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An application of the exponential rational function method to exact solutions to the Drinfeld–Sokolov system

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ABSTRACT

In the present study, we investigate solitary wave solutions to the Drinfeld–Sokolov system. This system describes the dispersive water waves in fluid mechanics and is known as an anisotropic integrable system along with a Lax extension of the famous Korteweg–De Vries equation. The pivotal aim of this study is to make much more contributions to the fluid mechanics by extracting various versions of traveling waves which are helpful to further understand the mechanism of propagation of the dispersive waves. The presented method used for achieving our main goal is the generalized exponential rational function method. Several different categories of solutions are all determined using a single structure in the utilized technique. After comparing with the existing methods, such as the Exp-function method, the generated exact wave solutions show the efficiency of the utilized method. For the sake of discovering more facts about the numerical nature of obtained wave solutions, several 2D diagrams have been shown. Further, the acquired results of this paper also pave the way for the use of the technique in solving other nonlinear models.

Introduction

Many practical phenomena around us are described by the language of differential equations. This great importance has led to many attempts to determine the exact solution or in some cases approximate solutions for these models. In some cases, investigating exact traveling wave solutions principally exact solitary wave solutions has a significant role in the study of physical phenomena. For example, exact N-soliton solutions to pKP-BKP and other NPDE equations have been obtained in [1–4] via the Hirota bilinear operation. Kuo extracted the resonant multi-soliton solutions by using the linear superposition principle [5–7]. Some exact solutions to the fifth-order potential Bogoyavlenskii–Schiff equation have been obtained in [8]. Refs. [9–26] show examples of the successful application of differential equations in solving some applied models. In [27] Gao et al. have employed a newly extended direct algebraic technique to acquire novel explicit solutions for the nonlinear Zoomeron. Also, some interesting applications of the fractional calculus in image processing through designing novel fractional masks have been presented in [28–30]. Moreover, Yel and her collaborators performed some complex simulations with dark–bright to the Hirota–Maccari system [31]. To the approaching numerical methods, Ghanbari derived the approaching approximate solutions to a model in forecasting the second wave of out-breaking COVID-19 in Iran [32]. McCue and his collaborators studied exact sharp-fronted traveling wave solutions of the Fisher–KPP equation in [33]. In [34], some new results for the time-fractional Phi-four equation using q-homotopy analysis transform method were presented. Optical solitons were also studied for various other models in nonlinear optics [35–40]. Further, the wave theory arising in phenomena in the framework of plasma, fluid dynamics, optical fibers, and in shallow water were studied in the literature [41–47]. On the other hand, in some problems, determining the exact solution for equations using existing methods is not possible and often we have to utilize numerical techniques in approximating the solution of such problems [48–56].

In the last decade, the research of finding new methods in solving solutions and seeking different types of solitary wave solutions has not slowed down. In this direction, there are some theoretical

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aspects and results in terms of establishing supplementary approaches in the literature. For example, decomposition method [57], sine–cosine method [58], Darboux transformation [59], a modified ϕ^6 method [60], the Hirota method [61], Lie group analysis [62], modified simple equation method [63], similarity reduced method, tanh method, and inverse scattering scheme [64]. Some other methodologies in the literature can be found in [65–71]. Recently, Ghanbari et al. proposed the so-called generalized exponential rational function method (GERFM) [72–77], to retrieve analytical solutions to general nonlinear partial differential equations. This technique has a general approach as some known methodologists, such as the transformed rational function technique [78], tanh–coth method, and what is considered in [79] as so-called seed solutions. The Ref. [80] has presented a general class of explicit solutions to the general Riccati equation, which could yield explicit traveling wave solutions to nonlinear PDEs via polynomial or rational function expansions.

In this paper, we study the Drinfeld–Sokolov (DS) system given by [81–83]

$$\begin{aligned} &u_t + \left(v^2\right)_x = 0,\\ &v_t - \delta v_{xxx} + 3\lambda u_x v + 3\sigma u v_x = 0. \end{aligned} \tag{1}$$

where $u = u(x,t)$, $v = v(x,t)$ and δ, λ, σ are nonzero constants. The system was developed by Drinfeld and Sokolov as an example of a system of nonlinear equations that possesses Lax pairs of a special form. This property has been proved in [84]. The nonlocal symmetry of the system was obtained, and infinitely many nonlocal symmetries are obtained by introducing the internal parameters [85].

Due to the high importance of this system, so far it has been studied using many different tools. Exp-function and its modification methods have been employed in [81] to obtain an exact solution of the nonlinear Drinfeld–Sokolov system. In [83], a variety of exact traveling wave solutions to the Drinfeld–Sokolov system is investigated by using the tanh method and the sine–cosine method. It is also notable that, Gao et al. applied the q-homotopy analysis transform method to find the numerical solutions for the fractional Drinfeld–Sokolov–Wilson equation with Mittag-Leffler law [86]. Novel solutions to the time-fractional of Drinfeld–Sokolov–Wilson system, which is based upon the Liouville–Caputo fractional integral (LCFI), the Caputo–Fabrizio fractional integral, and the Atangana–Baleanu fractional integral in the sense of the LCFI is constructed in [87] using the Adomian decomposition method. This article contains the following sections. The second section of the present paper discusses the analysis of the method in detail. In the third section, we will use the method to find the exact solutions of the DS system. In this section, we observe that our results differ from those solutions presented in the literature for the Drinfeld–Sokolov system. Further, several 2D numerical simulations corresponding to some of these solutions are depicted in this section. Finally, conclusions are presented in the last section of the article.

