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# Lump solutions to a generalized Kadomtsev–Petviashvili–Ito equation

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A (2+1)-dimensional generalized Kadomtsev–Petviashvili–Ito equation is introduced. Upon adding some second-order derivative terms, its various lump solutions are explicitly constructed by utilizing the Hirota bilinear method and calculated through the symbolic computation system Maple. Furthermore, two specific lump solutions are obtained with particular choices of the parameters and their dynamical behaviors are analyzed through three-dimensional plots and contour plots.

*Keywords*: Lump solutions; Hirota bilinear method; soliton; symbolic computation.

## 1. Introduction

In the past few decades, with the rapid development of nonlinear science, exact and numerical solutions of nonlinear partial differential equations (PDEs) have aroused great interest among many scientists and engineers.[1–6] Such as rogue wave solutions which are described an open water phenomenon, in which winds, currents and nonlinear phenomena. Soliton solutions, which are normally localized in the

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time and space, are exact ones determined by exponentially localized functions.[7–10] When the nonlinear differential equations with polynomial nonlinearity are studied, the existence of solitary solutions should be considered.[11] Compared to soliton solutions, we focus on another type of important exact solutions such as lump solutions, which are localized in directions only in space. Lump solutions are derived from solving integrable equations in (2+1)-dimensions,[12–14] and found out by taking long wave limits in soliton theory.[2,15,16] The main procedure in searching of lump solutions is to construct positive quadratic function solutions to the Hirota bilinear equations.[12] Then lump solutions to the corresponding nonlinear PDEs can be obtained through the logarithmic transformations, on account of the positive quadratic function solutions.

As is known, a (2+1)-dimensional partial differential equation can be transformed into a Hirota bilinear form through a depended variable transformation. Assume that $P$ is a polynomial in $x$, $y$ and $t$, then a Hirota bilinear equation can be defined by

$$P(D_x, D_y, D_t)f \cdot f = 0, \tag{1.1}$$

where $D_x, D_y, D_t$ are Hirota bilinear derivatives,[12]

$$\begin{aligned} &D_x^l D_y^m D_t^n f(x,y,t) \cdot g(x,y,t) \\ &\quad = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^m \\ &\qquad \times \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n f(x,y,t) \cdot g(x',y',t')|_{x'=x,y'=y,t'=t}, \end{aligned} \tag{1.2}$$

where $f, g$ are infinitely differentiable functions, and $l, m, n$ are non-negative integers. Then if $f$ is the solution to Eq. (1.2), the N-soliton solutions to the corresponding partial differential equation in (2+1)-dimensions can be given by[17]

$$f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right), \tag{1.3}$$

under the logarithm transformation $u = 2(\ln f)_x$ or $u = 2(\ln f)_{xx}$. Here $\sum_{\mu=0,1}$ denotes the sum of over all possibilities for $\mu_1, \ldots, \mu_N$ in $0, 1$, and

$$\begin{cases} \xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, & 1 \le i \le N, \\ e^{a_{ij}} = -\dfrac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, & 1 \le i < j \le N, \end{cases} \tag{1.4}$$

with the wave numbers $k_i, l_i$ and the frequencies $\omega_i$, $1 \le i \le N$ satisfying the corresponding dispersion relations, while the phases shifts $\xi_{i,0}$ being arbitrary.

For example, the Kadomtsev–Petviashvili (KP) equation[10]

$$(u_t + 6uu_x + u_{xxx})_x + \sigma u_{yy} = 0 \tag{1.5}$$

is a partial differential equation to describe nonlinear wave motion. It can be used to model water waves of long wavelength with weakly nonlinear restoring forces and frequency dispersion. When $\sigma = -1$, it is the famous KPI equation, which means the surface tension of water waves is strong compared to gravitational forces.[10] When $\sigma = +1$, (1.5) is known as the KPII equation, which describes the surface tension is weak.[10] The Ito equation in (2+1)-dimensions is usually written as[18,19]:

