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Painlevé analysis, soliton solutions and lump-type solutions of the (3+1)-dimensional generalized KP equation

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ABSTRACT

The Painlevé analysis is applied and the multi-soliton criterion is presented to test the integrability of the (3+1)-dimensional generalized KP equation derived from a Hirota bilinear equation. It is shown that the considered equation does not pass the well known Painlevé test and it is only integrable in a conditional sense. Solitary wave solutions are shown to interact each other like solitons in multiple wave collisions unless some additional conditions are imposed. Moreover, we analyze a class of analytical rational lump-type solutions in detail, which are generated from positive quadratic polynomial function and rationally localized in many directions in the space, based upon the Hirota bilinear form.

1. Introduction

Integrable dynamical systems as models of natural phenomena play a universal role. Since most of the nonlinear partial differential equations (PDEs) of mathematical physics are of 3+1 dimensions or even higher dimensions, it is of cardinal importance to see whether higher dimensional equations be integrable by linear techniques [1].

What we shall do in this paper is to discuss the integrability [2,3], investigate solitary wave solutions [4–6] and lump-type solutions [7–10] of the (3+1)-dimensional generalized Kadomtsev–Petviashvili (KP) equation

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} + 3u_{zz} = 0, \tag{1}$$

which is derived from a (3+1)-dimensional Hirota bilinear equation [7]

$$(D_t D_y - D_x^3 D_y - 3D_x^2 + 3D_z^2) ff = 0, \tag{2}$$

or equivalently,

$$ff_{ty} - f_t f_y + f_{xxx} f_y - ff_{xxxy} + 3(f_{xxy} f_x - f_{xx} f_{xy}) - 3(ff_{xx} - f_x^2) + 3(ff_{zz} - f_z^2) = 0, \tag{3}$$

under the transformation

$$u = 2\ln(f)_x. \tag{4}$$

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It is clear that if f solves Eq. (3), then u is a solution of Eq. (1) through the transformation (4).

In Section 2, Eq. (1) is tested for integrability through Painlevé analysis [11–13]. The criterion of Painlevé property is that the solutions of the PDEs should have no singularities other than poles. The result reveals that Eq. (1) does not pass the test.

Another test for integrability is performed in Section 3. A characteristic of integrability that integrable PDEs have in common is that they can possess N-soliton solutions for $N \geqslant 3$. Therefore, we would like to consider solitary wave solutions of Eq. (1) in detail and examine whether the equation admits multiple solitons which have mutual interactions without changing shape.

In Section 4, we present the lump-type solutions [14,15] for Eq. (1) and discuss the properties of these solutions. The most interesting novel feature of these solutions is that they decay to zero in many directions but not all directions in the space.

Section 5 is devoted to conclusions.

2. The Painlevé analysis

The Painlevé property implies integrability and the Painlevé test provides a way to identify integrability. A necessary condition for a PDE has the Painlevé property is when the solution of the PDE

$$u = \frac{1}{\phi^{\alpha}} \sum_{j=0}^{\infty} u_j \phi^j, \tag{5}$$

is single valued about the movable, singularity manifold. This singularity manifold $\phi = \phi(x, y, z, t)$ is introduced in the complex space of the independent variables x, y, z and t. Here α is an positive integer, and the coefficients $u_j = u_j(x, y, z, t)$ are analytic functions of the independent variables in a neighborhood of the singularity manifold $\phi = 0$ [1].

The solution (5) should contain N arbitrary functions for an Nth-order PDE, namely, ϕ and $N - 1$ of the coefficients u_j. The corresponding values of j are called resonances. That means resonances are those values of j at which it is possible to introduce free functions.

