanism and dynamical characteristics of associated problems [4,5,10–16]. In mathematical physics, abundant exact solutions have been derived to NLEEs, such as solitons, dromions, positons, complexitons, and lumps [1,17–19]. Two-dimensional solitons of the KP equation and their interaction have been studied in Ref. [20], while lump solutions to the KP equation have been given in Ref. [19,20]. Dromion is another type of localized solutions for the (2+1)-dimensional NLEEs, and the generalized dromion solutions of the (2+1)-dimensional KdV equation can be found in Ref. [17]. By using Wronskian technology, positon and complexiton solutions to integrable equations can be generated [18].

As a kind of special solutions to integrable NLEEs, soliton solutions are exponentially localized in certain directions, while lump solutions are rationally (algebraically) localized in all directions in the space [21–28]. With symbolic computation, lump solutions have been generated through searching for positive quadratic function solutions to the associated bilinear equations [19,27,28]. Study on the interaction solutions between lump and soliton or kink is of great importance in describing the interesting nonlinear phenomena [29–32]. Currently, many interaction solutions have been formulated by combining positive quadratic functions with other kinds of functions in bilinear equations [29,30]. It is obvious that lump is a type of rational solutions, and rogue wave solutions are a class of lump-like solutions, which attracted a great deal of attention from mathematicians and physicists worldwide, e.g., in oceanography, nonlinear optics and financial system [33–35].

The Hirota bilinear representation of NLEEs plays an important role in searching for soliton solutions, lump solutions and interaction solutions [27–30,36]. Based on a multivariate polynomial, a Hirota bilinear equation [37] has been proposed as

$$\left(D_t D_y - D_x^3 D_y - 3\,D_x^2 + 3\,D_z^2\right) f \cdot f = 0, \tag{1}$$

where $f = f(x, y, z, t)$ is a (3+1)-dimensional function, and the derivatives $D_t D_y$, $D_x^3 D_y$, D_x^2 and D_z^2 are the Hirota bilinear operators [36] defined by

$$\begin{aligned} &D_x^\alpha D_y^\beta D_z^\gamma D_t^\delta (f \cdot g)\\ &\quad= \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^\alpha \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^\beta \left(\frac{\partial}{\partial z} - \frac{\partial}{\partial z'}\right)^\gamma \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^\delta\\ &\qquad\times f(x, y, z, t) g(x', y', z', t')\Big|_{x'=x, y'=y, z'=z, t'=t}. \end{aligned}$$

Bell polynomial theories tell Eq. (1) is linked with the following (3+1)-dimensional NLEE

$$u_{yt} - u_{xxxy} - 3\,(u_x u_y)_x - 3\,u_{xx} + 3\,u_{zz} = 0, \tag{2}$$

through the dependent variable transformation

$$u = 2\Big[\ln f(x, y, z, t)\Big]_x = 2\frac{f_x(x, y, z, t)}{f(x, y, z, t)}, \tag{3}$$

where we demand that $f(x, y, z, t) \neq 0$.

By use of the linear superposition principle [37], two types of resonant N-wave solutions have been found and illustrated to Eq. (2). A bilinear Bäcklund transformation consisting of six bilinear equations and involving nine arbitrary parameters has been constructed, and nonresonant-typed one-, two-, and three-wave solutions have been obtained with multiple exponential function method [12]. By searching for positive quadratic function solutions to the associated bilinear equations [28], lump solutions have also been derived and discussed for the dimensionally reduced cases of Eq. (2), such as $z = y$, $z = t$, $y = x$, and $y = z$. However, we have not found lump solutions to the reductions $z = x$ and $y = t$. The higher-dimensional models, e.g., the (2+1)- and (3+1)-dimensional ones, are more applicable to describe the nonlinear phenomena in the real world. To reveal the complex physical mechanism described by higher-dimensional models, it is effective to consider the dimensional reductions. In summary, both soliton solutions and lump solutions have been studied for Eq. (2). How about the lump–soliton interaction for the dimensional reductions of Eq. (2)? The main contribution of this paper is to study the interaction phenomena (exactly interaction between lump and soliton) for Eq. (2). Moreover, the induced mechanism of rogue waves will be investigated in theory.

In this paper, we will search for the lump–soliton interaction solutions to the dimensionally reduced Hirota bilinear Eq. (1) via taking $z = t$ or $z = x$, and begin with

$$f = \xi_1^2 + \xi_2^2 + k \cosh \xi_3 + a_9, \tag{4}$$

where three linear wave variables are defined by

$$\begin{aligned} \xi_1 &= a_1 x + a_2 y + a_3 t + a_4,\\ \xi_2 &= a_5 x + a_6 y + a_7 t + a_8, \end{aligned}$$

$\xi_3 = k_1 x + k_2 y + k_3 t,$

while a_i $(1 \le i \le 9)$, k_i $(1 \le i \le 3)$ and k are all real parameters to be determined. Eq. (4) gives rise to a class of lump solutions with $\xi_3 = 0$ and generates a class of one-soliton solutions with $\xi_1 = \xi_2 = 0$. The exact solutions generated via the combined form of Eq. (4) are called interaction solutions, that is, interaction between lump and soliton [11,19,27–31].

2 Lump–soliton interaction for the reduction with $z = t$

The dimensionally reduced form of Eq. (2) with $z = t$ turns out to be

$$u_{yt} - u_{xxxy} - 3\,(u_x u_y)_x - 3\,u_{xx} + 3\,u_{tt} = 0, \tag{5}$$

the bilinear representation of which reads

$$\begin{aligned}&\left(D_t D_y - D_x^3 D_y - 3\,D_x^2 + 3\,D_t^2\right) f \cdot f\\ &\quad = 2\,\big[(f f_{ty} - f_t f_y + f_{xxx} f_y + 3\,f_{xxy} f_x\\ &\qquad -3\,f_{xx} f_{xy} - f f_{xxxy}\\ &\qquad -3\left(f f_{xx} - f_x^2\right) + 3\left(f f_{tt} - f_t^2\right)\big] = 0,\end{aligned} \tag{6}$$

through the dependent variable transformation

$$u = 2\Big[\ln f(x, y, t)\Big]_x = 2\frac{f_x(x, y, t)}{f(x, y, t)}. \tag{7}$$

The following two classes of solutions for the parameters are obtained by substituting f in Eq. (4) into Eq. (6):

$$\left\{\begin{aligned}&a_1 = \pm a_7,\ a_2 = 0,\ a_3 = 0,\ a_4 = a_4,\ a_5 = 0,\\ &a_6 = -6a_7,\ a_7 = a_7,\ a_8 = a_8,\ a_9 = \frac{1}{2}\sqrt{\frac{1}{3}\frac{k^2}{a_7^2}},\\ &k = k, k_1 = \varepsilon\left(\frac{1}{3}\right)^{\frac{1}{4}}, k_2 = -6k_1^3,\\ &k_3 = \left(\frac{1}{3}\right)^{-\frac{3}{4}}(-\varepsilon \pm \sqrt{2})\end{aligned}\right\}, \tag{8}$$