$$u_{tt} + u_{xxxt} + 6u_x u_t + 3uu_{xt} + 3u_{xx}v_t + \alpha u_y u_t + \beta u_x u_t = 0, \tag{1.6}$$

where $v_x = u$, the coefficients $\alpha$ and $\beta$ are arbitrary constants. It is reduced to the (1+1)-dimensional Ito equation by taking $\alpha = 0$ and $\beta = 0$. Then, Eqs. (1.5) and (1.6) possess the following Hirota bilinear forms:

$$(D_x D_t + D_x^4 + \sigma D_y^2) f \cdot f = 0 \tag{1.7}$$

and

$$(D_x D_t + D_x^3 D_t + \alpha D_y D_t + \beta D_x D_t) f \cdot f = 0, \tag{1.8}$$

through the transformation $u = 2(\ln f)_{xx}$.

We have known that the KPI equation possesses lump solutions[20]

$$u = \frac{4(a_1^2 + a_5^2) f - 8(a_1\xi_1 + a_5\xi_2)^2}{f^2},$$

where the functions of $\xi_1, \xi_2$ are given as follows:

$$\begin{cases} \xi_1 = a_1 x + a_2 y + \dfrac{a_1 a_2^2 - a_1 a_6^2 + 2a_2 a_5 a_6}{a_1^2 + a_5^2} t + a_4, \\[2ex] \xi_2 = a_5 x + a_6 y + \dfrac{2a_1 a_2 a_6 - a_2^2 a_5 + a_5 a_6^2}{a_1^2 + a_5^2} t + a_8 \end{cases}$$

and the corresponding positive quadratic function solution is

$$f = \left( a_1 x + a_2 y + \frac{a_1 a_2^2 - a_1 a_6^2 + 2a_2 a_5 a_6}{a_1^2 + a_5^2} t + a_4 \right)^2 + \left( a_5 x + a_6 y + \frac{2a_1 a_2 a_6 - a_2^2 a_5 + a_5 a_6^2}{a_1^2 + a_5^2} t + a_8 \right)^2 + \frac{3(a_1^2 + a_5^2)^3}{(a_1 a_6 - a_2 a_5)^2}$$

under the transformation $u = 2(\ln f)_{xx}$. In this kind of lump solutions, parameters of $a_1, a_2, a_4, a_5, a_6, a_8$ are arbitrary, and the condition $a_1 a_6 - a_2 a_5 \neq 0$ guarantees both analyticity and localization of the solutions in the $xy$-plane. It is obvious that the solutions are analytic if and only if the parameter $a_9 > 0$.

A lot of different types of solutions to integrable equations have been studied, for instance, soliton solutions, lump solutions, interaction solutions between lump solutions and solitons, lump-kinks, line-solitons, resonance solutions and other classes of solutions. What we all know is that the KP I equation possesses plenty of lump solutions,[20] and some special lump solutions are obtained from its soliton solutions.[21] Besides, a large number of other integrable equations which have been proven to possess lump solutions, such as the three-dimensional three-wave resonant interaction,[22] the Davey–Stewartson II equation,[15] the Ishimori-I equation,[23] the B-KP equation,[24,25] and the KP equation with a self-consistent source equation.[26] Meanwhile, many non-integrable equations also have been found that there exist lump solutions, among a few equations which are generalized KP, KP–Boussinesq, B-KP, Calogero–Bogoyavlenskii–Schiff, Sawada–Kotera and Bogoyavlensky–Konopelchenko equations in (2+1)-dimensions.[27–33] Therefore, we noticed the importance of finding lump solutions whether or not the equation is integrable and the key to constructing lump solutions by the Hirota bilinear method.

