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RESEARCH

New Optical Soliton Structures, Bifurcation Properties, Chaotic Phenomena, and Sensitivity Analysis of Two Nonlinear Partial Differential Equations

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Abstract
In this work, we provide new optical soliton structures of the Kadomtsev-Petviashvili equation in $(3+1)$-dimensional and the Jimbo-Miwa equation in $(3+1)$-dimensional together with some intriguing new analysis, chaotic phenomena, bifurcation properties, and sensitivity analysis. Since soliton structure with three analyses is a very interesting recent topic in nonlinear dynamics, we extract different chaotic structures, bifurcation analysis together with phase portrait and sensitivity of our mentioned nonlinear partial differential equations. Applications of the Kadomtsev-Petviashvili equation are in sonic waves, magneto sonic waves, superfluid, weakly nonlinear quasi-unidirectional waves, shallow water waves with weakly nonlinear restoring forces and frequency dispersion, plasma physics, etc. Advanced intellect could benefit from studying the Jimbo-Miwa equation, which addresses specific fascinating higher-dimensional waves in marine engineering, ocean sciences, various interesting physical structures in the areas of optics, acoustic, mathematical modeling, epidemics, circuit analysis, computational neuroscience, intergalactic modeling, etc. Due to the huge applications of the mentioned equations, there is a high demand to investigate with recently developed three analyses. Making use of the recently developed advanced strategy, the adaptive, compatible, further advanced closed-form solitary wave structures are harvested to the mentioned equations in the present manuscript. All these scientifically accomplished exact soliton structures, which take the forms of rational functions and trigonometric functions could assist in our comprehension of remarkable nonlinear challenging situations. In contrast to the present outcomes, our newly formed discoveries will exhibit unique features. The outcomes that were extracted confirm that the recommended technique is meticulously planned, intuitive, and advantageous for measuring the dynamic behavior of nonlinear evolution equations within contemporary science and technology.

Keywords Optical Soliton Solutions · Chaotic Phenomena · Bifurcation Properties · Sensitivity Analysis · Novel -expansion Approach · Kadomtsev-Petviashvili Equation · Jimbo-Miwa Equation

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1 Introduction

Nonlinear partial differential equations (NLPDEs) are associated with nonlinear physical configurations in a variety of subject areas that have subsequently emerged in a range of social, as well as scientific scenarios. Researchers from a broad spectrum of fields agree that this spacious idea can be used to meaningfully explain dramatic occurrences through various sources. Since such equations accurately capture the physical characteristics and actual aspects across a wide range of scientific fields, nano-technology, digital communication, modern engineering, and even the financial emergencies in the economy, the study of NLPDEs has recently become one of the most prominent multidisciplinary scientific research fields. In the fields of physics, mechanics, chemistry, biology, geophysics, hydrodynamics, oceanography, biochemistry, medical science, and finance sectors, NLPDEs may reflect an extensive variety of significant events and dynamic activities. Having soliton solutions for NLPDEs is one of its most significant features. Early in 1834, British scientist Scott Russell announced the initial detection of the solitary wave occurrence. The benefits of soliton comprise great commitment, excellent privacy, unchanging wave shape, and consistent speed. A highly important family of NLPDEs in modern times are indeed nonlinear evolution equations (NLEEs). By setting up suitable surroundings, the solutions of NLEEs enable investigators to plan and conduct simulations in search of suitable parameters in model equations. This is widely acknowledged that modern engineering as well as mathematical physics places a strong emphasis on precisely solving NLEEs through a variety of plentiful techniques. Since each new methodology leads to a new resolution, scientists in this area of study are constantly looking for innovative methods to apply. In light of this, an abundance of mathematicians and engineering teams have generated an extensive array of methodologies, comprising the (G'/G)-expansion method [1], the two variable $(G'/G, 1/G)$-expansion method [2–5], the Hirota method [6], the simple equation method [7], the modified auxiliary equation method [8], the higher degree B-spline algorithm [9], the Haar functions method [10], the $\left(G'/G_2\right)$-expansion method [11], the homogeneous balance method [12, 13], the function transformation method [14], the modified double sub-equation method [15], the binary bell polynomials method [16, 17], the Shehu transform scheme [18, 19], the Lie symmetric analysis [20], the Sardar-subequation technique [21–23], the Cole-Hopf transformation method [24, 25], the reductive perturbation method [26, 27], the variational iteration scheme [28], the method of Painleve analysis [29, 30], the conservation laws methods [31], the generalized exponential rational function approach [32], the collocation method [33, 34], the spectral Tau method [35], and there may be adequate others. Wang et al. [1] familiarized the $\left(G'/G\right)$-expansion methodology as an essential procedure to analyze closed soliton solutions of NLPDEs. The above-mentioned tactic is simpler to recognize and allows a class of NLPDEs to yield a significant amount of new innovative closed soliton solutions, that have further parameters. Numerous academics have used this method to investigate the exact solutions of different NLPDEs. In a former exploration, [3] practiced the two expansion algorithms to detect the rational, trigonometric, and hyperbolic solitary wave solutions for the fractional order Hirota-Satsuma coupled KdV system, which is highly advantageous for the arena of nonlinear analysis. A few years back, B. Hong et al. [36] produced some novel exact solutions to the two types of Schrödinger equations by effectively applying the $\left(\frac{G'}{G'+G+A}\right)$-expansion procedure. Scholars have proven the usefulness of the suggested method and expressed an attraction to understand this nonlinear phenomenon scientifically. To settle NLPDEs, this tactic was scarcely retained by investigators [37–39] and has been confirmed to be upright and operative.