A formal description of the technique

In the recent literature, many nonlinear models have been solved using the GERFM. For instance, please see [72–77]. These successful experiences encourage us to use the method in solving the DS system.

The general process of applying the method is outlined below for solving a nonlinear model such as

$$\mathcal{F}(\mathcal{U}, \mathcal{U}_x, \mathcal{U}_t, \mathcal{U}_{xx}, \ldots) = 0. \tag{2}$$

1. Taking $\mathcal{U} = \mathcal{U}(\zeta)$ and $\zeta = \kappa x - \omega t$ into account in Eq. (2), we attain
$$\mathcal{F}(\mathcal{U}, \kappa\mathcal{U}'_\zeta, -\omega\mathcal{U}'_\zeta, \kappa^2\mathcal{U}''_\zeta, \ldots) = 0, \tag{3}$$
where the values of κ and ω are two unknowns.
2. Now, we assume that the solution of the equation Eq. (3) follows the following general formulation
$$\mathcal{U}(\zeta) = a_0 + \sum_{k=1}^{n_0} a_k \mathcal{H}^k(\zeta) + \sum_{k=1}^{n_0} \frac{b_k}{\mathcal{H}^k(\zeta)}, \tag{4}$$
where
$$\mathcal{H}(\zeta) = \frac{\mu_1 exp(\nu_1\zeta) + \mu_2 exp(\nu_2\zeta)}{\mu_3 exp(\nu_3\zeta) + \mu_4 exp(\nu_4\zeta)}. \tag{5}$$
The unknowns in these structures, including $\mu_i, \nu_i (1 \le i \le 4)$, a_0, a_k and $b_k (1 \le k \le n_0)$, are determined later. To calculate n_0, we also use some known rules of balancing.
3. Inserting Eq. (4) into Eq. (3), we will arrive to a algebraic equation in terms of $\Lambda_i = e^{\nu_i\zeta}$ for $i = 1, \ldots, 4$. After setting each of the coefficients of different powers of ζ to zero, a system of nonlinear equations is characterized.
4. After solving this nonlinear system using symbolic computational packages such as Maple, and proper placement of the resulting solutions, analytical solutions for the original equation are introduced.

It is notable that the algorithm outlined in (4) and (5) is a particular case of the algorithm for the multiple exp-function method (refer to the Ref. [88] for more information on the multiple exp-function method).

Mathematical analysis for the DS system

Let us start with the following assumptions for the solutions

$$u(x,t) = \mathcal{U}(\zeta), v(x,t) = \mathcal{V}(\zeta), \quad \zeta = x - \omega t, \tag{6}$$

where ω is an arbitrary constant needs to be calculated.

If we use transformation (6) in Eq. (1), the following nonlinear system of ODEs is obtained [81]

$$\begin{aligned} &-\omega\mathcal{U}'(\zeta) + (\mathcal{V}^2(\zeta))' = 0,\\ &-\omega\mathcal{V}'(\zeta) - \delta\mathcal{V}'''(\zeta) + 3\lambda\mathcal{U}'(\zeta)\mathcal{V}(\zeta) + 3\sigma\mathcal{U}(\zeta)\mathcal{V}'(\zeta) = 0. \end{aligned} \tag{7}$$

From the first equation in (7), we obtain

$$\mathcal{U}(\zeta) = \frac{1}{\omega}\mathcal{V}^2(\zeta). \tag{8}$$

Substituting Eq. (8) into the second equation of the system and integrating, one gets

$$\omega\delta\mathcal{V}''(\zeta) - (2\lambda + \sigma)\mathcal{V}^3(\zeta) + \omega^2\mathcal{V}(\zeta) = 0. \tag{9}$$

Application of balancing principles between $\mathcal{V}''(\zeta)$ and $\mathcal{V}^3(\zeta)$ in Eq. (9) suggests $3n_0 = n_0+2$. So, we get $n_0 = 1$. Subsequently, the solution takes the following structure

$$\mathcal{V}(\zeta) = a_0 + a_1\mathcal{H}(\zeta) + \frac{b_1}{\mathcal{H}(\zeta)}. \tag{10}$$

where $\mathcal{H}(\zeta)$ is giving by (5).

By following the required steps for the method, the main results in this paper are as follows.