where $k > 0$, $a_7 \neq 0$, and $\varepsilon = \pm 1$; and

$$\left\{\begin{aligned}&a_1 = a_1,\ a_2 = 0,\ a_3 = 0,\ a_4 = a_4,\ a_5 = 0,\\ &a_6 = -\frac{3\left(a_1^2 + a_7^2\right)}{a_7},\ a_7 = a_7,\ a_8 = a_8,\ a_9 = \frac{k^2 k_1^2}{2a_1^2},\\ &k = k, k_1\\ &= \varepsilon\left[\frac{(3a_1^4 + 2a_1^2 a_7^2 + 3a_7^4) + \varepsilon\sqrt{\left(a_1^2 - a_7^2\right)^2\left(9a_1^4 + 14a_1^2 a_7^2 + 9a_7^4\right)}}{6\left(a_1^2 + a_7^2\right)^2}\right]^{\frac{1}{4}},\\ &k_2 = -\frac{2}{k_1}, k_3 = \frac{3\left(a_1^2 + a_7^2\right)k_1^4 - 2a_7^2}{3k_1\left(a_1^2 - a_7^2\right)}\end{aligned}\right\}, \tag{9}$$

where $k > 0$, $a_1 \neq 0$, $a_7 \neq 0$, $a_1^2 - a_7^2 \neq 0$, and $\varepsilon = \pm 1$.

The two sets of solutions for the parameters generate correspondingly the combined solutions to the bilinear Eq. (6), and then the resulting combined solutions present the interaction solutions to the dimensionally reduced Eq. (5), under the transformation in Eq. (7). The function f is well defined by the conditions $a_1 \neq 0$, $a_7 \neq 0$, and $a_1^2 - a_7^2 \neq 0$, while the positiveness of f and the analyticity of the interaction solution u is definitely guaranteed by $k > 0$ and $a_9 = \frac{1}{2}\sqrt{\frac{1}{3}\frac{k^2}{a_7^2}}$ or $a_9 = \frac{k^2 k_1^2}{2a_1^2}$. Because a line soliton is involved, the interaction solutions do not approach zero in all directions in the independent variable space, and a peak arises at finite times due to the existence of a lump wave.

Corresponding to Eqs. (8) and (9), the explicit formulas of solutions to Eq. (5) can be written, respectively, as

$$u = \frac{4(\pm a_7)(\pm a_7 x + a_4) + 2k\,\varepsilon\left(\frac{1}{3}\right)^{\frac{1}{4}}\sinh\left(\varepsilon\left(\frac{1}{3}\right)^{\frac{1}{4}}x - 6\,\varepsilon\left(\frac{1}{3}\right)^{\frac{3}{4}}y + \left(\frac{1}{3}\right)^{-\frac{3}{4}}(-\varepsilon \pm \sqrt{2})t\right)}{(\pm a_7 x + a_4)^2 + (-6a_7 y + a_7 t + a_8)^2 + k\cosh\left(\varepsilon\left(\frac{1}{3}\right)^{\frac{1}{4}}x - 6\,\varepsilon\left(\frac{1}{3}\right)^{\frac{3}{4}}y + \left(\frac{1}{3}\right)^{-\frac{3}{4}}(-\varepsilon \pm \sqrt{2})t\right) + \frac{1}{2}\sqrt{\frac{1}{3}\frac{k^2}{a_7^2}}}, \tag{10}$$

and

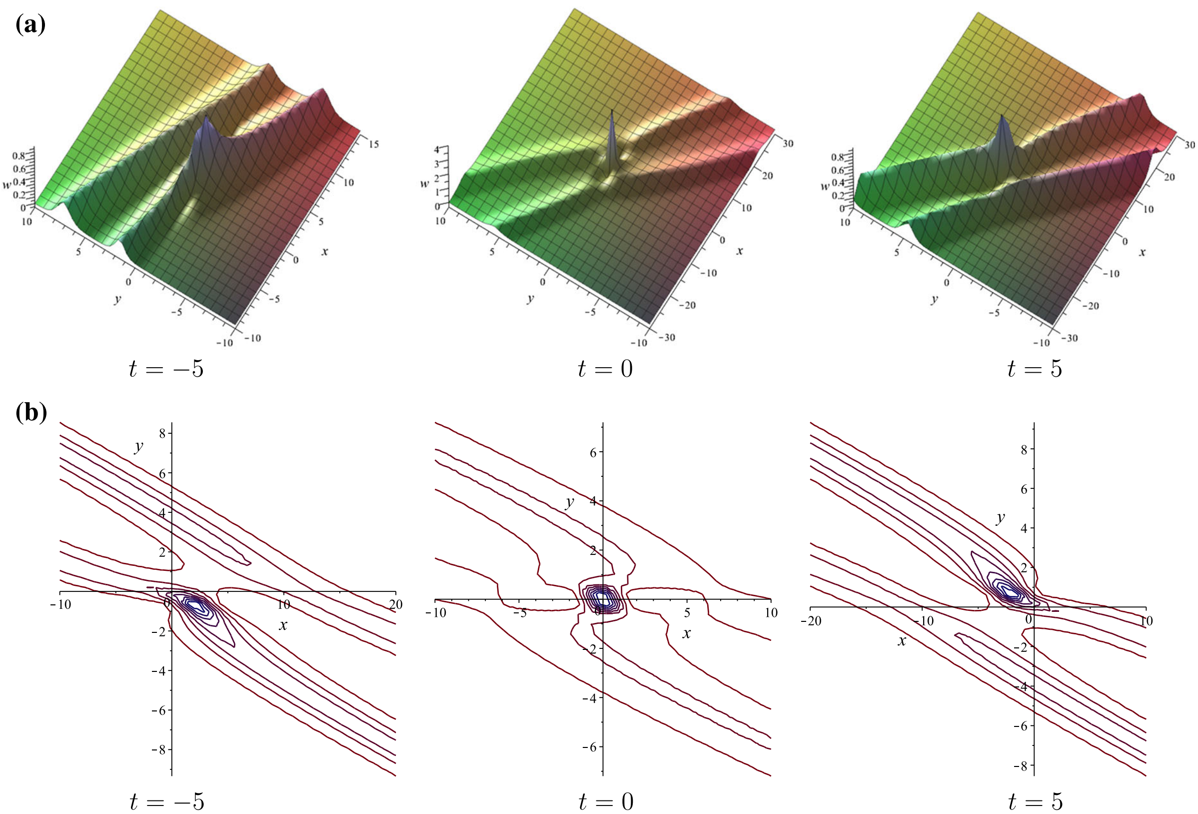

Fig. 1 **a** The three-dimensional plots of w_1 via Eq. (30) at the time $t=-5$, $t=0$ and $t=5$; **b** the corresponding contour plots of (**a**)