Ganie et al. [40] directed this system most freshly to evaluate the several soliton solutions of the (1 + 1)-dimensional integro-differential Ito equation and the (2 + 1)-dimensional integro-differential Sawada-Kotera equation which are more suited for nonlinear analysis. A variety of scholars have recently presented and analyzed some additional new soliton solutions [41–44].

The Kadomtsev-Petviashvili (KP) equation was first proposed in 1970 by two physicists Boris Borisovich Kadomtsev and Vladimir Iosifovich Petviashvili. It is a natural generalization of the Kortewege-de Vries (KdV) equation, and its integrability has been confirmed by the Painleve analysis. The (3 + 1)-dimensional KP equation was originally designed to analyze the dynamics of long wavelength and small amplitude solitary waves and to model shallow water waves with weakly nonlinear restoring forces and frequency dispersion, waves in ferromagnetic media, matter-wave pulses in Bose-Einstein condensates, superfluid, plasma physics, hydrodynamic, solid state physics, fiber optics, water engineering, and oceanography. In [45–49], certain exact solutions to the (3 + 1)-dimensional KP equation have been identified. In the KP hierarchy, the second equation referred to as the (3 + 1) -dimensional Jimbo-Miwa (JM) equation, depicts various inducing (3 + 1)-dimensional waves in physics, fiber optics, certain higher dimensional waves in ocean studies, marine engineering, plasma, acoustics, heat transfer, classical mechanics, electromagnetism, aerospace, and countless added disciplines. M. Jimbo and T. Miwa placed their starting announcement of this model equation, which is not Painleve integrable. The aforesaid model has been explored using a variety of techniques, and several intriguing exact soliton solutions were evaluated [49–53]. The ultimate objective of the entire investigation is to discover more recent, unique, and comprehensive closed-form solitary solutions for the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional JM equation. We are going to use the $\left(\frac{G'}{G'+G+A}\right)$-expansion mechanism to collect such analytic solitary solutions. To the best of our knowledge and skills, the closed-form soliton solutions of the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional JM equation have not progressed yet manipulating the $\left(\frac{G'}{G'+G+A}\right)$-expansion approach. Consequently, inspired by the discussions above, the ambition of our recent research work focuses on that to produce new closed-form solutions of the stated equations with the help of the proposed method. In the recent past, several analyses including bifurcation analysis, chaotic analysis, and sensitivity analysis have exercised different nonlinear physical models more and more [54–61]. Bifurcation analysis, which incorporates phase portraiture, is a technique used in dynamical systems theory to study how a system's behavior varies with changes in parameters. This analysis is understood by the dynamic feature innate to these above model equations. Chaotic behavior, in a general sense, refers to behavior that appears random or unpredictable, yet is governed by deterministic rules. It is characterized by extreme sensitivity to initial conditions, which means that small changes in the starting conditions can lead to drastically different outcomes over time. Chaos control is a concept in the field of nonlinear dynamics and control theory aimed at stabilizing chaotic systems. These systems, characterized by sensitive dependence on initial conditions and unpredictable long-term behavior, can be found in various scientific and engineering contexts, such as weather systems, stock markets, and biological processes. So, chaos theory has a wide range of applications, including weather forecasting, ecology, and financial markets. The method known as sensitivity analysis assesses how uncertainty in one or more inputs may affect uncertainty in the outputs. Here, we complete the sensitivity analysis in accordance with the Runge–Kutta method. The resulting solutions, assigned kink-shaped, parabolic-shaped soliton, singular periodic shaped, one-sided kink-shaped soliton, and flat kink-shaped solitons have proven more innovative, broadly applicable, further productive, managing, alongside responsible. If we compare our generated plentiful findings to the current results, they will have distinct characteristics compared to the results in the literature. In mathematical physics, modern engineering, and various other everyday activities, the characteristics of waves that have become apparent may be enhanced with the aid of these specific soliton solutions. A large number of NLPDEs could be supervised swiftly and modestly exercising the innovative $\left(\frac{G'}{G'+G+A}\right)$-expansion strategy. In the meantime, we show the developed figures including diverse analyses to demonstrate the numerous diverse wave natures, which are highly beneficial for learning more about NLDPEs.