Now, we apply the Painlevé test to Eq. (1). Substitution of (5) into Eq. (1) determines the value of $\alpha = 1$ by a leading order analysis. It is found that resonances occur at $j = -1, 1, 4$ and 6. Next step is to compute the coefficients u_2, u_3 and u_5 from the recursion relations and to verify the existence of the free functions u_1, u_4 and u_6. From the recursion relations, we find that $u_j, j = 0, 1, 2, \ldots,$

$$\begin{aligned}
&j = 0,\, u_0 = 2\phi_x,\\
&j = 1,\, u_1 = \mathit{free},\\
&j = 2,\, u_2 = \frac{3\phi_z^2 - 3\phi_x^2 - 3\phi_x\phi_{xxy} + 3\phi_{xy}\phi_{xx}}{6\phi_x^2\phi_y} + \frac{(\phi_t - \phi_{xxx})}{6\phi_x^2},\\
&j = 3,\, u_3 = \frac{(5\phi_{xx}\phi_y + 3\phi_x\phi_{xy})u_2}{4\phi_x^2\phi_y} + \frac{\phi_{4x} - \phi_{xt}}{12\phi_x^3} - \frac{\phi_{yt} + 3\phi_{zz} - 4\phi_{xxxy} - 9\phi_{xx}}{12\phi_x^2\phi_y}\\
&\qquad\qquad - \frac{6\phi_{xz}\phi_z + \phi_{xy}(\phi_t + 2\phi_{xxx})}{12\phi_x^3\phi_y},\\
&j = 4,\, u_4 = \mathit{free}.
\end{aligned}$$

The expression for u_5 is too long to present here. The compatibility conditions at $j = 1$ and 4 are satisfied identically. However, a resonance condition at the resonance $j = 6$

$$\begin{aligned}
&12u_2u_3\phi_x - 2(\phi_{xy}(18u_4\phi_x + 5u_{3,x}))_x - 2(u_3\phi_{xxy})_x - ((\frac{3}{2}u_2^2 + 2u_{3,x} + 18u_4\phi_x)_x\\
&+ 72u_5\phi_x^2 + 6u_{1,x}u_3 + 6u_{4,x}\phi_x)\phi_{xy} - 8(3u_4\phi_y + u_{3,y})\phi_{xxx} + 2u_3(3\phi_{zz} - 3\phi_{xx} + \phi_{yt})\\
&- 6(2u_2u_3 + 4u_5\phi_x + 5u_{4,x})\phi_{xx}\phi_y - 3((u_2^2)_y/2 + 2u_{1,y}u_3 + 6u_{4,y}\phi_x + 4u_{3,xy})\phi_{xx}\\
&+ 6(3u_4\phi_z + 2u_{3,z})\phi_z + 2(3u_4\phi_y + u_{3,y})\phi_t - 12u_{5,y}\phi_x^3 - 6((6u_2u_4 + 4u_3^2 + 2u_{5,x})\phi_y\\
&+ 3u_{1,y}u_4 + u_2u_{3,y} + 2u_{2,y}u_3 + 3u_4 + 2u_{4,xy})\phi_x^2 - 3u_{2,xx} + 3u_{2,zz} + u_{2,yt} - u_{2,xxxy}\\
&+ (2u_{3,t} - 2u_{3,xxx} - 3((u_2^2)_x/2 + 2u_{1,x}u_3)_x)\phi_y - 3(u_{1,x}u_{2,x})_y - 3u_{1,y}u_{2,xx} - 3u_{1,xx}u_{2,y}\\
&+ (-6(3u_{1,x}u_4 + 3u_2u_{3,x} + 4u_{2,x}u_3 + 2u_{4,xx})\phi_y - 3u_2u_{2,xy} - 6(u_{1,x}u_3)_y - 9u_{2,x}u_{2,y}\\
&- 12u_{1,y}u_{3,x} - 6(2u_3 + u_{3,xy})_x)\phi_x = 0,
\end{aligned}$$

is not satisfied automatically. Furthermore, if we reduce u_2, u_3 and u_5, the above resonance condition is found to involve the functions u_1 and ϕ, rather than an identity. Analysis allows one to conclude that, Eq. (1) does not possess the Painlevé property, in particular, is presumably not integrable.

3. Soliton solutions

Now we turn our attention to the integrability criterion related to solitary wave solutions. We take the attitude that the solutions of integrable PDEs can split apart into N-soliton for all values of N. Following this criterion, we look for the N-soliton solutions of Eq. (1).

