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Evolutionary behavior of rational wave solutions to the (4 + 1)-dimensional Boiti–Leon–Manna–Pempinelli equation

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Abstract

A nonlinear integrable model known as the (4 + 1)-dimensional Boiti–Leon–Manna–Pempinelli (4D-BLMP) equation is studied in the present paper. To this end, by considering the Hirota bilinear form of the model and utilizing the linear superposition method (LSM) along with symbolic computations, a group of rational wave solutions including multiple wave and positive (non-singular) compelexiton solutions is formally derived. The dynamical behavior of the solutions is also analyzed graphically by considering the special values of the involved parameters. The results of the current work reveal the existence of different wave structures to the 4D-BLMP equation and distinguish it from other models that do not possess non-singular compelexiton solutions.

Keywords: (4 + 1)-dimensional boiti–leon–manna–pempinelli equation, hirota bilinear form, linear superposition method, symbolic computations, multiple wave and positive compelexiton solutions

(Some figures may appear in colour only in the online journal)

1. Introduction

A wide variety of problems in modern scientific fields are described by nonlinear differential models. Although there are different strategies to deal with nonlinear differential models, a specific strategy is deriving the Hirota bilinear form and then applying the linear superposition principle along with symbolic computations to acquire rational wave solutions. Recently, the LSM has been used by many authors and has the theme of many research works [1–9]. For example, a

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group of rational wave solutions to the Hirota–Satsuma–Ito equation and the asymmetric Nizhnik–Novikov–Veselov equation were obtained respectively in [8, 9] by adopting the LSM along with symbolic computations.

Due to the efficiency of the LSM along with symbolic computations, our goal in the present paper is extracting a series of rational wave solutions to the following (4 + 1)-dimensional Boiti–Leon–Manna–Pempinelli equation [10]

$$\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial^2 u}{\partial t\partial z}+\frac{\partial^2 u}{\partial t\partial s}+\alpha\left(\frac{\partial^4 u}{\partial y\partial x^3}+\frac{\partial^4 u}{\partial z\partial x^3}+\frac{\partial^4 u}{\partial s\partial x^3}\right)+\beta\left(\frac{\partial u}{\partial x}\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 u}{\partial z\partial x}+\frac{\partial^2 u}{\partial s\partial x}\right)+\frac{\partial^2 u}{\partial x^2}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}+\frac{\partial u}{\partial s}\right)\right)=0. \tag{1}$$

The above nonlinear model has been introduced by Xu and Wazwaz [10] and has been studied using different approaches. Xu and Wazwaz demonstrated that the above new model has the Painlevé property and derived its bilinear representation, bilinear Bäcklund transformation, Lax pair, and infinite conservation laws by means of the Bell polynomial method.

It is worth mentioning that when $u=u(x, y, t)$, $\alpha=1$, $\beta=-3$, then equation (1) reduces to the following (2 + 1)-dimensional BLMP equation [11–13]

$$\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial^4 u}{\partial y\partial x^3}-3\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y\partial x}-3\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial y}=0. \tag{2}$$

Furthermore, for $u=u(x, y, z, t)$, $\alpha=1$, and $\beta=-3$, equation (1) reduces to the following (3 + 1)-dimensional BLMP equation [14–16]

$$\frac{\partial^2 u}{\partial t\partial y}+\frac{\partial^2 u}{\partial t\partial z}+\frac{\partial^4 u}{\partial y\partial x^3}+\frac{\partial^4 u}{\partial z\partial x^3}-3\left(\frac{\partial u}{\partial x}\left(\frac{\partial^2 u}{\partial y\partial x}+\frac{\partial^2 u}{\partial z\partial x}\right)+\frac{\partial^2 u}{\partial x^2}\left(\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}\right)\right)=0. \tag{3}$$

The above (2 + 1) and (3 + 1)-dimensional BLMP equations have been investigated through effective methods. For example, Luo [11] derived multi-soliton solutions of the equation (2) by means of the Hirota's bilinear method. Mabrouk and Kassem [12] acquired group similarity solutions of the model (2) through a two-parameter group transformation. Kumar and Tiwari [13] obtained soliton solutions of the equation (2) using the Lie symmetry method. Li and Ma [14] procured multiple-lump waves and interaction solutions of the model (3) with the use of ansatz methods. Liu *et al* [15] reported resonant soliton and complexiton solutions of the equation (3) through the linear superposition method. Luo [16] gained double-periodic soliton solutions of the model (3) using an extended homoclinic test technique. More articles are found in [17–28].

