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RESEARCH ARTICLE

Interaction solutions to Hirota-Satsuma-Ito equation in $(2+1)$-dimensions

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Abstract Abundant exact interaction solutions, including lump-soliton, lump-kink, and lump-periodic solutions, are computed for the Hirota-Satsuma-Ito equation in (2+1)-dimensions, through conducting symbolic computations with Maple. The basic starting point is a Hirota bilinear form of the Hirota-Satsuma-Ito equation. A few three-dimensional plots and contour plots of three special presented solutions are made to shed light on the characteristic of interaction solutions.

Keywords Symbolic computation, lump solution, interaction solution
MSC 35Q51, 35Q53, 37K40

1 Introduction

In the classical theory of differential equations, the main aim of Cauchy problems is to determine the existence of a solution for a differential equation, which satisfies given initial data. Laplace's method aims to solve Cauchy problems for linear ordinary differential equations, and the Fourier transform method, for linear partial differential equations. In modern soliton theory, the isomonodromic and inverse scattering transform methods have been introduced for solving Cauchy problems for nonlinear ordinary and partial differential equations [1,6,41].

Usually, only constant-coefficient and linear differential equations are solv-

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able explicitly, and it is extremely difficult to determine exact solutions to variable-coefficient or nonlinear differential equations. Nevertheless, the Hirota bilinear method provides an efficient approach to soliton solutions [2,10]. Solitons are a kind of analytic solutions exponentially localized, historically found for nonlinear integrable equations. Let a polynomial P determine a Hirota bilinear form

$$P(D_x, D_y, D_t)f \cdot f = 0,$$

where D_x, D_y, and D_t are Hirota's bilinear derivatives [10], for a given partial differential equation with a dependent variable $u = u(x, y, t)$. Within the Hirota bilinear formulation, soliton solutions can often be determined through

$$u = 2(\log f)_{xx} \text{ or } u = 2(\log f)_x, \quad f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right),$$

where $\sum_{\mu=0,1}$ denotes the sum over all possibilities for $\mu_1, \mu_2, \ldots, \mu_N$ taking either 0 or 1, and the wave variables and the phase shifts are given by

$$\xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, \quad 1 \leqslant i \leqslant N,$$

and

$$\mathrm{e}^{a_{ij}} = -\frac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, \quad 1 \leqslant i < j \leqslant N,$$

with k_i, l_i, and ω_i satisfying the corresponding dispersion relation and $\xi_{i,0}$ being arbitrary translation shifts.

It is shown that there exists another kind of interesting analytic solutions called lumps, originated from solving soliton equations in $(2+1)$-dimensions (see, e.g., [37,38,43]). Lumps are a class of rational analytic function solutions that are localized in all directions in space. Taking long wave limits of N-soliton solutions can generate special lumps [42]. Positon and complexiton solutions add valuable insights into the diversity of exact solutions to nonlinear integrable equations [22,47]. Recent studies also tell that there exist interaction solutions (see, e.g., [36]) between two different kinds of solutions to integrable equations. Particularly, integrable equations in $(2+1)$-dimensions exhibit the remarkable richness of interaction solutions (see, e.g., [35]), which can be used to describe various wave phenomena in sciences. The KP I equation possesses lump solutions [24], among which are special ones generated from soliton solutions [39]. Other integrable equations which possess lump solutions include the three-dimensional three-wave resonant interaction [13], the BKP equation [7,49], the Davey-Stewartson equation II [42], the Ishimori-I equation [12], and the KP equation with a self-consistent source [54]. The most important step in getting lumps is to determine positive quadratic function solutions to bilinear equations [37], based on which some general analyses on lumps were made (see, e.g., [37] for Hirota bilinear equations and [38] for generalized bilinear equations).

In this paper, we would like to consider the Hirota-Satsuma-Ito (HSI) equation in $(2+1)$-dimensions. It is known that the Hirota-Satsuma shallow water wave equation [10]

$$u_t = u_{xxt} + 3uu_t - 3u_x v_t - u_x, \quad v_x = -u, \tag{1}$$

has a bilinear form

$$(D_t D_x^3 - D_t D_x - D_x^2) f \cdot f = 0, \tag{2}$$

under the logarithmic transformation $u = 2(\log f)_{xx}$. An integrable $(2+1)$-dimensional extension of the Hirota-Satsuma equation reads [9]:

$$w_t = u_{xxt} + 3uu_t - 3u_x v_t + u_x, \quad w_x = -u_y, \quad v_x = -u, \tag{3}$$

and its derivative form, called the HSI equation in $(2+1)$-dimensions and passing the three-soliton test [9],

$$P(u) = u_{xx} + u_{ty} + 3(u_t u_x)_x + u_{txxx} = 0, \tag{4}$$

has a bilinear form under the logarithmic transformation $u = 2(\log f)_x$:

$$B(f) = (D_t D_x^3 + D_t D_y + D_x^2) f \cdot f = 0. \tag{5}$$

Actually, under $u = 2(\log f)_x$, we have the relation $P(u) = (B(f)/f^2)_x$.