Substituting the quantities

$$u = \exp(\xi_i), \quad \xi_i = k_i x + l_i y + m_i z + c_i t, \tag{6}$$

into the linear terms of Eq. (1), we obtain the dispersion relation

$$c_i = k_i^3 + \frac{3(k_i^2 - m_i^2)}{l_i}, \quad i = 1, 2, \ldots. \tag{7}$$

As a result, the dispersion variable ξ_i reads

$$\xi_i = k_i x + l_i y + m_i z + (k_i^3 + \frac{3(k_i^2 - m_i^2)}{l_i})t. \tag{8}$$

3.1. One-soliton solution

The form of one-soliton is

$$f = 1 + \exp(\xi_1).$$

This function f is a solution of Eq. (3) with the dispersion variable (8) in the case $i = 1$. The one-soliton solution of Eq. (1) is given by

$$u = \frac{2k_1 \exp \xi_1}{1 + \exp \xi_1},$$

through Eq. (4).

3.2. Two-soliton solution

The situation for two-soliton we consider is

$$f = 1 + \exp(\xi_1) + \exp(\xi_2) + A_{12} \exp(\xi_1 + \xi_2),$$

with ξ_1 and ξ_2 satisfy dispersion variable (8). If the value of phase shift A_{12} satisfies

$$A_{12} = \frac{k_1k_2l_1l_2(k_1 - k_2)(l_1 - l_2) - (k_1l_2 - l_1k_2)^2 + (l_1m_2 - m_1l_2)^2}{k_1k_2l_1l_2(k_1 + k_2)(l_1 + l_2) - (k_1l_2 - l_1k_2)^2 + (l_1m_2 - m_1l_2)^2},$$

then f is a solution of Eq. (3). Moreover, the exact two-soliton solution of Eq. (1) is

$$u = \frac{2(k_1 \exp \xi_1 + k_2 \exp(\xi_2) + A_{12}(k_1 + k_2) \exp(\xi_1 + \xi_2))}{1 + \exp(\xi_1) + \exp(\xi_2) + A_{12} \exp(\xi_1 + \xi_2)}.$$

3.3. Three-soliton solution

Now, we postulate an explicit form of three-soliton solution as

$$\begin{aligned} f = {} & 1 + \exp(\xi_1) + \exp(\xi_2) + \exp(\xi_3) + A_{12} \exp(\xi_1 + \xi_2) + A_{13} \exp(\xi_1 + \xi_3) \\ & + A_{23} \exp(\xi_2 + \xi_3) + A_{12}A_{13}A_{23} \exp(\xi_1 + \xi_2 + \xi_3), \end{aligned} \tag{9}$$

here,

$$A_{ij} = \frac{k_ik_jl_il_j(k_i - k_j)(l_i - l_j) - (k_il_j - l_ik_j)^2 + (l_im_j - m_il_j)^2}{k_ik_jl_il_j(k_i + k_j)(l_i + l_j) - (k_il_j - l_ik_j)^2 + (l_im_j - m_il_j)^2}.$$

Furthermore, the formula (9) is a solution of Eq. (3) only if the equality

$$\begin{aligned} & (l_1B_1 + l_2B_2 + l_3B_3 + l_1l_2l_3(k_1 + k_2)(k_1 + k_3)(k_2 + k_3) \\ & (l_1 + l_2 + l_3))A_{12}A_{13}A_{23} + (-l_1B_1 - l_2B_2 + l_3B_3 \\ & + l_1l_2l_3(k_1 + k_2)(k_1 - k_3)(k_2 - k_3)(l_1 + l_2 - l_3))A_{12} \\ & + (-l_1B_1 + l_2B_2 - l_3B_3 - l_1l_2l_3(k_1 - k_2)(k_1 + k_3)(k_2 - k_3) \\ & (l_1 - l_2 + l_3))A_{13} + (l_1B_1 - l_2B_2 - l_3B_3 - l_1l_2l_3(k_1 - k_2) \\ & (k_1 - k_3)(k_2 + k_3)(l_1 - l_2 - l_3))A_{23} = 0, \end{aligned}$$

vanishes identically, with

$$B_1 = (l_2m_3 - l_3m_2)^2 - (k_2l_3 - k_3l_2)^2,$$
$$B_2 = (l_1m_3 - l_3m_1)^2 - (k_1l_3 - k_3l_1)^2,$$
$$B_3 = (l_1m_2 - l_2m_1)^2 - (k_1l_2 - k_2l_1)^2.$$

Therefore, the formula (9) is not a solution of Eq. (3). Eq. (1) possesses neither three-soliton solution nor N-soliton solutions for any $N > 3$. It is obvious that Eq. (1) do not have solitary wave solutions with elastic collisions.