Set 1: Supposing $\mu = [-1, -1, 1, -1]$ and $\nu = [1, -1, 1, -1]$ yields

$$\mathcal{H}(\zeta) = -\frac{\cosh(\zeta)}{\sinh(\zeta)}. \tag{11}$$

Subset 1: For $\sigma + 2\lambda > 0$,

$$\omega = 2\delta, a_0 = 0, a_1 = 0, b_1 = \frac{2\delta^2}{\sqrt{2\lambda + \sigma}}.$$

After considering these results in Eqs. (10) and (11), it reads

$$\mathcal{V}(\zeta) = -\frac{2\delta}{\sqrt{2\lambda + \sigma}\coth(\zeta)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_1(x,t) = -\frac{2\delta}{\sqrt{2\lambda + \sigma}\coth(x - 2\delta t)}. \tag{12}$$

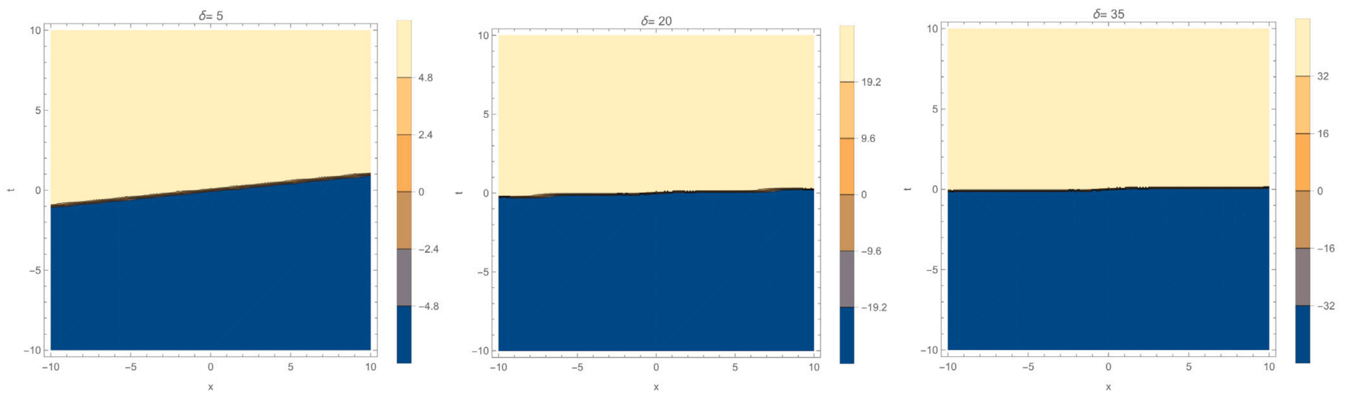

Fig. 1. 2D contour plot corresponding to $v_1(x,y)$ for $\lambda = 0.7, \sigma = 0.5$ different δ's.

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 1.

Subset 2: For $\sigma < -2\lambda$,

$$\omega = -4\delta, a_0 = 0, a_1 = \frac{4\delta}{\sqrt{-4\lambda - 2\sigma}}, b_1 = -\frac{4\delta}{\sqrt{-4\lambda - 2\sigma}}.$$

After considering these results in Eqs. (10) and (11), it reads

$$\mathscr{V}(\zeta) = -\frac{4\delta}{\sqrt{-4\lambda - 2\sigma}\sinh(\zeta)\cosh(\zeta)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_2(x,t) = -\frac{4\delta}{\sqrt{-4\lambda - 2\sigma}\sinh(x+4\delta t)\cosh(x+4\delta t)}. \tag{13}$$

Subset 3: For $\sigma + 2\lambda > 0$,

$$\omega = 8\delta, a_0 = 0, a_1 = \frac{4\delta}{\sqrt{2\lambda + \sigma}}, b_1 = \frac{4\delta}{\sqrt{2\lambda + \sigma}}.$$

After considering these results in Eqs. (10) and (11), it reads

$$\mathscr{V}(\zeta) = -\frac{4\delta\left((\coth(\zeta))^2 + 1\right)}{\sqrt{2\lambda + \sigma}\coth(\zeta)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_3(x,t) = -\frac{4\delta\left((\coth(x-8\delta t))^2 + 1\right)}{\sqrt{2\lambda + \sigma}\coth(x-8\delta t)}. \tag{14}$$

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 2.

Set 2: Supposing $\mu = [1, -3, -1, 1]$ and $\nu = [1, -1, 1, -1]$ yields

$$\mathscr{H}(\zeta) = \frac{\cosh(\zeta) - 2\sinh(\zeta)}{\sinh(\zeta)}. \tag{15}$$

Subset 1: For $\sigma + 2\lambda > 0$,

$$\omega = 2\delta, a_0 = -\frac{4\delta}{\sqrt{2\lambda + \sigma}}, a_1 = 0, b_1 = -\frac{6\delta}{\sqrt{2\lambda + \sigma}}.$$

After considering these results in Eqs. (10) and (15), it reads

$$\mathscr{V}(\zeta) = -\frac{2\delta(-\sinh(\zeta) + 2\cosh(\zeta))}{\sqrt{2\lambda + \sigma}(\cosh(\zeta) - 2\sinh(\zeta))}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_4(x,t) = -\frac{2\delta(-\sinh(x-2\delta t) + 2\cosh(x-2\delta t))}{\sqrt{2\lambda + \sigma}(\cosh(x-2\delta t) - 2\sinh(x-2\delta t))}. \tag{16}$$

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 3.