This paper aims to search for interaction solutions including lump-soliton, lump-kink, and lump-periodic solutions to the HSI equation in (2+1)-dimensions (4), through symbolic computations with Maple. The Hirota bilinear form is the starting point for our search (see, e.g., [21,33,37,38,62] for other equations). For three special presented solutions, a few three-dimensional plots and contour plots will be made via the Maple plot tool, to shed light on the characteristic of interaction solutions. A few concluding remarks will be given in the last section.

2 Interaction solutions

A search for positive quadratic solutions to the bilinear equation (5) generates a class of lump solutions to the HSI equation in $(2+1)$-dimensions (4):

$$u = 2(\log f)_x, \quad f = (a_1 x + a_2 y + a_3 t + a_4)^2 + (a_5 x + a_6 y + a_7 t + a_8)^2 + a_9, \tag{6}$$

where

$$\begin{cases} a_2 = -\dfrac{a_1^2 a_3 + 2a_1 a_5 a_7 - a_3 a_5^2}{a_3^2 + a_7^2}, \\ a_6 = \dfrac{a_1^2 a_7 - 2a_1 a_3 a_5 - a_5^2 a_7}{a_3^2 + a_7^2}, \\ a_9 = -\dfrac{3(a_1^2 + a_5^2)(a_3^2 + a_7^2)(a_1 a_3 + a_5 a_7)}{(a_1 a_7 - a_3 a_5)^2}, \end{cases} \tag{7}$$

and all other a_i's are arbitrary. It is easy to see that

$$a_1a_6 - a_2a_5 = \frac{(a_1^2+a_5^2)(a_1a_7-a_3a_5)}{a_3^2+a_7^2}, \tag{8}$$

and thus, the conditions of $a_1a_3 + a_5a_7 < 0$ and $a_1a_7 - a_3a_5 \neq 0$ guarantee that $u = 2(\log f)_{xx}$ will present lump solutions to the HSI equation in $(2+1)$-dimensions (4).

In what follows, we would like to compute interaction solutions between lumps and another kind of exact solutions, including solitons, kinks, and periodic solutions. To begin with, suppose that the wave variables are defined by

$$\begin{cases} \xi_1 = a_1x + a_2y + a_3t + a_4, \\ \xi_2 = a_5x + a_6y + a_7t + a_8, \\ \xi_3 = a_9x + a_{10}y + a_{11}t + a_{12}, \end{cases} \tag{9}$$

and adopt an ansatz

$$f = \xi_1^2 + \xi_2^2 + g(\xi_3) + a_{13}, \tag{10}$$

where a_i, $1 \leqslant i \leqslant 13$, are constant parameters, and g is a given test function. We will only study three special kinds of lump interactions with homoclinic, heteroclinic, and periodic (sinusoidal and cosinusoidal) test functions.

2.1 Lump-soliton solutions

If we take a choice with a hyperbolic test function

$$f = \xi_1^2 + \xi_2^2 + \cosh(\xi_3) + a_{13}, \tag{11}$$

then upon setting $a_{10} = 0$, we can have the following two solutions for the parameters:

$$\Big\{a_1 = ba_5,\ a_2 = -ba_5a_9^2,\ a_3 = 0,\ a_6 = a_5a_9^2,$$
$$a_7 = \frac{2a_5}{a_9^2},\ a_{10} = 0,\ a_{11} = -\frac{1}{a_9},\ a_{13} = -\frac{32a_5^4 - a_9^4}{4a_5^2a_9^2}\Big\}, \tag{12}$$

where $b^2 - 3 = 0$, and

$$\Big\{a_1 = -ba_5,\ a_2 = 2ba_5a_9^2,\ a_3 = \frac{a_5}{3ba_9^2},\ a_6 = 0,$$
$$a_7 = \frac{a_5}{a_9^2},\ a_{10} = 0,\ a_{11} = -\frac{1}{a_9},\ a_{13} = -\frac{32a_5^4 - 9a_9^4}{12a_5^2a_9^2}\Big\}, \tag{13}$$

where $3b^2 - 1 = 0$.

