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Interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation for nonlinear waves

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ABSTRACT

In this paper, we focus on the interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation. With symbolic computation, two types of interaction solutions including lump-kink and lump-soliton ones are derived through mixing two positive quadratic functions with an exponential function, or two positive quadratic functions with a hyperbolic cosine function in the bilinear equation. The completely non-elastic interaction between a lump and a stripe is presented, which shows the lump is drowned or shallowed by the stripe. The interaction between lump and soliton is also given, where the lump moves from one branch to the other branch of the soliton. These phenomena exhibit the dynamics of nonlinear waves and the solutions are useful for the study on interaction behavior of nonlinear waves in shallow water, plasma, nonlinear optics and Bose–Einstein condensates.

1. Introduction

As is known, solitons possess many special characteristics of nonlinear waves [1] and are widely used to describe the nonlinear phenomena in such fields as shallow water [2,3], plasma [4–7], nonlinear optics [8], and Bose-Einstein condensates [9]. Due to its vital application, the theoretical analysis on soliton solutions to nonlinear evolution equations is of great importance [1,10,11]. Many effective methods have been employed to solve nonlinear evolution equations, for example, the Hirota bilinear method [12], the exp-function method [13,14], the homotopy perturbation method [15], the variational iteration method [16], the Adomian decomposition method [16] and the Galerkin method [17,18]. As a type of rational solution, lump solutions are different from soliton solutions. Lump solutions are localized in all directions in the space. In 2006, lump solutions were studied with the variable separation method [19]. In 2015, lump solutions were constructed to the KP equation via substituting the positive quadratic function to bilinear equation [20]. Moreover, lump or multi-lump solutions

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to the Boussinesq [21], the KPI equation [20,22,23], the BKP equation [24] and the potential-YTSF equation [25] have been obtained.

Recently, the interaction between lump solutions and soliton solutions has attracted more and more attention (see, [26,27] and references therein). Interaction behavior of nonlinear waves appearing in many different systems in nature can be described and illustrated with the interaction solutions [28]. Interaction solutions are valuable in analyzing the nonlinear dynamics of waves in shallow water and can be used for forecasting the appearance of rogue waves [26,28,29]. It generally hold that the rogue waves turn at the interaction location of a lump with a two-soliton wave [28,29].

Lump dynamics has been studied for the following (2+1)-dimensional nonlinear evolution equation [30]

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{yy} = 0, \tag{1}$$

which enjoys the Hirota bilinear form as

$$(D_t D_y - D_x^3 D_y - 3D_x^2 + 3D_y^2) f \cdot f = 0, \tag{2}$$

through the dependent variable transformation $u = 2\big[\ln f(x, y, t)\big]_x$, where the D-operator [12] is defined by

$$D_x^m D_y^n D_t^p (f \cdot g) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^p f(x, y, t) g(x', y', t') \Big|_{x'=x, y'=y, t'=t}.$$

The coefficients of each term in some nonlinear evolution equations reflex different physical meaning or background, such as medium inhomogeneity, different boundary conditions, or different external force [2,3,6,7]. In order to show the influence of the coefficients on the wave interaction and investigate some more general cases, we will focus on the interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation as

$$\begin{aligned} &(D_t D_y + c_1 D_x^3 D_y + c_2 D_y^2) f \cdot f \\ &\quad = 2[f_{yt} f - f_y f_t + c_1 (f_{xxxy} f - 3 f_{xxy} f_x + 3 f_{xy} f_{xx} - f_y f_{xxx}) + c_2 (f_{yy} f - f_y^2)] = 0, \end{aligned} \tag{3}$$

which is linked with the following equation

$$u_{yt} + c_1 \left[u_{xxxy} + 3(2u_x u_y + u_{xy} u) + 3u_{xx} \int_{-\infty}^{x} u_y \mathrm{d}x' \right] + c_2 u_{yy} = 0, \tag{4}$$

through the dependent variable transformation $u = 2\big[\ln f(x, y, t)\big]_{xx}$, where c_1 and c_2 are arbitrary real constants.

With symbolic computation, two types of interaction solutions including lump-kink and lump-soliton ones will be derived to Eq. (4). The outline of this paper is as follows: In Section 2, we will analyze the interaction between a lump and a stripe by considering a mixed solution of two positive quadratic functions with an exponential function. The dynamic behaviors of the interaction solutions will be exhibited. In Section 3, we will discuss the interaction between a lump and a two-soliton by considering a mixed solution of two positive quadratic functions with a hyperbolic cosine function, and generate two cases of interaction solutions. We will display the propagation behaviors with some figures and study the dynamics with limitation analysis of these solutions. The last section is our concluding remarks.

2. Interaction solutions of lump-kink type

In this section, we will focus on computing interaction solutions between lump and stripe to Eq. (4) by making a combination of two positive quadratic functions with an exponential function as

$$f = g^2 + h^2 + ke^l + a_9, \tag{5}$$

where three wave variables are defined by

$$\begin{aligned} g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \\ l &= k_1 x + k_2 y + k_3 t, \end{aligned}$$

while a_i $(1 \le i \le 9)$ and k_j $(1 \le j \le 3)$ are real constants to be determined, and $k > 0$ is a real constant.

Case 1

$$\left\{ a_1 = -\frac{a_5 a_6}{a_2}, a_2 = a_2, a_3 = -a_2 c_2, a_4 = a_4, a_5 = a_5, a_6 = a_6, a_7 = -a_6 c_2, \right.$$

$$\left. a_8 = a_8, a_9 = a_9, k_1 = k_1, k_2 = 0, k_3 = -c_1 k_1^3 \right\},$$

which needs to satisfy the condition

$$a_2 \neq 0, \tag{6}$$

to make the corresponding solution f be well-defined, the condition

$$a_9 > 0, \tag{7}$$

to guarantee the positiveness of f, the condition

$$a_5 \neq 0, \tag{8}$$

to realize the localization of u in all directions in the (x, y)-plane, and the condition

$$k_1 \neq 0, \tag{9}$$

to ensure the interaction solutions be obtained.