$$u=\frac{4a_1(a_1x+a_4)+2k\,k_1\sinh\left(k_1x-\frac{2}{k_1}y+\frac{3\,(a_1^2+a_7^2)k_1^4-2a_7^2}{3\,k_1(a_1^2-a_7^2)}t\right)}{(a_1x+a_4)^2+\left(-\frac{3(a_1^2+a_7^2)}{a_7}y+a_7t+a_8\right)^2+k\cosh\left(k_1x-\frac{2}{k_1}y+\frac{3\,(a_1^2+a_7^2)k_1^4-2a_7^2}{3\,k_1(a_1^2-a_7^2)}t\right)+\frac{k^2k_1^2}{2a_1^2}},\tag{11}$$

where k_1 can be found in Eq. (9).

We will choose two special sets of the parameters corresponding to Eqs. (8) and (9) to get two specific interaction solutions to the dimensionally reduced Eq. (5), respectively, as u_1 and u_2, which lead to another forms of the associated interaction solutions w_1 and w_2 with introducing the transformation $w=u_x$. The detailed parameters and formulas are given in "Appendix A."

The three-dimensional plots and contour plots of w_1 and w_2 are shown in Figs. 1 and 2, respectively.

3 Lump–soliton interaction for the reduction with $z=x$

The dimensionally reduced form of Eq. (2) with $z=x$ turns out to be

$$u_{yt}-u_{xxxy}-3\,(u_xu_y)_x=0,\tag{12}$$

the bilinear representation of which reads

$$\begin{aligned}(D_tD_y-D_x^3D_y)f\cdot f\\ &=2\left[\left(ff_{ty}-f_tf_y+f_{xxx}f_y\right.\right.\\ &\quad\left.\left.+3\,f_{xxy}f_x-3\,f_{xx}f_{xy}-ff_{xxxy}\right)\right]=0,\end{aligned}\tag{13}$$

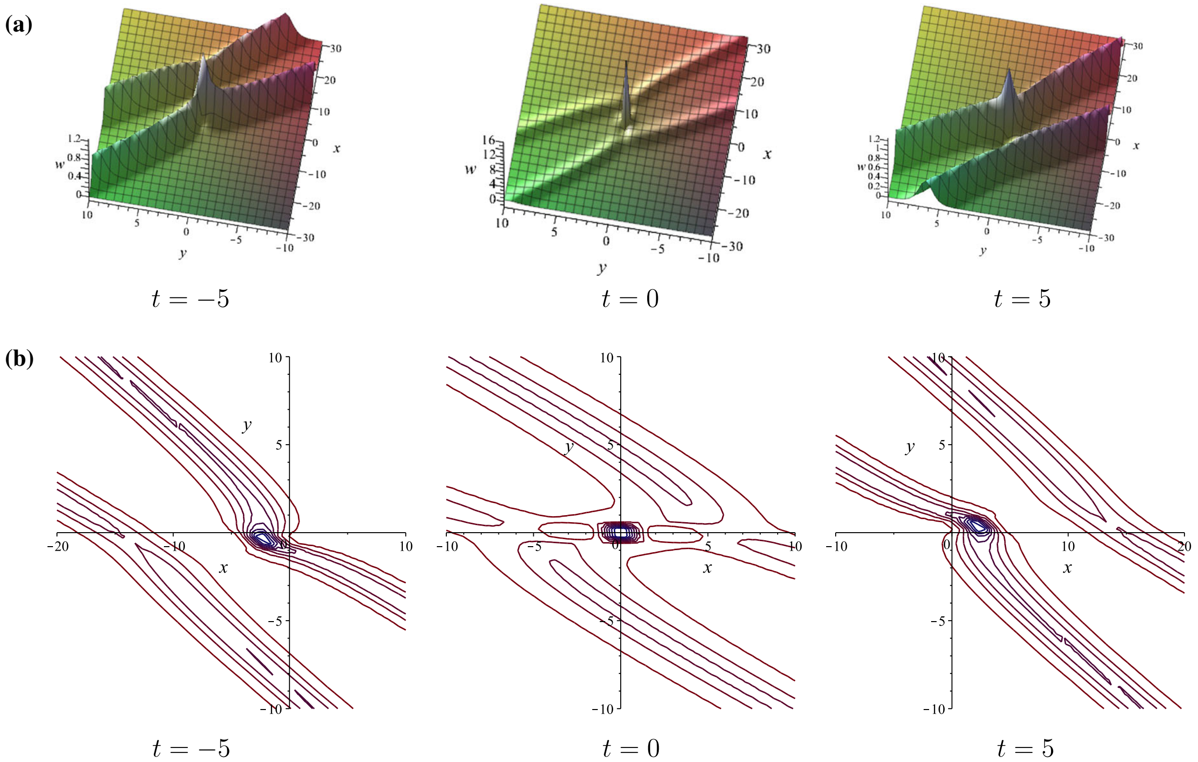

Fig. 2 **a** The three-dimensional plots of w_2 via Eq. (33) at the time $t=-5$, $t=0$ and $t=5$; **b** the corresponding contour plots of (**a**)

through the dependent variable transformation Eq. (7).

Substitution of f in Eq. (4) into Eq. (13) leads to the following two classes of solutions for the parameters:

$$\Big\{a_1=-\frac{a_5a_6}{a_2},\ a_2=a_2,\ a_3=0,\ a_4=a_4,\ a_5=a_5,\ a_6=a_6,\ a_7=0,\ a_8=a_8,\ a_9=a_9,\ k=k,\ k_1=k_1,\ k_2=0,\ k_3=k_1^3\Big\}, \tag{14}$$

where $k>0$, $a_9>0$, and $a_5(a_6^2+a_2^2)\neq 0$; and

$$\Big\{a_1=a_1,\ a_2=0,\ a_3=0,\ a_4=a_4,\ a_5=0,\ a_6=a_6,\ a_7=0,\ a_8=a_8,\ a_9=a_9,\ k=k,\ k_1=k_1,\ k_2=0,\ k_3=k_1^3\Big\}, \tag{15}$$

where $k>0$, $a_9>0$, and $a_1a_6\neq 0$. Parameters given in both Eqs. (14) and (15) can lead to lump–soliton interaction solutions for the reduction with $z=x$, which can be shown in an explicit form, respectively, as

$$u=\frac{4\left(-\frac{a_5a_6}{a_2}\right)\left(-\frac{a_5a_6}{a_2}x+a_2y+a_4\right)+4a_5\ (a_5x+a_6y+a_8)+2kk_1\sinh\left(k_1x+k_1^3t\right)}{\left(-\frac{a_5a_6}{a_2}x+a_2y+a_4\right)^2+(a_5x+a_6y+a_8)^2+k\cosh\left(k_1x+k_1^3t\right)+a_9}, \tag{16}$$