If we take a choice with another hyperbolic test function

$$f = \xi_1^2 + \xi_2^2 + \sinh(\xi_3) + a_{13}, \tag{14}$$

then

$$a_{13} = -\frac{32a_5^4 + a_9^4}{4a_5^2 a_9^2} \quad \text{or} \quad a_{13} = -\frac{32a_5^4 + 9a_9^4}{12a_5^2 a_9^2}, \tag{15}$$

and all other parameters do not change. This set of parameters generates a class of singular interaction solutions to the HSI equation in $(2+1)$-dimensions (4) through $u = 2(\log f)_x$.

For a special case of the first solution (12) with

$$a_4 = 1, \quad a_5 = 1, \quad a_8 = -1, \quad a_9 = 3, \quad a_{12} = 2,$$

three three-dimensional plots and contour plots of the corresponding interaction solution are made via Maple plot tools, to shed light on the characteristic of interaction solutions, in Figure 1.

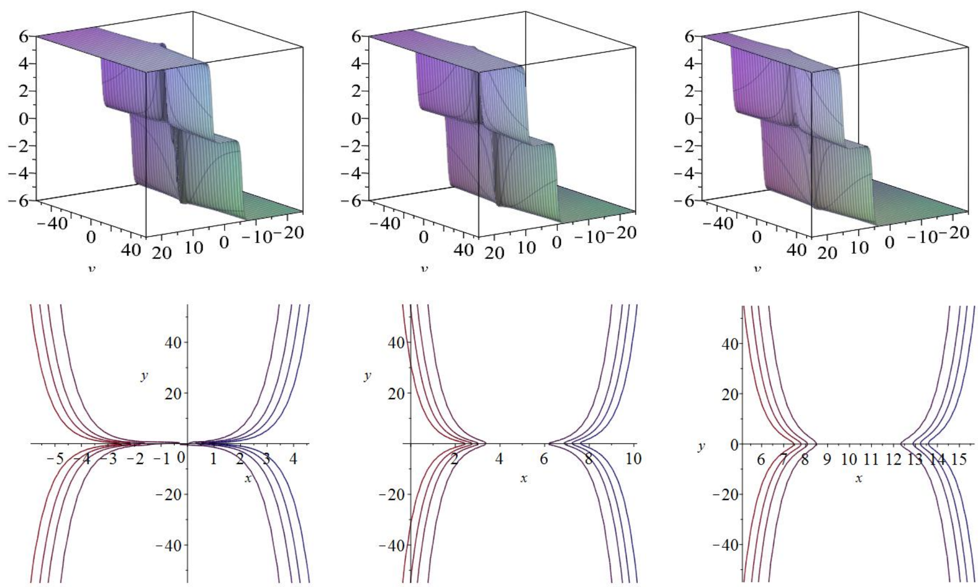

Figure 1: Profiles of u when $t = 0, 50, 100$: 3d plots (top) and contour plots (bottom)

2.2 Lump-kink solutions

If we take a choice with an exponential test function

$$f = \xi_1^2 + \xi_2^2 + \mathrm{e}^{\xi_3} + a_{13}, \tag{16}$$

then we can have the following two solutions for the parameters:

$$\Big\{a_1 = \frac{3}{2}\,a_7 a_9^2,\ a_2 = -\frac{3}{2}\,a_5 a_9^2,\ a_3 = -\frac{2a_5}{3a_9^2},$$
$$a_6 = \frac{9}{4}\,a_7 a_9^4,\ a_{10} = \frac{1}{2}\,a_9^3,\ a_{11} = -\frac{2}{3a_9},\ a_{13} = 0\Big\}, \tag{17}$$

and

$$\Big\{a_2 = \frac{a_1(4a_1^2 - 9a_7^2a_9^4)}{6a_7^2a_9^2},\ a_3 = 0,\ a_5 = -\frac{4a_1^2 - 9a_7^2a_9^4}{12a_7a_9^2},$$
$$a_6 = -\frac{16a_1^4 - 216a_1^2a_7^2a_9^4 + 81a_7^4a_9^8}{144a_7^3a_9^4},$$
$$a_{10} = \frac{4a_1^2 - 3a_7^2a_9^4}{12a_7^2a_9},\ a_{11} = -\frac{12a_7^2a_9^3}{4a_1^2 + 9a_7^2a_9^4},$$
$$a_{13} = \frac{(4a_1^2 - 9a_7^2a_9^4)(16a_1^4 + 72a_1^2a_7^2a_9^4 + 81a_7^4a_9^8)}{576a_1^2a_7^2a_9^6}\Big\}. \tag{18}$$