Case 2

$$\left\{ a_1 = a_1, a_2 = 0, a_3 = 0, a_4 = a_4, a_5 = 0, a_6 = a_6, a_7 = a_7, \right.$$
$$\left. a_8 = a_8, a_9 = a_9, k_1 = k_1, k_2 = 0, k_3 = -c_1 k_1^3 \right\},$$

which needs to satisfy the conditions

$$a_1 a_6 \neq 0, a_9 > 0, k_1 \neq 0. \tag{10}$$

Case 3

$$\left\{ a_1 = -\frac{a_5 a_6}{a_2}, a_2 = a_2, a_3 = -a_2 c_2, a_4 = a_4, a_5 = a_5, a_6 = a_6, a_7 = -a_6 c_2, \right.$$
$$\left. a_8 = a_8, a_9 = a_9, k_1 = 0, k_2 = k_2, k_3 = -c_2 k_2 \right\},$$

which needs to satisfy the conditions

$$a_2 \neq 0, a_9 > 0, a_5 \neq 0, k_2 \neq 0. \tag{11}$$

Case 4

$$\left\{ a_1 = a_1, a_2 = 0, a_3 = 0, a_4 = a_4, a_5 = 0, a_6 = a_6, a_7 = a_7, \right.$$
$$\left. a_8 = a_8, a_9 = a_9, k_1 = 0, k_2 = k_2, k_3 = \frac{a_7 k_2}{a_6} \right\},$$

which needs to satisfy the conditions

$$a_1 a_6 \neq 0, a_9 > 0, k_2 \neq 0. \tag{12}$$

Substituting the four cases of parameters into the function f, we can get the interaction solutions to Eq. (4).

For any fixed value of $t = t_0$, the extremum points of the lump can be obtained. The extremum point of the lump locates at

$$\left(x = \frac{a_2 a_7 t_0 - a_3 a_6 t_0 + a_2 a_8 - a_4 a_6}{a_1 a_6 - a_2 a_5}, y = -\frac{a_1 a_7 t_0 - a_3 a_5 t_0 + a_1 a_8 - a_4 a_5}{a_1 a_6 - a_2 a_5} \right), \tag{13}$$

where the maximum of the amplitude of the lump is attained as $\frac{4(a_1^2 + a_5^2)}{a_9}$.

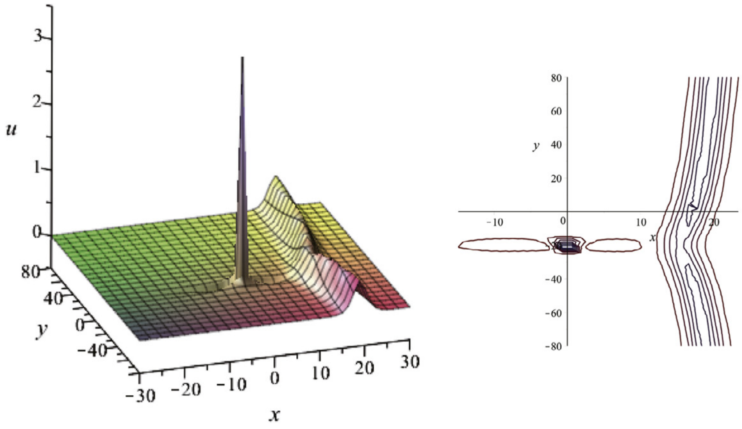
Substituting suitable values of a_i $(1 \leq i \leq 9)$, k, k_j $(1 \leq j \leq 3)$ and c_p $(p = 1, 2)$ into the resulting solutions, we get various of exact interaction solutions to Eq. (4).

The parameters are arbitrary but need to satisfy the corresponding conditions, that is, Constraints (6)–(12). To exhibit the interaction process clearly, we have to choose suitable parameters for the simulation because the parameters determine the location and height of the waves. After trying many times, we take $a_4 = a_8 = 0$ and choose $a_1 = 2, a_2 = -15, a_3 = 3, a_4 = 0, a_5 = -10, a_6 = -3, a_7 = \frac{3}{5}, a_8 = 0, a_9 = 120, k = 120, k_1 = 1, k_2 = 0, k_3 = \frac{1}{10}, c_1 = -\frac{1}{10}, c_2 = \frac{1}{5}$ in Case 1, and $a_1 = 2, a_2 = 0, a_3 = 0, a_4 = 0, a_5 = 0, a_6 = 5, a_7 = \frac{40}{9}, a_8 = 0, a_9 = 10, k = 120, k_1 = \frac{1}{2}, k_2 = 0, k_3 = \frac{1}{8}, c_1 = -1, c_2 = -\frac{8}{9}$ in Case 2 to simulate the interaction.

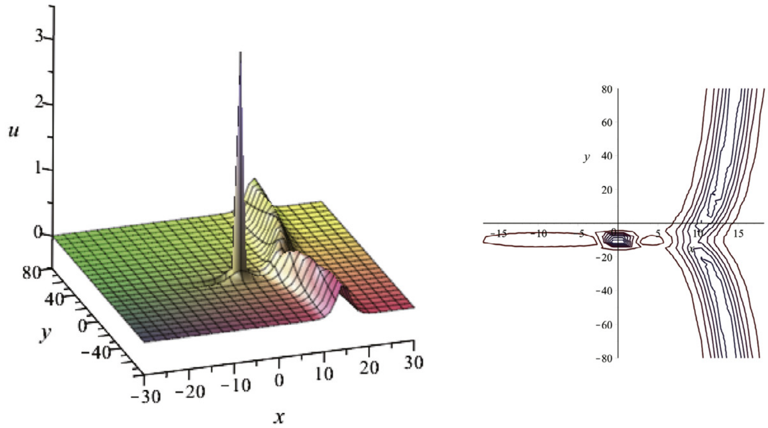
As an example, Figs. 1 and 2 show, respectively, the interaction phenomena between a lump and a stripe with the parameters given above.