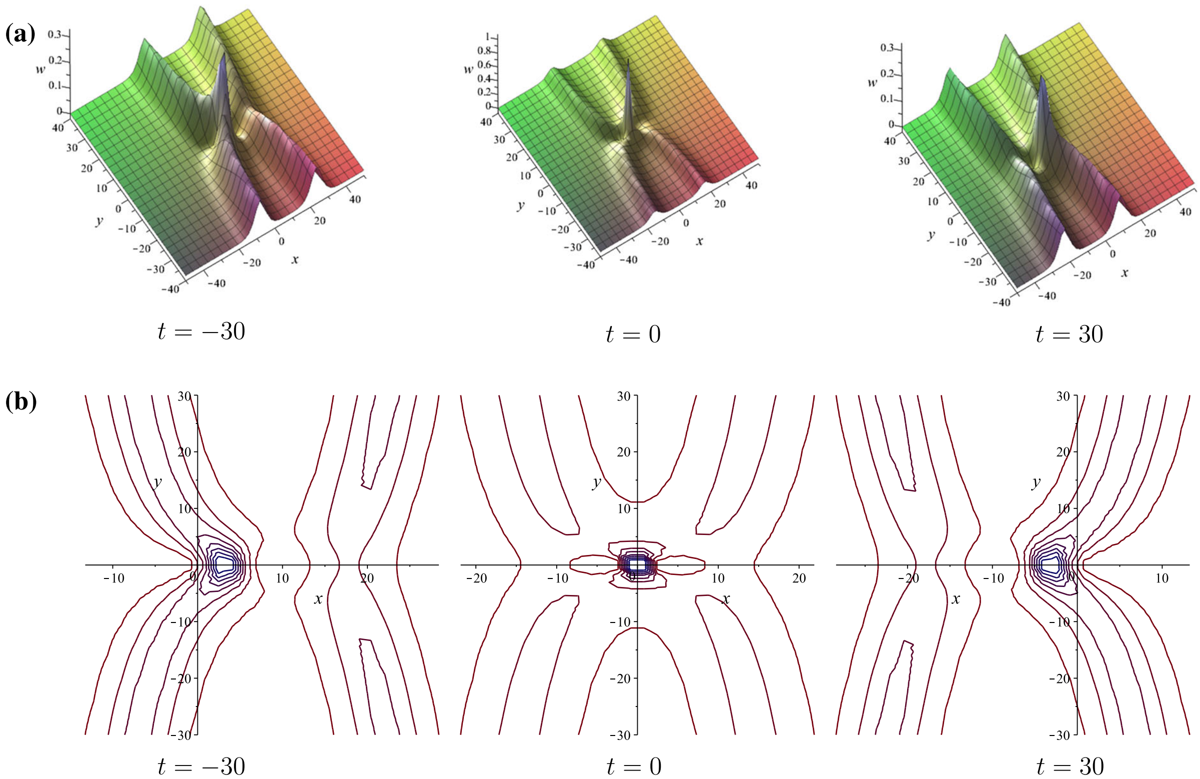

Fig. 3 **a** The three-dimensional plots of w_3 via Eq. (38) at the time $t=-30$, $t=0$ and $t=30$; **b** the corresponding contour plots of (**a**)

and

$$u=\frac{4a_1(a_1x+a_4)+2kk_1\sinh\left(k_1x+k_1^3t\right)}{(a_1x+a_4)^2+(a_6y+a_8)^2+k\cosh\left(k_1x+k_1^3t\right)+a_9}. \tag{17}$$

With two special sets of the parameters, we can obtain lump–soliton interaction solutions u_3 and u_4 with w_3 and w_4. The details can be seen in "Appendix B." Three three-dimensional plots and contour plots of solutions w_3 and w_4 are shown in Figs. 3 and 4, respectively.

Furthermore, we can obtain some parameters for exact solutions of other type, for example,

$$\Big\{a_1=0,\ a_2=a_2,\ a_3=0,\ a_4=a_4,\ a_5=a_5,\ a_6=0,\ a_7=0,\ a_8=a_8,\ a_9=a_9,\ k=k,\ k_1=0,\ k_2=k_2,\ k_3=0\Big\}, \tag{18}$$

where $k>0$, $a_9>0$; and

$$\Big\{a_1=a_1,\ a_2=0,\ a_3=a_3,\ a_4=a_4,\ a_5=a_5,\ a_6=0,\ a_7=a_7,\ a_8=a_8,\ a_9=a_9,\ k=k,\ k_1=k_1,\ k_2=0,\ k_3=k_3\Big\}, \tag{19}$$

where $k>0$, $a_9>0$, and $a_1a_7-a_3a_5\neq 0$; and

$$\Big\{a_1=a_1,\ a_2=-\frac{a_5a_6}{a_1},\ a_3=0, a_4=a_4,\ a_5=a_5,\ a_6=a_6,\ a_7=0,\ a_8=a_8,\ a_9=a_9,\ k=k,\ k_1=0,\ k_2=k_2,\ k_3=0\Big\}, \tag{20}$$