It is worthy of note that by using the Painlevé analysis under the transformation

$$u=6\frac{\alpha}{\beta}(\ln f)_x,$$

the equation (1) can be written in its Hirota bilinear form as follows [10]

$$(D_yD_t+D_zD_t+D_sD_t+\alpha\,(D_yD_x^3+D_zD_x^3+D_sD_x^3))f.f=0, \tag{4}$$

in which f is an unknown to be computed and 'D' is the Hirota's derivative operator. The organization of this paper is as follows: In section 2, key ideas of the linear superposition method are presented. In section 3, a group of rational wave solutions including multiple wave and positive compelexiton solutions to the model is formally derived. Concluding remarks are given in the last section.

2. Linear superposition method

Suppose that a nonlinear differential model can be converted to the following Hirota bilinear equation [1, 2, 6]

$$P(D_{x_1}, D_{x_2}, \ldots, D_{x_M})f.f=0, \tag{5}$$

under a specific transformation where P is a polynomial satisfying

$$P(0, 0, \ldots, 0)=0.$$

Now, consider the N-wave functions

$$f_i=e^{\eta_i},\quad \eta_i=k_{1i}x_1+k_{2i}x_2+\ldots+k_{Mi}x_M,\quad 1\leqslant i\leqslant N,$$

where k_{ji}, $1\leqslant i\leqslant N$, $1\leqslant j\leqslant M$ are unknowns that must be computed later. By inserting the N-wave function

$$f=\varepsilon_1 f_1+\varepsilon_2 f_2+\ldots+\varepsilon_N f_N=\varepsilon_1 e^{\eta_1}+\varepsilon_2 e^{\eta_2}+\ldots+\varepsilon_N e^{\eta_N},$$

into the left hand side of equation (5) and considering the properties of Hirota's bilinear operators, we find

$$P(D_{x_1}, D_{x_2}, \ldots, D_{x_M})f.f=2\sum_{1\leqslant j<i\leqslant N}\varepsilon_i\varepsilon_j P(\mathrm{k}_i-\mathrm{k}_j)e^{\eta_i+\eta_j},$$

$$\mathrm{k}_i=\{k_{1i}, k_{2i}, \ldots, k_{Mi}\},$$

and therefore, f solves the Hirota bilinear equation (5) if and only if

$$P(\mathrm{k}_i-\mathrm{k}_j)=0,\quad 1\leqslant j<i\leqslant N.$$

Now, by considering the relation

$$k_{li}=a_l k_i^{n_l},\quad 1\leqslant l\leqslant M,\quad 1\leqslant i\leqslant N,$$

we obtain

$$P(k_{1i}-k_{1j}, k_{2i}-k_{2j}, \ldots, k_{Mi}-k_{Mj})=P(a_1(k_i^{n_1}-k_j^{n_1}),\times a_2(k_i^{n_2}-k_j^{n_2}), \ldots, a_M(k_i^{n_M}-k_j^{n_M})). \tag{6}$$

It should be noted that $n_l,\ 1 \leqslant l \leqslant M$ are derived by balancing powers in (6) and $a_l,\ 1 \leqslant l \leqslant M$ are selected such that

$$P(a_1(k_i^{n_1} - k_j^{n_1}),\ a_2(k_i^{n_2} - k_j^{n_2}),\ldots,a_M(k_i^{n_M} - k_j^{n_M})) = 0.$$

After finding unknowns, rational wave solutions of the nonlinear differential model are formally constructed.

3. Rational wave solutions of the 4D-BLMP equation

In this section, the linear superposition method along with symbolic computations is utilized to extract a group of rational wave solutions including multiple wave and positive compelexiton solutions of the 4D-BLMP equation.