The goal of this paper is to investigate lump solutions for the following generalized KP–Ito equation combined with some new second-order terms:

$$\alpha(6uu_x + u_{xxx})_x + \beta(u_{xxxt} + 6u_xu_t + 3uu_{xt} + 3u_{xx}v_t) + \delta_1u_yu_t + \delta_2u_{xx} + \delta_3u_xu_t + +\delta_4u_xu_y + \delta_5u_{yy} + \delta_6u_{tt} = 0, \quad (1.9)$$

where $v_x = u$, the coefficients $\alpha$ and $\beta$ satisfy $\alpha^2 + \beta^2 \neq 0$, but $\delta_i, 1 \leq i \leq 6$, are arbitrary constants. It reduces to the KP equation by choosing $\beta = 0, \alpha = 1$, $\delta_1 = \delta_2 = \delta_4 = \delta_6 = 0$, and the Ito equation by taking $\alpha = 0, \beta = \delta_6 = 1, \delta_2 = \delta_4 = \delta_5 = 0$. Based on a bilinear transformation, the new equation enjoys a Hirota bilinear form. Lump solutions can be worked out through symbolic computation with Maple. Finally, three-dimensional plots and contour plots of these solutions are exhibited and their dynamic behaviors are studied. Some conclusions are given in the last section.

## 2. Bilinear Form and Lump Solutions

In this section, we will produce some classes of the lump solutions by utilizing the Hirota bilinear method. We substitute the logarithmic transformation $u = 2(\ln f)_{xx}$ and $v = 2(\ln f)_x$ into (1.9), it is obvious that the generalized KP-Ito equation possesses the following Hirota bilinear form:

$$(\alpha D_x^4 + \beta D_x^3D_t + \delta_1D_yD_t + \delta_2D_x^2 + \delta_3D_xD_t + \delta_4D_xD_y + \delta_5D_y^2 + \delta_6D_t^2)f \cdot f = 0, \quad (2.1)$$

equivalently,

$$\alpha(2ff_{xxxx} - 8f_xf_{xxx} + 6f_{xx}^2) + \beta((2ff_{xxxt} - 2f_{xxx}f_t - 6f_xf_{xxt} + 6f_{xx}f_{xt})) + \delta_1(2ff_{yt} - 2f_yf_t) + \delta_2(2ff_{xx} - 2f_x^2) + \delta_3(2ff_{xt} - 2f_xf_t) + \delta_4(2ff_{xy} - 2f_xf_y) + \delta_5(2ff_{yy} - 2f_y^2) + \delta_6(2ff_{tt} - 2f_t^2) = 0. \quad (2.2)$$

Therefore, if $f$ solves the bilinear equation (2.2), then $u = 2(\ln f)_{xx}$ solves Eq. (1.9).

Since Eq. (1.9) has the bilinear form (2.1), it is critical to look for the quadratic function solution. We suppose the quadratic solution $f$ is expressed as

$$f = (a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 + a_9, \quad (2.3)$$

where $a_i$, $i = 1, 2, \ldots, 9$, are arbitrary real constants to be determined. We plug (2.3) into (2.2), and then a system of nonlinear algebraic equations on the parameters can be solved by Maple symbolic computations. A set of solutions for the parameters are exhibited as follows:

**Case 1.**

$$a_2 = \frac{b}{4\delta_5 c_1}, \quad a_3 = \frac{a_1 c_2}{c_1}, \quad a_6 = -\frac{a_5\delta_4 + a_7\delta_1}{2\delta_5},$$
$$a_9 = -\frac{12\delta_5(a_1^2 + a_5^2)(\alpha c_3 + \beta c_4)}{c_5},$$

