Physica Scripta

PAPER

Study of lump solutions to an extended Calogero-Bogoyavlenskii-Schiff equation involving three fourth-order terms

To cite this article: Qingxian Chen *et al* 2020 *Phys. Scr.* **95** 095207

IOP Publishing | Physica Scripta
Phys. Scr. 95 (2020) 095207 (7pp) | https://doi.org/10.1088/1402-4896/abaad5

Study of lump solutions to an extended Calogero-Bogoyavlenskii-Schiff equation involving three fourth-order terms

Qingxian Chen[1,4], Wen-Xiu Ma[2,3,4,5,7] and Yehui Huang[4,6]

[1] School of Information, Zhejiang University of Finance & Economics Dongfang College, Zhejiang 314408, People's Republic of China
[2] Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, People's Republic of China
[3] Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia
[4] Department of Mathematics and Statistics, University of South Florida, Tampa, FL 33620, United States of America
[5] School of Mathematics, South China University of Technology, Guangzhou 510640, People's Republic of China
[6] School of Mathematics and Physics, North China Electric Power University, Beijing, 102206, People's Republic of China

E-mail: cqingxian@foxmail.com, mawx@cas.usf.edu and yhhuang@ncepu.edu.cn

Received 30 June 2020, revised 21 July 2020
Accepted for publication 30 July 2020
Published 10 August 2020

Abstract

A new generalized fourth-order nonlinear differential equation originating from the Calogero-Bogoyavlenskii-Schiff equation with an extra term $D_xD_y^2D_t$ is studied. In terms of the coefficients of this combined nonlinear equation, a class of lump solutions is constructed by the Hirota bilinear method and calculated through the symbolic computation system of Maple. Furthermore, the affection of the extended item on the solution is explored. Two particular lump solutions with special choices of the involved parameters are generated and plotted, as illustrative examples.

Keywords: Lump solution, Hirota bilinear method, soliton, symbolic computation, CBS equation

(Some figures may appear in colour only in the online journal)

1. Introduction

Originated from solving integrable equations, Hirota bilinear forms are introduced and used to present soliton solutions. Soliton solutions are exponentially localized solutions in certain directions. Compared with soliton solutions, lump solutions are a kind of rational function solutions, which are localized in all directions in space [1]. By taking long wave limits, some special lump solutions can be obtained from solitons of integrable equations [2], including the KP equation [3].

In recent years, lots of works have been done on soliton solutions andlump solutions to integrable equations. Some other studies haven be made, which present insightful results contributing to the basis of the related theory.

These equations include the Davey-Stewarton equation II [2], the three-dimensional three-wave resonant interaction [4], the Ishimori-I equation [5], the BKP equation [6], the CBS equation [7], and so on.

Hirota bilinear theory plays an important role in the study of soliton theory and lump solutions. Generally, through a dependent variable transformation, an integrable partial differential equation in (2+1)-dimensions can be mapped into a Hirota bilinear form:

$$P(D_x, D_y, D_t)f\cdot f = 0, \quad (1)$$

[7] Author to whom any correspondence should be addressed.

where P is a polynomial, and D_x, D_y, D_t are the Hirota bilinear derivatives [8], defined by

$$D_x^l D_y^n D_t^m f(x, y, t) \cdot g(x, y, t) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m \times f(x, y, t) \cdot g(x', y', t')|_{x'=x, y'=y, t'=t}. \quad (2)$$

When f solves (2), with the form as

$$f = \sum_{\mu=0,1} \exp\left(\sum_{i=1}^{N} \mu_i \xi_i + \sum_{i<j} \mu_i \mu_j a_{ij}\right), \quad (3)$$

where $\sum_{\mu=0,1}$ denotes the sum over all possibilities for $\mu_1, \cdots, \mu_N$ in 0, 1, and

$$\begin{cases} \xi_i = k_i x + l_i y - \omega_i t + \xi_{i,0}, & 1 \leqslant i \leqslant N, \\ e^{a_{ij}} = -\frac{P(k_i - k_j, l_i - l_j, \omega_j - \omega_i)}{P(k_i + k_j, l_i + l_j, \omega_j + \omega_i)}, & 1 \leqslant i < j \leqslant N, \end{cases} \quad (4)$$

with k_i, l_i, ω_i satisfying the dispersion relation, and $\xi_{i,0}$ being arbitrary shifts, it presents the N-soliton solution in (2+1)-dimensions to the corresponding PDE under the transformations: $u = 2(lnf)_x$ or $u = 2(lnf)_{xx}$.