Set 3: Supposing $\mu = [-2, 0, -1, 1]$ and $\nu = [1, -1, 1, -1]$ yields

$$\mathscr{H}(\zeta) = \frac{\cosh(\zeta) + \sinh(\zeta)}{\sinh(\zeta)}. \tag{17}$$

Subset 1: For $\sigma + 2\lambda > 0$,

$$\omega = 2\delta, a_0 = \frac{2\delta}{\sqrt{2\lambda + \sigma}}, a_1 = -\frac{2\delta}{\sqrt{2\lambda + \sigma}}, b_1 = 0.$$

After considering these results in Eqs. (10) and (17), it reads

$$\mathscr{V}(\zeta) = -\frac{2\delta\cosh(\zeta)}{\sqrt{2\lambda + \sigma}\sinh(\zeta)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_5(x,t) = -\frac{2\delta\cosh(x-2\delta t)}{\sqrt{2\lambda + \sigma}\sinh(x-2\delta t)}. \tag{18}$$

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 4.

Set 4: Supposing $\mu = [2-i, -2-i, -1, 1]$ and $\nu = [i, -i, i, -i]$ yields

$$\mathscr{H}(\zeta) = \frac{-2\sin(\zeta) + \cos(\zeta)}{\sin(\zeta)}. \tag{19}$$

Subset 1: For $\sigma < -2\lambda$,

$$\omega = -2\delta, a_0 = -\frac{4\delta}{\sqrt{-\sigma - 2\lambda}}, a_1 = 0, b_1 = -\frac{10\delta}{\sqrt{-\sigma - 2\lambda}}.$$

After considering these results in Eqs. (10) and (19), it reads

$$\mathscr{V}(\zeta) = -\frac{2\delta(2\cos(\zeta) + \sin(\zeta))}{\sqrt{-\sigma - 2\lambda}(-2\sin(\zeta) + \cos(\zeta))}.$$

In this case, Eq. (1) possesses the following wave soliton solution

$$v_6(x,t) = -\frac{2\delta(2\cos(x+2\delta t) + \sin(x+2\delta t))}{\sqrt{-\sigma - 2\lambda}(-2\sin(x+2\delta t) + \cos(x+2\delta t))}. \tag{20}$$

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 5.

Set 5: Supposing $\mu = [-1-i, 1-i, -1, 1]$ and $\nu = [i, -i, i, -i]$ yields

$$\mathscr{H}(\zeta) = \frac{\cos(\zeta) + \sin(\zeta)}{\sin(\zeta)}. \tag{21}$$

Subset 1: For $\sigma < -2\lambda$,

$$\omega = -2\delta, a_0 = -\frac{2\delta}{\sqrt{-\sigma - 2\lambda}}, a_1 = 0, b_1 = \frac{4\delta}{\sqrt{-\sigma - 2\lambda}}.$$

After considering these results in Eqs. (10) and (21), it reads

$$\mathscr{U}(\zeta) = -\frac{2\delta(\cos(\zeta) - \sin(\zeta))}{\sqrt{-\sigma - 2\lambda}(\cos(\zeta) + \sin(\zeta))}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_7(x,t) = -\frac{2\delta(\cos(x+2\delta t) - \sin(x+2\delta t))}{\sqrt{-\sigma - 2\lambda}(\cos(x+2\delta t) + \sin(x+2\delta t))}. \tag{22}$$

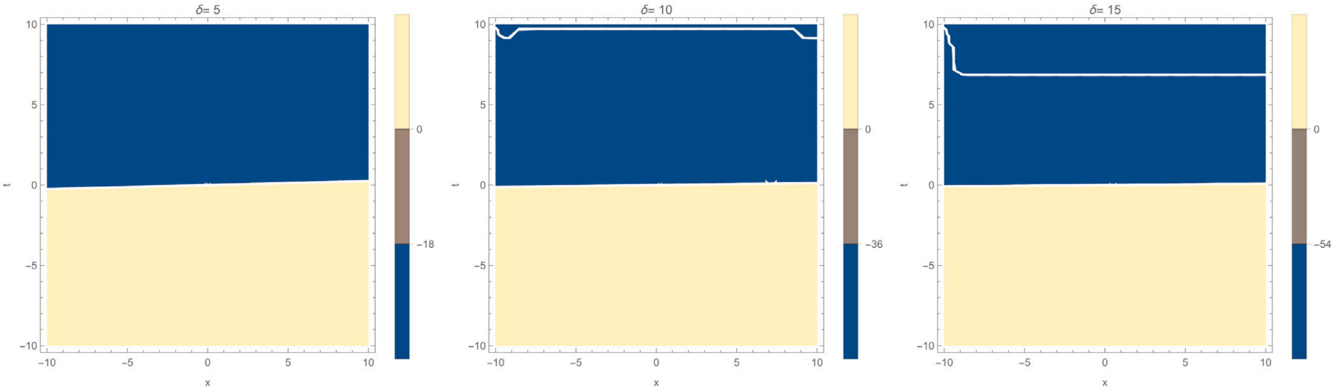

Fig. 2. 2D contour plot corresponding to $v_3(x,y)$ for $\lambda = 1.3, \sigma = 0.9$ different δ's.