where $b$ is a solution of the equation

$$\begin{aligned} b^2 &+ (16a_1a_5\delta_1\delta_2\delta_5 - 8a_1a_5\delta_3\delta_4\delta_5 + 8a_1a_7\delta_1\delta_3\delta_5 - 16a_1a_7\delta_4\delta_5\delta_6)b \\ &- 64(a_5^2\delta_3^2 + 4a_5a_7\delta_3\delta_6 + 4\delta_6^2a_7^2 - 4\delta_2\delta_6a_1^2 - \delta_3^2a_1^2)(a_5^2\delta_2 + a_5a_7\delta_3 + a_7^2\delta_6)\delta_5^3 \\ &+ 16[(4\delta_1\delta_2\delta_3 + \delta_3^2\delta_4)a_5^3 + (8\delta_2\delta_6\delta_1 + 5\delta_3^2\delta_1 + 4\delta_3\delta_4\delta_6)a_7a_5^2 \\ &+ (16\delta_1\delta_3\delta_6 + 4\delta_4\delta_6^2)a_7^2a_5 - a_1^2\delta_4(8\delta_2\delta_6 - 2\delta_3^2)a_5 \\ &- \delta_1(8\delta_2\delta_6a_1^2 - 2\delta_3^2a_1^2 - 12\delta_6^2a_7^2)a_7](a_5\delta_4 + a_7\delta_1)\delta_5^2 \\ &- 16(a_5\delta_4 + a_7\delta_1)^2[(\delta_1^2\delta_2 + \delta_1\delta_3\delta_4)a_5^2 + 2a_7\delta_1(\delta_1\delta_3 + \delta_4\delta_6)a_5 \\ &+ 3\delta_1^2\delta_6a_7^2 - a_1^2(\delta_1^2\delta_2 - \delta_1\delta_3\delta_4 + \delta_4^2\delta_6)]\delta_5 + 4\delta_1^2(a_5\delta_4 + a_7\delta_1)^4 = 0 \end{aligned}$$

and the above involved five constants $c_i$, $i = 1, 2, 3, 4, 5$, are defined as follows:

$$\begin{cases} c_1 = a_5\delta_1\delta_4 - 2a_5\delta_3\delta_5 + a_7\delta_1^2 - 4a_7\delta_5\delta_6, \\ c_2 = 4a_5\delta_2\delta_5 - a_5\delta_4^2 - a_7\delta_1\delta_4 + 2a_7\delta_3\delta_5, \\ c_3 = (a_1^2 + a_5^2)c_1, \\ c_4 = a_1^2c_1 + a_5a_7c_2, \\ c_5 = c_1(a_5c_2 - a_7c_1). \end{cases} \tag{2.4}$$

Therefore, besides $a_1a_6 - a_2a_5 \neq 0$, the other condition assures that the non-singularity of the lump solution is $c_1(a_5c_2 - a_7c_1) \neq 0$, and it should also satisfy the constraint condition $a_9 > 0$ to ensure that $f$ is positive.

**Case 2.**

$$a_3 = -\frac{a_1\delta_4 + 2a_2\delta_5}{\delta_1}, \quad a_6 = \frac{b}{\delta_1 c_1}, \quad a_7 = -\frac{a_5c_2}{c_1},$$
$$a_9 = -\frac{3\delta_1(a_1^2 + a_5^2)(\alpha c_3 + \beta c_4)}{c_5},$$

where $b$ needs to satisfy the equation

$$\begin{aligned} \delta_5b^2 &- (2a_1a_5\delta_1^3\delta_2 - 2a_1a_5\delta_1^2\delta_3\delta_4 + 2a_1a_5\delta_1\delta_4^2\delta_6 - 2a_2a_5\delta_1^2\delta_3\delta_5 + 4a_2a_5\delta_1\delta_4\delta_5\delta_6)b \\ &- a_2^2(a_1^2\delta_2 - \delta_5a_2^2 - a_5^2\delta_2)\delta_1^6 - \delta_3(2a_1^3\delta_2 - a_1^2a_2\delta_4 - 4a_1a_2^2\delta_5 + a_2a_5^2\delta_4)a_2\delta_1^5 \\ &- [a_1^4\delta_2\delta_3^2 - (4\delta_2\delta_6 + 2\delta_3^2)\delta_4a_2a_1^3 - (8\delta_2\delta_5\delta_6a_2^2 - \delta_4^2\delta_6a_2^2 + 5\delta_3^2\delta_5a_2^2 \end{aligned}$$

