Results in Physics 20 (2021) 103711

Contents lists available at ScienceDirect

Results in Physics

journal homepage: www.elsevier.com/locate/rinp

# Nonsingular complexiton solutions and resonant waves to an extended Jimbo–Miwa equation

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ARTICLE INFO

*Keywords:*
Jimbo–Miwa equation
Bilinear form
Nonsingular complexiton solutions
Resonant wave solutions
Symbolic computations

ABSTRACT

The goal of this letter is to consider an extended Jimbo–Miwa equation which exists nonsingular complexiton solutions in (3+1)-dimensions. A few types of two-wave and complexiton solutions are developed through symbolic computations with Maple, including resonant two-wave solutions and nonsingular complexiton solutions. Nonsingular complexitons to the generalized Jimbo–Miwa equation should be new, though there exist plenty of studies in the literature. A few specific cases are derived to illustrate the remarkable richness of explicit solutions for the considered equation.

## 1. Introduction

The research of exact solutions is an important part to nonlinear differential equations and has become a hot topic. The determination of exact solutions, particularly the multi-soliton solutions, can help us to comprehend various qualitative and quantitative characteristics of nonlinear phenomena modeled by these evolution equations. For typical soliton equations, there possess multiple-soliton solutions, constructed from combinations of multiple exponential waves in terms of their Hirota bilinear formulations. Such equations contain the Korteweg–de Vries (KdV) equation, the Kadomtsev–Petviashvili (KP) equation and the Sawada–Kotera equation [1]. It is recognized by applying the multiple exp-function algorithm and the simplified Hirota's approach that many higher-dimensional equations can also possess multi-wave soliton type solutions, which include the generalized KP, B-type Kadomtsev–Petviashvili (BKP) equations and the (3+1)-dimensional Jimbo–Miwa equation [2–5]. Many kinds of effective methods have been built for the investigation of soliton solutions, such as the Wronskian formula [6], the inverse scattering transform [7] and the Hirota bilinear approach [1,8]. Enlarging the diversity of solitons, complexiton solutions or periodic-soliton solutions have been introduced in previous papers [9,10]. Some recent works have been carried out on the complexiton solutions which involve two classes of transcendental functions, namely, exponential and trigonometric functions [11–15]. The Wronskian method is highly designed to seek complexiton solutions for soliton equations [10], but the complexiton solutions derived from the Wronskian formulation are extremely complicated to higher-dimensional equations. The linear superposition principle also provides a useful tool for finding complexiton solutions [11]. Moreover, complexiton solutions may be generated from solitons via extending the real parameters to the complex field [9,14,15].

The second member of the famous KP hierarchy is the Jimbo–Miwa equation:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1.1}$$

which is applied to describing the propagation of three-dimensional nonlinear waves with a weak dispersion [16]. More recent studies exhibit that this equation exists diverse exact solutions [2,17–20]. The exp-function algorithm allowed us to construct two- and three-wave solutions and traveling wave solutions [2], and the linear superposition principle presented resonant multiple wave solutions [20]. A direct symbolic calculation yielded abundant lump-type solutions and interaction solutions [18,19]. Multiple periodic-soliton solutions generated from multiple line-soliton solutions were established through the Hirota bilinear approach in Refs. [21,22]. As a generalization of Eq. (1.1), Wazwaz presented the two extended Jimbo–Miwa equations as follows [23]:

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3(u_{xz} + u_{yz} + u_{zz}) = 0, \tag{1.2}$$

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2(u_{xt} + u_{yt} + u_{zt}) - 3u_{xz} = 0. \tag{1.3}$$

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https://doi.org/10.1016/j.rinp.2020.103711
Received 9 August 2020; Received in revised form 2 December 2020; Accepted 8 December 2020
Available online 11 December 2020

Through the simplified Hirota's method, multi-solitons of distinct physical structures were explored to each extended Jimbo–Miwa equation. By applying the Maple computer algebra system, lump and lump–kink solutions were obtained for the Jimbo–Miwa (1.1) and two extended Jimbo–Miwa equations equation (1.2) and (1.3) [24]. It demonstrates that the theoretical studies and extensions on the well-known Jimbo–Miwa equation (1.1) in (3+1)-dimensions draw much attention of researchers in various areas of natural science [24–28]. This motivates us to discuss an extended Jimbo–Miwa equation for the propagation of three-dimensional nonlinear waves.