Based on the Hirota bilinear form and processed by symbolic computation system of Maple, one of the authors (Ma) has presented the lump solutions of the KPI equation [1]: $u = 2(lnf)_{xx}$, where

$$f = \left(a_1 x + a_2 y + \frac{a_1 a_2^2 - a_1 a_6^2 + 2a_2 a_5 a_6}{a_1^2 + a_5^2} t + a_4\right)^2 + \left(a_5 x + a_6 y + \frac{2a_1 a_2 a_6 - a_2^2 a_5 + a_5 a_6^2}{a_1^2 + a_5^2} t + a_8\right)^2 + \frac{3(a_1^2 + a_5^2)^3}{(a_1 a_6 - a_2 a_5)^2}. \quad (5)$$

The condition $a_1 a_6 - a_2 a_5 \neq 0$ guarantees the expression (5) makes sense.

Here, the construction of a positive quadratic rational function is a key step to find lump solutions to nonlinear partial differential equations and it can be applied to general Hirota bilinear equations which contain even order terms [9].

Following this approach, lump solutions and related issues for more nonlinear equations have been studied, for example, BKP equation [10], SK equation [11], gBK equation [12], KPI equation with a self-consistent source[13], gCBS equation [14] etc. Lump solutions with higher-order rational dispersion relations have been studied in [15]. Furthermore, [16] formulates a combined nonlinear PDE involved two Fourth-order terms and all possible second-order terms and tries to extend the Hirota bilinear approach to more general nonlinear equations.

The Calogero-Bogoyavlenskii-Schiff (CBS) equation

$$u_t + uu_y + \frac{1}{2} u_x \partial_x^{-1} u_y + \frac{1}{4} u_{xxy} = 0. \quad (6)$$

was constructed by Bogoyavlenskii and Schiff in different ways [7]. It can be constructed by using the modified Lax formalism or by reducing the self-dual Yang-Mills equation.

In this paper, we extend the CBS equation (6) to following generalized nonlinear partial differential equation, which contains three fourth-order terms and all linear second-order terms:

$$\begin{aligned} P(u) = {} & \alpha[3(u_x u_t)_x + u_{xxxt}] + \beta[3(u_x u_y)_x + u_{xxxy}] \\ & + \theta(u_t s_y + 2u_y s_t + u_{tyy})_x \\ & + \gamma_1 u_{yt} + \gamma_2 u_{xx} + \gamma_3 u_{xt} \\ & + \gamma_4 u_{xy} + \gamma_5 u_{yy} + \gamma_6 u_{tt} = 0, \end{aligned} \quad (7)$$

where $s_x = u_y$, the coefficients α, β and θ satisfy $\alpha^2 + \beta^2 + \theta^2 \neq 0$, but γ_i, $1 \leqslant i \leqslant 6$, are arbitrary constants.

This (2+1)-dimensional nonlinear equation (7) possesses a Hirota bilinear form:

$$\begin{aligned} B(f) = {} & (\alpha D_x^3 D_t + \beta D_x^3 D_y + \theta D_x D_y^2 D_t + \gamma_1 D_y D_t \\ & + \gamma_2 D_x^2 + \gamma_3 D_x D_t \\ & + \gamma_4 D_x D_y + \gamma_5 D_y^2 + \gamma_6 D_t^2) f \cdot f = 0, \end{aligned} \quad (8)$$

under the logarithmic transformation

$$u = 2(lnf)_x = \frac{2f_x}{f}, \quad (9)$$

and we have the relation $P(u) = \left(\frac{B(f)}{f^2}\right)_x$.

If we get a solution to the Hirota bilinear form equation (8), we can generate the corresponding solution to the nonlinear equation (7).

The equation (8) with $\theta = 0$ has been studied in [16]. In comparison with the equation in [16], $D_x D_y^2 D_t$ is an extended-term and so we assume $\theta \neq 0$ here. In this paper, we will explore lump solutions of equation (7) and discuss the effect of the extended-term on the solutions.

In section 2, we construct lump solutions to equation(7) via a positive quadratic form and calculate them through the Maple symbolic system. In section 3, we take special values for the coefficients in equation (8), to give specific lump solutions to the equation in the corresponding cases, and get their graphs through Maple. The conclusion is given in section 4.

2. Lump solutions of a generalized fourth-order PDE with an extra term

In this section, we will explore lump solutions of the combined fourth-order equation (7) with $\theta \neq 0$. Because the equation (7) has the bilinear form (8), we construct the solution of equation (8) in a positive quadratic form as follows:

$$f = (a_1 x + a_2 y + a_3 t + a_4)^2 + (a_5 x + a_6 y + a_7 t + a_8)^2 + a_9, \quad (10)$$

where a_i, $1 \leqslant i \leqslant 9$, are the real constant parameters to be determined.

We insert the formula (10) into equation (8) to yield a system of algebraic equations on the parameters a_i, $1 \leqslant i \leqslant 9$, and try to solve them. Most of these computations are processed through the Maple symbolic computation system except for some necessary sorting and simplification of the result. In order to facilitate the calculation and expression of the results, we need to make some setting for γ_i, $1 \leqslant i \leqslant 6$, in equation (8), or equation (7).