This paper's remnants are arranged in the following fashion: an introduction has been furnished in Sect. 1. The elementary notion of the $\left(\frac{G'}{G'+G+A}\right)$-expansion approach is outlined in Sect. 2. The procedure is applied in detail in Sect. 3 with the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional JM equation. Bifurcation analysis is well-appointed in Sect. 4. In Sect. 5, chaotic nature is provided. Sensitivity analysis has also been equipped in Sect. 6. In Sect. 7, the chosen closed soliton solutions to these model equations have been graphically depicted. Lastly, a conclusion has been gathered in Sect. 8.

2 Method Highlight

In this portion of the research work, we furnish an extensive summary of the methodology we considered to explore for the best outcomes for the NLPDEs. We are going to address NLPDEs with a total of four variables $x,\ y,\ z$ and t having $w = w(x,\ y,\ z,\ t)$ namely,

$$P\left(w,\ w_x,\ w_{xy},\ w_{xz},\ w_{xx},\ w_t,\ w_{tx},\ w_{ty},\ w_{tz}, w_y,\ w_{yy},\ w_z,\ w_{zz}\cdots\right) = 0, \tag{2.1}$$

therein, the P is a polynomial constructed from the partial derivative of $w = w(x,\ y,\ z,\ t)$ and their corresponding value. The recommended method is outlined in the stages that eventually arise.

Stage 1: Contemplate a brand-new variable η that is the arrangement of all the self-monitoring variables $x,\ y, z$ along with t,

$$w(x,\ y,\ z,\ t) = w(\eta), \eta = x + y + z - mt, \tag{2.2}$$

where m holds for constant. Equation (2.1) can be reshaped directly into an ordinary differential equation (ODE) through the application of Eq. (2.2)

$$P_1\left(w,\ ww',\ w'',\ ww'',\ w''',\cdots\right) = 0, \tag{2.3}$$

,

where P_1 governs a polynomial of w and its numerous ordinary derivatives utilizing the independent variable η.

Stage 2: The structure that is displayed beneath can be utilized for assembling the solution to Eq. (2.3)

$$w(\eta) = \sum{}_{k=0}^{Q} c_k\left(\frac{G'}{G'+G+A}\right)^k, \tag{2.4}$$

,

here, Q might have been regulated using the homogeneous balancing concept, wherein it represents the degree of the polynomial. Nonetheless, the collection of algebraic equations arising from the proposed method can be applied to readily compute the coefficients c_k $(k = 0,\ 1,\ 2,\ \cdots\ ,\ Q)$. Additionally, $G = G(\eta\ )$ sustains the ensuing ODE

$$G'' + RG' + TG + AT = 0, \tag{2.5}$$

Where, $C_k(k = 0, 1, 2, \cdots , Q),\ A, R, T$ stay unchanged.