4. Lump-type solutions

Although Eq. (1) is non-integrable on its own in the sense of both Painlevé property and multi-soliton criterion, we can still look for some exact solutions. In what follows, we study lump-type solutions [16–21] of Eq. (1) through searching for positive quadratic function solution

$$f = g^2 + h^2 + c_0, \tag{10}$$

with

$$g = a_1x + a_2y + a_3z + a_4t + a_5,$$
$$h = b_1x + b_2y + b_3z + b_4t + b_5,$$

to the bilinear Eq. (3), where a_i, b_i, c_0, $1 \leqslant i \leqslant 5$, are real parameters to be determined. We note that all the solutions obtained through Eq. (10) do not satisfy the criterion of localizing in all directions in space, therefore, they are not lump solutions. However, they are rationally localized in many directions in space, so this kind of solution is called lump-type solution [8]. Substitution of Eq. (10) into Eq. (3) shows that f is a solution of the equation provided that some relations of the parameters hold. Two sets of them are analyzed in detail.

The first set of constraining relations among the parameters is

$$\{a_1 = -\frac{b_1b_2}{a_2}, a_4 = \frac{3(b_1^2b_2^2 - a_2^2a_3^2)}{a_2^3}, b_3 = \frac{b_1(a_2^2 + b_2^2)}{a_2^2} + \frac{a_3b_2}{a_2},$$
$$b_4 = -\frac{3(a_2a_3 + b_1b_2)(2a_2^2b_1 + a_2a_3b_2 + b_1b_2^2)}{a_2^4}\},$$

which needs to satisfy $a_2 \neq 0$ to make the corresponding solution f to be well defined. The condition $c_0 > 0$ guarantees the positiveness of f. In addition, the determinant condition $\{\Delta_1 = a_1b_2 - a_2b_1 = -\frac{b_1}{a_2}(a_2^2 + b_2^2) \neq 0, \Delta_2 = a_1b_3 - a_3b_1 = -\frac{b_1(a_2^2+b_2^2)(a_2a_3+b_1b_2)}{a_2^3} \neq 0, \Delta_3 = a_2b_3 - a_3b_2 = \frac{b_1(a_2^2+b_2^2)}{a_2} \neq 0\}$ means that directions of (a_1, a_2) with (b_1, b_2) in the xy plane, (a_1, a_3) with (b_1, b_3) in the xz plane and (a_2, a_3) with (b_2, b_3) in the yz plane are not parallel.

These parameters in this set yield the positive quadratic function solution of f as

$$f = (-\frac{b_1b_2}{a_2}x + a_2y + a_3z + \frac{3(b_1^2b_2^2 - a_2^2a_3^2)}{a_2^3}t + a_5)^2 + (b_1x + b_2y$$
$$+ (\frac{b_1(a_2^2 + b_2^2)}{a_2^2} + \frac{a_3b_2}{a_2})z - \frac{3(a_2a_3 + b_1b_2)(2a_2^2b_1 + a_2a_3b_2 + b_1b_2^2)}{a_2^4}t + b_5)^2 + c_0,$$

which leads to the lump-type solution

$$u = \frac{4(-\frac{b_1b_2}{a_2}g + b_1h)}{g^2 + h^2 + c_0}, \tag{11}$$

the functions of g and h are given as follows,

$$g = -\frac{b_1b_2}{a_2}x + a_2y + a_3z + \frac{3(b_1^2b_2^2 - a_2^2a_3^2)}{a_2^3}t + a_5,$$
$$h = b_1x + b_2y + (\frac{b_1(a_2^2 + b_2^2)}{a_2^2} + \frac{a_3b_2}{a_2})z$$
$$- \frac{3(a_2a_3 + b_1b_2)(2a_2^2b_1 + a_2a_3b_2 + b_1b_2^2)}{a_2^4}t + b_5,$$

to the (3+1)-dimensional generalized KP Eq. (1) under the transformation (4).