According to the expressions of the functions f, g, h and l, the asymptotic property of the lump and stripe waves can be analyzed. When $t \to -\infty$ and $k_3 > 0$, we have

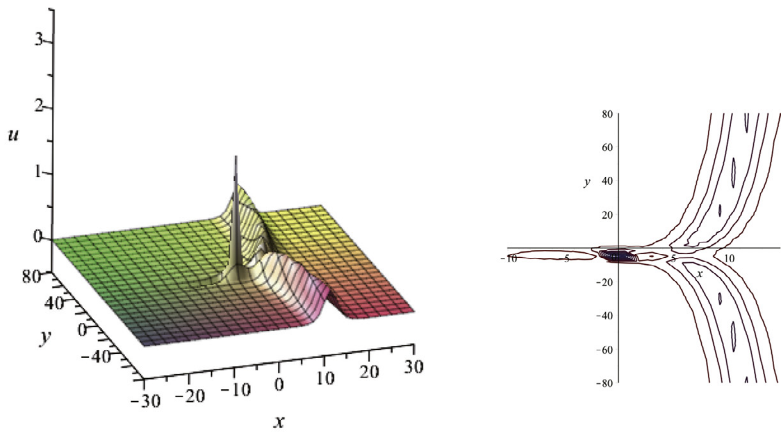
$$\lim_{t \to -\infty} f = \lim_{t \to -\infty} (g^2 + h^2 + ke^l + a_9) = g^2 + h^2 + a_9,$$



(*a*) t=-100

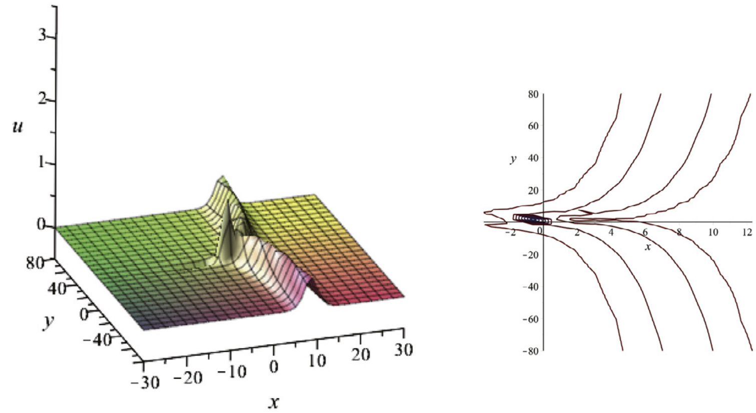


(*b*) t=-50

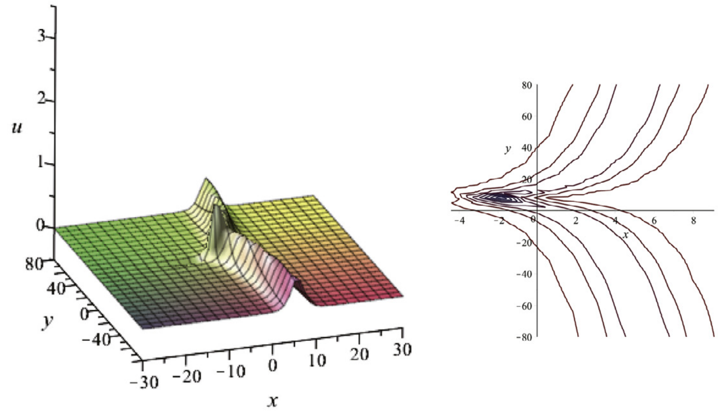


(*c*) t=-20

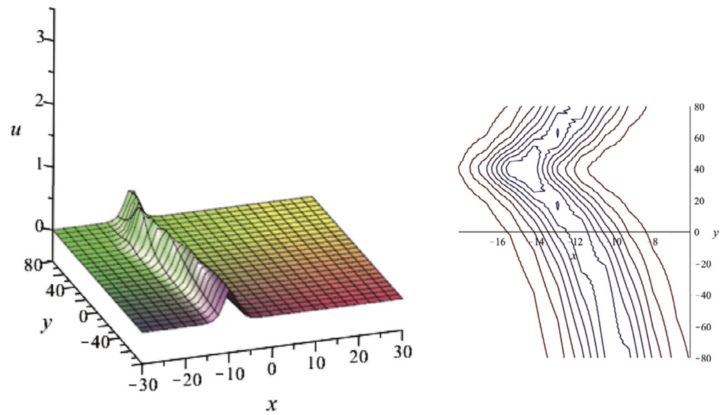
Fig. 1. Interaction behavior between a lump and a stripe in Case 1: 3d plots (left) and contour plots (right).



