Nonsingular complexiton solutions and resonant waves to an extended Jimbo–Miwa equation

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A B S T R A C T

The goal of this letter is to consider an extended Jimbo–Miwa equation which exists nonsingular complexiton solutions in (3+1)-dimensions. A few types of two-wave and complex solutions are developed through symbolic computations with Maple, including resonant two-wave solutions and nonsingular complexiton solutions. Nonsingular complexitons to the generalized Jimbo–Miwa equation should be new, though there exist plenty of studies in the literature. A few specific cases are derived to illustrate the remarkable richness of explicit solutions for the considered equation.

1. Introduction

The research of exact solutions is an important part to nonlinear differential equations and has become a hot topic. The determination of exact solutions, particularly the multi-soliton solutions, can help us to comprehend various qualitative and quantitative characteristics of nonlinear phenomena modeled by these evolution equations. For typical soliton equations, there possess multiple-soliton solutions, constructed from combinations of multiple exponential waves in terms of their Hirota bilinear formulations. Such equations contain the Korteweg–de Vries (KdV) equation, the Kadomtsev–Petviashvili (KP) equation and the Sawada–Kotera equation [1]. It is recognized by applying the multiple exp-function algorithm and simplified Hirota’s approach that many higher-dimensional equations can also possess multiwave soliton type solutions, which include the generalized KP, B-type Kadomtsev–Petviashvili (BKP) equations and the (3+1)-dimensional Jimbo–Miwa equation [2–5]. Many kinds of effective methods have been built for the investigation of soliton solutions, such as the Hirota bilinear approach [1,8], the inverse scattering transform [7] and the Hirota bilinear approach [1,8]. Enlarging the diversity of solitons, complexiton solutions or periodic-soliton solutions have been introduced in previous papers [9,10]. Some recent works have been carried out on the complexiton solutions which involve two classes of transcendental functions, namely, exponential and trigonometric functions [11–15].

The Wronskian method is highly designed to seek complexiton solutions for soliton equations [10], but the complexiton solutions derived from the Wronskian formulation are extremely complicated to higher-dimensional equations. The linear superposition principle also provides a useful tool for finding complexiton solutions [11]. Moreover, complexiton solutions may be generated from solitons via extending the real parameters to the complex field [9,14,15].

The second member of the famous KP hierarchy is the Jimbo–Miwa equation:

\[ u_{xxx} + 3u_x u_{xx} + 3u_x u_{xy} + 2u_{xt} - 3u_{xz} = 0, \]

(1.1)

which is applied to describing the propagation of three-dimensional nonlinear waves with a weak dispersion [16]. More recent studies exhibit that this equation exists diverse exact solutions [2,17–20]. The exp-function algorithm allowed us to construct two- and three-wave solutions and traveling wave solutions [2], and the linear superposition principle presented resonant multiple wave solutions [20]. A direct symbolic calculation yielded abundant lump-type solutions and interaction solutions [18,19]. Multiple periodic-soliton solutions generated from multiple line-soliton solutions were established through the Hirota bilinear approach in Refs. [21,22]. As a generalization of Eq. (1.1), Wazwaz presented the two extended Jimbo–Miwa equations as follows [23]:

\[ u_{xxx} + 3u_x u_{xx} + 3u_x u_{xy} + 2u_{xt} - 3(u_{xz} + u_{xx} + u_{xx} + u_{xz}) = 0, \]

(1.2)

\[ u_{xxx} + 3u_x u_{xx} + 3u_x u_{xy} + 2(u_{xt} + u_{xt} + u_{xx} + u_{xz}) = 0. \]

(1.3)
Through the simplified Hirota’s method, multi-solitons of distinct physical structures were explored to each extended Jimbo–Miwa equation. By applying the Maple computer algebra system, lump and lump–kink solutions were obtained for the Jimbo–Miwa (1.1) and two extended Jimbo–Miwa equations equation (1.2) and (1.3) [24]. It demonstrates that the theoretical studies and extensions on the well-known Jimbo–Miwa equation (1.1) in (3+1)-dimensions draw much attention of researchers in various areas of natural science [24–28]. This motivates us to discuss an extended Jimbo–Miwa equation for the propagation of three-dimensional nonlinear waves.