Let us begin with a new extended Jimbo–Miwa equation in the research and development process of nonlinear physical phenomena, read as

$$u_{xxxy}+\chi(u_xu_y)_x+\rho_1u_{xy}+\rho_2u_{xz}+\rho_3u_{yt}+\rho_4u_{yy}=0, \tag{1.4}$$

where $\chi$ is a non-zero constant coefficient and $\rho_i, 1\le i\le 4$, are all arbitrary real constants, but the constants $\rho_2$ and $\rho_3$ satisfy $\rho_2\rho_3\ne 0$. When $\chi=3, \rho_2=-3, \rho_3=2$ and the other $\rho_i$'s are zero, the nonlinear evolution equation (1.4) reduces to the Jimbo–Miwa equation (1.1). The above equation (1.4) is also an extension of the $(3+1)$-dimensional generalized BKP equation introduced in Ref. [2]

$$u_{ty}-u_{xxxy}-3(u_xu_y)_x+3u_{xz}=0. \tag{1.5}$$

By the transformation

$$u=\frac{6}{\chi}(\ln f)_x, \tag{1.6}$$

this equation (1.4) has a Hirota bilinear formulation

$$(D_x^3D_y+\rho_1D_xD_y+\rho_2D_xD_z+\rho_3D_yD_t+\rho_4D_y^2)f\cdot f=0, \tag{1.7}$$

where $D_x, D_y, D_z$ and $D_t$ are Hirota differential operators [1]. Equivalently, we have

$$\begin{aligned}&(f_{xxxy}+\rho_1f_{xy}+\rho_2f_{xz}+\rho_3f_{yt}+\rho_4f_{yy})f-f_{xxx}f_y+3f_{xx}f_{xy}\\&\quad-3f_xf_{xxy}-\rho_1f_xf_y-\rho_2f_xf_z-\rho_3f_yf_t-\rho_4f_y^2=0.\end{aligned} \tag{1.8}$$

It is shown that Eq. (1.4) has the same degree and dimension as the Jimbo–Miwa equation (1.1). Therefore, this equation (1.4) could be widely used to describing a large number of phenomena in fluid dynamics, plasma mechanics and other fields, according to the physical nature of the Jimbo–Miwa equation. There are some studies on Wronskian and Grammian formulations to the Hirota bilinear equation (1.7) by using the KP hierarchy reduction [29,30]. By applying the linear superposition principle, the Hirota bilinear form (1.7) has been derived to possess the following $N$-wave solution [30]:

$$f=\sum_{i=1}^N\varepsilon_if_i=\sum_{i=1}^N\varepsilon_ie^{k_ix+\delta_1k_i^2y+(\delta_2k_i^4+\delta_3k_i^2)z+(\delta_4k_i^3+\delta_5k_i^2)t}, \tag{1.9}$$

where the $\varepsilon_i$s and $k_i$s are arbitrary constants, but $\delta_i, 2\le i\le 5$, satisfy the following system

$$\delta_2=\frac{-3\delta_1}{\rho_2},\ \delta_3=\frac{-\rho_1\delta_1}{\rho_2},\ \delta_4=\frac{2}{\rho_3},\ \delta_5=\frac{-\rho_4\delta_1}{\rho_3}, \tag{1.10}$$

with $\delta_1$ is a free parameter not to be zero. Each exponential wave $f_i$ satisfies the corresponding nonlinear dispersion relation in this solution.

This paper aims to look for a few classes of exact solutions for the extended Jimbo–Miwa equation (1.4) through symbolic computations with Maple. Based on the associated Hirota bilinear formulation (1.7), we would like to explore one-wave and two-wave solutions by Maple symbolic computations in Section 2. A kind of resonant solutions can be computed from the resulting two-wave solutions with particular phase shifts. In Section 3, under the help of Maple, a general type of complexiton solutions will be determined. In addition, we will show that the extended Jimbo–Miwa equation (1.4) exists a class of nonsingular complexiton solutions. Section 4 gives our conclusions and remarks.

## 2. Resonant solutions

### 2.1. One-wave type solutions

We now consider some special expressions with $u$ and $f$ to establish one-wave type solutions for the bilinear extended Jimbo–Miwa equation (1.7). Conducting symbolic computations with Maple, the following three exact solutions for the bilinear extended Jimbo–Miwa equation (1.7) can be obtained :

$$f=e^{a_1g_1(x)+a_2y+a_3g_2(z)+a_4g_3(t)+a_5}, \tag{2.1}$$

$$\begin{aligned}&f=h_1\left(a_1x+a_2y+a_3z+\frac{4a_1^3a_2-\rho_1a_1a_2-\rho_2a_1a_3-\rho_4a_2^2}{\rho_3a_2}t+a_5\right),\\&h_1=\sin\ \text{or}\ \cos,\end{aligned} \tag{2.2}$$