(*d*) t=10

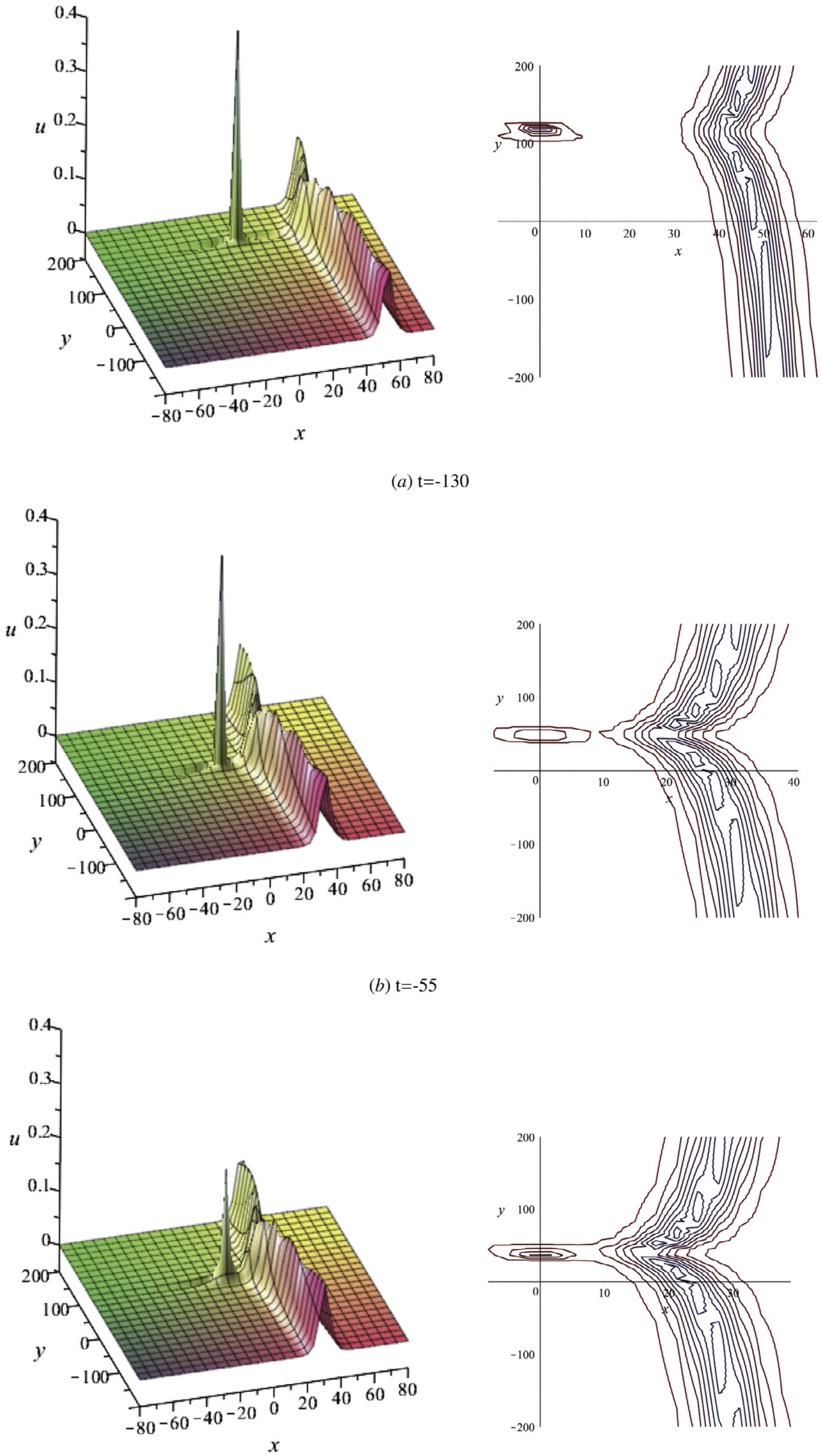


(*e*) t=40



(*f*) t=200

Fig. 1. Continued

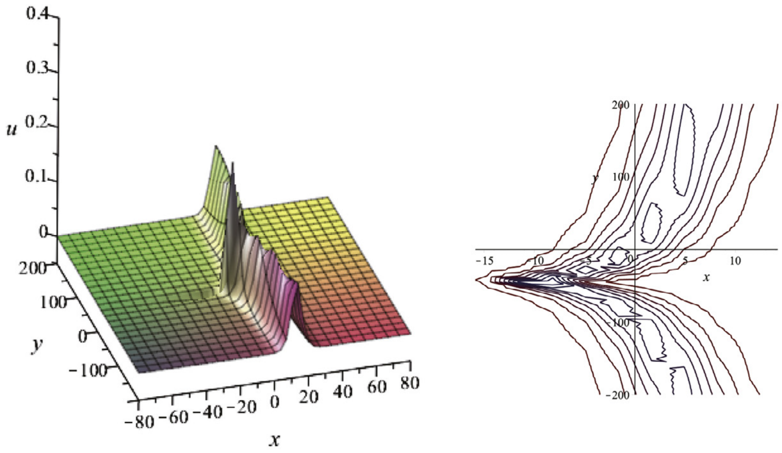


(*a*) t=-130

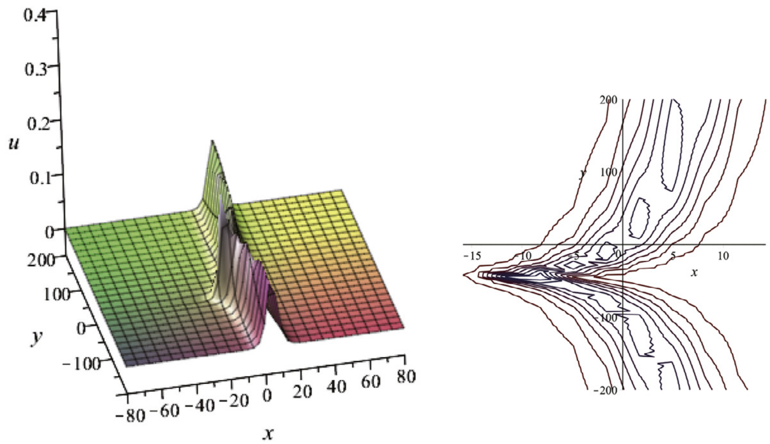
(*b*) t=-55

(*c*) t=-45

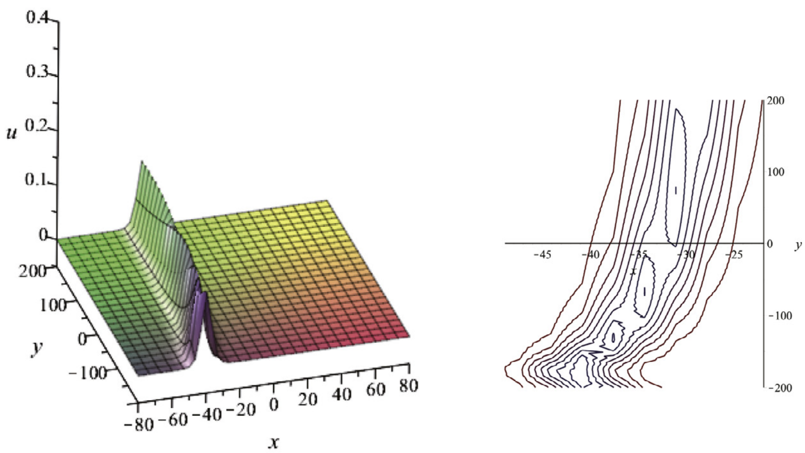
Fig. 2. Interaction behavior between a lump and a stripe in Case 2: 3d plots (left) and contour plots (right).



(*d*) t=20



(*e*) t=50



(*f*) t=200

Fig. 2. Continued

which means only lump wave exists and the stripe wave disappears. When $t \to -\infty$ and $k_3 = 0$, both lump and the stripe waves exist, but the lump wave plays the dominant role. When $t \to -\infty$ and $k_3 < 0$,

$$\lim_{t\to-\infty} ke^l = +\infty, \ \lim_{t\to-\infty} g^2 = +\infty, \ \lim_{t\to-\infty} h^2 = +\infty,$$

we need to compare g^2, h^2 and ke^l. It is proved that

$$\lim_{t\to\pm\infty} \frac{g^2}{h^2} = \frac{a_3^2}{a_7^2} = constant, \tag{14}$$

so g^2 and h^2 have the same order, we just compare g^2 and ke^l. Hereby, both lump and the stripe waves exist, but the stripe wave plays the dominant role in terms of the following result

$$\lim_{t\to-\infty} \frac{g^2}{ke^l} = 0.$$

Similar analysis can be given correspondingly to the case of $t \to +\infty$. We conclude that when $t \to -\infty$ and $k_3 > 0$, as well as $t \to +\infty$ and $k_3 < 0$, only lump wave exists, while when $t \to \pm\infty$ and $k_3 = 0$, both lump wave and stripe wave exist, but the lump wave plays the dominant role. When $t \to -\infty$ and $k_3 < 0$, as well as $t \to +\infty$ and $k_3 > 0$, both waves exist, but the lump is swallowed or drowned by the stripe.