$$
\begin{aligned}
&+ 4\delta_6 a_5^2\delta_2^2 - a_5^2\delta_2\delta_3^2)a_1^2 + 8\delta_6 a_1 a_2^3\delta_4\delta_5 + (12\delta_6 a_2^2\delta_5^2 + 8\delta_2\delta_5\delta_6 a_5^2 \\
&- \delta_4^2\delta_6 a_5^2 - 2\delta_3^2\delta_5 a_5^2)a_2^2]\delta_1^4 + (a_1\delta_4 + 2a_2\delta_5)[(4\delta_2\delta_6 + \delta_3^2)a_1^2 \\
&- 6a_1a_2\delta_4\delta_6 - 16\delta_6 a_2^2\delta_5 - a_5^2(4\delta_2\delta_6 - \delta_3^2)]\delta_3 a_1\delta_1^3 \\
&- (a_1\delta_4 + 2a_2\delta_5)^2[(4\delta_2\delta_6 + 5\delta_3^2)a_1^2 - 4a_1a_2\delta_4\delta_6 - 12\delta_6 a_2^2\delta_5 \\
&- a_5^2(4\delta_2\delta_6 - \delta_3^2)]\delta_6\delta_1^2 + 8a_1\delta_3\delta_6^2(a_1\delta_4 + 2a_2\delta_5)^3\delta_1 \\
&- 4\delta_6^3(a_1\delta_4 + 2a_2\delta_5)^4 = 0
\end{aligned}
$$

and the above involved five constants $c_i$, $i = 1, 2, 3, 4, 5$, are defined as follows:

$$
\begin{cases}
c_1 = a_1\delta_1\delta_3 - 2a_1\delta_4\delta_6 + a_2\delta_1^2 - 4a_2\delta_5\delta_6, \\
c_2 = 2a_1\delta_1\delta_2 - a_1\delta_3\delta_4 + a_2\delta_1\delta_4 - 2a_2\delta_3\delta_5, \\
c_3 = \delta_1(a_1^2 + a_5^2)c_1, \\
c_4 = 2\delta_5\delta_6(a_1\delta_4 + 2a_2\delta_5) - \delta_1\delta_5(a_1\delta_3 + a_2\delta_1) - a_1^2\delta_4 c_1 - a_5^2\delta_1 c_2, \\
c_5 = c_1(a_1^2\delta_1^2\delta_2 - a_1^2\delta_1\delta_3\delta_4 + a_1^2\delta_4^2\delta_6 - 2a_1a_2\delta_1\delta_3\delta_5 \\
\qquad + 4a_1a_2\delta_4\delta_5\delta_6 - a_2^2\delta_1^2\delta_5 + 4a_2^2\delta_5^2\delta_6)
\end{cases}
\tag{2.5}
$$

which needs to satisfy the conditions $c_5 \neq 0$ and $a_9 > 0$, to make sure that the corresponding solutions $f$ are well defined and positive.

## 3. Two Specific Lump Solutions and Their Dynamic Behaviors

In this section, special choices of the involved parameters in the Hirota bilinear equation (2.1) will be adopted to achieve the corresponding lump solutions, and then the dynamic behaviors of the solutions be discussed.

First, by assuming

$$\alpha = 1, \quad \beta = 1, \quad \delta_1 = \delta_3 = \delta_4 = 1, \quad \delta_2 = \delta_5 = \delta_6 = 0$$

and selecting a special choice for the parameters:

$$a_1 = 1, \quad a_3 = 3, \quad a_4 = 4, \quad a_5 = 1, \quad a_7 = -1, \quad a_8 = 3,$$

we get the value of other parameters: $a_2 = -1$, $a_6 = -\frac{1}{2}$, $a_9 = 24$. Substituting all the parameters $a_i$, $1 \leq i \leq 9$, into the formula (2.3), we obtain a kind of positive quadratic function solutions to Eq. (2.2)

$$f_1 = (x - y + 3t + 4)^2 + \left(x - \frac{y}{2} - t + 3\right)^2 + 24. \tag{3.1}$$