$$\begin{aligned}&f=h_2\left(a_1x+a_2y+a_3z-\frac{4a_1^3a_2+\rho_1a_1a_2+\rho_2a_1a_3+\rho_4a_2^2}{\rho_3a_2}t+a_5\right),\\&h_2=\sinh\ \text{or}\ \cosh,\end{aligned} \tag{2.3}$$

where $g_i, 1\le i\le 3$, are arbitrary functions, $h_1$ is the sine or cosine function, $h_2$ is hyperbolic sine or hyperbolic cosine function, and the parameters $a_i, 1\le i\le 5$, are arbitrary. Although all kinds of solutions above are fascinating solutions of the bilinear extended Jimbo–Miwa equation (1.7), they cannot yield nontrivial exact solutions to the extended Jimbo–Miwa equation (1.4) via the transformation (1.6).

### 2.2. Soliton and resonant solutions

In what follows, to investigate two-wave solutions of Eq. (1.4) , we suppose

$$\begin{aligned}&u=\frac{6}{\chi}(\ln f)_x,\ f=\varepsilon_1e^{\theta_1}+\varepsilon_2e^{\theta_2}+A\varepsilon_1\varepsilon_2e^{\theta_1}e^{\theta_2}+a_9,\\&\theta_1=a_1x+a_2y+a_3z+a_4t,\ \theta_2=a_5x+a_6y+a_7z+a_8t,\end{aligned} \tag{2.4}$$

where $\varepsilon_i\ne 0, i=1,2$, are arbitrary, and $a_i, 1\le i\le 9$, and $A$ are parameters to be computed. Substituting $f$ into (1.8), and applying Maple symbolic computations, we have three cases of solutions for the parameters $A$ and $a_i, 1\le i\le 9$, as follows:

Case 1

$$\begin{aligned}a_4=&-\frac{1}{\rho_3(a_2-a_6)}\left[(a_1-a_5)^3(a_2-a_6)+\rho_1(a_1-a_5)(a_2-a_6)\right.\\&\left.+\rho_2(a_1-a_5)(a_3-a_7)+\rho_3(a_6a_8-a_2a_8)+\rho_4(a_2-a_6)^2\right],\\a_9=&\,0,A=0,\end{aligned} \tag{2.5}$$

where $a_2-a_6\ne 0$ and the other parameters are arbitrary.

Case 2

$$\begin{aligned}a_4&=-\frac{a_1^3a_2+\rho_1a_1a_2+\rho_2a_1a_3+\rho_4a_2^2}{\rho_3a_2},\\a_8&=-\frac{a_5^3a_6+\rho_1a_5a_6+\rho_2a_5a_7+\rho_4a_6^2}{\rho_3a_6},\ A=\frac{c_1}{c_2},\end{aligned} \tag{2.6}$$

with the constants $c_i, i=1,2$, are defined by

$$\begin{cases}c_1=3a_1a_2a_5a_6(a_2-a_6)(a_1-a_5)+\rho_2(a_2a_7-a_3a_6)(a_1a_6-a_2a_5),\\c_2=a_9\left[3a_1a_2a_5a_6(a_2+a_6)(a_1+a_5)+\rho_2(a_2a_7-a_3a_6)(a_1a_6-a_2a_5)\right],\end{cases} \tag{2.7}$$

where $a_2a_6a_9\ne 0, c_1c_2\ne 0$, and the other parameters are arbitrary.

Case 3

$$\begin{aligned}a_3=&\frac{3a_1a_2a_5(a_1-a_5)(a_2-a_6)}{\rho_2(a_1a_6-a_2a_5)}+\frac{a_2a_7}{a_6},\\a_4=&-\frac{1}{\rho_3(a_1a_6-a_2a_5)}\left[a_1^2(a_1^2a_6+2a_1a_2a_5-3a_5^2a_2-3a_1a_5a_6+3a_6a_5^2)\right]\\&-\frac{\rho_1a_1a_6+\rho_2a_1a_7+\rho_4a_2a_6}{\rho_3a_6},\end{aligned}$$

$$a_8 = -\frac{a_5^3 a_6 + \rho_1 a_5 a_6 + \rho_2 a_5 a_7 + \rho_4 a_6^2}{\rho_3 a_6}, A = 0, \tag{2.8}$$

where $a_6 a_9(a_1 a_6 - a_2 a_5) \neq 0$ and the other parameters are arbitrary.

It is direct to see, through taking the dependent variable transformation (1.6), that the resulting exact solutions (2.5) and (2.6) yield one-soliton and two-soliton solutions, respectively, and the third class of exact solutions (2.8) can exhibit resonant phenomena. Furthermore, for the two-soliton solutions (2.6) with (2.7), note that the following special choice of parameters:

$$a_9 = 1,\ a_1 a_5 = 0 \quad \text{or} \quad a_9 = 1,\ a_1 a_6 + a_2 a_5 = 0, \tag{2.9}$$

leads to the phase shift $A = 1$.