To show the localized behavior of the presented lump-type solution clearly, 3D plots, contour plots and curves with particular choices of the involved parameters in the potential function u are plotted in Fig. 1.

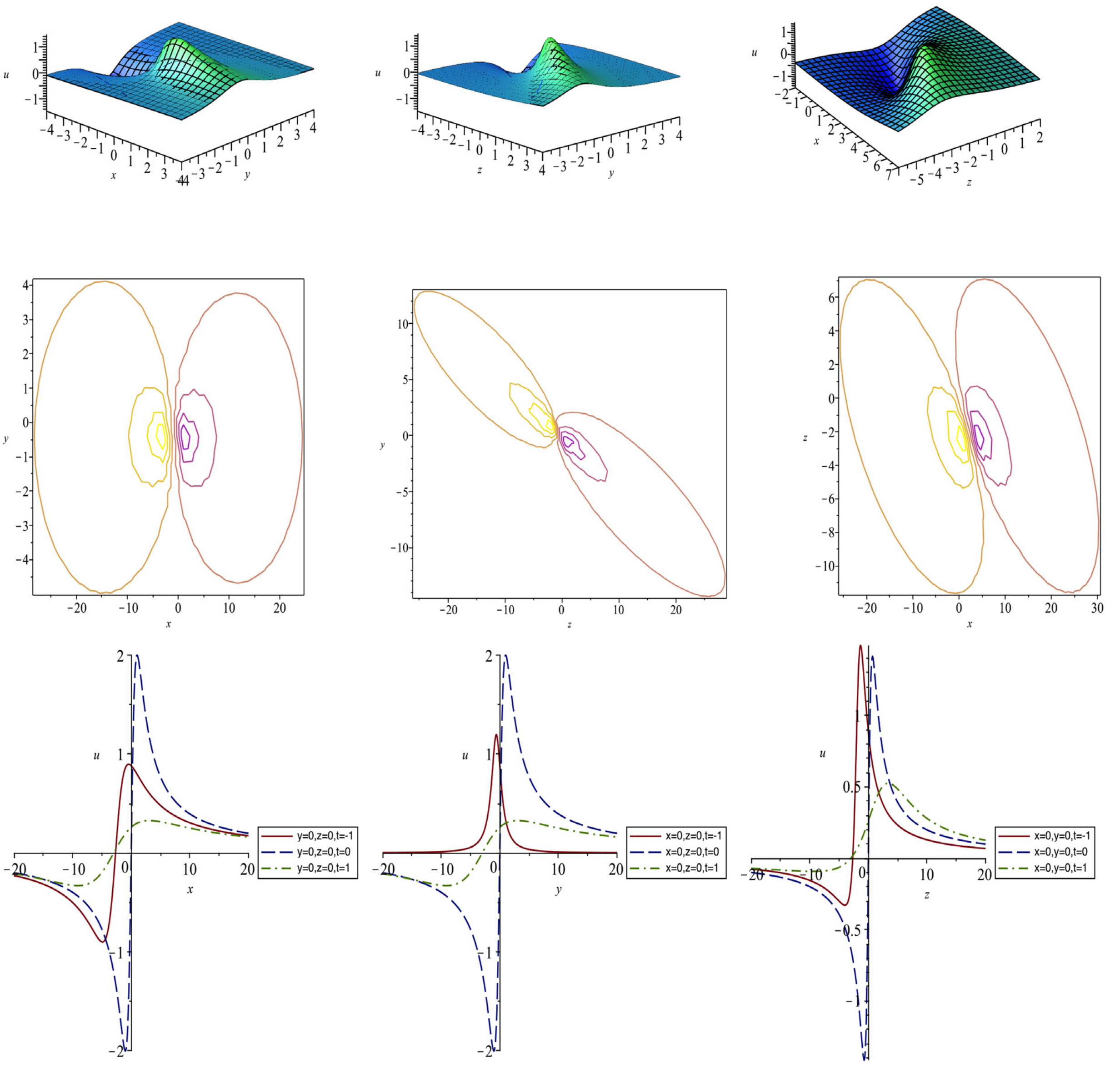

Fig. 1. Profiles of lump-type solution of u via Eq. (11) with parameters $\{a_2 = 3, a_3 = 1, a_5 = 3, b_1 = 1, b_2 = 1, b_5 = 3, c_0 = 2\}$ at $\{t = 0, z = 1\}$, $\{t = 0, x = 1\}$, $\{t = 0, y = 1\}$. 3D plots (top), contour plots (middle) and curve plots (bottom), correspondingly.

The second set of constraining conditions on the parameters is

$$\{a_4 = \frac{3a_1^2}{a_2} - \frac{3(a_2(a_3^2 - b_3^2) + 2a_3b_2b_3)}{a_2^2 + b_2^2}, b_1 = \frac{a_1b_2}{a_2},$$
$$b_4 = \frac{3a_1^2b_2}{a_2^2} - \frac{3(b_2(b_3^2 - a_3^2) + 2a_2a_3b_3)}{a_2^2 + b_2^2}, c_0 = \frac{a_1^3(a_2^2 + b_2^2)^3}{a_2^3(a_2b_3 - a_3b_2)^2}\}.$$

This set of parameters generates quadratic function solution defined by Eq. (10), to the bilinear Eq. (3), and further the resulting quadratic function solution yields lump-type solution,

$$u = \frac{4(a_1g + \frac{a_1b_2}{a_2}h)}{g^2 + h^2 + \frac{a_1^3(a_2^2+b_2^2)^3}{a_2^3(a_2b_3-a_3b_2)^2}}, \tag{12}$$