It is also clear that

$$\lim_{t\to\pm\infty} u = \lim_{t\to\pm\infty} 2(\ln f)_{xx} = \lim_{t\to\pm\infty} \left[\frac{2(2a_1^2 + 2a_5^2 + kk_1^2 e^l)}{g^2 + h^2 + ke^l + a_9} - \frac{2(2a_1 g + 2a_5 h + kk_1 e^l)^2}{(g^2 + h^2 + ke^l + a_9)^2} \right] = 0,$$

where $k > 0$ is an arbitrary constant.

When choosing the parameters given in Case 1, the solution of u can be rewritten as

$$\begin{aligned} u = {} & \frac{2(120e^{x+\frac{1}{10}t} + 208)}{(2x - 15y + 3t)^2 + (-10x - 3y + \frac{3}{5}t)^2 + 120e^{x+\frac{1}{10}t} + 120} \\ & - \frac{2(120e^{x+\frac{1}{10}t} + 208x)^2}{\left((2x - 15y + 3t)^2 + (-10x - 3y + \frac{3}{5}t)^2 + 120e^{x+\frac{1}{10}t} + 120\right)^2}, \end{aligned}$$

which shows the stripe wave has the speed $v_s = -\frac{1}{10}$ along the x-axis and the speed along the y-axis is zero, while the lump wave has the speed $v_l = \frac{1}{5}$ along the y-axis and the speed along the x-axis is zero. Because the stripe wave is exponentially localized in certain direction and its speed along the x-axis is faster than the lump wave, as the lump moves from the negative direction of y-axis to the positive direction, the stripe will finally catch up with the lump and interact with it. After the collision, the lump is swallowed or drowned by the stripe and the waves have a common speed.

As can be seen in both Figs. 1 and 2, when $t = -100$ in Fig. 1(a) and when $t = -130$ in Fig. 2(a), the wave is consist of two separate parts: the lump wave and the stripe wave. Then the lump moves closely to the stripe and begins to interact with the stripe. The two waves collide over a period and the amplitudes, shapes and velocities of both waves change. When $t = 200$ in Figs. 1(f) and 2(f), the lump is swallowed or drowned by the stripe and the amplitude of the stripe turns higher than its original one, which presents the completely non-elastic interaction between the two different waves. This kind of interaction solution can be used in the fields of shallow water waves, plasma, nonlinear optics, Bose–Einstein condensates and so on [26,31].

3. Interaction solutions of lump-soliton type

In this section, we will pay attention to the interaction solutions between lump and soliton to Eq. (4) by making a combinations of two positive quadratic functions and a hyperbolic cosine function. We suppose f is in the form of

$$f = g^2 + h^2 + \cosh(l) + a_9, \tag{15}$$

and three wave variables are defined by

$$\begin{aligned} g &= a_1 x + a_2 y + a_3 t + a_4, \\ h &= a_5 x + a_6 y + a_7 t + a_8, \\ l &= k_1 x + k_2 y + k_3 t, \end{aligned}$$

where a_i $(1 \le i \le 9)$ and k_j $(1 \le j \le 3)$ are real constants to be determined.

With symbolic computation, we obtain two cases of parameters:

Case 1

$$\left\{a_1=-\frac{a_5a_6}{a_2}, a_2=a_2, a_3=-a_2c_2, a_4=a_4, a_5=a_5, a_6=a_6, a_7=-a_6c_2,\right.$$

$$\left.a_8=a_8, a_9=a_9, k_1=k_1, k_2=0, k_3=-c_1k_1^3\right\},$$

which needs to satisfy the conditions

$$a_2\neq 0, a_9>0, a_5\neq 0, k_1\neq 0. \tag{16}$$

Case 2

$$\left\{a_1=a_1, a_2=0, a_3=0, a_4=a_4, a_5=0, a_6=a_6, a_7=a_7,\right.$$

$$\left.a_8=a_8, a_9=a_9, k_1=k_1, k_2=0, k_3=-c_1k_1^3\right\},$$

which needs to satisfy the conditions

$$a_1a_6\neq 0, a_9>0, k_1\neq 0. \tag{17}$$

Substituting the two cases of parameters into the function f, we can get the interaction solutions to Eq. (4) through the transformation $u=2[\ln f(x,y,t)]_{xx}$.

Figs. 3 and 4 illustrate, respectively, the interaction phenomena between a lump and a two-soliton wave with the parameters $a_1=-\frac{1}{2}, a_2=3, a_3=3, a_4=0, a_5=\frac{3}{2}, a_6=1, a_7=1, a_8=0, a_9=1, k_1=1, k_2=0, k_3=1, c_1=-1, c_2=-1$ in Case 1, and $a_1=\frac{3}{2}, a_2=0, a_3=0, a_4=0, a_5=0, a_6=2, a_7=-\frac{2}{3}, a_8=0, a_9=1, k_1=\frac{4}{5}, k_2=0, k_3=\frac{64}{125}, c_1=-1, c_2=\frac{1}{3}$ in Case 2.

For any fixed value of $t=t_0$, the extremum points of the lump can be obtained. The extremum point of the lump locates at

$$\left(x=\frac{a_2a_7t_0-a_3a_6t_0+a_2a_8-a_4a_6}{a_1a_6-a_2a_5}, y=-\frac{a_1a_7t_0-a_3a_5t_0+a_1a_8-a_4a_5}{a_1a_6-a_2a_5}\right), \tag{18}$$

where the maximum of the amplitude of the lump is attained as $\frac{4(a_1^2+a_5^2)}{a_9}$.

When $t\to-\infty$ and $k_3>0$, we have

$$\lim_{t\to-\infty}\cosh(l)=\lim_{t\to-\infty}\frac{e^l+e^{-l}}{2}=\lim_{t\to-\infty}\frac{e^{-l}}{2}=+\infty, \lim_{t\to-\infty}g^2=+\infty, \lim_{t\to-\infty}h^2=+\infty.$$

We have proved that when $t\to\pm\infty$, g^2 and h^2 have the same order, and we need to compare g^2 and $\cosh(l)$ in virtue of

$$\lim_{t\to-\infty}\frac{g^2}{\cosh(l)}=2\lim_{t\to-\infty}\frac{g^2}{e^{-l}}=0, \tag{19}$$

which means both waves exist, but the soliton dominates the waves.

When $t\to-\infty$ and $k_3=0$, both waves exist, but the lump wave plays the dominant role.