First, let $\gamma_6 = 0$. Equation (8) becomes:

$$B(f) = (\alpha D_x^3 D_t + \beta D_x^3 D_y + \theta D_x D_y^2 D_t + \gamma_1 D_y D_t + \gamma_2 D_x^2 + \gamma_3 D_x D_t + \gamma_4 D_x D_y + \gamma_5 D_y^2) f \cdot f = 0. \tag{11}$$

After some calculations, we obtain a set of solutions for the parameters:

$$\begin{cases} a_3 = -\dfrac{b_1}{(a_2\gamma_1 + a_1\gamma_3)^2 + (a_6\gamma_1 + a_5\gamma_3)^2}, \\ a_7 = -\dfrac{b_2}{(a_2\gamma_1 + a_1\gamma_3)^2 + (a_6\gamma_1 + a_5\gamma_3)^2}, \\ a_9 = \dfrac{3(a_1^2 + a_5^2)(b_3\alpha - b_4\beta) + b_5\theta}{(a_1a_6 - a_2a_5)^2(\gamma_1^2\gamma_2 - \gamma_1\gamma_3\gamma_4 + \gamma_3^2\gamma_5)}, \end{cases} \tag{12}$$

and all the other a_i are arbitrary. The above-involved constants b_i,$1 \leqslant i \leqslant 5$, are defined as follows:

Directly comparing our results with the context in [16], where $\gamma_6 = 0$, we have some new terms contained with θ in the denominator of a_9.

Secondly, let $\gamma_5 = 0$. Equation (8) becomes:

$$B(f) = (\alpha D_x^3 D_t + \beta D_x^3 D_y + \theta D_x D_y^2 D_t + \gamma_1 D_y D_t + \gamma_2 D_x^2 + \gamma_3 D_x D_t + \gamma_4 D_x D_y + \gamma_6 D_t^2) f \cdot f = 0. \tag{14}$$

After some calculations, we obtain:

$$\begin{cases} a_2 = -\dfrac{c_1}{(a_3\gamma_1 + a_1\gamma_4)^2 + (a_7\gamma_1 + a_5\gamma_4)^2}, \\ a_6 = -\dfrac{c_2}{(a_3\gamma_1 + a_1\gamma_4)^2 + (a_7\gamma_1 + a_5\gamma_4)^2}, \\ a_9 = \dfrac{3(a_1^2 + a_5^2)(c_3\alpha + c_4\beta)}{(a_1a_7 - a_3a_5)^2(\gamma_1^2\gamma_2 - \gamma_1\gamma_3\gamma_4 + \gamma_4^2\gamma_6)} \\ - \dfrac{c_5\theta}{(a_1a_7 - a_3a_5)^2[(a_3\gamma_1 + a_1\gamma_4)^2 + (a_7\gamma_1 + a_5\gamma_4)^2](\gamma_1^2\gamma_2 - \gamma_1\gamma_3\gamma_4 + \gamma_4^2\gamma_6)}, \end{cases} \tag{15}$$

where all the other a_i are arbitrary constants. The involved constants c_i,$1 \leqslant i \leqslant 5$, are defined as follows:

$$\begin{cases} b_1 = [(a_1^2a_2 + 2a_1a_5a_6 - a_2a_5^2)\gamma_2 + a_1(a_2^2 + a_6^2)\gamma_4 + a_2(a_2^2 + a_6^2)\gamma_5]\gamma_1 \\ \quad + [a_1(a_1^2 + a_5^2)\gamma_2 + a_2(a_1^2 + a_5^2)\gamma_4 + (a_1a_2^2 + 2a_2a_5a_6 - a_1a_6^2)\gamma_5]\gamma_3, \\ b_2 = [(-a_1^2a_6 + 2a_1a_2a_5 + a_5^2a_6)\gamma_2 + a_5(a_2^2 + a_6^2)\gamma_4 + a_6(a_2^2 + a_6^2)\gamma_5]\gamma_1 \\ \quad + [a_5(a_1^2 + a_5^2)\gamma_2 + a_6(a_1^2 + a_5^2)\gamma_4 + (-a_2^2a_5 + 2a_1a_2a_6 + a_5a_6^2)\gamma_5]\gamma_3, \\ b_3 = (a_1^2 + a_5^2)(a_1a_2 + a_5a_6)(\gamma_1\gamma_2 + \gamma_3\gamma_4) + (a_1^2 + a_5^2)(a_2^2 + a_6^2)\gamma_1\gamma_4 \\ \quad + (a_1^2 + a_5^2)^2\gamma_2\gamma_3 + (a_2^2 + a_6^2)(a_1a_2 + a_5a_6)\gamma_1\gamma_5 \\ \quad + [(a_1a_2 + a_5a_6)^2 - (a_1a_6 - a_2a_5)^2]\gamma_3\gamma_5, \\ b_4 = (a_1a_2 + a_5a_6)[(a_2\gamma_1 + a_1\gamma_3)^2 + (a_6\gamma_1 + a_5\gamma_3)^2], \\ b_5 = [3(a_1a_2 + a_5a_6)^2 - (a_1a_6 - a_2a_5)^2][(a_1a_2 + a_5a_6)\gamma_1\gamma_2 + (a_2^2 + a_6^2)\gamma_3\gamma_5] \\ \quad + [3(a_1a_2 + a_5a_6)^2 + (a_1a_6 - a_2a_5)^2][(a_2^2 + a_6^2)\gamma_1\gamma_4 + (a_1^2 + a_5^2)\gamma_2\gamma_3] \\ \quad + 3(a_2^2 + a_6^2)(a_1a_2 + a_5a_6)[(a_2^2 + a_6^2)\gamma_1\gamma_5 + (a_1^2 + a_5^2)\gamma_3\gamma_4]. \end{cases} \tag{13}$$