We were able to govern the following two probable outputs after solving Eq. (2.5), which belong.

State 1: whenever $H = R^2 - 4T > 0$,

$$\left(\frac{G'}{G' + G + A}\right) = \frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}}.$$

State 2: whenever $H = R^2 - 4T < 0$,

$$\left(\frac{G'}{G' + G + A}\right) = \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)}.$$

It is now straightforward to arrive at the polynomial of $\left(\frac{G'}{G' + G + A}\right)$ by employing Eq. (2.4) in Eq. (2.3).

Stage 3: It is now straightforward to arrive at the polynomial of $\left(\frac{G'}{G' + G + A}\right)$ by employing Eq. (2.4) in Eq. (2.3) and one way for gathering the algebraic equations for $c_k,\ A,\ R,\ T$ is to allocate the coefficient to zero. By swapping the values of c_k's and m, we may rapidly get the expected solutions to the given NLPDEs after finalizing all of these algebraic computations.

3 Applications

In this section, we offer the standard, sophisticated, updated, extensively useful closed traveling wave soliton solutions for the KP equation and the JM equation aided by the novel innovative $\left(\frac{G'}{G' + G + A}\right)$-expansion approach.

3.1 The Kadomtsev-Petviashvili Equation

Boris Borisovich Kadomtsev and Vladimir Iosifovich Petviashvili first demonstrated the KP equation in 1970. The KP equation is featured in several areas of plasma physics, solid state physics, dust acoustic waves, fiber optics, water engineering, weakly nonlinear quasi-unidirectional waves, oceanography, the surface waves, and internal waves in straits or channels, shallow-water waves when the viscosity and the surface tension are less. The (3 + 1)-dimensional KP equation has been exercised to effigy more dispersion impact in nonlinear analysis. Here, several more recent closed-form soliton solutions to the (3 + 1)

-dimensional KP model will be explored. So, the (3 + 1)-dimensional Kadomtsev-Petviashvili equation ensues here [49],

$$\left(w_t + 6ww_x + w_{xxx}\right)_x - 3w_{yy} - 3w_{zz} = 0. \tag{3.1.1}$$

The modification shown in Eq. (2.2) undergoes execution in this most current portion, additionally, Eq. (3.1.1) could have been profoundly changed into an ordinary differential equation as

$$\left(-mw' + 6ww' + w'''\right)' - 6w'' = 0, \tag{3.1.2}$$

here, prime symbolizes an ordinary derivative concerning η. Integrating Eq. (3.1.2) two times produces,

$$-mw - 6w + 3w^2 + w'' = 0, \tag{3.1.3}$$

here, the value of the integral constant is assumed zero. At this moment, the homogeneous balance procedure delivers us $Q = 2$. Accordingly, we could compose the Eq. (2.4)

$$w(\eta) = \sum{}_{k=0}^{2} c_k \left(\frac{G'}{G' + G + A}\right)^k = c_0 + c_1\left(\frac{G'}{G' + G + A}\right) + c_2\left(\frac{G'}{G' + G + A}\right)^2. \tag{3.1.4}$$

It is essential to derive the constants $m,\ c_0,\ c_1$ and c_2.

Assortment − 1:

$$\begin{aligned} m &= R^2 - 4T - 6,\ c_0 = -2T + 2RT - 2T^2,\ c_1 \\ &= 4T - 2R - 6RT + 4T^2 + 2R^2,\ c_2 \\ &= -2 - 4T + 4R - 2T^2 + 4RT - 2R^2. \end{aligned}$$

Assortment − 2:

$$\begin{aligned} m &= -R^2 + 4T - 6,\ c_0 = -\frac{1}{3}R^2 - \frac{2}{3}T + 2RT - 2T^2,\ c_1 \\ &= 4T - 2R - 6RT + 4T^2 + 2R^2,\ c_2 = -2 - 4T + 4R - 2T^2 + 4RT - 2R^2. \end{aligned}$$