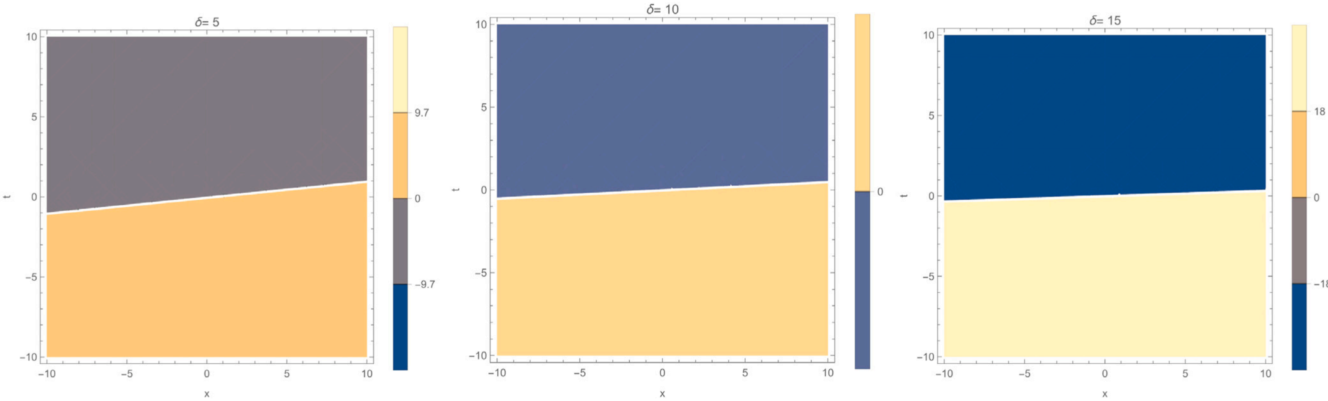

Fig. 3. 2D contour plot corresponding to $v_4(x,y)$ for $\lambda = 1.2, \sigma = 0.1$ different δ's.

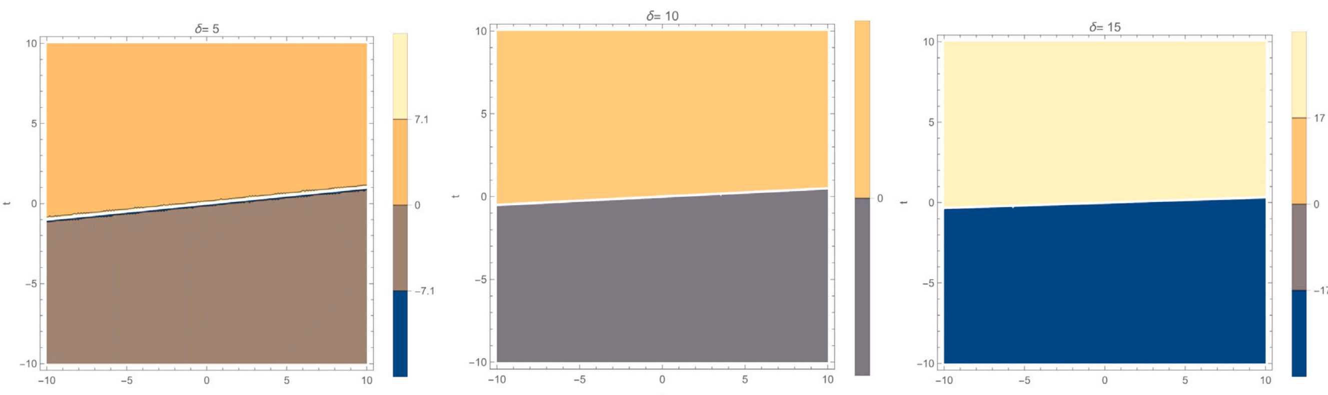

Fig. 4. 2D contour plot corresponding to $v_5(x,y)$ for $\lambda = 1.2, \sigma = 0.2$ different δ's.

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 6.

Set 6: Supposing $\mu = [-2-i, 2-i, -1, 1]$ and $\nu = [1, -1, 1, -1]$ yields

$$\mathscr{H}(\zeta) = \frac{\cos(\zeta) + 2\sin(\zeta)}{\sin(\zeta)}. \tag{23}$$

Subset 1: For $\sigma < -2\lambda$,

$$\omega = -2\delta, a_0 = -\frac{4\delta}{\sqrt{-\sigma - 2\lambda}}, a_1 = 0, b_1 = \frac{10\delta}{\sqrt{-\sigma - 2\lambda}}.$$

After considering these results in Eqs. (10) and (23), it reads

$$\mathscr{V}(\zeta) = -\frac{2\delta(-\sin(\zeta) + 2\cos(\zeta))}{\sqrt{-\sigma - 2\lambda}(\cos(\zeta) + 2\sin(\zeta))}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_8(x,t) = -\frac{2\delta(-\sin(x + 2\delta t) + 2\cos(x + 2\delta t))}{\sqrt{-\sigma - 2\lambda}(\cos(x + 2\delta t) + 2\sin(x + 2\delta t))}. \tag{24}$$