An illustrative example is given according to the above results. Taking $\chi = 3, \rho_1 = 1, \rho_2 = -3, \rho_3 = 2, \rho_4 = 2$, and then, from (1.4), we arrive at the following specific Jimbo–Miwa type equation

$$u_{xxxy} + 3(u_x u_y)_x + u_{xy} - 3u_{xz} + 2u_{yt} + 2u_{yy} = 0. \tag{2.10}$$

This equation has its bilinear form

$$(D_x^3 D_y + D_x D_y - 3D_x D_z + 2D_y D_t + 2D_y^2) f \cdot f = 0, \tag{2.11}$$

under the dependent variable transformation $u = 2(\ln f)_x$.

The nonlinear equation (2.10) has a class of special two-soliton solutions

$$u = \frac{2\left[a_1 \varepsilon_1 e^{\theta_1} + a_2 \varepsilon_2 e^{\theta_2} + (a_1 + a_2)\varepsilon_1 \varepsilon_2 e^{\theta_1+\theta_2}\right]}{1 + \varepsilon_1 e^{\theta_1} + \varepsilon_2 e^{\theta_2} + \varepsilon_1 \varepsilon_2 e^{\theta_1+\theta_2}} \tag{2.12}$$

with

$$\theta_1 = a_1 x + k a_1 y + a_3 z - \frac{k a_1^4 + k a_1^2 - 3a_1 a_3 + 2k^2 a_1^2}{2k a_1} t,$$
$$\theta_2 = a_5 x - k a_5 y + a_7 z + \frac{-k a_5^4 - k a_5^2 - 3a_5 a_7 + 2k^2 a_5^2}{2k a_5} t, \tag{2.13}$$

where $k, a_1, a_5$ are arbitrary nonzero parameters and the other parameters are arbitrary. Also, let us choose the following two special sets of parameters:

$$a_1 = 2, a_2 = 1, a_3 = -3, a_5 = 5, a_6 = 2, a_7 = -1, a_8 = 2, a_9 = 0,$$
$$\varepsilon_1 = 1, \varepsilon_2 = 1,$$

and

$$a_1 = 1, a_2 = 2, a_5 = 2, a_6 = -1, a_7 = 1, a_9 = 1, \varepsilon_1 = 3, \varepsilon_2 = 5,$$

then Eq. (2.10) possesses a one-soliton solution

$$u = \frac{2(2e^{2x+y-3z+9t} + 5e^{5x+2y-z+2t})}{e^{2x+y-3z+9t} + e^{5x+2y-z+2t}}, \tag{2.14}$$

and a resonant two-wave solution

$$u = \frac{2(3e^{x+2y-\frac{22}{5}z-\frac{63}{10}t} + 10e^{2x-y+z-7t})}{3e^{x+2y-\frac{22}{5}z-\frac{63}{10}t} + 5e^{2x-y+z-7t} + 1}, \tag{2.15}$$

respectively. Three-dimensional plots of these two solutions are made in Fig. 1. It is easy to observe that the solution (2.14) is the kink-shape solitary wave, which spreads without any temporal evolution in size or shape and phase speed is amplitude-dependent. In the resonant two-wave solution (2.15) , a big soliton is grown from interacting solitons and its amplitude is larger than any amplitude of solitons before resonance in the interaction region, since the interaction terms vanish.

## 3. Nonsingular complexiton solutions

As we stated in §1, basic approaches to complexiton solutions include the Wronskian formula [10], the linear superposition principle [11,12] and the Hirota perturbation technique [14,15]. In this section, our main concern is to get a general kind of complexiton solutions, particularly nonsingular complexiton solutions via symbolic calculations with Maple for the extended Jimbo–Miwa equation (1.4) in (3+1)-dimensions.