For a special case of the second solution (18) with

$$a_1 = 2,\quad a_4 = 1,\quad a_7 = -1,\quad a_8 = -1,\quad a_9 = 1,\quad a_{12} = 2,$$

three three-dimensional plots and contour plots of the corresponding interaction solution are made via Maple plot tools, to shed light on the characteristic of interaction solutions, in Figure 2.

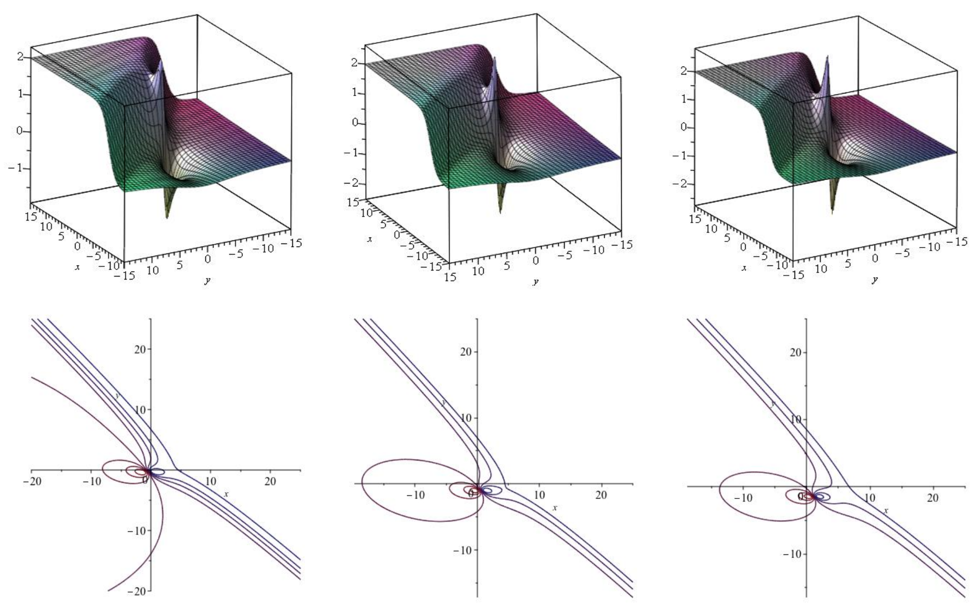

Figure 2: Profiles of u when $t = 0, 3, 5$: 3d plots (top) and contour plots (bottom)

2.3 Lump-periodic solutions

If we take a choice with a trigonometric test function

$$f = \xi_1^2 + \xi_2^2 + g(\xi_3) + a_{13},\quad g = \sin \text{ or } \cos, \tag{19}$$

then upon setting $a_{10} = 0$, we can have the following two solutions for the parameters:

$$\Big\{ a_1 = ba_5,\ a_2 = ba_5a_9^2,\ a_3 = 0,\ a_6 = -a_5a_9^2,\\ a_7 = -\frac{2a_5}{a_9^2},\ a_{10} = 0,\ a_{11} = \frac{1}{a_9},\ a_{13} = \frac{32a_5^4 - a_9^4}{4a_5^2a_9^2}\Big\}, \tag{20}$$

where $b^2 - 3 = 0$, and

$$\Big\{ a_1 = ba_5,\ a_2 = 2ba_5a_9^2,\ a_3 = \frac{a_5}{3ba_9^2},\ a_6 = 0,\\ a_7 = -\frac{a_5}{a_9^2},\ a_{10} = 0,\ a_{11} = \frac{1}{a_9},\ a_{13} = \frac{32a_5^4 - 9a_9^4}{12a_5^2a_9^2}\Big\}, \tag{21}$$

where $3b^2 - 1 = 0$.

For a special case of the first solution (20) with

$$a_5 = -1, \quad a_4 = -1, \quad a_8 = -1, \quad a_9 = 1, \quad a_{12} = -2, \quad g = \sin,$$

three three-dimensional plots and contour plots of the corresponding interaction solution are made via Maple plot tools, to shed light on the characteristic of interaction solutions, in Figure 3.