Let us begin with a new extended Jimbo–Miwa equation in the research and development process of nonlinear physical phenomena, read as

\[ u_{xxyy} + x(u_u)_{x} + \rho_1 u_x + \rho_2 u_{xx} + \rho_3 u_{xy} + \rho_4 u_{yy} = 0, \]

(1.4)

where \( x \) is a non-zero constant coefficient and \( \rho_i, 1 \leq i \leq 4 \), are all arbitrary real constants, but the constants \( \rho_1 \) and \( \rho_2 \) satisfy \( \rho_1 \rho_2 \neq 0 \). When \( x = 3, \rho_1 = -3, \rho_2 = 2 \) and the other \( \rho_i \)'s are zero, the nonlinear evolution equation (1.4) reduces to the Jimbo–Miwa equation (1.1). The above equation (1.4) is also an extension of the (3 + 1)-dimensional generalized BKP equation introduced in Ref. [2]

\[ u_{yy} - u_{xxyy} - 3(u_u)_{x} + 3u_z = 0. \]

(1.5)

By the transformation

\[ u = \frac{6}{x} \log f, \]

(1.6)

the equation (1.4) has a Hirota bilinear formulation

\[ (D^3_x + \rho_1 D_x + \rho_2 D_x^2 + \rho_3 D_x + \rho_4 D_x^2)f \cdot f = 0, \]

(1.7)

where \( D, D_x, D_y \) and \( D_z \) are Hirota differential operators [1]. Equivalently, we have

\[ (f_{xxyy} + \rho_1 f_{xy} + \rho_2 f_{xx} + \rho_3 f_{yy} + \rho_4 f_{xy})f - f_{xxx} f_y + 3f_{xx} f_{yy} - 3f_{xfxx} - \rho_1 f_{xfy} - \rho_2 f_{xx} f_y - \rho_3 f_{xy} f_y - \rho_4 f_{yx} f_y = 0. \]

(1.8)

It is shown that Eq. (1.4) has the same degree and dimension as the Jimbo–Miwa equation (1.1). Therefore, this equation (1.4) could be widely used to describing a large number of phenomena in fluid dynamics, plasma mechanics and other fields, according to the physical nature of the Jimbo–Miwa equation. There are some studies on Wronskian and Grammian formulations to the Hirota bilinear equation (1.7) by using the KP hierarchy reduction [29,30]. By applying the linear superposition principle, the Hirota bilinear form (1.7) has been derived to possess the following N-wave solution [30]:

\[ f = \sum_{n=1}^{N} \epsilon_i f_i = \sum_{n=1}^{N} \epsilon_i e^{i\delta_i x + \lambda_i k_i^1 x + \lambda_i k_i^2 y + \lambda_i k_i^3 z + \lambda_i k_i^4 t + \lambda_i k_i^5 \tau}, \]

(1.9)

where the \( \epsilon_i \)’s and \( k_i \)’s are arbitrary constants, but \( \delta_i, 2 \leq i \leq 5 \), satisfy the following system

\[ \delta_i = -\frac{3\delta_i}{\rho_i}, \quad \delta_1 = -\frac{\rho_1 \delta_1}{\rho_2}, \quad \delta_4 = \frac{2}{\rho_3}, \quad \delta_5 = -\frac{\rho_4 \delta_1}{\rho_5}, \]

(1.10)

with \( \delta_1 \) is a free parameter not to be zero. Each exponential wave \( f_i \) satisfies the corresponding nonlinear dispersion relation in this solution.

This paper aims to look for a few classes of exact solutions for the extended Jimbo–Miwa equation (1.4) through symbolic computations with Maple. Based on the associated Hirota bilinear formulation (1.7), we would like to explore one-wave and two-wave solutions by Maple symbolic computations in Section 2. A kind of resonant solutions can be computed from the resulting two-wave solutions with particular phase shifts. In Section 3, under the help of Maple, a general type of complexiton solutions will be determined. In addition, we will show that the extended Jimbo–Miwa equation (1.4) exists a class of nonsingular complexiton solutions. Section 4 gives our conclusions and remarks.
where \( a_0a_0(a_0 - a_0a_0) \neq 0 \) and the other parameters are arbitrary.

It is direct to see, through taking the dependent variable transformation (1.6), that the resulting exact solutions (2.5) and (2.6) yield one-soliton and two-soliton solutions, respectively, and the third class of exact solutions (2.8) can exhibit resonant phenomena. Furthermore, for the two-soliton solutions (2.6) with (2.7), note that the following special choice of parameters:

\[
a_0 = 1, a_1a_2 = 0 \quad \text{or} \quad a_0 = 1, a_1a_2 + a_0a_5 = 0,
\]

leads to the phase shift \( A = 1 \).