Multiple wave solution: According to the linear superposition method [1–9], a polynomial corresponding to the Hirota bilinear equation (4) is considered as

$$P(x,\ y,\ z,\ s,\ t) = yt + zt + st$$
$$+\alpha(yx^3 + zx^3 + sx^3).$$

The weights of the independent variables are defined as $(n_1,\ n_2,\ n_3,\ n_4,\ n_5) = (1,\ -1,\ -1\ -\ 1,\ 3)$ such that the polynomial is homogeneous of degree 2. Inserting the N-wave function

$$f = \varepsilon_1 e^{\eta_1} + \varepsilon_2 e^{\eta_2} + \ldots + \varepsilon_N e^{\eta_N},$$
$$\eta_i = a_1 k_i x + a_2 k_i^{-1} y + a_3 k_i^{-1} z + a_4 k_i^{-1} s + a_5 k_i^3 t, \quad 1 \leqslant i \leqslant N,$$

into equation (4) and considering the properties of Hirota's bilinear operators yields

$$\begin{aligned} &P(a_1(k_i - k_j),\ a_2(k_i^{-1} - k_j^{-1}),\\ &\quad\times a_3(k_i^{-1} - k_j^{-1}),\ a_4(k_i^{-1} - k_j^{-1}),\ a_5(k_i^3 - k_j^3))\\ &= a_2 a_5 (k_i^{-1} - k_j^{-1})(k_i^3 - k_j^3) + a_3 a_5 (k_i^{-1} - k_j^{-1})\\ &\quad\times(k_i^3 - k_j^3) + a_4 a_5 (k_i^{-1} - k_j^{-1})(k_i^3 - k_j^3) + \alpha a_1^3 a_2\\ &\quad\times(k_i^{-1} - k_j^{-1})(k_i - k_j)^3 + \alpha a_1^3 a_3 (k_i^{-1} - k_j^{-1})\\ &\quad\times(k_i - k_j)^3 + \alpha a_1^3 a_4 (k_i^{-1} - k_j^{-1})(k_i - k_j)^3 = 0. \end{aligned} \tag{7}$$

By expanding equation (7) and setting the coefficients of the resulting expression to zero, one derives

$$a_2 a_5 + a_3 a_5 + a_4 a_5 = 0.$$

By solving the above algebraic equation, we get

$$a_2 = -(a_3 + a_4).$$

Now, by considering $a_1 = 1$, one can acquire the following multiple wave solution for the 4D-BLMP equation

$$u = 6\frac{\alpha}{\beta}(\ln f)_x,$$

in which

$$f = \varepsilon_1 e^{\eta_1} + \varepsilon_2 e^{\eta_2} + \ldots + \varepsilon_N e^{\eta_N}, \quad \eta_i = k_i x - (a_3 + a_4) k_i^{-1} y + a_3 k_i^{-1} z + a_4 k_i^{-1} s + a_5 k_i^3 t, \quad 1 \leqslant i \leqslant N,$$

and $k_i \neq 0,\ \ i = 1,\ldots,N$.

The 3D and density graphs of u for the special values of the involved parameters have been presented in figure 1. Actually, figure 1 exhibits the 3D and density graphs of 1, 2, and 3-wave solutions.

Positive compelexiton solution: In order to extract the positive compelexiton solution of the 4D-BLMP equation, we first consider

$$\begin{aligned} f_1 &= \varepsilon_1 e^{\eta_1} + \varepsilon_2 e^{\eta_2} + \ldots + \varepsilon_N e^{\eta_N},\\ \eta_i &= k_i x - (a_3 + a_4) k_i^{-1} y + a_3 k_i^{-1} z\\ &\quad + a_4 k_i^{-1} s + a_5 k_i^3 t, \quad 1 \leqslant i \leqslant N,\\ f_2 &= \varepsilon_1 e^{-\eta_1} + \varepsilon_2 e^{-\eta_2} + \ldots + \varepsilon_N e^{-\eta_N},\\ -\eta_i &= (-k_i)x - (a_3 + a_4)(-k_i)^{-1} y\\ &\quad + a_3(-k_i)^{-1} z + a_4(-k_i)^{-1} s\\ &\quad + a_5(-k_i)^3 t \quad 1 \leqslant i \leqslant N, \end{aligned}$$