The closed-form soliton solutions with reference to assortment − 1:

Consequence − 1: Permitting as $H = R^2 - 4T > 0$,

$$\begin{aligned} w(\eta) =& \left(-2T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right) \\ & \left\{\frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}}\right\} \\ & + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right) \\ & \left\{\frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}}\right\}^2. \end{aligned} \tag{3.1.5}$$

In the meantime, make use of the modification $w(x, y, z, t) = w(\eta)$ coupled with $\eta = x + y + z - mt$ in Eq. (3.1.5), to organize the soliton solution of Eq. (3.1.1),

$$w(x, y, z, t) = \left(-2T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right) \left\{ \frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}} \right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right) \left\{ \frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}} \right\}^2. \tag{3.1.6}$$

Consequence −2: Permitting as $H = R^2 - 4T < 0$,

$$w(\eta) = \left(-2T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\}^2. \tag{3.1.7}$$

In a similar fashion, operate the reformation $w(x, y, z, t) = w(\eta)$ along with $\eta = x + y + z - mt$ in Eq. (3.1.7), to establish the soliton solution of Eq. (3.1.1),

$$w(x, y, z, t) = \left(-2T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\}^2. \tag{3.1.8}$$

The closed-form soliton solutions with orientation to assortment −2:
Consequence −1: Permitting as $H = R^2 - 4T > 0$,

$$w(\eta) = \left(-\frac{1}{3}R^2 - \frac{2}{3}T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right)\left\{\frac{B_1\left(R+\sqrt{H}\right) + B_2\left(R-\sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R+\sqrt{H}-2\right) + B_2\left(R-\sqrt{H}-2\right)e^{\sqrt{H}\eta}}\right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right)\left\{\frac{B_1\left(R+\sqrt{H}\right) + B_2\left(R-\sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R+\sqrt{H}-2\right) + B_2\left(R-\sqrt{H}-2\right)e^{\sqrt{H}\eta}}\right\}^2. \tag{3.1.9}$$

Instantly, manipulate the conversion $w(x, y, z, t) = w(\eta)$ coupled with $\eta = x + y + z - mt$ in Eq. (3.1.9), to establish the closed-form soliton solution of Eq. (3.1.1),

$$w(x, y, z, t) = \left(-\frac{1}{3}R^2 - \frac{2}{3}T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right)\left\{\frac{B_1\left(R+\sqrt{H}\right) + B_2\left(R-\sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R+\sqrt{H}-2\right) + B_2\left(R-\sqrt{H}-2\right)e^{\sqrt{H}\eta}}\right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right)\left\{\frac{B_1\left(R+\sqrt{H}\right) + B_2\left(R-\sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R+\sqrt{H}-2\right) + B_2\left(R-\sqrt{H}-2\right)e^{\sqrt{H}\eta}}\right\}^2. \tag{3.1.10}$$

Consequence -2: Permitting as $H = R^2 - 4T < 0$,

$$w(\eta) = \left(-\frac{1}{3}R^2 - \frac{2}{3}T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right)\left\{\frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)}\right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right)\left\{\frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)}\right\}^2. \tag{3.1.11}$$

At this instant, manage the reformation $w(x, y, z, t) = w(\eta)$ along with $\eta = x + y + z - mt$ in Eq. (3.1.11), to launch the soliton solution of Eq. (3.1.1),

$$w(x, y, z, t) = \left(-\frac{1}{3}R^2 - \frac{2}{3}T + 2RT - 2T^2\right) + \left(4T - 2R - 6RT + 4T^2 + 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\} + \left(-2 - 4T + 4R - 2T^2 + 4RT - 2R^2\right) \left\{ \frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)} \right\}^2. \tag{3.1.12}$$