An illustrative example is given according to the above results. Taking \( \chi = 3, \rho_1 = 1, \rho_2 = -3, \rho_3 = 2, \rho_4 = 2 \), and then, from (1.4), we arrive at the following specific Jimbo–Miwa type equation

\[
u_{xxxy} + 3(a_0u_x) + u_{yy} - 3u_{x} + 2u_{x2} + 2u_{y} = 0.
\]

This equation has its bilinear form

\[
(D_x^3D_y + D_xD_y - 3D_xD_y + 2D_xD_y + 2D_y^3)f \cdot f = 0,
\]

under the dependent variable transformation \( u = 2(\ln f)_x \).

The nonlinear equation (2.10) has a class of special two-soliton solutions

\[
\frac{2[x_1e^{\rho_1^5} + a_0e^{\rho_2}](a_1 + a_0e^{\rho_2^5})}{1 + x_1e^{\rho_1^5} + x_2e^{\rho_2} + x_1^2e^{\rho_2^5} + x_2^2e^{\rho_2^5}}
\]

with

\[
\theta_1 = a_1x + ka_1y + a_2 - \frac{k^3a_1 + k^2a_1^2 - 3a_1a_2 + 2k^2a_2^2}{2k^2a_2},
\]

\[
\theta_2 = a_2x - ka_2y + a_2 - \frac{k^2a_2^3 - 3a_2a_3 + 2k^2a_3^2}{2k^2a_3},
\]

where \( k, a_1, a_2 \) are arbitrary nonzero parameters and the other parameters are arbitrary. Also, let us choose the following two special sets of parameters:

\[
\theta_1 = 2, \theta_2 = 2, a_1 = 1, a_2 = 1, a_3 = 5, a_4 = 2, a_7 = -1, a_9 = 2, a_0 = 0, \epsilon_1 = 1, \epsilon_2 = 1,
\]

\[
a_1 = 1, a_2 = 2, a_3 = 2, a_5 = -1, a_7 = 1, a_9 = 3, \epsilon_1 = 3, \epsilon_2 = 5,
\]

and then Eq. (2.10) possesses a one-soliton solution

\[
\frac{2(2e^{x_2 + y_2} - 3e^{x_2 + y_2 - 3}) + 5e^{x_2 + y_2 - 3}e^{x_2 + y_2}}{e^{x_2 + y_2} + 3e^{x_2 + y_2 - 3}e^{x_2 + y_2 - 3}}.
\]

and a resonant two-wave solution

\[
\frac{2(3e^{x_2 + y_2} + 6e^{x_2 + y_2 - 3} + 10e^{y_2 - x_2} + 7)}{3e^{x_2 + y_2} + 6e^{x_2 + y_2 - 3} + 5e^{y_2 - x_2} + 7}.
\]

respectively. Three-dimensional plots of these two solutions are made in Fig. 1. It is easy to observe that the solution (2.14) is the kink-shape solitary wave, which spreads without any temporal evolution in size or shape and phase speed is amplitude-dependent. In the resonant two-wave solution (2.15), a big soliton is generated from interacting solitons and its amplitude is larger than any amplitude of solitons before resonance in the interaction region, since the interaction terms vanish.

### 3. Nonisolating complexon solutions

As we stated in §1, basic approaches to complexon solutions include the Wronskian formula [10], the linear superposition principle [11,12] and the Hirota perturbation technique [14,15]. In this section, our main concern is to get a general kind of complexon solutions, particularly nonisolating complexon solutions via symbolic calculations with Maple for the extended Jimbo–Miwa equation (1.4) in (3+1)-dimensions.

#### 3.1. Complexon solutions

We begin with an ansatz for complexon solutions:

\[
f = 1 + 2e^{\rho_1^5}h(\theta_1) + e^{\rho_2}h(\theta_2), \quad h = \sin or \cos,
\]

where

\[
\theta_1 = a_1x + a_2y + a_3z + a_4t, \quad \theta_2 = a_1x + a_6y + a_7z + a_8t + a_9,
\]

and \( \epsilon_1, \epsilon_2 \) and \( \theta_1, \theta_2 \) are parameters to be computed. Plugging this expression into the bilinear form (1.8) generates a set of nonlinear algebraic equations in terms of the parameters \( \theta_1, \theta_2, \epsilon_1, \epsilon_2, \) and \( \theta_1, \theta_2 \).