which $k_i \neq 0,\ \ i = 1,\ldots,N$. Obviously, f_1 and f_2 are the solutions for the equation (4). Thus, a linear combination of f_1 ad f_2 as

$$\frac{f_1 + f_2}{2} = \sum_{i=1}^{N} \varepsilon_i \cosh\left(k_i x - (a_3 + a_4) k_i^{-1} y + a_3 k_i^{-1} z + a_4 k_i^{-1} s + a_5 k_i^3 t\right),$$

is also a solution of the equation (4). On the other hand, by considering the nonzero real numbers $k_{N+1},\ k_{N+2},\ \cdots,\ k_{N+M}$ and owing to the new functions

$$\begin{aligned} f_1 &= \varepsilon_{N+1} e^{\eta_{N+1}} + \varepsilon_{N+2} e^{\eta_{N+2}} + \ldots + \varepsilon_{N+M} e^{\eta_{N+M}},\\ \eta_i &= (Ik_i)x - (a_3 + a_4)(Ik_i)^{-1} y + a_3 (Ik_i)^{-1} z\\ &\quad + a_4 (Ik_i)^{-1} s + a_5 (Ik_i)^3 t\\ &= I(k_i x + (a_3 + a_4) k_i^{-1} y\\ &\quad - a_3 k_i^{-1} z - a_4 k_i^{-1} s - a_5 k_i^3 t), \end{aligned}$$