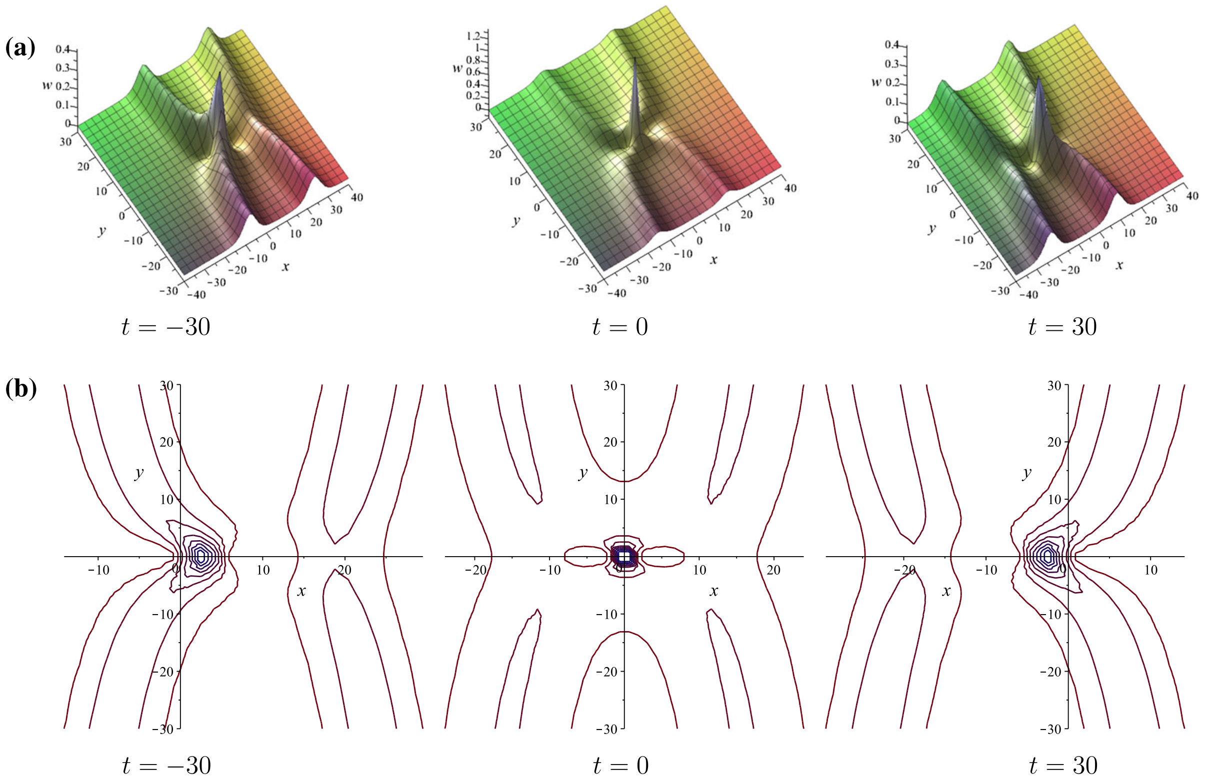

Fig. 4 **a** The three-dimensional plots of w_4 via Eq. (39) at the time $t = -30$, $t = 0$ and $t = 30$; **b** the corresponding contour plots of (**a**)

where $k > 0$ and $a_9 > 0$.

Parameters given in Eqs. (18), (19) and (20) give rise to some other explicit solutions for the reduction with $z = x$, respectively, as

$$u = \frac{4a_5(a_5x + a_8)}{(a_2y + a_4)^2 + (a_5x + a_8)^2 + k\cosh(k_2y) + a_9}, \tag{21}$$

$$u = \frac{4a_1(a_1x + a_3t + a_4) + 4a_5\,(a_5x + a_7t + a_8) + 2kk_1\sinh\Big(k_1x + k_3t\Big)}{(a_1x + a_3t + a_4)^2 + (a_5x + a_7t + a_8)^2 + k\cosh\Big(k_1x + k_3t\Big) + a_9}, \tag{22}$$

and

$$u = \frac{4a_1\left(a_1x - \frac{a_5a_6}{a_1}y + a_4\right) + 4a_5\,(a_5x + a_6y + a_8)}{(a_1x - \frac{a_5a_6}{a_1}y + a_4)^2 + (a_5x + a_6y + a_8)^2 + k\cosh(k_2y) + a_9}. \tag{23}$$

We will choose three special sets of the parameters corresponding to Eqs. (18), (19) and (20) to get three other type specific exact solutions to the dimensionally reduced Eq. (5), respectively, as u_5, u_6 and u_7 with w_5, w_6 and w_7 by introducing the transformation $w = u_x$. The detailed parameters and formulas are given in "Appendix C." The three-dimensional plots and contour plots of solutions w_5, w_6 and w_7 are shown in Figs. 5, 6 and 7, respectively.

4 Discussions

Summarily, we have derived seven classes of exact solutions to the dimensionally reduced Eqs. (5) and (12),

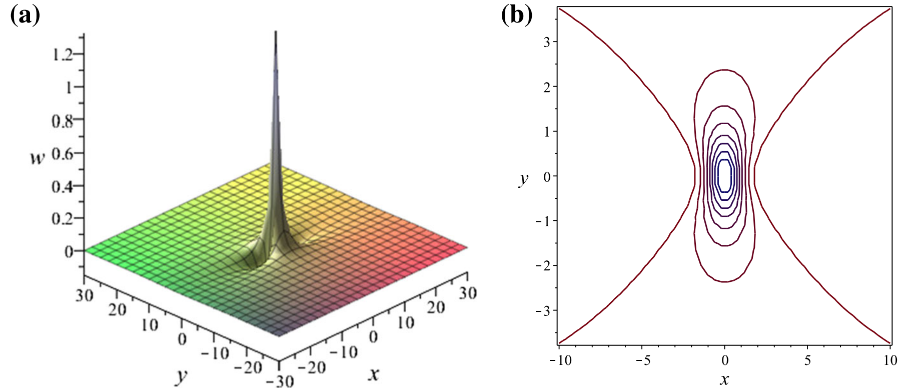

Fig. 5 **a** The three-dimensional plot of w_5 via Eq. (44); **b** the corresponding contour plot of (**a**)

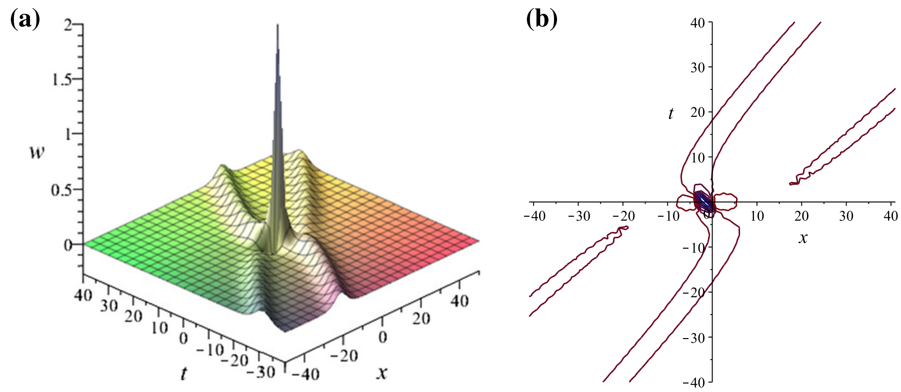

Fig. 6 **a** The three-dimensional plot of w_6 via Eq. (46); **b** the corresponding contour plot of (**a**)