Solving the algebraic system with Maple, a direct analysis provides us with two cases of solutions for the parameters as follows:

**Case 1**

\[
\begin{align*}
\epsilon_2 &= 0, \\
\theta_2 &= -\frac{3a_6(a_5^2 + a_6^2)(a_1^2 + a_2^2)}{\rho_2(\rho_2a_1 - a_2a_5)} + a_0a_5, \\
a_0 &= 2a_1a_2a_5(3a_5^2 + a_6^2) + a_0(a_1^2 + 3a_2^2) - \frac{\rho_2a_1 - a_2a_5}{\rho_2a_5}, \\
\theta_1 &= -\frac{3a_6a_0a_5 - \rho_2a_1a_2a_5 - a_0a_5}{\rho_2a_5}.
\end{align*}
\]

**Case 2**

\[
\begin{align*}
\epsilon_2 &= -\frac{3a_5(a_6^2 + a_7^2)(a_5^2 + a_6^2)}{\rho_1(\rho_1a_6 - a_7a_5)} + a_0a_5, \\
\theta_2 &= a_0a_5a_6(a_5^2 + a_6^2) + a_0(a_1^2 + a_2^2) + a_0(a_1^2 + a_2^2), \\
\theta_1 &= a_0a_5(a_6^2 + a_7^2) + 3a_0a_5(a_5^2 + a_6^2) + a_0(a_1^2 + a_2^2) + a_0(a_1^2 + a_2^2).
\end{align*}
\]

where \( \theta_1, \theta_2 \) and \( \epsilon_1, \epsilon_2 \) are parameters to be computed.

By the logarithm transformation (1.6), we get a type of complexon solutions for the extended Jimbo–Miwa equation (1.4) as

\[
u = \frac{12(2e^{x_1^5}h(\theta_2) + a_1x e^{x_1^5}h(\theta_1) + a_1x e^{x_1^5}h(\theta_2))}{(1 + 2e^{x_1^5}h(\theta_2) + e^{x_1^5}h(\theta_2))}, \quad h = \sin or \cos, (3.5)
\]

where \( \theta_1, \theta_2 \) and \( \epsilon_1, \epsilon_2 \) are parameters to be computed. The complexon solutions (3.5) given in this way could be found to be singular since \(-f \) defined by (3.1) with (3.2) has zeros.

According to our presented solution (3.4) and taking \( a_1 = 2, a_2 = 3, a_3 = 2, a_5 = -1, a_7 = 1, a_9 = 4, a_0 = 2, \epsilon_1 = 3, h = \cos \), the extended Jimbo–Miwa equation (2.10) has a special complexon solution as follows:

\[
u = \frac{1}{1 + 6e^{2x_1^5} - 5e^{2x_1^5} - 3e^{2x_1^5} \cos(2x_1^5) - 5e^{2x_1^5} \cos(2x_1^5) - 3e^{2x_1^5} \cos(2x_1^5)},
\]

where

\[
\begin{align*}
\rho_1 &= 2[12e^{2x_1^5} + 3e^{2x_1^5} \cos(2x_1^5) - 5e^{2x_1^5} \cos(2x_1^5) - 3e^{2x_1^5} \cos(2x_1^5)] - \frac{20}{3}e^{2x_1^5} - 3e^{2x_1^5} - 6e^{2x_1^5} - 6e^{2x_1^5}.
\end{align*}
\]

Three-dimensional graphics of this complexon solution are exhibited, which show some singularities of the solution, in Fig. 2.
or complexiton solutions, were obtained for two higher-dimensional fifth-order nonlinear integrable systems by Wazwaz [15]. Generally, it is essential to explore nonsingular complexiton solutions for nonlinear differential equations because they could describe complicated nonlinear physical phenomena.

To search for nonsingular complexiton solutions, let us now take
\[ f = 1 + 2e^{i\psi} h(\theta_1) + \psi^2 e^{i\phi}, \quad h = \sin \text{ or } \cos, \]
where \( \theta_i, i = 1, 2 \), are defined by (3.2) and \( \psi \) is a nonzero real parameter. A similar direct symbolic calculation yields two cases of solutions for the parameters as follows:

**Case 1**
\[
\gamma = \pm 1, \quad \alpha_1 = \frac{-a_2 a_6}{a_1}, \quad \alpha_2 = \frac{3a_1^2 a_5^2 a_6 - a_4^2 a_6 - a_1 a_5 a_6 - 3a_1 a_2^2 a_5 + a_1 a_4 a_5^2}{a_3 a_1 a_6}, \quad \alpha_3 = \frac{-3a_1^2 a_5 a_6 + a_4^2 a_6 - a_1 a_5 a_6 + 2a_1 a_3 - a_4 a_6}{a_3 a_1 a_6},
\]
where \( a_1 a_6 \neq 0 \) and the other parameters are arbitrary.