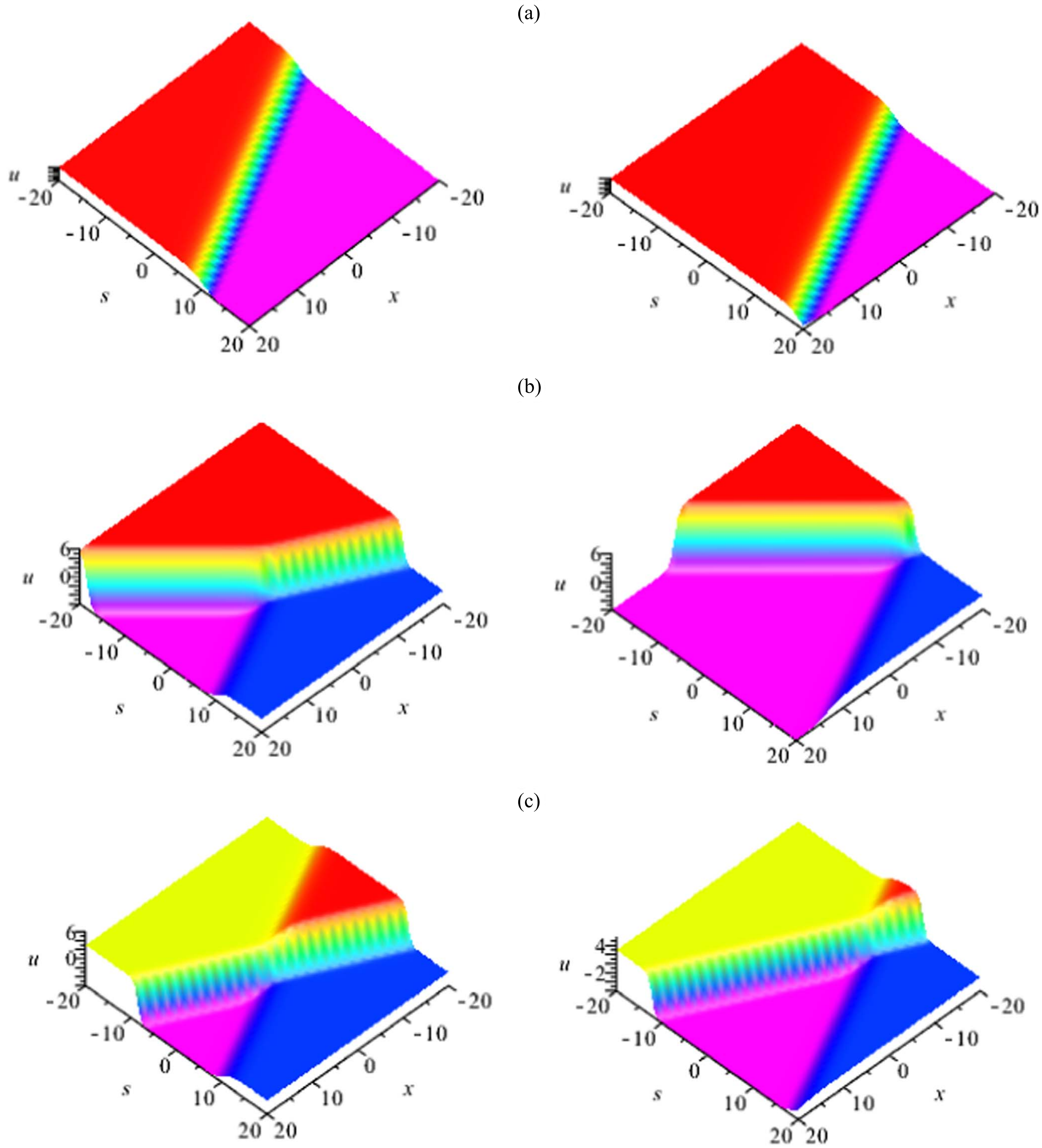
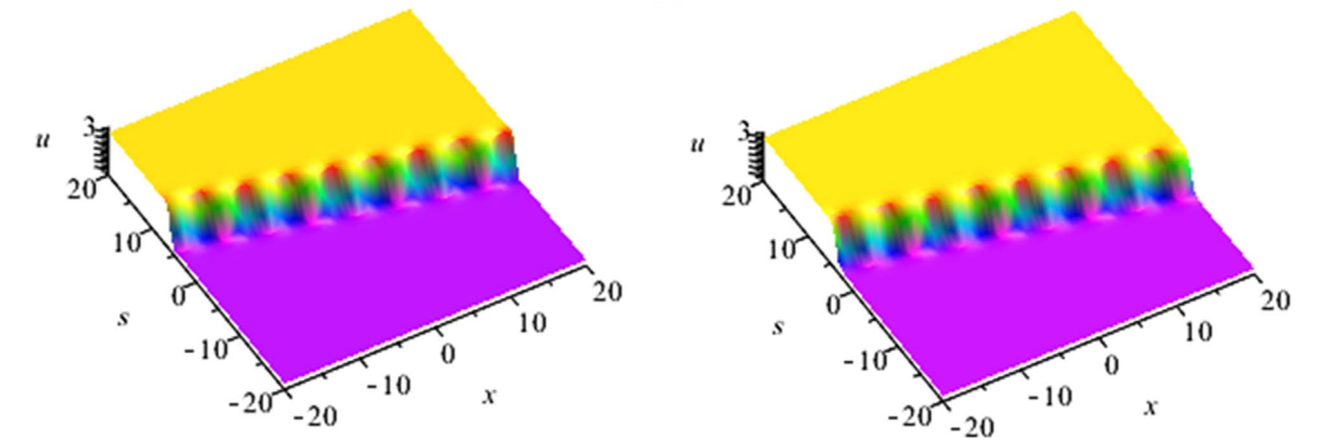


Figure 1. 3D and density plots of u for (a) $N = 2$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\alpha = 1$, $\beta = 1$, $k_1 = 1$, $k_2 = 0.5$, $a_3 = -1.5$, $a_4 = 1$, $a_5 = 1$, $y = 1$, $z = 1$ when $t = 0$ and $t = 10$ (Left to Right); (b) $N = 3$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 3$, $\alpha = 1$, $\beta = -1$, $k_1 = 1$, $k_2 = 0.5$, $k_3 = -1$, $a_3 = -1.5$, $a_4 = 1$, $a_5 = 1$, $y = 1$, $z = 1$ when $t = 0$ and $t = 15$ (Left to Right); (c) $N = 4$, $\varepsilon_1 = 1$, $\varepsilon_2 = 2$, $\varepsilon_3 = 3$, $\varepsilon_4 = 4$, $\alpha = 1$, $\beta = -1$, $k_1 = 1$, $k_2 = 0.5$, $k_3 = -1$, $k_4 = -0.5$, $a_3 = -1.5$, $a_4 = 1$, $a_5 = 1$, $y = 1$, $z = 1$ when $t = 0$ and $t = 10$ (Left to Right).