When $t\to-\infty$ and $k_3<0$, the results

$$\lim_{t\to-\infty}\cosh(l)=\lim_{t\to-\infty}\frac{e^l}{2}=+\infty, \lim_{t\to-\infty}g^2=+\infty, \lim_{t\to-\infty}h^2=+\infty,$$

$$\lim_{t\to-\infty}\frac{g^2}{\cosh(l)}=2\lim_{t\to-\infty}\frac{g^2}{e^l}=0,$$

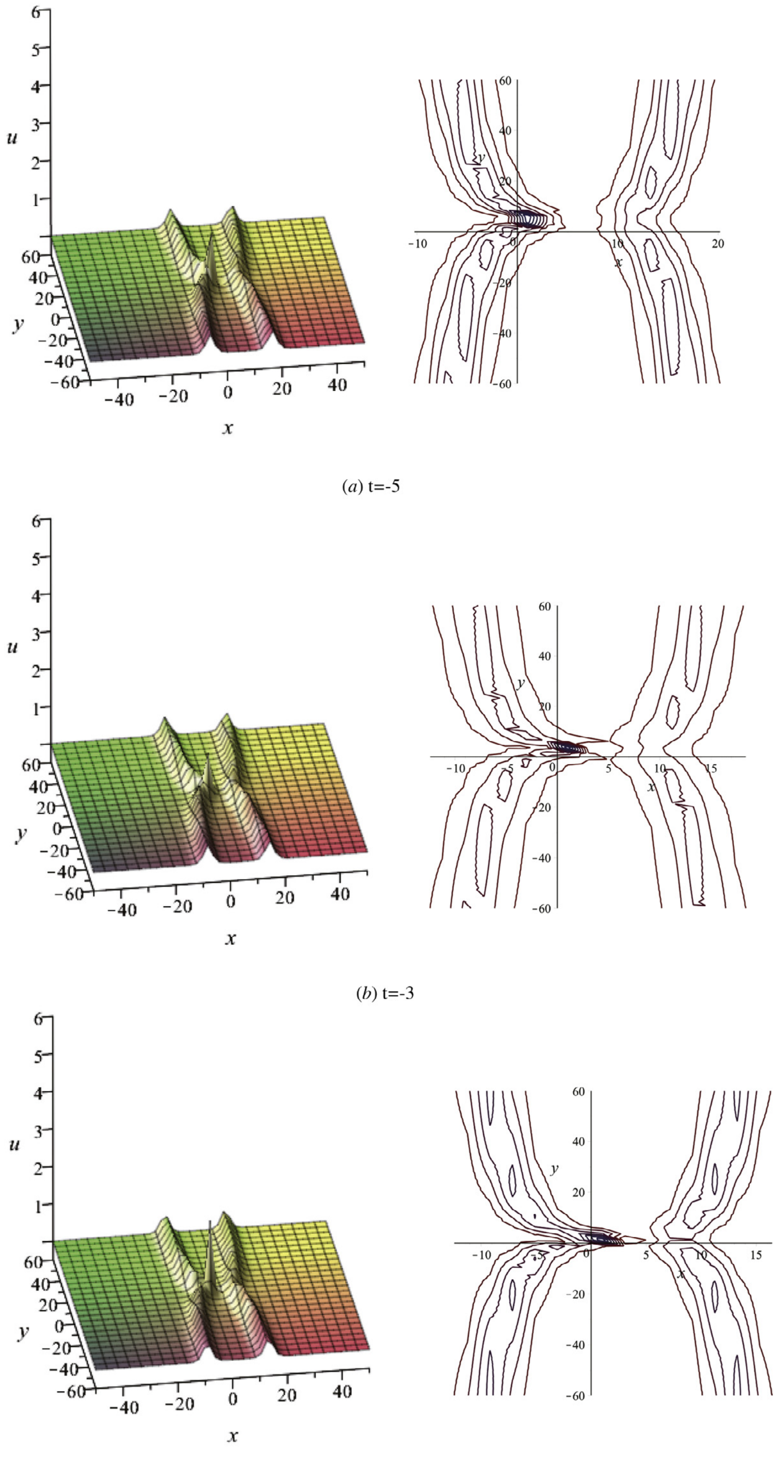
shows the soliton plays the dominant role.

Based on the similar analysis when $t\to+\infty$, we conclude that when $t\to\pm\infty$ and $k_3\neq 0$, both the lump and soliton waves exist, but the soliton plays the dominant role, while when $k_3=0$, the lump dominates the waves.

It is also clear that

$$\lim_{t\to\pm\infty}u=\lim_{t\to\pm\infty}2(\ln f)_{xx}=\lim_{t\to\pm\infty}\left[\frac{2(2a_1^2+2a_5^2+k_1^2\cosh(l))(g^2+h^2+\cosh(l)+a_9)-2(2a_1g+2a_5h+k_1\sinh(l))^2}{(g^2+h^2+\cosh(l)+a_9)^2}\right]=0. \tag{20}$$

When choosing the parameters given in Case 1, the solution of u can be rewritten as