$$\begin{cases} c_1 = [(a_1^2a_3 + 2a_1a_5a_7 - a_3a_5^2)\gamma_2 + a_1(a_3^2 + a_7^2)\gamma_3 + a_3(a_3^2 + a_7^2)\gamma_6]\gamma_1 \\ \quad + [a_1(a_1^2 + a_5^2)\gamma_2 + a_3(a_1^2 + a_5^2)\gamma_3 + (a_1a_3^2 + 2a_3a_5a_7 - a_1a_7^2)\gamma_6]\gamma_4, \\ c_2 = [(-a_1^2a_7 + 2a_1a_3a_5 + a_5^2a_7)\gamma_2 + a_5(a_3^2 + a_7^2)\gamma_3 + a_7(a_3^2 + a_7^2)\gamma_6]\gamma_1 \\ \quad + [a_5(a_1^2 + a_5^2)\gamma_2 + a_7(a_1^2 + a_5^2)\gamma_3 + (-a_3^2a_5 + 2a_1a_3a_7 + a_5a_7^2)\gamma_6]\gamma_4, \\ c_3 = -(a_1a_3 + a_5a_7)[(a_3\gamma_1 + a_1\gamma_4)^2 + (a_7\gamma_1 + a_5\gamma_4)^2], \\ c_4 = (a_1^2 + a_5^2)(a_1a_3 + a_5a_7)(\gamma_1\gamma_2 + \gamma_3\gamma_4) + (a_1^2 + a_5^2)(a_3^2 + a_7^2)\gamma_1\gamma_3 \\ \quad + (a_1^2 + a_5^2)^2\gamma_2\gamma_4 + (a_3^2 + a_7^2)(a_1a_3 + a_5a_7)\gamma_1\gamma_6 \\ \quad + [(a_1a_3 + a_5a_7)^2 - (a_1a_7 - a_3a_5)^2]\gamma_4\gamma_6, \\ c_5 = d_1[2d_3^2\gamma_1\gamma_2^2\gamma_4 + d_3d_4(\gamma_1^2\gamma_2^2 + 2\gamma_2\gamma_4^2\gamma_6) + 2d_3d_5(\gamma_1^2\gamma_2\gamma_3 + \gamma_3\gamma_4^2\gamma_6) \\ \quad + d_4d_5(2\gamma_1^2\gamma_2\gamma_6 + \gamma_4^2\gamma_6^2) + 2d_5^2\gamma_1\gamma_4\gamma_6^2] + d_2[2d_3^2\gamma_2\gamma_3\gamma_4^2 \\ \quad + 4d_3d_4\gamma_1\gamma_2\gamma_3\gamma_4 + 2d_3d_5\gamma_1\gamma_3^2\gamma_4 + 4d_4d_5\gamma_1\gamma_3\gamma_4\gamma_6 + 2d_5^2\gamma_1^2\gamma_3\gamma_6] \\ \quad + 3d_4[d_3^3\gamma_2^2\gamma_4^2 + d_3^2d_5\gamma_3^2\gamma_4^2 + d_3d_5^2\gamma_1^2\gamma_3^2 + d_5^3\gamma_1^2\gamma_6^2] \\ \quad + [a_1^4(3a_3^4 + a_7^4) + 4a_1a_3a_5a_7(3a_1^2a_3^2 - a_1^2a_7^2 + 6a_1a_3a_5a_7 \\ \quad - a_3^2a_5^2 + 3a_5^2a_9^2) + a_5^4(a_3^4 + 3a_7^4)]\gamma_1\gamma_2\gamma_4\gamma_6. \end{cases} \tag{16}$$

The parameters d_i, $1 \leqslant i \leqslant 5$, involved in c_5 are defined as follows:

$$\begin{cases} d_1 = [3(a_1a_3 + a_5a_7)^2 - (a_1a_7 - a_3a_5)^2], \\ d_2 = [3(a_1a_3 + a_5a_7)^2 + (a_1a_7 - a_3a_5)^2], \\ d_3 = (a_1^2 + a_5^2), \\ d_4 = (a_1a_3 + a_5a_7), \\ d_5 = (a_3^2 + a_7^2). \end{cases} \tag{17}$$

We find that a_2 and a_6 are consistent with earlier findings in paper [16]. Although there is an extended-term $D_xD_y^2D_t$ in the equation (7), the coefficient θ, as well as α and β, the other coefficients of the fourth-order terms in the nonlinear PDE (7), just appears in a_9. So in each case, when γ_6 or γ_5 is zero, the extension only affects a_9, the constant term of the positive quadratic form of solution (10).