(a)



(b)

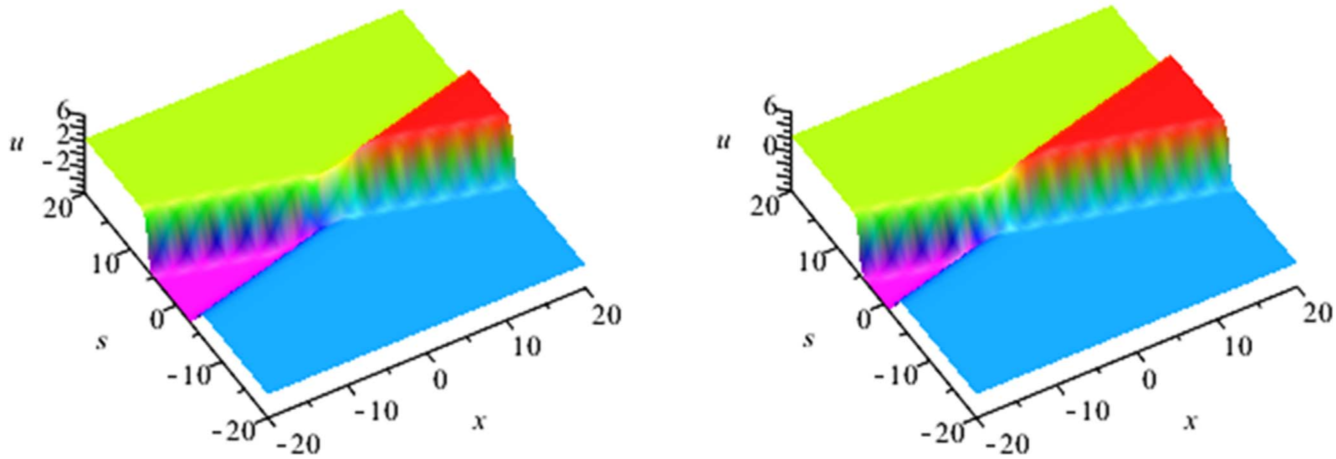


Figure 2. 3D and density plots of u for (a) $M = 1$, $N = 1$, $\varepsilon_1 = 1$, $\varepsilon_2 = 0.4$, $\alpha = 1$, $\beta = 1$, $k_1 = 0.5$, $k_2 = 1$, $a_3 = -1$, $a_4 = 1$, $a_5 = 1$, $y = 1$, $z = 1$ when $t = 0$ and $t = 30$ (Left to Right); (b) $M = 2$, $N = 2$, $\varepsilon_1 = 1.1$, $\varepsilon_2 = 0.9$, $\varepsilon_3 = 0.3$, $\varepsilon_4 = 0.9$, $\alpha = 1$, $\beta = 1$, $k_1 = 1$, $k_2 = 0.4$, $k_3 = 0.8$, $k_4 = 0.6$, $a_3 = 1$, $a_4 = 1.5$, $a_5 = 0.5$, $y = 1$, $z = 1$ when $t = 0$ and $t = 10$ (Left to Right).

$$f_2 = \varepsilon_{N+1}e^{-\eta_{N+1}} + \varepsilon_{N+2}e^{-\eta_{N+2}} + \dots + \varepsilon_{N+M}e^{-\eta_{N+M}}, \quad -\eta_i = I((-k_i)x + (a_3 + a_4)(-k_i)^{-1}y - a_3(-k_i)^{-1}z - a_4(-k_i)^{-1}s - a_5(-k_i)^3 t),$$

are the solutions for the equation (4), we find that a linear combination of f_1 ad f_2 as