Through the logarithmic transformation: $u = 2(\ln f)_{xx}$, the resulting class of positive quadratic function solutions yields a class of lump solutions to the generalized KP-Ito equation (1.9), which is generated as follows:

$$
\begin{aligned}
u_1 = &\frac{16}{(x - y + 3t + 4)^2 + \left(x - \frac{y}{2} - t + 3\right)^2 + 24} \\
&- \frac{4(4x - 3y + 4t + 14)^2}{\left((x - y + 3t + 4)^2 + \left(x - \frac{y}{2} - t + 3\right)^2 + 24\right)^2}.
\end{aligned}
\tag{3.2}
$$

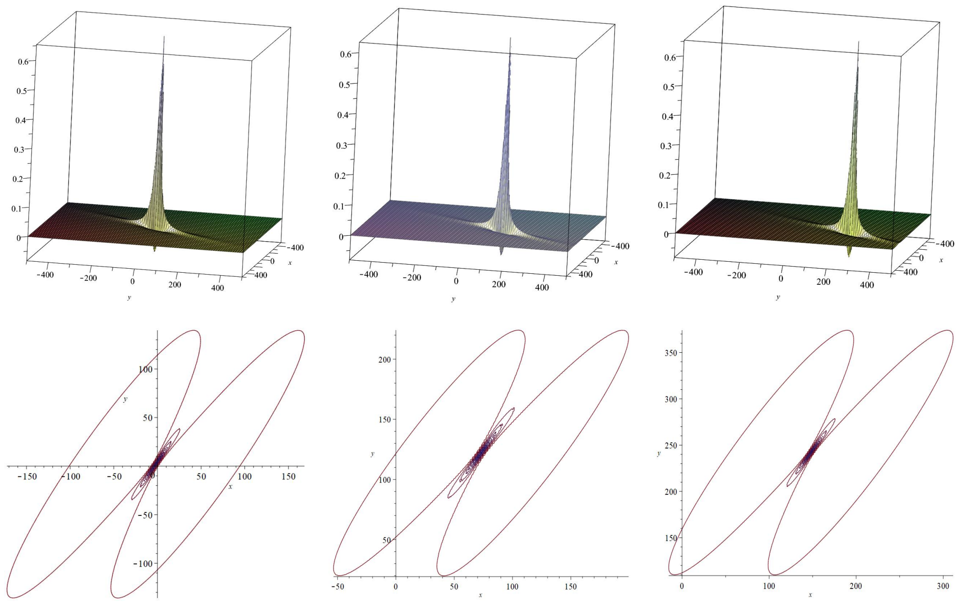

Fig. 1. (Color online) Profiles of $u_1$ when $t = 0, 15, 30$: 3d plots (top) and contour plots (bottom).

The three-dimensional plots and contour plots of the above lump solutions (3.2) are exhibited in Fig. 1, which are made via Maple plot tools.

Second, by taking

$$\alpha = 1, \quad \beta = 1, \quad \delta_1 = \delta_2 = \delta_4 = 1, \quad \delta_3 = \delta_5 = \delta_6 = 0$$

and choosing suitable values for the parameters

$$a_1 = -1, \quad a_2 = -3, \quad a_4 = 6, \quad a_5 = -1, \quad a_6 = -1, \quad a_8 = -3,$$

we have $a_3 = \frac{6}{5}$, $a_7 = \frac{8}{5}$, $a_9 = 12$. In the same way, we can obtain the corresponding positive quadratic solution as follows:

$$f_2 = \left(x + 3y - \frac{6}{5}t - 6\right)^2 + \left(x + y - \frac{8}{5}t + 3\right)^2 + 12, \tag{3.3}$$

which produces another class of lump solutions to the generalized KP-Ito equation:

$$u_2 = \frac{16}{\left(x + 3y - \frac{6}{5}t - 6\right)^2 + \left(x + y - \frac{8}{5}t + 3\right)^2 + 12} - \frac{16\left(2x + 4y - \frac{14}{5}t - 3\right)^2}{\left(\left(x + 3y - \frac{6}{5}t - 6\right)^2 + \left(x + y - \frac{8}{5}t + 3\right)^2 + 12\right)^2}. \tag{3.4}$$

Then the lump solutions' three-dimensional plots and contour plots are displayed in Fig. 2, which are made via Maple plot tools.