Set 7: Supposing $\mu = [1-i, -1-i, -1, 1]$ and $\nu = [i, -i, i, -i]$ yields

$$\mathscr{H}(\zeta) = \frac{\cos(\zeta) - \sin(\zeta)}{\sin(\zeta)}. \tag{25}$$

Subset 1: For $\sigma < -2\lambda$,

$$\omega = -2\delta, a_0 = -\frac{2\delta}{\sqrt{-\sigma - 2\lambda}}, a_1 = 0, b_1 = -\frac{4\delta}{\sqrt{-\sigma - 2\lambda}}.$$

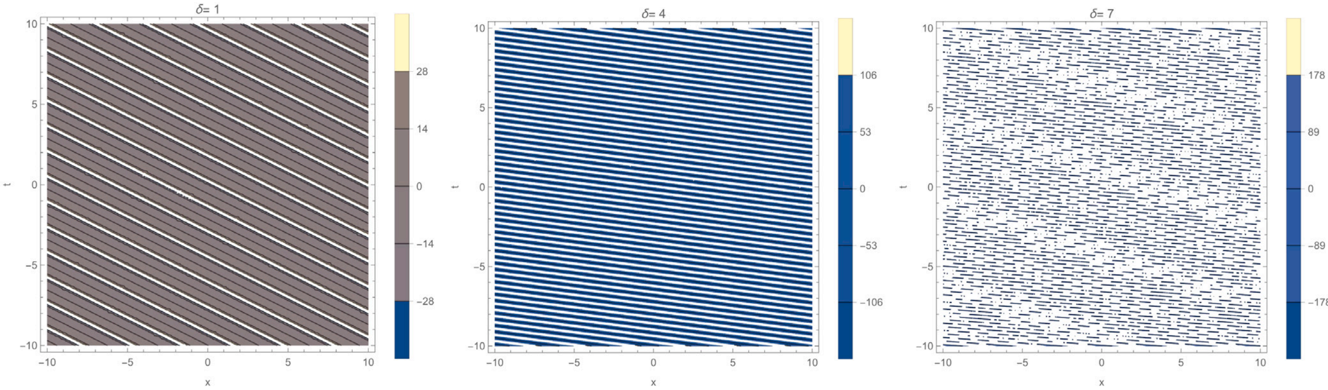

Fig. 5. 2D contour plot corresponding to $v_6(x,y)$ for $\lambda=0.3, \sigma=-0.7$ different δ's.

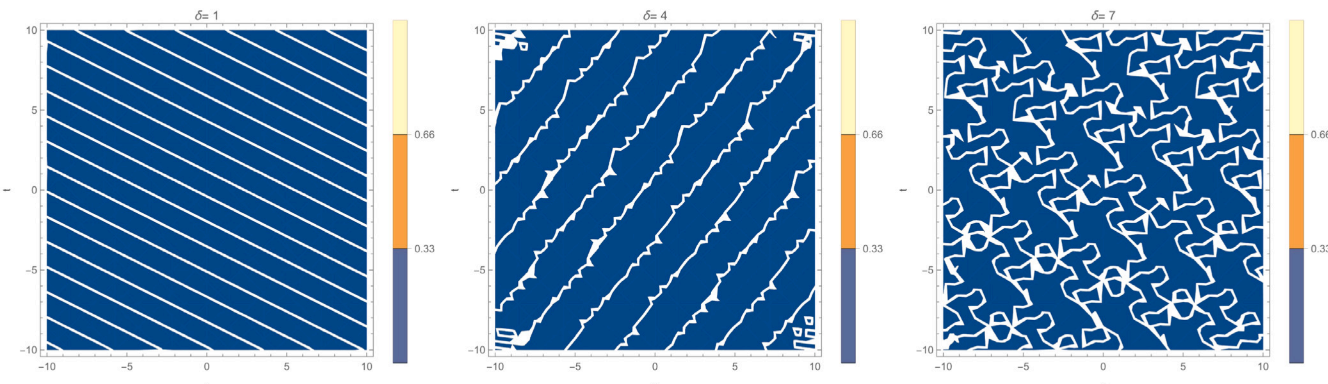

Fig. 6. 2D contour plot corresponding to $v_7(x,y)$ for $\lambda=0.5, \sigma=-0.9$ different δ's.

After considering these results in Eqs. (10) and (25), it reads

$$\mathscr{V}(\zeta)=-\frac{2\delta(\cos(\zeta)+\sin(\zeta))}{\sqrt{-\sigma-2\lambda}(\cos(\zeta)-\sin(\zeta))}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_9(x,t)=-\frac{2\delta(\cos(x+2\delta t)+\sin(x+2\delta t))}{\sqrt{-\sigma-2\lambda}(\cos(x+2\delta t)-\sin(x+2\delta t))}. \tag{26}$$

Set 8: Supposing $\mu=[-3,-2,1,1]$ and $\nu=[0,1,0,1]$ yields

$$\mathscr{H}(\zeta)=\frac{-3-2e^{\zeta}}{1+e^{\zeta}}. \tag{27}$$

Subset 1: For $\sigma+2\lambda>0$,

$$\omega=\frac{\delta}{2}, a_0=-\frac{5\delta}{2\sqrt{2\lambda+\sigma}}, a_1=0, b_1=-\frac{6\delta}{\sqrt{2\lambda+\sigma}}.$$

After considering these results in Eqs. (10) and (27), it reads

$$\mathscr{V}(\zeta)=\frac{\delta\left(2e^{\zeta}-3\right)}{2\sqrt{2\lambda+\sigma}\left(3+2e^{\zeta}\right)}.$$

In this case, Eq. (1) possesses the following wave soliton solution

$$v_{10}(x,t)=\frac{\delta\left(2e^{\left(x-\frac{\delta}{2}t\right)}-3\right)}{2\sqrt{2\lambda+\sigma}\left(3+2e^{\left(x-\frac{\delta}{2}t\right)}\right)}. \tag{28}$$

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 7.