According to formula (5), for the case of $\gamma_5 = 0$, we need to check $a_1a_6 - a_2a_5 \neq 0$ to make sure the solution f defined by (10) with parameters (15) to generate a lump solution. That is:

$$a_1a_6 - a_2a_5 = \frac{(a_1a_7 - a_3a_5)[(a_1^2 + a_5^2)(\gamma_1\gamma_2 - \gamma_3\gamma_4) - (a_3^2 + a_7^2)\gamma_1\gamma_6 - (a_1a_3 + a_5a_7)\gamma_4\gamma_6]}{(a_3\gamma_1 + a_1\gamma_4)^2 + (a_7\gamma_1 + a_5\gamma_4)^2}.$$

So it follows that $a_1a_6 - a_2a_5 \neq 0$ if and only if

$$\begin{cases} a_1a_7 - a_3a_5 \neq 0, \ \gamma_1^2 + \gamma_4^2 \neq 0, \\ (a_1^2 + a_5^2)(\gamma_1\gamma_2 - \gamma_3\gamma_4) - (a_3^2 + a_7^2)\gamma_1\gamma_6 - (a_1a_3 + a_5a_7)\gamma_4\gamma_6 \neq 0. \end{cases} \tag{18}$$

Besides, as it showed in formula (5), a_9 should be positive to guarantee that the positive quadratic form (10) with the above resulting parameters (15) will present lump solutions.

3. Two specific lump solutions and their profiles

In this section, we will take special sets of values for the coefficients in the combined fourth-order nonlinear equation (7) to obtain the corresponding lump solutions and study the dynamics of those solutions.

First, we take:

$$\alpha = 1,\ \beta = 0,\ \theta = 1,\ \gamma_3 = \gamma_5 = 1,\ \gamma_1 = \gamma_2 = \gamma_4 = \gamma_6 = 0. \tag{19}$$

equation (7) is reduced to:

$$3(u_xu_t)_x + u_{xxxt} + (u_ts_y + 2u_ys_t + u_{tyy})_x + u_{xt} + u_{yy} = 0, \tag{20}$$

which has a Hirota bilinear form:

$$(D_x^3D_t + D_xD_y^2D_t + D_xD_t + D_y^2)f \cdot f = 0. \tag{21}$$

Substitute the above values (19) into the resulting parameters (12). Then, take the free parameters as follows:

$$a_1 = 1,\ a_2 = -3,\ a_4 = 6,\ a_5 = -1,\ a_6 = 1,\ a_8 = -3, \tag{22}$$

to get: $a_3 = -7$, $a_7 = -1$, $a_9 = 128$. Putting all the a_i, $1 \leqslant i \leqslant 9$, into the formula (10), we obtain the corresponding positive quadratic form solution of the equation (21)

$$f_1 = (x - 3y - 7t + 6)^2 + (-x + y - t - 3)^2 + 128. \tag{23}$$

By the logarithmic transformation: $u = 2(lnf)_x$, the lump solution of the special fourth-order nonlinear equation (20) is generated as follows:

$$u_1 = \frac{4(2x - 4y - 6t + 9)}{(x - 3y - 7t + 6)^2 + (-x + y - t - 3)^2 + 128}. \tag{24}$$

The three-dimensional plots and contour plots of this lump solution (24) are showed in figure 1, which are made via Maple plot tools.

Secondly, we take:

$$\alpha = 1,\ \beta = 1,\ \theta = 1,\ \gamma_4 = \gamma_6 = 1,\ \gamma_1 = \gamma_2 = \gamma_3 = \gamma_5 = 0. \tag{25}$$

to get another fourth-order nonlinear equation:

$$3(u_xu_t)_x + u_{xxxt} + 3(u_xu_y)_x + u_{xxxy} + (u_ts_y + 2u_ys_t + u_{tyy})_x + u_{xy} + u_{tt} = 0. \tag{26}$$

which has a Hirota bilinear form:

$$(D_x^3D_t + D_x^3D_y + D_xD_y^2D_t + D_xD_y + D_t^2)f \cdot f = 0. \tag{27}$$

Substitute the above values (25) into the resulting parameters (15) and associated with the special value of the free parameters:

$$a_1 = 1,\ a_3 = -3,\ a_4 = 4,\ a_5 = 1,\ a_7 = -1,\ a_8 = 3, \tag{28}$$

we get $a_2 = -7$, $a_6 = 1$, $a_9 = 250$.