### 3.1. Complexiton solutions

We begin with an ansatz for complexiton solutions:

$$f = 1 + 2\varepsilon_1 e^{\theta_1} h(\theta_2) + \varepsilon_2 e^{2\theta_1}, \quad h = \sin \text{ or } \cos, \tag{3.1}$$

where

$$\theta_1 = a_1 x + a_2 y + a_3 z + a_4 t,\ \theta_2 = a_5 x + a_6 y + a_7 z + a_8 t + a_9, \tag{3.2}$$

and $\varepsilon_1$, $\varepsilon_2$ and $a_i, 1 \le i \le 9$, are parameters to be computed. Plugging this expression into the bilinear form (1.8) generates a set of nonlinear algebraic equations in terms of the parameters $a_i, 1 \le i \le 9, \varepsilon_1$ and $\varepsilon_2$. Solving the algebraic system with Maple, a direct analysis provides us with two cases of solutions for the parameters as follows:

Case 1

$$\varepsilon_2 = 0,\ a_3 = \frac{3a_5(a_2^2 + a_6^2)(a_1^2 + a_5^2)}{\rho_2(a_1 a_6 - a_2 a_5)} + \frac{a_2 a_7}{a_6},$$
$$a_4 = -\frac{2a_1 a_2 a_5(3a_2^2 + a_1^2) + a_6(a_1^4 + 3a_5^4)}{\rho_3(a_1 a_6 - a_2 a_5)} - \frac{\rho_1 a_1 a_6 + \rho_2 a_1 a_7 + \rho_4 a_2 a_6}{\rho_3 a_6},$$
$$a_8 = \frac{4a_5^3 a_6 - \rho_1 a_5 a_6 - \rho_2 a_5 a_7 - \rho_4 a_6^2}{\rho_3 a_6}, \tag{3.3}$$

where $\varepsilon_1 a_6(a_1 a_6 - a_2 a_5) \neq 0$ and the other parameters are arbitrary.

Case 2

$$a_3 = \frac{(a_2^2 + a_6^2)(3a_1^2 a_5 - a_5^3 + \rho_1 a_5 + \rho_3 a_8 + \rho_4 a_6) + \rho_2 a_7(a_1 a_2 + a_5 a_6)}{\rho_2(a_1 a_6 - a_2 a_5)},$$
$$a_4 = \frac{-1}{\rho_3(a_1 a_6 - a_2 a_5)}\left[(a_1^2 + a_5^2)(a_1^2 a_6 - a_5^2 a_6 + 2a_1 a_2 a_5 + \rho_1 a_6 + \rho_2 a_7)\right.$$
$$\left. + \rho_3 a_8(a_1 a_2 + a_5 a_6) + \rho_4(a_5 a_6^2 - a_2^2 a_5 + 2a_1 a_2 a_6)\right],$$
$$\varepsilon_2 = \frac{-\varepsilon_1^2(4a_5^3 a_6 - \rho_1 a_5 a_6 - \rho_2 a_5 a_7 - \rho_3 a_6 a_8 - \rho_4 a_6^2)}{a_5 a_6(3a_1^2 - a_5^2) + 3a_1 a_2(a_1^2 + a_5^2) + a_5(\rho_1 a_6 + \rho_2 a_7) + a_6(\rho_3 a_8 + \rho_4 a_6)}, \tag{3.4}$$

where $\varepsilon_2(a_1 a_6 - a_2 a_5) \neq 0$ and the other parameters are arbitrary given that all the terms in (3.4) will make sense.

By the logarithm transformation (1.6), we get a type of complexiton solutions for the extended Jimbo–Miwa equation (1.4) as

$$u = \frac{12(a_1 \varepsilon_1 e^{\theta_1} h(\theta_2) + a_5 \varepsilon_1 e^{\theta_1} h_{\theta_2}(\theta_2) + a_1 \varepsilon_2 e^{2\theta_1})}{\chi(1 + 2\varepsilon_1 e^{\theta_1} h(\theta_2) + \varepsilon_2 e^{2\theta_1})}, \quad h = \sin \text{ or } \cos, \tag{3.5}$$

where $\theta_i, i = 1, 2$, the involved parameters are defined by the above result, and $h_{\theta_2}$ denotes the derivative of $h$ with respect to $\theta_2$. In general, the complexiton solutions (3.5) given in this way could be found to be singular since $f$ defined by (3.1) with (3.2) has zeros.

According to our presented solution (3.4) and taking $a_1 = 2, a_2 = -3, a_5 = 2, a_6 = -1, a_7 = -1, a_8 = 4, a_9 = 2, \varepsilon_1 = 3, h = \cos$, the extended Jimbo–Miwa equation (2.10) has a special complexiton solution as follows:

$$u = \frac{p_1}{1 + 6e^{2x-3y-18z+31t}\cos(2x - y - z + 4t + 2) - \frac{5}{3}e^{4x-6y-36z+62t}}, \tag{3.6}$$