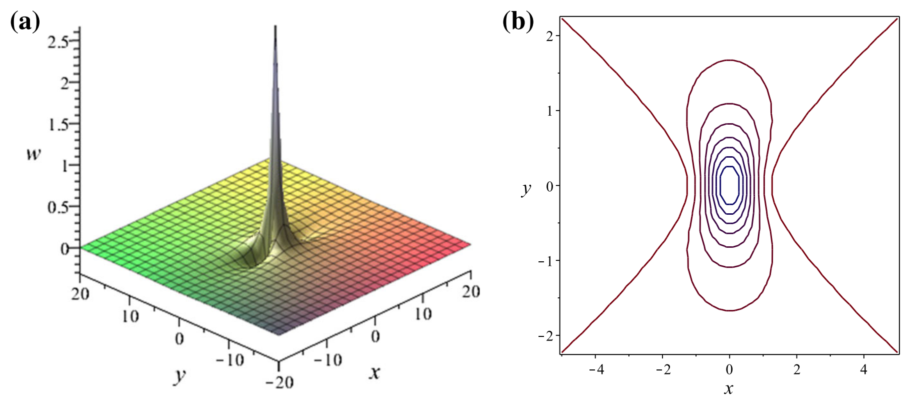

Fig. 7 **a** The three-dimensional plot of w_7 via Eq. (48); **b** the corresponding contour plot of (**a**)

among which u_1 (w_1), u_2 (w_2), u_3 (w_3) and u_4 (w_4) are lump–soliton interaction solutions. We find the rogue waves appear during the collision between the twins of solitons and the lump. The collision properties of the lump and soliton can be analyzed and simulated (see Figs. 1, 2, 3 and 4). Based on Eq. (4), we get all critical points of the function f, at a fixed time t, as

$$x(t) = \frac{(a_2a_7 - a_3a_6)t + (a_2a_8 - a_4a_6)}{a_1a_6 - a_2a_5}, \; y(t) = -\frac{(a_1a_7 - a_3a_5)t + (a_1a_8 - a_4a_5)}{a_1a_6 - a_2a_5}, \tag{24}$$

which is the global extreme value point and describes the moving path of the invisible lump. Further, we can get the centerline of the twins of solitons as

$$k_1x + k_2y + k_3t = 0. \tag{25}$$

Therefore, the rogue wave will appear at the time

$$t=\frac{(a_4a_6-a_2a_8)k_1+(a_1a_8-a_4a_5)k_2}{(a_2a_7-a_3a_6)k_1+(a_3a_5-a_1a_7)k_2+(a_1a_6-a_2a_5)k_3}, \tag{26}$$

and the place

$$\left(\frac{(a_4a_7-a_3a_8)k_2+(a_2a_8-a_4a_6)k_3}{(a_2a_7-a_3a_6)k_1+(a_3a_5-a_1a_7)k_2+(a_1a_6-a_2a_5)k_3},\right.$$
$$\left.-\frac{(a_4a_7-a_3a_8)k_1+(a_1a_8-a_4a_5)k_3}{(a_2a_7-a_3a_6)k_1+(a_3a_5-a_1a_7)k_2+(a_1a_6-a_2a_5)k_3}\right), \tag{27}$$

and the maximum amplitude of the rogue wave can be obtained by calculating the value of u when the lump arrive at the center of the twin soliton.

Attention should be paid to solutions u_5 (w_5), u_6 (w_6) and u_7 (w_7), which are special cases of lump–soliton interaction solutions. As a result of the vanishment of the variable t in the solutions u_5 (w_5) and u_7 (w_7), they look like lump-type structures (see Figs. 5 and 7). However, the variable y vanishes in the solution u_6 (w_6), and the rogue wave appears (Its amplitude is giant compared with background waves, see Fig. 6). This case is similar to the rogue waves in nonintegrable KdV-type systems described in Ref. [32].

5 Concluding remarks

It is generally defined the algebraic solution localized in space as a lump solution, and the two-dimensional lump solutions are well studied in the literature. Currently, attention has been attracted on interaction solutions for two-dimensional NLEEs. Based on symbolic computation and bilinear method, we have investigated lump–soliton interaction phenomenon for (3+1)-dimensional NLEEs with dimensional reduction. The models are Eqs. (5) and (12), and the solutions in terms of u are given in Eqs. (29), (32), (36), (37), (43), (45) and (47), and of w are given in Eqs. (30), (33), (38), (39), (44), (46) and (48).

The collision between lump and soliton has been analyzed and simulated. When the lump is induced by a bounded twin soliton, the rogue wave turns up (see Figs. 1, 2, 3 and 4), which can only be visible at an instant time. Based on the solutions, it is easy to find the amplitude, the place and the arrival time of the rogue waves.

We hope our work shed some light on the investigation of rogue waves in oceanography, fluid dynamics and nonlinear optics.

Acknowledgements This work is supported by the National Key R&D Program of China (2017YFC0820700), the National Natural Science Foundation of China under Grant No. 61602034, the Fundamental Research Funds for the Central Universities of China, the Foundation of Beijing Engineering and Technology Research Center for Convergence Networks and Ubiquitous Services and the Open Fund of IPOC (BUPT) under Grant No. IPOC2016B008.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest concerning the publication of this manuscript.

Appendix A

Corresponding to Eqs. (8), we choose the parameters

$$\Big\{a_1=1,\ a_2=0,\ a_3=0,\ a_4=0,\ a_5=0,$$
$$a_6=-6,\ a_7=1,\ a_8=0,\ a_9=\frac{\sqrt{3}}{6},\ \varepsilon=1,$$
$$k=1,\ k_1=3^{-\frac{1}{4}},\ k_2=-2\cdot 3^{\frac{1}{4}},\ k_3=(\sqrt{2}-1)3^{\frac{1}{4}}\Big\}, \tag{28}$$

to get the interaction solutions u_1 with w_1 as

$$u_1=\frac{4x+2\cdot 3^{-\frac{1}{4}}\sinh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)}{x^2+(-6y+t)^2+\cosh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)+\frac{\sqrt{3}}{6}}, \tag{29}$$

and

$$w_1 = \frac{2\left[2+3^{-\frac{1}{2}}\cosh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)\right]}{x^2+(-6y+t)^2+\cosh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)+\frac{\sqrt{3}}{6}} - \frac{2\left[2x+3^{-\frac{1}{4}}\sinh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)\right]^2}{\left[x^2+(-6y+t)^2+\cosh\left(3^{-\frac{1}{4}}x-2\cdot 3^{\frac{1}{4}}y+3^{\frac{1}{4}}\left(\sqrt{2}-1\right)t\right)+\frac{\sqrt{3}}{6}\right]^2}. \quad (30)$$