**Case 2**
\[
\gamma = \frac{a_1^2 a_5(4\psi^2 - 1) + 3a_1 a_2(a_1 a_6 + a_2 a_5) + 3a_1 a_6}{a_3 a_2(\psi^2 - 1)}, \quad \alpha_2 = \frac{\rho_3 a_2(\psi^2 - 1)}{a_3 a_2}, \quad \alpha_3 = \frac{3(a_1^2 + a_2^2)(a_1^2 + a_2^2)(\psi^2 a_1 a_2 + a_2 a_6)}{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)} + \frac{a_1 a_6}{a_2}, \quad \alpha_4 = \frac{a_1 a_6}{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)}
\]
where \( \alpha_2, \alpha_3 \neq 0 \) and the other parameters are arbitrary.

Fig. 1. (a) The two-wave solution (2.12) with special parameters: \( \alpha_1 = 2, \alpha_2 = 3, \alpha_3 = -1, \alpha_4 = 2, k = 4, \epsilon_1 = 1, \epsilon_2 = 2, z = 1, t = 0 \). (b) The one-soliton wave (2.14) with \( z = 1, \epsilon = 1 \). (c) The resonant two-wave solution (2.15) with \( z = 1, \epsilon = 0 \).

Fig. 2. The nonlinear wave propagation determined by the solution (3.6) with parameters: (a) \( z = 1, \epsilon = 0 \), (b) \( z = 1, \epsilon = 0.5 \), (c) \( z = 1, \epsilon = 1 \).

\[ x = \frac{1}{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)} \theta_1 + \frac{a_1 a_6}{a_2}, \quad \alpha_3 = \frac{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)}{\rho_3 a_2}, \quad \alpha_4 = \frac{a_1 a_6}{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)}
\]

\[ \theta_1 = \frac{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)}{a_2}, \quad \alpha_3 = \frac{\rho_3 a_2(\psi^2 - 1)(\alpha_1 a_6 - a_2 a_5)}{a_2}, \quad \alpha_4 = \frac{a_1 a_6}{a_2}
\]

where \( a_2 a_5 \neq 0 \) and \( \theta_1 \) indicates the derivative of \( \theta \) with respect to \( x \). This class of solutions describes a stationary wave periodic in \( x \) with period \( 2\pi/|a_1| \) and exponentially decaying along the propagating direction \( y \). The condition for guaranteeing nonsingular complexiton solutions is \( |\gamma| > 1 \).

For the extended Jimbo–Miwa equation (2.10), a typical spatial structure of nonsingular complexiton solutions (3.11) is depicted in Fig. 3. It indicates an inclined sequence of algebraic solitons. In Refs. [9, 15, 21], this solution is also called the nonsingular periodic soliton.
The parameters are chosen with $a_2 = 2, a_5 = -1, a_9 = 8, h = \sin$ and $\gamma = 2$. As shown in Fig. 3, this solution (3.11) represents a kink solitary wave which is periodic along the propagating direction and analytical without any singularity. The wave number of this kink solitary solution determines its amplitude. It is easy to observe that the solution possesses one bottom hump and upper peak in each periodic unit.

We point out that the kind of complexiton solutions defined by (3.10) with (3.7) and (3.8) can also be generated from the special two-soliton solutions (2.12) with (2.13) to Eq. (2.10), through extending the parameters to the complex field [21]. The theoretical solutions as well as numerical studies are beneficial in investigating the nonlinear wave propagation in any natural varied instance due to the variation of the parameters.

4. Concluding remarks

To conclude, we discussed an extended Jimbo–Miwa equation (1.4) in (3+1)-dimensions, and computed a few types of two-wave solutions and nonsingular complexiton solutions, via Maple symbolic computations. Resonant and complexiton solutions can be generated from the resulting two-wave solutions with particular phase shifts. Hirota bilinear formulations play a leading part in establishing two-wave and the resulting two-wave solutions with particular phase shifts. Hirota equations. Resonant and complexiton solutions can be generated from the special two-soliton solutions (2.12) with (2.13) to Eq. (2.10), through extending the parameters to the complex field [21]. The theoretical solutions as well as numerical studies are beneficial in investigating the nonlinear wave propagation in any natural varied instance due to the variation of the parameters.

CRediT authorship contribution statement

Li Cheng: Writing - original draft, Visualization. Yi Zhang: Supervision, Methodology. Wen-Xiu Ma: Supervision, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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