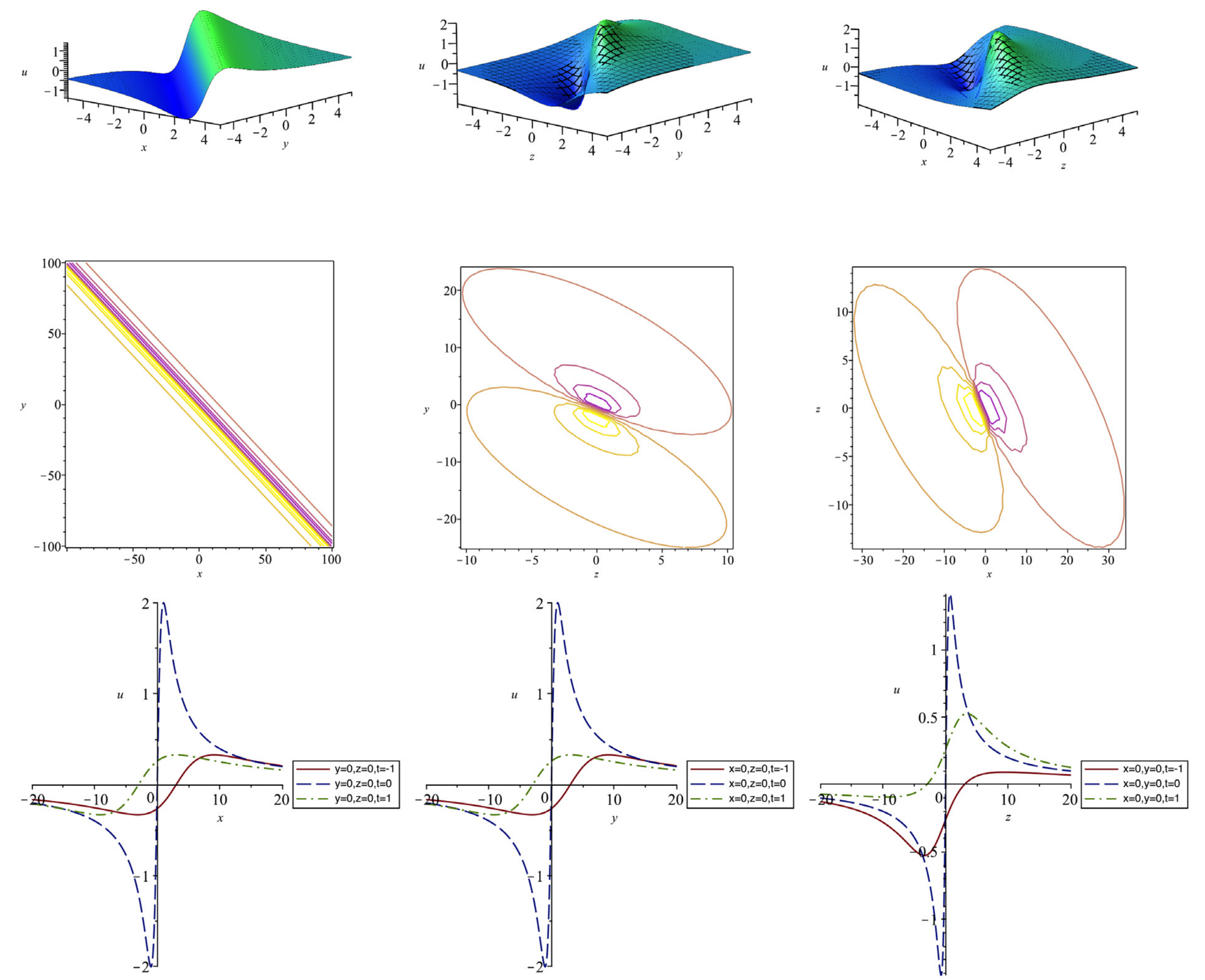

Fig. 2. Profiles of lump-type solution u of Eq. (12) with parameters $\{a_1 = 1, a_2 = 1, a_3 = 3, a_5 = 3, b_2 = 2, b_3 = 1, b_5 = 4\}$ at $\{t = 0, z = 1\}$, $\{t = 0, x = 1\}$, $\{t = 0, y = 1\}$. 3D plots (top), contour plots (middle) and curve plots (bottom), correspondingly.

here, the functions g and h satisfy

$$g = a_1x + a_2y + a_3z + \left(\frac{3a_1^2}{a_2} - \frac{3(a_2(a_3^2 - b_3^2) + 2a_3b_2b_3)}{a_2^2 + b_2^2}\right)t + a_5,$$

$$h = \frac{a_1b_2}{a_2}x + b_2y + b_3z + \left(\frac{3a_1^2b_2}{a_2^2} - \frac{3(b_2(b_3^2 - a_3^2) + 2a_2a_3b_3)}{a_2^2 + b_2^2}\right)t + b_5.$$

If the conditions $a_2 \neq 0$ and $a_2b_3 - a_3b_2 \neq 0$ are satisfied, this class of rational function solutions is well defined. If we choose the parameters guaranteeing $a_1a_2 > 0$, the analyticity of the rational solution can be achieved. The conditions $a_2b_3 - a_3b_2 \neq 0$ and $a_1b_3 - a_3b_1 \neq 0$ guarantee that directions (a_2, a_3) with (b_2, b_3) and (a_1, a_3) with (b_1, b_3) are not parallel in the yz plane and xz plane, while another corresponding determinant condition $a_1b_2 - a_2b_1 = 0$ indicates that two directions (a_1, a_2) and (b_1, b_2) in the xy plane are parallel. See plots in Fig. 2.

5. Summary

In this paper, we have carried out the Painlevé test and the multi-soliton analysis for a (3+1)-dimensional generalized KP equation. Both the Painlevé property and the multi-soliton criterion show that Eq. (1) is not completely integrable. Our result shows that Eq. (1) is only integrable in a conditional sense and its solitary wave solution can be split apart into three solitons, while an additional quantity has to vanish identically. Such a characteristic is generally considered to be incompatible with integrability.

In addition, based on the Hirota bilinear formula, we have analyzed positive quadratic function solution and thereby lump-type solutions of the corresponding generalized KP equation. The conditions imposed on the parameters have been explicitly presented to guarantee the well definedness, the positiveness and the localization of the solutions. Those solutions can help us recognize the solvability characteristics of the considered partial differential equations.

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