Set 9: Supposing $\mu=[-1,-2,1,1]$ and $\nu=[1,0,1,0]$ yields

$$\mathscr{H}(\zeta)=\frac{-e^{\zeta}-2}{e^{\zeta}+1}. \tag{29}$$

Subset 1: For $\sigma+2\lambda>0$,

$$\omega=\frac{\delta}{2}, a_0=\frac{3\delta}{2\sqrt{2\lambda+\sigma}}, a_1=0, b_1=\frac{2\delta}{\sqrt{2\lambda+\sigma}}.$$

After considering these results in Eqs. (10) and (29), it reads

$$\mathscr{V}(\zeta)=-\frac{\delta\left(e^{\zeta}-2\right)}{2\sqrt{2\lambda+\sigma}\left(e^{\zeta}+2\right)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_{11}(x,t)=-\frac{\delta\left(e^{\left(x-\frac{\delta}{2}t\right)}-2\right)}{2\sqrt{2\lambda+\sigma}\left(e^{\left(x-\frac{\delta}{2}t\right)}+2\right)}. \tag{30}$$

Set 10: Supposing $\mu=[2,1,1,1]$ and $\nu=[1,0,1,0]$ yields

$$\mathscr{H}(\zeta)=\frac{2e^{\zeta}+1}{e^{\zeta}+1}. \tag{31}$$

Subset 1: For $\sigma+2\lambda>0$,

$$\omega=\frac{\delta}{2}, a_0=\frac{3\delta}{2\sqrt{2\lambda+\sigma}}, a_1=0, b_1=-\frac{2\delta}{\sqrt{2\lambda+\sigma}}.$$

After considering these results in Eqs. (10) and (31), it reads

$$\mathscr{V}(\zeta)=\frac{\delta\left(2e^{\zeta}-1\right)}{2\sqrt{2\lambda+\sigma}\left(2e^{\zeta}+1\right)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_{12}(x,t)=\frac{\delta\left(2e^{\left(x-\frac{\delta}{2}t\right)}-1\right)}{2\sqrt{2\lambda+\sigma}\left(2e^{\left(x-\frac{\delta}{2}t\right)}+1\right)}. \tag{32}$$

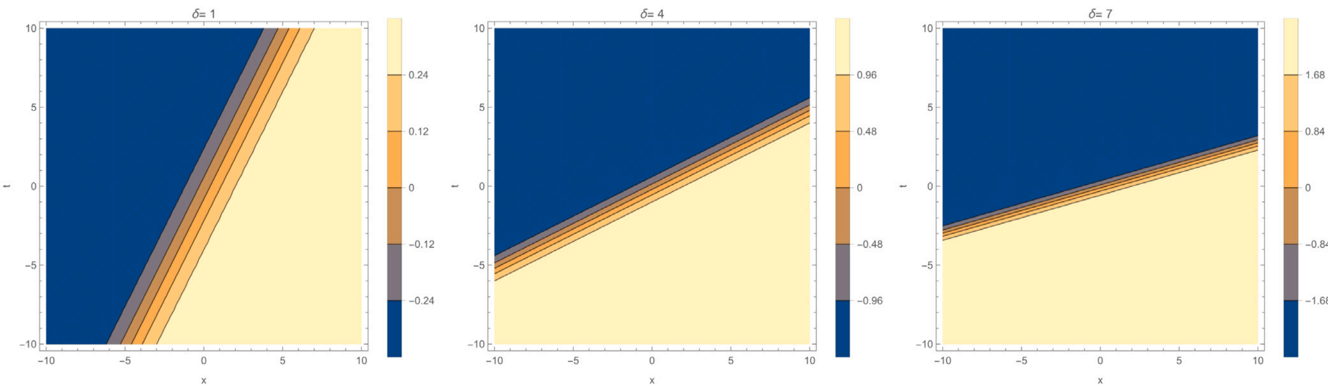

Fig. 7. 2D contour plot corresponding to $v_{10}(x,y)$ for $\lambda = 0.5, \sigma = 0.9$ different δ's.