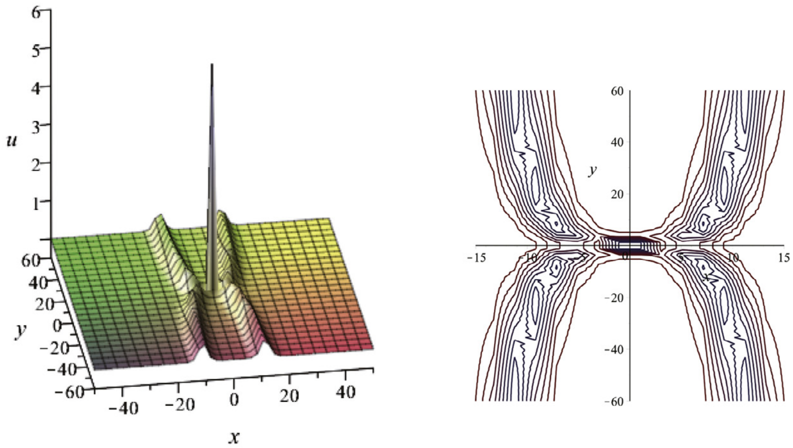


(a) t=-5

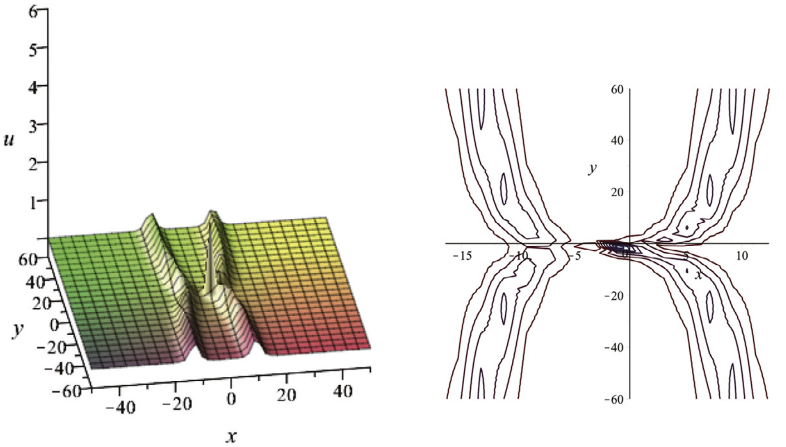
(b) t=-3

(c) t=-2

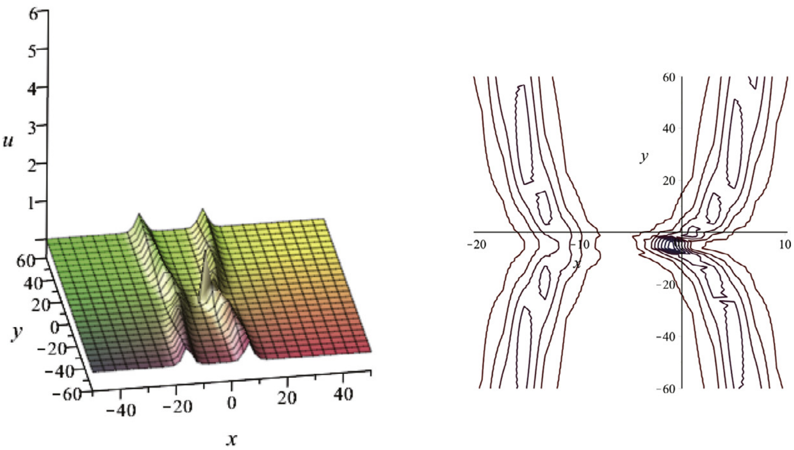
Fig. 3. Interaction behavior between a lump and a two-soliton wave in Case 1: 3d plots (left) and contour plots (right).