$$\frac{f_1 + f_2}{2} = \sum_{i=N+1}^{N+M} \varepsilon_i \cos(k_i x + (a_3 + a_4)k_i^{-1}y - a_3 k_i^{-1}z - a_4 k_i^{-1}s - a_5 k_i^3 t),$$

is a solution of the equation (4). Now, it is clear that the new expression

$$f = \sum_{i=1}^{N} \varepsilon_i \cosh(k_i x - (a_3 + a_4)k_i^{-1}y + a_3 k_i^{-1}z + a_4 k_i^{-1}s + a_5 k_i^3 t) + \sum_{i=N+1}^{N+M} \varepsilon_i \cos(k_i x + (a_3 + a_4)k_i^{-1}y - a_3 k_i^{-1}z - a_4 k_i^{-1}s - a_5 k_i^3 t),$$

$$\varepsilon_i \in \mathbb{R},$$

when $\varepsilon_i > 0$ for $i = 1, 2, \dots, N$ and $\sum_{i=1}^{N} \varepsilon_i > \sum_{i=N+1}^{N+M} |\varepsilon_i|$ is a positive function and thus, the positive compelexiton solution can be written as

$$u = 6\frac{\alpha}{\beta}(\ln f)_x,$$

which the function f has been defined above. The 3D and density graphs of u for the special values of the involved parameters have been given in figure 2. Truly, figure 2 shows the 3D and density graphs of positive compelexiton solutions.

Compelexiton solution: Finally, the following compelexiton solution for the 4D-BLMP equation can be constructed

$$
\begin{aligned}
u &= 6\frac{\alpha}{\beta}(\ln f)_x,\\
f &= \sum_{i=1}^{N} e^{\eta_{i,1}}(\varepsilon_{i,1}\cos(\eta_{i,2}) + \varepsilon_{i,2}\sin(\eta_{i,2})),\\
&\times k_i = k_{1i} + Ik_{2i},\quad \eta_i = k_i x - (a_3 + a_4)k_i^{-1}y\\
&+ a_3 k_i^{-1}z + a_4 k_i^{-1}s + a_5 k_i^3 t = \eta_{i,1} + I\eta_{i,2},\\
&\varepsilon_{i,1},\ \varepsilon_{i,2},\ k_{1i},\ k_{2i} \in \mathbb{R}.
\end{aligned}
$$

In a special case, when $N = 2$, $k_1 = -1 - I$, and $k_2 = 1 + I$, we find

$$
\begin{aligned}
\eta_1 = \eta_{1,1} + I\eta_{1,2} &= -x + \frac{1}{2}(a_3 + a_4)y - \frac{1}{2}a_3 z\\
&- \frac{1}{2}a_4 s + 2a_5 t + I\left(-x - \frac{1}{2}(a_3 + a_4)y\right.\\
&\left. + \frac{1}{2}a_3 z + \frac{1}{2}a_4 s - 2a_5 t\right),
\end{aligned}
$$

$$
\begin{aligned}
\eta_2 = \eta_{2,1} + I\eta_{2,2} &= x - \frac{1}{2}(a_3 + a_4)y + \frac{1}{2}a_3 z\\
&+ \frac{1}{2}a_4 s - 2a_5 t + I\left(x + \frac{1}{2}(a_3 + a_4)y\right.\\
&\left. - \frac{1}{2}a_3 z - \frac{1}{2}a_4 s + 2a_5 t\right).
\end{aligned}
$$