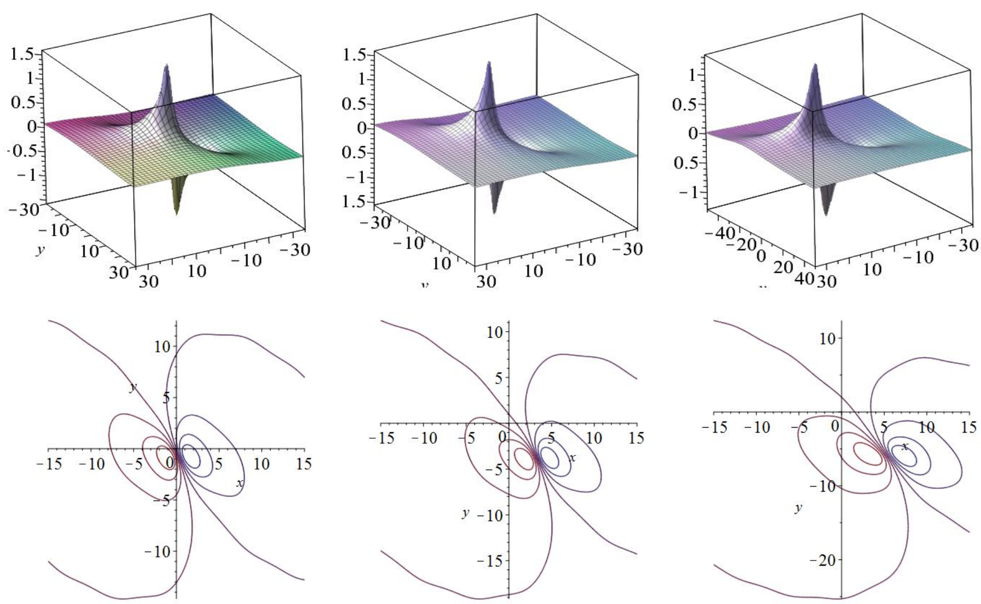

Figure 3: Profiles of u when $t = 0, 3, 5$: 3d plots (top) and contour plots (bottom)

All the interaction solutions generated above provide a valuable supplement to the existing theories on soliton solutions and dromion-type solutions, developed through powerful existing techniques including the Hirota perturbation approach, the Riemann-Hilbert approach, symmetry reductions, and symmetry constraints (see, e.g., [4,5,15–18,20,27,46,61]).

3 Concluding remarks

We have studied the $(2+1)$-dimensional Hirota-Satsuma-Ito equation to explore diverse interaction solutions, through symbolic computations with Maple. The results enrich the theory of solitons, providing a new example of $(2+1)$-dimensional nonlinear integrable equations, which possess abundant interaction solutions. Three-dimensional plots and contour plots of the three specially chosen interaction solutions were made by using the plot tool in Maple.

On one hand, recent studies show that many nonintegrable equations possess lump solutions, which include $(2+1)$-dimensional generalized KP, BKP, KP-Boussinesq, Sawada-Kotera, and Bogoyavlensky-Konopelchenko equations (see, e.g., [3,19,31,33,34,40,55,56,58]). Diversity of lump solutions supplements exact solutions generated from different kinds of combinations (see, e.g., [32,45,48]), and yields the corresponding Lie-Bäcklund symmetries, which can be used to determine conservation laws by symmetries and adjoint symmetries [11,23,26]. On the other hand, some other studies exhibit diverse interaction solutions for many integrable equation in $(2+1)$-dimensions. Those include lump-soliton interaction solutions (see, e.g., [35,50,52,53]) and lump-kink interaction solutions (see, e.g., [14,44,59,60]). In the $(3+1)$-dimensional case, a class of lump-type solutions, being rationally localized in almost all directions in space, were constructed for the integrable Jimbo-Miwa equations. Various such solutions were computed for the (3+1)-dimensional Jimbo-Miwa equation (see, e.g., [25,51,57]) and the $(3+1)$-dimensional Jimbo-Miwa like equation [8]. Some other recent studies also demonstrate the remarkable richness of lumps and interaction solutions in the case of linear partial differential equations in $(3+1)$-dimensions [28,29] and $(4+1)$-dimensions [30].

We conjecture that the existence of interaction solutions with diverse features would strongly reflect complete integrability of partial differential equations. Naturally, it is interesting to search for lump solutions and interaction solutions to partial differential equations of all orders and dimensions. The other interesting problem is to characterize either linear or nonlinear partial differential equations which exhibit the existence of diverse lump solutions and interaction solutions.

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