(*d*) t=0

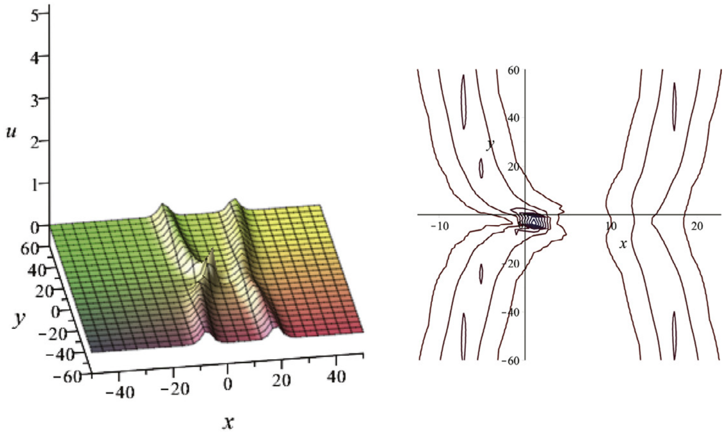


(*e*) t=2

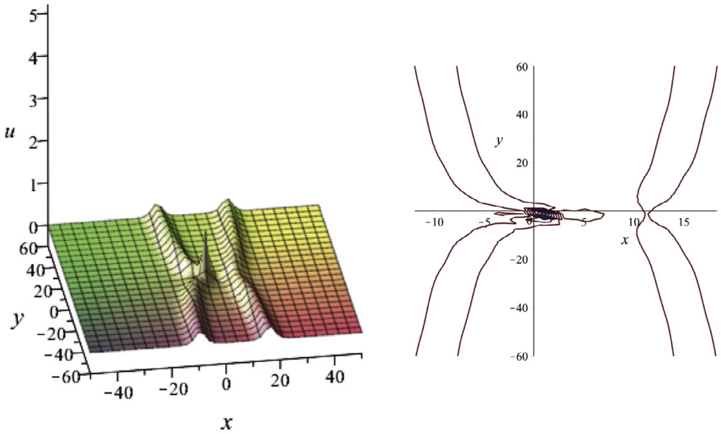


(*f*) t=5

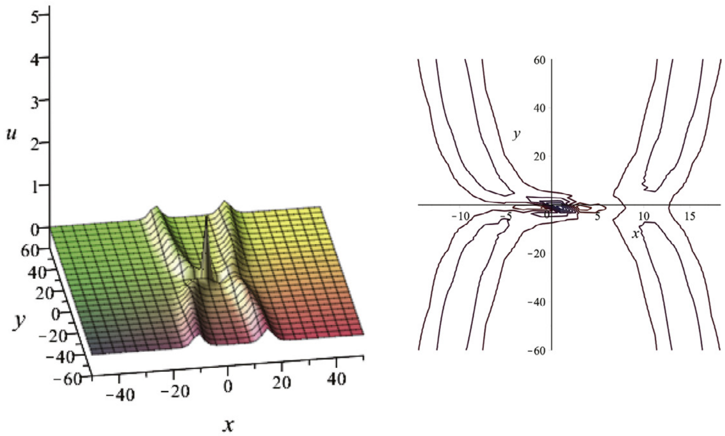
Fig. 3. Continued



(a) t=-8

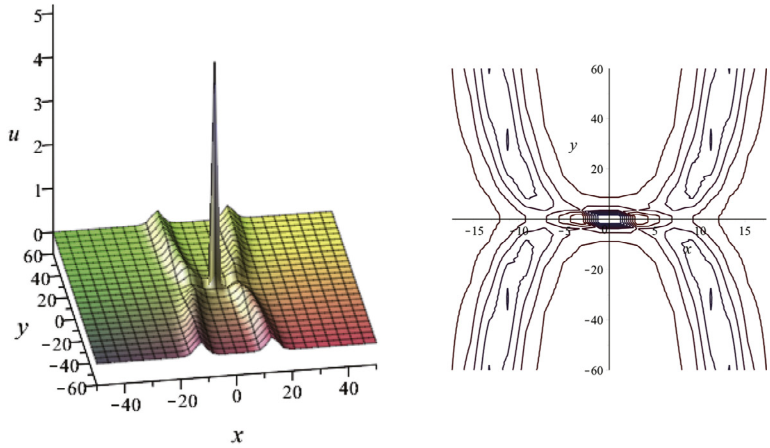


(b) t=-5

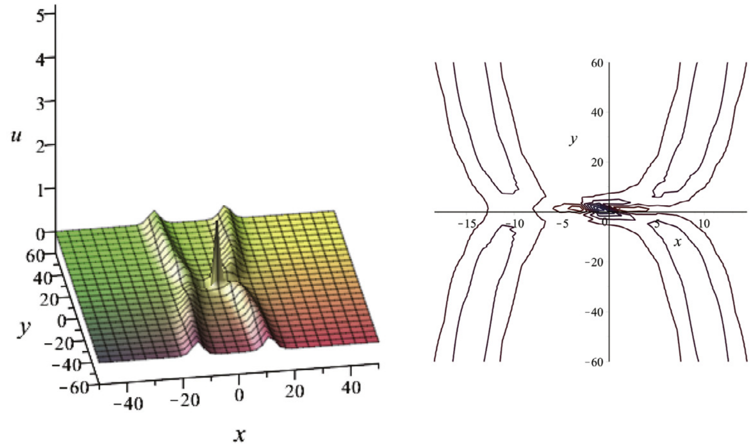


(c) t=-3

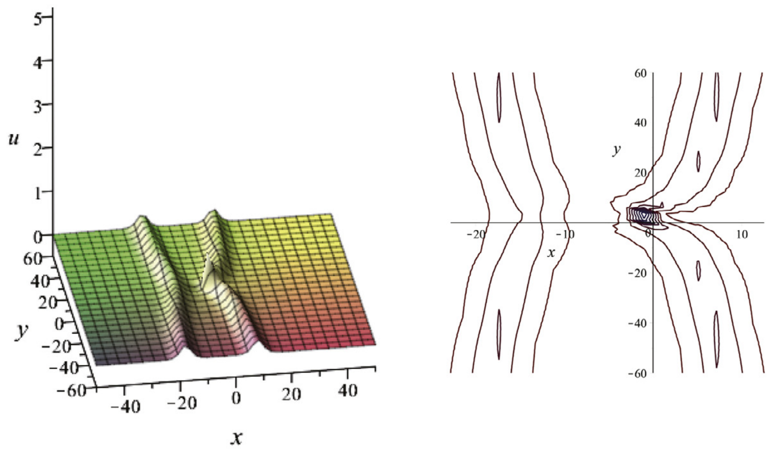
Fig. 4. Interaction behavior between a lump and a two-soliton wave in Case 2: 3d plots (left) and contour plots (right).



(d) t=0



(e) t=3



(f) t=8

Fig. 4. Continued

$$u = \frac{2(\cosh(x+t)+5)}{(-\frac{1}{2}x+3y+3t)^2+(\frac{3}{2}x+y+t)^2+\cosh(x+t)+1} - \frac{2(\sinh(x+t)+5x)^2}{\left((-\frac{1}{2}x+3y+3t)^2+(\frac{3}{2}x+y+t)^2+\cosh(x+t)+1\right)^2}.$$

The characteristic lines of the soliton wave are

$$l_1 : x+t+b_1 = 0,$$
$$l_2 : x+t+b_2 = 0,$$

where $b_1 \neq b_2, b_1$ and b_2 are two constants. It is easy to find that two branches of the soliton wave are parallel in (x, t)-plane and they coexist with a common speed.

When $t \to -\infty$, the lump wave nearly disappears and the soliton dominates the waves, the two branches of which are of the common speed $v_s = -1$ along the x-axis and the speed along the y-axis is zero. As time goes on, the lump wave gradually appears from the left branch of the soliton with the speed $v_l = -1$ along the y-axis and the speed along the x-axis is zero. With the soliton waves moving from the positive direction of the x-axis to the negative direction, the lump leaves gradually from the left branch of the soliton to the right branch. Then the lump will interact with the soliton and goes together with the right branch. When $t \to +\infty$, the lump wave nearly disappears again and the soliton waves play the dominant role.

As can be seen in both Figs. 3 and 4, when $t = -5$ in Fig. 3(a) and $t = -8$ in Fig. 4(a), the wave is consist of two parts, including a lump wave and a two-soliton wave. The lump appears from one branch of the two-soliton wave and begins to move to the other one. The amplitude of the lump changes with the variable t, and especially when $t = 0$, the lump locates in the middle of the two branches of the soliton waves. Then the lump continues moving, until it attaches to the other branch of the two-soliton wave. The process of interaction changes the amplitudes, shapes and velocities of both waves. This type of interaction solutions provide a method to forecast the appearance of rogue waves, such as financial rogue wave, optical rogue wave and plasma rogue wave, through analyzing the relations between lump wave part and soliton wave part [29].

4. Concluding remarks

Besides of finding lump solutions to the (2+1)-dimensional nonlinear evolution equations, we have focused on the interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation. With symbolic computation, two types of interaction solutions including lump-kink and lump-soliton ones have been derived to Eq. (4), and the relations among all the parameters and the coefficients have been obtained. We have analyzed the asymptotic properties of the interaction solutions and shown the interaction process via four sets of figures under some selections of parameters. The phenomena are useful in understanding the propagation of nonlinear waves.

We have obtained four cases of interaction solutions between lump and stripe by considering mixing two positive quadratic functions with an exponential function. The interaction process show that the lump moves closely to the stripe and begins to interact with the stripe. They collide over a period and the amplitudes, shapes and velocities of both waves change. When $t \to +\infty$, the lump is swallowed or drowned by the stripe, which presents the completely non-elastic interaction between the two different type of waves. This kind of interaction solution can be used in the fields of shallow water so as to reveal the propagation and interaction between stripe wave and lump wave.

We have also obtained two cases of interaction solutions between lump and two-soliton by considering mixing two positive quadratic functions with a hyperbolic cosine function. It is noted that the cases we discussed are under the situation $k_2 = 0$. The interaction process is as follows: The lump moves from one branch to the other branch of the two-soliton. When $t = 0$, the lump locates in between of the two branches of the soliton wave. Both the lump and the soliton change their amplitudes, shapes and velocities when they interact. The interaction solution of this type is valuable in forecasting the appearance of rogue waves through analyzing the relations between soliton wave and lump wave.

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