Therefore, the compelexiton solution can be written as

$$
u = 6\frac{\alpha}{\beta}(\ln f)_x,
$$

which

$$
\begin{aligned}
f &= e^{-x+\frac{1}{2}(a_3+a_4)y-\frac{1}{2}a_3 z-\frac{1}{2}a_4 s+2a_5 t}\\
&\times [\varepsilon_{1,1}\cos\left(-x - \tfrac{1}{2}(a_3 + a_4)y + \tfrac{1}{2}a_3 z + \tfrac{1}{2}a_4 s - 2a_5 t\right)\\
&+ \varepsilon_{1,2}\sin\left(-x - \tfrac{1}{2}(a_3 + a_4)y + \tfrac{1}{2}a_3 z + \tfrac{1}{2}a_4 s - 2a_5 t\right)]\\
&+ e^{x-\frac{1}{2}(a_3+a_4)y+\frac{1}{2}a_3 z+\frac{1}{2}a_4 s-2a_5 t}\\
&\times\ [\varepsilon_{2,1}\cos\left(x + \tfrac{1}{2}(a_3 + a_4)y - \tfrac{1}{2}a_3 z - \tfrac{1}{2}a_4 s + 2a_5 t\right)\\
&+\ \varepsilon_{2,2}\sin\left(x + \tfrac{1}{2}(a_3 + a_4)y - \tfrac{1}{2}a_3 z - \tfrac{1}{2}a_4 s + 2a_5 t\right)].
\end{aligned}
$$

It should be mentioned that 'Compelexiton Solutions' were introduced in [29] and mean a kind of explicit exact solutions involving two elementary functions 'trigonometric functions and exponential functions'.

Remark 1. It is worth noting that when $u = u(x, y, s, t)$, then equation (1) reduces to the following (3 + 1)-dimensional BLMP equation

$$
\begin{aligned}
&\frac{\partial^2 u}{\partial t\partial y} + \frac{\partial^2 u}{\partial t\partial s} + \alpha\left(\frac{\partial^4 u}{\partial y\partial x^3} + \frac{\partial^4 u}{\partial s\partial x^3}\right)\\
&+ \beta\left(\frac{\partial u}{\partial x}\left(\frac{\partial^2 u}{\partial y\partial x} + \frac{\partial^2 u}{\partial s\partial x}\right) + \frac{\partial^2 u}{\partial x^2}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial s}\right)\right)\\
&= 0.
\end{aligned}
$$

Now, one can obtain the following multiple wave and positive compelexiton solutions to the 3D-BLMP equation (which are in agreement with the solutions reported in [15])

$$
\begin{aligned}
&u = 6\frac{\alpha}{\beta}(\ln f)_x,\\
&f = \varepsilon_1 e^{\eta_1} + \varepsilon_2 e^{\eta_2} + \ldots + \varepsilon_N e^{\eta_N},\\
&\eta_i = k_i x + a_2 k_i^{-1}y - a_2 k_i^{-1}s + a_4 k_i^3 t,\\
&1 \leqslant i \leqslant N,\quad k_i \neq 0,\quad i = 1,\ldots,N,
\end{aligned}
$$

and

$$
\begin{aligned}
u &= 6\frac{\alpha}{\beta}(\ln f)_x,\\
f &= \sum_{i=1}^{N}\varepsilon_i\cosh(k_i x + a_2 k_i^{-1}y - a_2 k_i^{-1}s + a_4 k_i^3 t)\\
&+ \sum_{i=N+1}^{N+M}\varepsilon_i\cos(k_i x - a_2 k_i^{-1}y + a_2 k_i^{-1}s - a_4 k_i^3 t),\\
&\varepsilon_i > 0 \text{ for } i = 1, 2,\ldots,N \text{ and } \sum_{i=1}^{N}\varepsilon_i > \sum_{i=N+1}^{N+M}|\varepsilon_i|.
\end{aligned}
$$

Remark 2. The correctness of the rational wave solutions listed in the present paper was checked by substituting each solution back into its corresponding equation. Maple package was formally utilized to handle the required computations.

4. Conclusion

A nonlinear integrable model called the (4 + 1) dimensional Boiti–Leon–Manna–Pempinelli equation was successfully studied in the current paper. Through the use of the Hirota bilinear form of the model and adopting the linear superposition principle along with symbolic computations, a group of rational wave solutions including multiple wave and positive compelexiton solutions was formally obtained. The properties and physical structure of the solutions were analyzed graphically by considering the special choices of the involved parameters, providing some useful information. The positive compelexiton solution presented herein is of great importance; it does not contain any singularity and such a key characteristic distinguishes the model from other models that do not possess non-singular complexiton solutions. It is corroborated that the current work presents comprehensive results about the existence of different wave structures to the 4D-BLMP equation.

Conflict of interest

The authors declare that they have no conflict of interest.

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