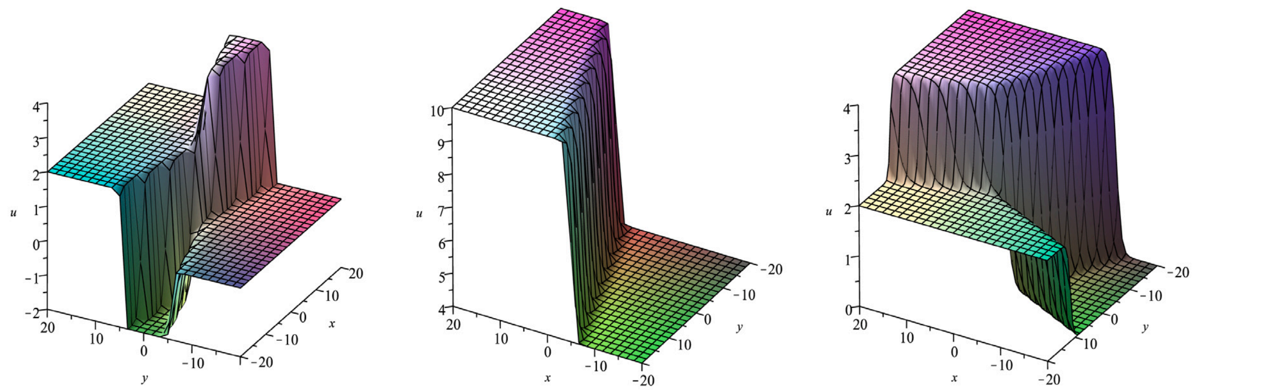
where

$$p_1 = 2\left[12e^{2x-3y-18z+31t}\cos(2x - y - z + 4t + 2)\right.$$
$$\left. - 12e^{2x-3y-18z+31t}\sin(2x - y - z + 4t + 2) - \frac{20}{3}e^{4x-6y-36z+62t}\right].$$

Three-dimensional graphics of this complexiton solution are exhibited, which show some singularities of the solution, in Fig. 2.

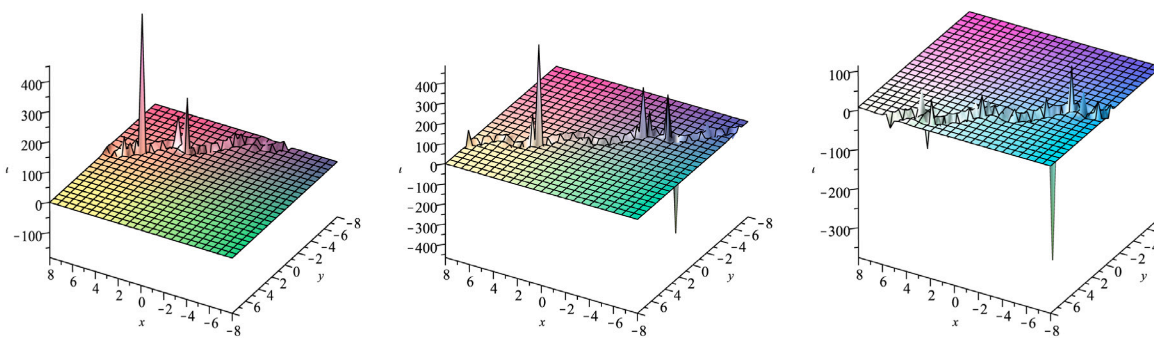
### 3.2. Nonsingular complexiton solutions

It is known that complexiton solutions emerge as singular and nonsingular. For higher-dimensional nonlinear evolution equations and the coupled integrable equations [13,15,31], there exist nonsingular complexiton solutions, and the nonsingular periodic-soliton solutions,



(a) (b) (c)

**Fig. 1.** (a) The two-wave solution (2.12) with special parameters: $a_1 = 2, a_3 = 3, a_5 = -1, a_7 = 2, k = 4, \varepsilon_1 = 1, \varepsilon_2 = 2, z = 1, t = 0$. (b) The one-soliton wave (2.14) with $z = 1, t = 1$. (c) The resonant two-wave solution (2.15) with $z = 1, t = 0$.



(a) (b) (c)

**Fig. 2.** The nonlinear wave propagation determined by the solution (3.6) with parameters: (a) $z = 1, t = 0$, (b) $z = 1, t = 0.5$, (c) $z = 1, t = 1$.

or complexiton solutions, were obtained for two higher-dimensional fifth-order nonlinear integrable systems by Wazwaz [15]. Generally, it is essential to explore nonsingular complexiton solutions for nonlinear differential equations because they could describe complicated nonlinear physical phenomena.