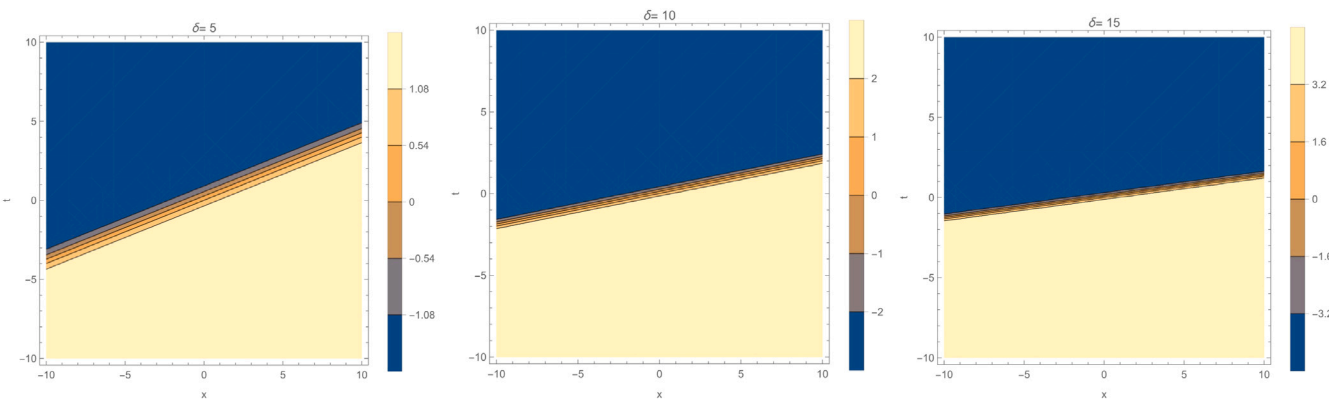

Fig. 8. 2D contour plot corresponding to $v_{12}(x,y)$ for $\lambda = 0.9, \sigma = 0.5$ different δ's.

The graphical depictions related to this solution in 2D contour plots are provided in Fig. 8.

Set 11: Supposing $\mu = [3,2,1,1]$ and $\nu = [1,0,1,0]$ yields

$$\mathscr{H}(\zeta) = \frac{3e^{\zeta}+2}{e^{\zeta}+1}. \tag{33}$$

Subset 1: For $\sigma + 2\lambda > 0$,

$$\omega = \frac{\delta}{2}, a_0 = \frac{5\delta}{2\sqrt{2\lambda+\sigma}}, a_1 = 0, b_1 = -\frac{6\delta}{\sqrt{2\lambda+\sigma}}.$$

After considering these results in Eqs. (10) and (33), it reads

$$\mathscr{V}(\zeta) = \frac{\delta\left(3e^{\zeta}-2\right)}{2\sqrt{2\lambda+\sigma}\left(3e^{\zeta}+2\right)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_{13}(x,t) = \frac{\delta\left(3e^{\left(x-\frac{\delta}{2}t\right)}-2\right)}{2\sqrt{2\lambda+\sigma}\left(3e^{\left(x-\frac{\delta}{2}t\right)}+2\right)}. \tag{34}$$

Set 12: Supposing $\mu = [-1,0,1,1]$ and $\nu = [0,1,0,1]$ yields

$$\mathscr{H}(\zeta) = -\frac{1}{e^{\zeta}+1}. \tag{35}$$

Subset 1: For $\sigma + 2\lambda > 0$,

$$\omega = \frac{\delta}{2}, a_0 = -\frac{\delta}{2\sqrt{2\lambda+\sigma}}, a_1 = -\frac{\delta}{\sqrt{2\lambda+\sigma}}, b_1 = 0.$$

After considering these results in Eqs. (10) and (35), it reads

$$\mathscr{V}(\zeta) = -\frac{\delta\left(e^{\zeta}-1\right)}{2\sqrt{2\lambda+\sigma}\left(e^{\zeta}+1\right)}.$$

In this way, the following analytical solution of Eq. (1) is extracted

$$v_{14}(x,t) = -\frac{\delta\left(e^{\left(x-\frac{\delta}{2}t\right)}-1\right)}{2\sqrt{2\lambda+\sigma}\left(e^{\left(x-\frac{\delta}{2}t\right)}+1\right)}. \tag{36}$$

Remark 1. Using $u(x,t) = \frac{1}{\omega}v^2(x,t)$, another solution for the system is obtained.

Remark 2. To ensure the correctness of the obtained solutions, we have put them all in the original equation and found that they are all correct.

Conclusion

In this survey, by utilizing the generalized exponential rational function method along with twelve categories of assumed solution forms fourteen new exact traveling wave solutions are formally extracted to the Drinfeld–Sokolov system. After comparing with the existing literature the presented results are brand new. Moreover, these obtained exact solutions can be naturally considered as the standard to the approaching solutions. Further, to better understand the dynamic behavior of solutions to the dispersive waves, various numerical simulations have been added to the paper, including solitary and periodic waves. We have checked the correctness of all the solutions and the satisfaction of the original equation for all the results presented in this contribution. It can be observed that the proposed method is more powerful and introduces various novel solutions which have not reported in the past literature. We deduce that our method is explicit, reliable, and straightforward which is helpful to solve complicated nonlinear models in various branches of nonlinear science. Our achievements can be considered as efficient and new results for the considered equation. These achievements have several applications in physical sciences. Numerical simulations related to this article have been performed in Wolfram Mathematica 12.0.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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