Corresponding to Eqs. (9), we choose the parameters

$$\Big\{a_1=2,\ a_2=0,\ a_3=0,\ a_4=0,\ a_5=0,\ a_6=-15,\ a_7=1,\ a_8=0,\ a_9=\frac{1}{8},\ \varepsilon=1,\ k=1,\ k_1=1,\ k_2=-2,\ k_3=\frac{13}{9}\Big\}, \quad (31)$$

to get the interaction solutions u_2 with w_2 as

$$u_2 = \frac{16x+2\sinh(x-2y+\frac{13}{9}t)}{4x^2+(-15y+t)^2+\cosh(x-2y+\frac{13}{9}t)+\frac{1}{8}}, \quad (32)$$

and

$$w_2 = \frac{2\left[8+\cosh\left(x-2y+\frac{13}{9}t\right)\right]}{4x^2+(-15y+t)^2+\cosh\left(x-2y+\frac{13}{9}t\right)+\frac{1}{8}} - \frac{2\left[8x+\sinh\left(x-2y+\frac{13}{9}t\right)\right]^2}{\left[4x^2+(-15y+t)^2+\cosh\left(x-2y+\frac{13}{9}t\right)+\frac{1}{8}\right]^2}. \quad (33)$$

Appendix B

Corresponding to the parameters given in Eqs. (14) and (15), we choose the following two special sets of the parameters:

$$\Big\{a_1=-1,\ a_2=2,\ a_3=0,\ a_4=0,\ a_5=1,\ a_6=2,\ a_7=0,\ a_8=0,\ a_9=2,\ k=10,\ k_1=-\frac{1}{2},\ k_2=0,\ k_3=-\frac{1}{8}\Big\}, \quad (34)$$

and

$$\Big\{a_1=1,\ a_2=0,\ a_3=0,\ a_4=0,\ a_5=0,\ a_6=2,\ a_7=0,\ a_8=0,\ a_9=1,\ k=3,\ k_1=-\frac{1}{2},\ k_2=0,\ k_3=-\frac{1}{8}\Big\}, \quad (35)$$

which generate two interaction solutions to the dimensionally reduced Eq. (12), respectively, as

$$u_3 = \frac{8x-10\sinh\left(-\frac{1}{2}x-\frac{1}{8}t\right)}{(-x+2y)^2+(x+2y)^2+10\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+2}, \quad (36)$$

and

$$u_4 = \frac{4x-3\sinh\left(-\frac{1}{2}x-\frac{1}{8}t\right)}{x^2+4y^2+3\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+1}, \quad (37)$$

Associated with Eqs. (36) and (37), the derivative form of the interaction solutions can be obtained with $w = u_x$, respectively, as

$$w_3 = \frac{8+5\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)}{(-x+2y)^2+(x+2y)^2+10\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+2} - \frac{2\left[4x-5\sinh\left(-\frac{1}{2}x-\frac{1}{8}t\right)\right]^2}{\left[(-x+2y)^2+(x+2y)^2+10\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+2\right]^2}, \quad (38)$$

and

$$w_4 = \frac{4+\frac{3}{2}\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)}{x^2+4y^2+3\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+1} - \frac{2\left[2x-\frac{3}{2}\sinh\left(-\frac{1}{2}x-\frac{1}{8}t\right)\right]^2}{\left[x^2+4y^2+3\cosh\left(-\frac{1}{2}x-\frac{1}{8}t\right)+1\right]^2}. \quad (39)$$

Appendix C

Corresponding to the parameters given in Eqs. (18), (19) and (20), we choose the following three special sets of the parameters:

$$\Big\{a_1 = 0,\ a_2 = 2,\ a_3 = 0,\ a_4 = 0,\ a_5 = 1,\ a_6 = 0,\ a_7 = 0,\ a_8 = 0,\ a_9 = 1,\ k = 2,\ k_1 = 0,\ k_2 = \frac{1}{2},\ k_3 = 0\Big\}, \tag{40}$$

$$\Big\{a_1 = 1,\ a_2 = 0,\ a_3 = 2,\ a_4 = 2,\ a_5 = 1,\ a_6 = 0,\ a_7 = -1,\ a_8 = 2,\ a_9 = 1,\ k = 2,\ k_1 = -\frac{1}{2},\ k_2 = 0,\ k_3 = \frac{1}{2}\Big\}, \tag{41}$$

and

$$\Big\{a_1 = 1,\ a_2 = -2,\ a_3 = 0,\ a_4 = 0,\ a_5 = 1,\ a_6 = 2,\ a_7 = 0,\ a_8 = 0,\ a_9 = 1,\ k = 2,\ k_1 = 0,\ k_2 = \frac{1}{2},\ k_3 = 0\Big\}, \tag{42}$$

to derive the exact solutions in the form of u and w, respectively, as

$$u_5 = \frac{4x}{4y^2 + x^2 + 2\cosh\left(\frac{1}{2}y\right) + 1}, \tag{43}$$

$$w_5 = \frac{4}{4y^2 + x^2 + 2\cosh\left(\frac{1}{2}y\right) + 1} - \frac{8x^2}{[4y^2 + x^2 + 2\cosh\left(\frac{1}{2}y\right) + 1]^2}, \tag{44}$$

$$u_6 = \frac{8x + 4t - 2\sinh\left(-\frac{1}{2}x + \frac{1}{2}t\right) + 16}{(x + 2t + 2)^2 + (x - t + 2)^2 + 2\cosh\left(-\frac{1}{2}x + \frac{1}{2}t\right) + 1}, \tag{45}$$

$$w_6 = \frac{8 + \cosh\left(-\frac{1}{2}x + \frac{1}{2}t\right)}{(x + 2t + 2)^2 + (x - t + 2)^2 + 2\cosh\left(-\frac{1}{2}x + \frac{1}{2}t\right) + 1} - \frac{2\left[4x + 2t - \sinh\left(-\frac{1}{2}x + \frac{1}{2}t\right) + 8\right]^2}{\left[(x + 2t + 2)^2 + (x - t + 2)^2 + 2\cosh\left(-\frac{1}{2}x + \frac{1}{2}t\right) + 1\right]^2}, \tag{46}$$

and

$$u_7 = \frac{8x}{(x - 2y)^2 + (x + 2y)^2 + 2\cosh\left(\frac{1}{2}y\right) + 1}, \tag{47}$$

$$w_7 = \frac{8}{(x - 2y)^2 + (x + 2y)^2 + 2\cosh\left(\frac{1}{2}y\right) + 1} - \frac{32x^2}{\left[(x - 2y)^2 + (x + 2y)^2 + 2\cosh\left(\frac{1}{2}y\right) + 1\right]^2}. \tag{48}$$