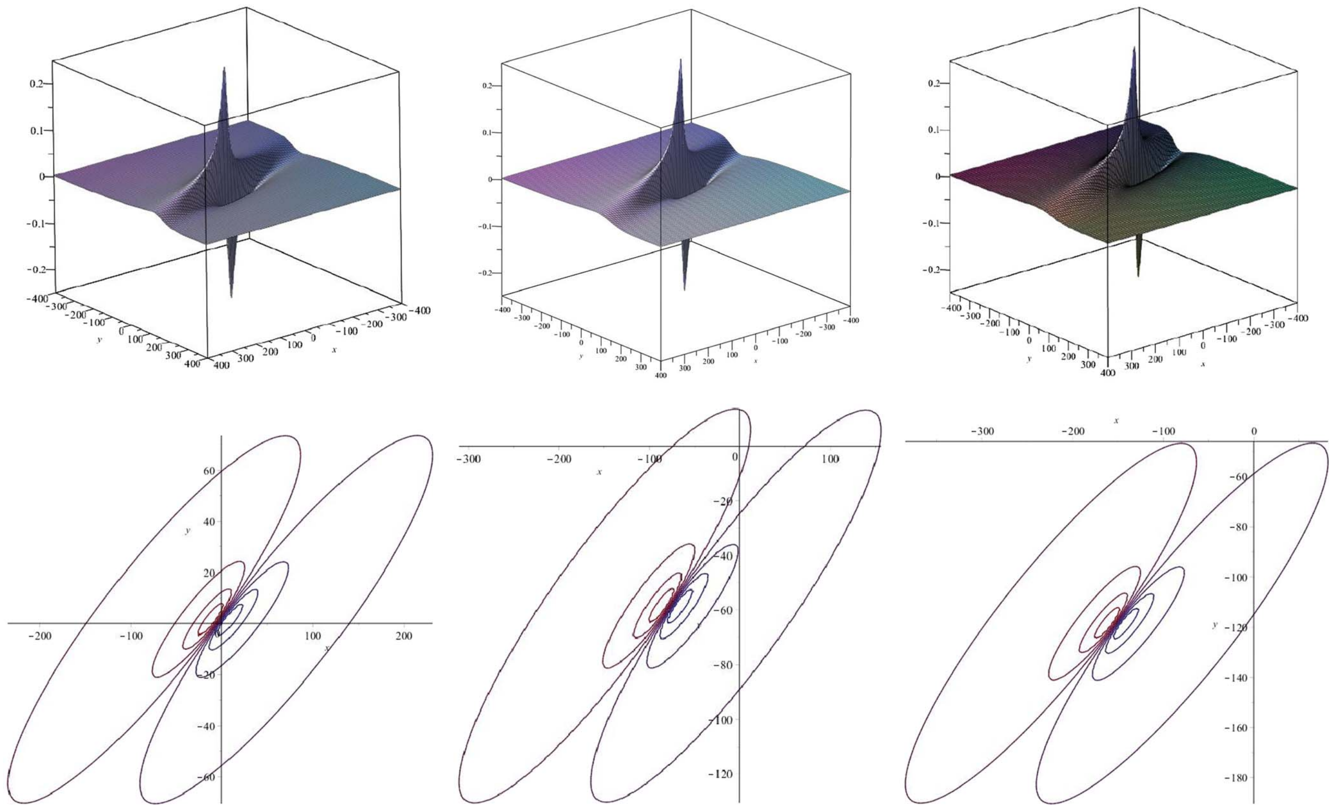

Figure 1. Profiles of u_1 when $t = 0,\ 15,\ 30$: 3d plots (top) and contour plots (bottom).