To search for nonsingular complexiton solutions, let us now take

$$f = 1 + 2e^{\theta_1} h(\theta_2) + \gamma^2 e^{2\theta_1}, \quad h = \sin \text{ or } \cos, \tag{3.7}$$

where $\theta_i, i = 1, 2$, are defined by (3.2) and $\gamma$ is a nonzero real parameter. A similar direct symbolic calculation yields two cases of solutions for the parameters as follows:

Case 1

$$\begin{aligned} &\gamma = \pm 1,\ a_2 = \frac{-a_5 a_6}{a_1}, \\ &a_4 = \frac{3a_1^2 a_5^2 a_6 - a_1^4 a_6 - \rho_1 a_1^2 a_6 - \rho_2 a_1^2 a_7 + \rho_4 a_5 a_6^2}{\rho_3 a_1 a_6}, \\ &a_8 = \frac{-3a_1^2 a_5 a_6 + a_5^3 a_6 - \rho_1 a_5 a_6 + \rho_2 a_1 a_3 - \rho_4 a_6^2}{\rho_3 a_6}, \end{aligned} \tag{3.8}$$

where $a_1 a_6 \neq 0$ and the other parameters are arbitrary.

Case 2

$$\begin{aligned} a_4 = &-\frac{a_1^3 a_2(4\gamma^2 - 1) + 3a_1 a_5(a_1 a_6 + a_2 a_5) + 3a_5^3 a_6}{\rho_3 a_2(\gamma^2 - 1)} \\ &- \frac{\rho_1 a_1 a_2 + \rho_2 a_1 a_3 + \rho_4 a_2^2}{\rho_3 a_2}, \\ a_7 = &\frac{3(a_1^2 + a_5^2)(a_2^2 + a_6^2)(\gamma^2 a_1 a_2 + a_5 a_6)}{\rho_2 a_2(\gamma^2 - 1)(a_1 a_6 - a_2 a_5)} + \frac{a_3 a_6}{a_2}, \\ a_8 = &\frac{-1}{\rho_3 a_2(\gamma^2 - 1)(a_1 a_6 - a_2 a_5)} \\ &\times \left[a_5^2(4a_1 a_2 a_5 a_6 - a_2^2 a_5^2 + 3a_1^2 a_2^2 + 3a_5^2 a_6^2 + 3a_6^2 a_1^2) \right. \\ &\left. + a_2 \gamma^2 (a_2 a_5^4 + 3a_2 a_1^4 + 6a_1^3 a_5 a_6 + 2a_5^3 a_1 a_6)\right] \\ &- \frac{\rho_1 a_2 a_5 + \rho_2 a_3 a_5 + \rho_4 a_2 a_6}{\rho_3 a_2}, \end{aligned} \tag{3.9}$$

where $a_2(\gamma^2 - 1)(a_1 a_6 - a_2 a_5) \neq 0$.

Note that $f$ in (3.7) is a positive function if the parameter $|\gamma| > 1$ holds, and so, a kind of nonsingular complexiton solutions to the extended Jimbo–Miwa equation (1.4) may be derived:

$$u = \frac{6}{\chi} \frac{a_1 h(\theta_2) + a_5 h_{\theta_2}(\theta_2) + a_1 \gamma^2 e^{\theta_1}}{\sqrt{\gamma^2}\cosh(\theta_1 + \ln\sqrt{\gamma^2}) + h(\theta_2)}, \quad h = \sin \text{ or } \cos, \tag{3.10}$$

where $\theta_i, i = 1, 2$, are defined by (3.2), the involved parameters are defined by (3.9) and $h_{\theta_2}$ denotes the derivative of $h$ with respect to $\theta_2$. Let us take $a_1 = a_3 = a_6 = 0$ in case 2, then the solutions (3.10) become

$$u = \frac{6}{\chi} \frac{h_x\left(a_5 x + \frac{a_5^3 - \rho_1 a_5}{\rho_3} t + a_9\right)}{\sqrt{\gamma^2}\cosh\left(a_2 y - \frac{\rho_4 a_2}{\rho_3} t + \ln\sqrt{\gamma^2}\right) + h\left(a_5 x + \frac{a_5^3 - \rho_1 a_5}{\rho_3} t + a_9\right)},$$
$$h = \sin \text{ or } \cos, \tag{3.11}$$

where $a_2 a_5 \neq 0$ and $h_x$ indicates the derivative of $h$ with respect to $x$. This class of solutions describes a stationary wave periodic in $x$ with period $2\pi/|a_5|$ and exponentially decaying along the propagating direction $y$. The condition for guaranteeing nonsingular complexiton solutions is $|\gamma| > 1$.

For the extended Jimbo–Miwa equation (2.10), a typical spatial structure of nonsingular complexiton solutions (3.11) is depicted in Fig. 3. It indicates an inclined sequence of algebraic solitons. In Refs. [9, 15,21], this solution is also called the nonsingular periodic soliton.