References

1. Wazwaz, A.-M., El-Tantawy, S.A.: New (3 + 1)-dimensional equations of Burgers type and Sharma–Tasso–Olver type: multiple-soliton solutions. Nonlinear Dyn. **87**, 2457 (2017)
2. Manakov, S.V., Zakharov, V.E., Bordag, L.A., Matveev, V.B.: Two-dimensional solitons of the Kadomtsev–Petviashvili equation and their interaction. Phys. Lett. A **63**, 205 (1977)
3. Zabusky, N.J., Kruskal, M.D.: Interaction of solitons in a collisionless plasma and the recurrence of initial states. Phys. Rev. Lett. **15**, 240 (2008)
4. Kadomtsev, B.B., Petviashvili, V.I.: On the stability of solitary waves in weakly dispersing media. Sov. Phys. Dokl. **15**, 539 (1970)
5. Ablowitz, M.J., Clarkson, P.A.: Solitons, Nonlinear Evolution Equations and Inverse Scattering. Cambridge University Press, Cambridge (1991)
6. Chow, K.W., Lai, W.C., Shek, C.K., Tso, K.: Positon-like solutions of nonlinear evolution equations in (2 + 1) dimensions. Chaos Solitons Fractals **9**, 1901 (1998)
7. Khatera, A.H., Malfliet, W., Callebautc, D.K., Kamel, E.S.: Travelling wave solutions of some classes of nonlinear evolution equations in (1 + 1) and (2 + 1) dimensions. J. Comput. Appl. Math. **140**, 469 (2002)
8. Lü, X., Ma, W.X., Zhou, Y., Khalique, C.M.: Rational solutions to an extended Kadomtsev–Petviashvili-like equation with symbolic computation. Comput. Math. Appl. **71**, 1560 (2016)
9. Lü, X., Ma, W.X., Chen, S.T., Khalique, C.M.: A note on rational solutions to a Hirota–Satsuma-like equation. Appl. Math. Lett. **58**, 13 (2016)
10. Yin, Y.H., Ma, W.X., Liu, J.G., Lü, X.: Diversity of exact solutions to a (3 + 1)-dimensional nonlinear evolution equation and its reduction. Comput. Math. Appl. (2018). https://doi.org/10.1016/j.camwa.2018.06.020
11. Lü, X., Wang, J.P., Lin, F.H., Zhou, X.W.: Lump dynamics of a generalized two-dimensional Boussinesq equation in shallow water. Nonlinear Dyn. **91**, 1249 (2018)
12. Gao, L.N., Zi, Y.Y., Ma, W.X., Lü, X.: Bäklund transformation, multiple wave solutions and lump solutions to a (3 + 1)-dimensional nonlinear evolution equation. Nonlinear Dyn. **89**, 2233 (2017)

13. Lü, X., Ma, W.X., Yu, J., Khalique, C.M.: Solitary waves with the Madelung fluid description: a generalized derivative nonlinear Schrödinger equation. Commun. Nonlinear Sci. Numer. Simul. **31**, 40 (2016)
14. Wang, D.S., Yin, Y.B.: Symmetry analysis and reductions of the two-dimensional generalized Benney system via geometric approach. Comput. Math. Appl. **71**, 748 (2016)
15. Lü, X., Lin, F.H.: Soliton excitations and shape-changing collisions in alphahelical proteins with interspine coupling at higher order. Commun. Nonlinear Sci. Numer. Simul. **32**, 241 (2016)
16. Lin, F.H., Chen, S.T., Qu, Q.X., Wang, J.P., Zhou, X.W., Lü, X.: Resonant multiple wave solutions to a new (3 + 1)-dimensional generalized Kadomtsev–Petviashvili equation: linear superposition principle. Appl. Math. Lett. **78**, 112 (2018)
17. Lou, S.Y.: Generalized dromion solutions of the (2 + 1)-dimensional KdV equation. J. Phys. A **28**, 7227 (1995)
18. Ma, W.X.: Wronskian solutions to integrable equations. Discrete Contin. Dyn. Syst. Suppl. **2009**, 506 (2009)
19. Ma, W.X.: Lump solutions to the Kadomtsev–Petviashvili equation. Phys. Lett. A **379**, 1975 (2015)
20. Manakov, S.V., Zakharov, V.E., Bordag, L.A., Matveev, V.B.: Two-dimensional solitons of the Kadomtsev–Petviashvili equation and their interaction. Phys. Lett. A **63**, 205 (1977)
21. Johnson, R.S., Thompson, S.: A solution of the inverse scattering problem for the Kadomtsev–Petviashvili equation by the method of separation of variables. Phys. Lett. A **66**, 279 (1978)
22. Satsuma, J., Ablowitz, M.J.: Two-dimensional lumps in nonlinear dispersive systems. J. Math. Phys. **20**, 1496 (1979)
23. Kaup, D.J.: The lump solutions and the Bäcklund transformation for the three-dimensional three-wave resonant interaction. J. Math. Phys. **22**, 1176 (1981)
24. Gilson, C.R., Nimmo, J.J.C.: Lump solutions of the BKP equation. Phys. Lett. A **147**, 472 (1990)
25. Imai, K.: Dromion and lump solutions of the Ishimori-I equation. Prog. Theor. Phys. **98**, 1013 (1997)
26. Fokas, A.S., Pelinovsky, D.E., Sulem, C.: Interaction of lumps with a line soliton for the DSII equation. Physica D **152–153**, 189 (2001)
27. Lü, X., Chen, S.T., Ma, W.X.: Constructing lump solutions to a generalized Kadomtsev–Petviashvili–Boussinesq equation. Nonlinear Dyn. **86**, 523 (2016)
28. Lü, X., Ma, W.X.: Study of lump dynamics based on a dimensionally reduced Hirota bilinear equation. Nonlinear Dyn. **85**, 1217 (2016)
29. Yang, J.Y., Ma, W.X.: Abundant interaction solutions of the KP equation. Nonlinear Dyn. **89**, 1539 (2017)
30. Tang, Y.N., Tao, S.Q., Guan, Q.: Lump solitons and the interaction phenomena of them for two classes of nonlinear evolution equations. Comput. Math. Appl. **72**, 2334 (2016)
31. Jia, M., Lou, S. Y.: A Novel Type of Rogue Waves with Predictability in Nonlinear Physics. arXiv:1710.06604
32. Lou, S.Y., Lin, J.: Rogue waves in nonintegrable KdV-Type systems. Chin. Phys. Lett. **35**, 050202 (2018)
33. Müller, P., Garrett, C., Osborne, A.: Roguewaves. Oceanography **18**, 66 (2005)
34. Solli, D.R., Ropers, C., Koonath, P., Jalali, B.: Optical rogue waves. Nature **450**, 1054 (2007)
35. Yan, Z.Y.: Financial rogue waves. Commun. Theor. Phys. **54**, 947 (2010)
36. Hirota, R.: The Direct Method in Soliton Theory. Cambridge University Press, Cambridge (2004)
37. Gao, L.N., Zhao, X.Y., Zi, Y.Y., Yu, J., Lü, X.: Resonant behavior of multiple wave solutions to a Hirota bilinear equation. Comput. Math. Appl. **72**, 1225 (2016)