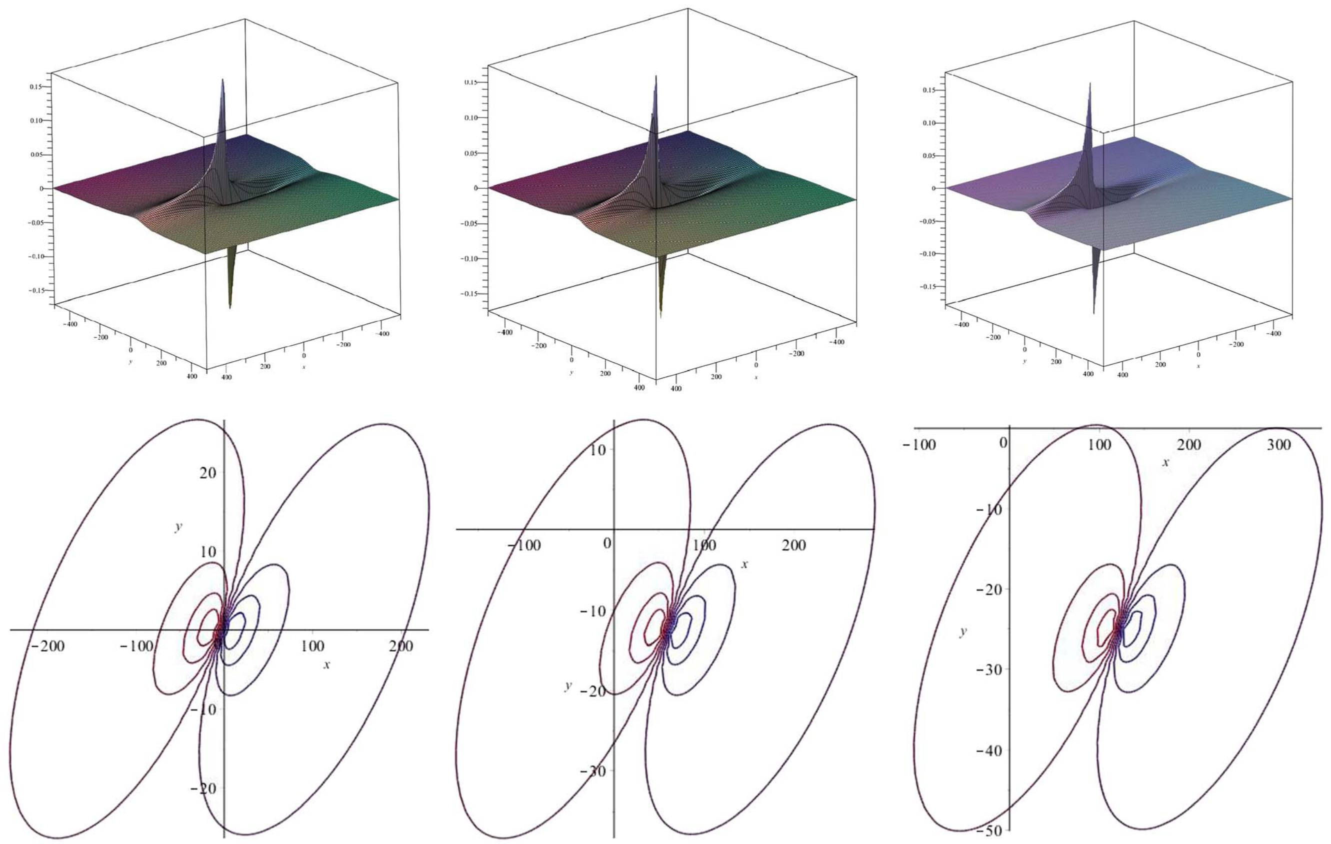

Figure 2. Profiles of u_2 when $t = 0,\ 50,\ 100$: 3d plots (top) and contour plots (bottom).

The corresponding f defined by (10) reads as follows:

$$f_2 = (x - 7y - 3t + 4)^2 + (x + y - t + 3)^2 + 250, \tag{29}$$

which provides a lump solution of the special fourth-order nonlinear equation (27):

$$u_2 = \frac{4(2x - 6y - 4t + 7)}{(x - 7y - 3t + 4)^2 + (x + y - t + 3)^2 + 250}. \tag{30}$$

Figure 2 displays the three-dimensional plots and contour plots of this lump solution made through Maple.

4. Conclusion

In this paper, we studied a special fourth-order nonlinear differential equation originated from the Calogero-Bogoyavlenskii-Schiff equation with three fourth-order terms. Two special cases of this nonlinear equation (7), $\gamma_5 = 0$ and $\gamma_6 = 0$, were studied by the Hirota bilinear method. Through the symbolic computations with the Maple, we obtained abundant lump solutions and found out that the coefficient of the extended term just affects a_9, the constant term of the positive quadratic form of solution (10). Under the setting of γ_5 and γ_6, we determined the other coefficients of the equation (7) to obtain the related specific lump solution and presented their profiles via the Maple plot tools. Our research has enlarged the category of nonlinear PDEs that possess lump solutions and tried to figure out the relation between the lump solutions and the nonlinear terms contained in the new equation. Still, more work needs to be done on the diversity of γ_i, $1 \leqslant i \leqslant 6$, in this fourth-order nonlinear differential equation (7) because of the limitation of the computation capability of Maple and the difficulty in presenting a simplified expression for the result getting from Maple.

However, it is well known that interaction solutions between lump solutions and soliton solutions can describe more nonlinear phenomena [17, 18] and various studies have shown the existence of interaction solutions between lumps and other kinds of exact solutions to nonlinear integrable equation [19–24], even in (3+1) dimension [25–27] and linear wave equation [28]. Since the interaction properties involve much more complicated mathematical computations, further investigation on related issues is needed.

Acknowledgments

This work is supported in part by the Fundamental Research Funds of the Central Universities with the [grant number 2020MS043] and the National Natural Science Foundation of China [grant number 11 975 145 and 11 972 291].