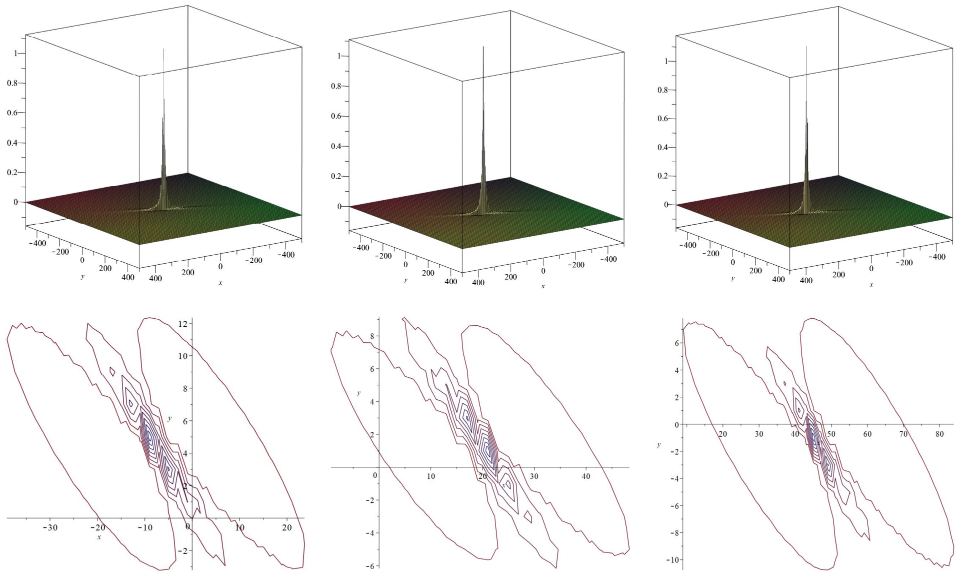

Fig. 2. (Color online) Profiles of $u_2$ when $t = 0, 15, 30$: 3d plots (top) and contour plots (bottom).

## 4. Conclusions

In this paper, a generalized KP-Ito equation has been considered. We presented two sets of complex lump solutions by using the Hirota bilinear method and symbolic computation. Then, we obtained two kinds of simple lump solutions through selecting appropriate values of the parameters for the positive quadratic function. At the same time, we used Maple to give some three-dimensional plots and contour plots of two specific lump solutions in order to make it easy to understand the changes of the lump solutions with time. All the above results provided us with abundant new exact solutions. It is necessary to point out that only two specific solutions are given in this paper, but there are many other more complex exact solutions such as other kinds of lump solutions and lump-kink solutions that we have not presented here. If someone is interested, you can also continue to discuss and research.

This study shows that the category of nonlinear partial differential equations with lump solutions is very broad and rich. It also attempts to point out the relation between the lump solutions and the nonlinear terms contained in the new generalized equation. It is a common knowledge that many nonlinear phenomena can be described by interaction solutions between lump solutions and soliton solutions[30,34] and a lot of different studies have shown the existence of interaction solutions between lump solutions and lump-kink solutions and other kinds of exact solutions to linear wave equations,[35] nonlinear integrable equations[36–39] as well as in (3+1)-dimensions.[40–42] Since the interaction properties involve much more complicated mathematical computations, the further research for interaction solutions to other generalized bilinear differential equations is becoming more interesting and meaningful.

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