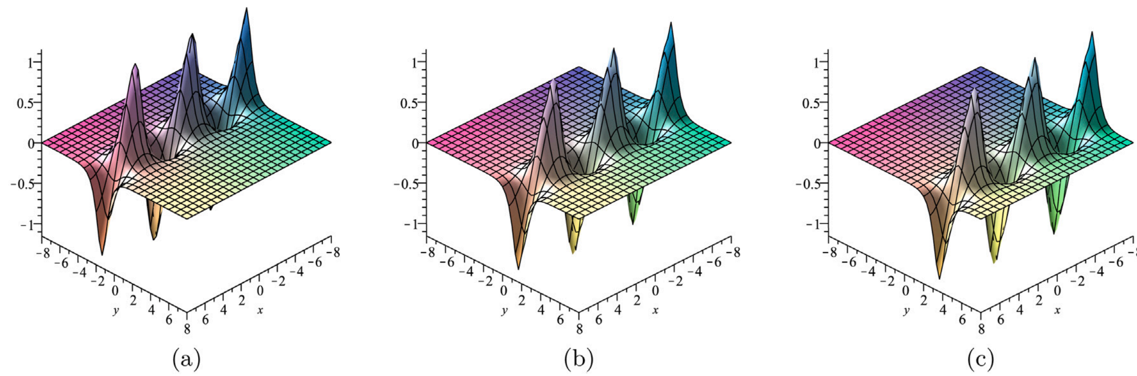


**Fig. 3.** For Eq. (2.10), the nonlinear wave propagation determined by the expression (3.11) with parameters: $a_2 = 2, a_5 = -1, a_9 = 8, \gamma = 2, h = \sin$ and (a) $t = -1$, (b) $t = 2$, (c) $t = 4$.

The parameters are chosen with $a_2 = 2, a_5 = -1, a_9 = 8, h = \sin$ and $\gamma = 2$. As shown in Fig. 3, this solution (3.11) represents a kink solitary wave which is periodic along the propagating direction and analytical without any singularity. The wave number of this kink solitary solution determines its amplitude. It is easy to observe that the solution possesses one bottom hump and upper peak in each periodic unit.

We point out that the kind of complexiton solutions defined by (3.10) with (3.7) and (3.8) can also be generated from the special two-soliton solutions (2.12) with (2.13) to Eq. (2.10), through extending the parameters to the complex field [21]. The theoretical solutions as well as numerical studies are beneficial in investigating the nonlinear wave propagation in any natural varied instance due to the variation of the parameters.

## 4. Concluding remarks

To conclude, we discussed an extended Jimbo–Miwa equation (1.4) in (3+1)-dimensions, and computed a few types of two-wave solutions and nonsingular complexiton solutions, via Maple symbolic computations. Resonant and complexiton solutions can be generated from the resulting two-wave solutions with particular phase shifts. Hirota bilinear formulations play a leading part in establishing two-wave and complexiton solutions. Nonsingular complexitons to the Jimbo–Miwa equation (1.4) should be new, though there exist plenty of studies in the literature. Our results enlarge the type of nonlinear higher-dimensional equations which exist nonsingular complexiton solutions.

We remark that the following invertible linear transformation:

$$x' = x,\ y' = \delta_1 y + \delta_3 z + \delta_5 t,\ z' = \delta_2 z,\ t' = \delta_4 t, \tag{4.1}$$

where $\delta_i, 1 \le i \le 5$, are defined by (1.10), can transform (1.7) into

$$(D_{x'}^3 D_{y'} - 3D_{x'} D_{z'} + 2D_{y'} D_{t'}) f \cdot f = 0, \tag{4.2}$$

which yields the Hirota bilinear formulation of the Jimbo–Miwa equation (1.1). Recent studies demonstrate that there exists a class of compelling exact solutions called lumps, stemmed from solving integrable systems [32–36]. Typical examples include the second KPI equation and the combined fourth-order equation [37,38]. It is recognized that linear partial differential equations exist lump solutions as well [39]. Lump-type solutions, which are rationally localized in almost all directions in space, were given to the Jimbo–Miwa equation [19] in (3+1)-dimensions. Therefore, it is worth studying lump-type and interaction solutions between lumps and other classes of explicit solutions to the extended Jimbo–Miwa equation (1.4). It should be noted that the kind of nonsingular complexiton solutions (3.10) may be reduced to lump-type solutions by using the long wave limit procedure [40]. Another interesting problem is to investigate integrable properties to nonlinear multi-dimensional equations [41]. We hope that more research will be done in the future.

## CRediT authorship contribution statement

**Li Cheng:** Writing - original draft, Visualization. **Yi Zhang:** Supervision, Methodology. **Wen-Xiu Ma:** Supervision, Methodology.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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