ORCID iDs

Qingxian Chen https://orcid.org/https://orcid.org/0000-0003-0292-7128
Wen-Xiu Ma https://orcid.org/0000-0001-5309-1493

References

[1] Ma W X 2015 Lump solutions to the Kadomtsev-Petviashvili equation *Phys. Lett.* A **379** 1975–8
[2] Satsuma J and Ablowitz M J 1979 Two-dimensional lumps in nonlinear dispersive systems *J. Math. Phys.* **20** 1496–503
[3] Kadomtsev B B and Petviashvili V I 1970 On the stability of solitary waves in weakly dispersing media *Sov. Phys. Dokl.* **15** 539–41
[4] Kaup D J 1981 The lump solutions and the Bäcklund transformation for the three-dimensional three-wave resonant interaction *J. Math. Phys.* **22** 1176–81
[5] Imai K 1997 Dromion and lump solutions of the Ishimori-I equation *Prog. Theor. Phys.* **98** 1013–23
[6] Gilson C R and Nimmo J J C 1990 Lump solutions of the BKP equation *Phys. Lett.* A **147** 472–6
[7] Bruzon M S, Gandarias M L, Muriel C, Ramierez J, Saez S and Romero F R 2003 The Calogero-Bogoyavlenskii-Schiff equation in 2 + 1 dimensions *Theoret. Math. Phys.* **137** 1367–77
[8] Hirota R 2004 *The Direct Method in Soliton Theory* (Cambridge: Cambridge University Press) (https://doi.org/10.1017/CBO9780511543043)
[9] Ma W X and Zhou Y 2018 Lump solutions to nonlinear differential equations via Hirota bilinear forms *J. Differential Equations* **264** 2633–59
[10] Yang J Y and Ma W X 2016 Lump solutions of the BKP equation by symbolic computation *Int. J. Mod. Phys.* B **30** 1640028
[11] Zhang H Q and Ma W X 2017 Lump solutions to the (2+1)-dimensional Sawada-Kotera equations *Nonlinear Dynam* **87** 2305–10
[12] Chen S T and Ma W X 2018 Lump solutions to a generalized Bogoyavlensky-Konopelchenko equation *Front. Math. China* **13** 525–34
[13] Yong X L, Ma W X, Huang Y H and Liu Y 2018 Lump solutions to the Kadomtsev-Petviashvili I equation with a self-consistent source *Comput. Math. Appl.* **75** 3414–9
[14] Chen S T and Ma W X 2018 Lump solutions of a generalized Calogero-Bogoyavlenskii-Schiff equation *Comput. Math. Appl.* **76** 1680–5
[15] Ma W X and Zhang L Q 2020 Lump solutions with higher-order rational dispersion relations *Pramana—J. Phys.* **94** 43
[16] Ma W X 2019 A search for lump solutions to a combined fourth-order nonlinear PDE in (2+1)-dimensions *J. Appl. Anal. Comput.* **9** 1319–32
[17] Yang J Y, Ma W X and Qin Z Y 2018 Lump and lump-soliton to the (2+1)-dimensional Ito equation *Anal. Math. Phys.* **8** 427–36
[18] Ma W X 2019 Interaction solutions to the Hirota-Satsuma-Ito equation in (2+1)-dimensions *Front. Math. China* **14** 619–29
[19] Tang Y N, Tao S Q and Qing G 2016 Lump solutions and interaction phenomenon of them for two classes of nonlinear evolution equations *Comput. Math. Appl.* **72** 2334–42
[20] Ma W X, Zhou Y and Dougherty R 2016 Lump-type solutions to nonlinear differential equations derived from

generalized bilinear equations *Int. J. Mod. Phys.* B **30** 1640018

[21] Zhang J B and Ma W X 2017 Mixed lump-kink solutions to the BKP equation *Comput. Math. Appl.* **74** 591–6

[22] Kofane T C, Fokou M, Mohamadou A and Yomba E 2017 Lump solutions and interaction phenomenon to the third-order nonlinear evolution equation *Eur. Phys. J. Plus* **132** 465–73

[23] Zhao H Q and Ma W X 2017 Mixed lump-kink solutions to the KP equation *Comput. Math. Appl.* **74** 1399–405

[24] Ma W X, Yong X L and Zhang H Q 2018 Diversity of interaction solutions to the (2+1)-dimensional Ito equation *Comput. Math. Appl.* **75** 289–95

[25] Yang J Y and Ma W X 2017 Abundant lump-type solutions of the Jimbo-Miwa equation in (3+1)-dimensional *Comput. Math. Appl.* **73** 220–5

[26] Ma W X 2017 Lump-type solutions to the (3+1)-dimensional Jimbo-Miwa equation *Int. J. Nonlinear Sci. Number. Simul.* **17** 355–9

[27] Qi F H, Ma W X, Qu Q X and Wang P 2020 Lump-type and interaction solutions to an extended (3+1)-dimensional Jimbo-Miwa equation *Int. J. Mod. Phys.* B **34** 2050043

[28] Ma W X 2019 Lump and interaction solutions to linear PDEs in 2 + 1 dimensions via symbolic computation *Mod. Phys. Lett.* B **33** 1950457