3.2 The Jimbo-Miwa Equation

In 1983, Jimbo and Miwa primarily revealed the (3 + 1)-dimensional Jimbo-Miwa equation. The esteemed Kadomtsev-Petviashvili hierarchy of integrable systems is the originating point of this model equation. Significantly, this model is deficient in the Painleve characteristic. Abundant fascinating waves in physics, fiber optics, certain higher dimensional waves in ocean studies, marine engineering, plasma, acoustics, classical mechanics, fluid dynamics, electromagnetism, and aerospace have been analyzed by the (3 + 1)-dimensional JM equation. To elevate the likelihood of implementing this JM equation, the model's analytical solution nonetheless has to be established. Accordingly, the (3 + 1)-dimensional Jimbo-Miwa equation evolves herein [49],

$$w_{xxxy} + 3w_y w_{xx} + 3w_x w_{xy} + 2w_{yt} - 3w_{xz} = 0. \tag{3.2.1}$$

Equation (3.2.1) could potentially be restored as an ordinary differential equation if we carried out a transformation in Eq. (2.2) like

$$w^{(4)} + 6w' w'' - 2mw'' - 3w'' = 0, \tag{3.2.2}$$

where, the superscript means the ordinary differentiation associating η. Integrate single time the Eq. (3.2.2) and dropping the integrating constant to zero,

$$w''' + 3\left(w'\right)^2 - 2mw' - 3w' = 0. \tag{3.2.3}$$

The homogeneous balance technique in this particular situation yields $Q = 1$. It would be thus straightforward to formulate the Eq. (2.4),

$$w(\eta) = \sum{}_{k=0}^{1} c_k \left(\frac{G'}{G' + G + A}\right)^k = c_0 + c_1\left(\frac{G'}{G' + G + A}\right). \tag{3.2.4}$$

For the constants m, c_0 and c_1, a computation is necessary. After a successful calculation, we see

$$m = \frac{1}{2}R^2 - \frac{3}{2} - 2T,\ c_0 = c_0,\ c_1 = 2(T - R + 1).$$

The closed-form soliton solutions with this consideration:

Circumstances-1: Accepting for $H = R^2 - 4T > 0$,

$$w(\eta) = c_0 + 2(T - R + 1)\left\{\frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}}\right\}. \tag{3.2.5}$$

Now, utilize the adjustment $w(x, y, z, t) = w(\eta)$ besides $\eta = x + y + z - mt$ in Eq. (3.2.5), to form the closed soliton solution of Eq. (3.2.1),

$$w(x, y, z, t) = c_0 + 2(T - R + 1)\left\{\frac{B_1\left(R + \sqrt{H}\right) + B_2\left(R - \sqrt{H}\right)e^{\sqrt{H}\eta}}{B_1\left(R + \sqrt{H} - 2\right) + B_2\left(R - \sqrt{H} - 2\right)e^{\sqrt{H}\eta}}\right\}. \tag{3.2.6}$$

Circumstances-2: Accepting for $H = R^2 - 4T < 0$,

$$w(\eta) = c_0 + 2(T - R + 1)\left\{\frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)}\right\}. \tag{3.2.7}$$

Analogously, we apply the changes $w(x, y, z, t) = w(\eta)$ and $\eta = x + y + z - mt$ in Eq. (3.2.7), to sort out the closed soliton solution of Eq. (3.2.1),

$$w(x, y, z, t) = c_0 + 2(T - R + 1)\left\{\frac{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left(RB_1 - B_2\sqrt{-H}\right)}{sin\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_2 + B_1\sqrt{-H}\right) + cos\left(\frac{\sqrt{-H}\eta}{2}\right)\left((R-2)B_1 - B_2\sqrt{-H}\right)}\right\}. \tag{3.2.8}$$

4 Bifurcation Analysis

The analysis of how a system's state shifts qualitatively when a parameter has been altered is known as bifurcation theory. In this part, bifurcation analysis alongside phase portraits is observed for the (3 + 1)-dimensional Kadomtsev-Petviashvili equation and the (3 + 1) -dimensional Jimbo-Miwa equation.

4.1 Bifurcation Analysis of the Kadomtsev-Petviashvili Equation

This subsection is designed for bifurcation analysis of the stated equation. To implement this analysis, we apply the Galilean transformation to Eq. (3.1.3) which yields a dynamical system,

$$\begin{cases} \frac{dw(\eta)}{d\eta} = T \\ \frac{dT(\eta)}{d\eta} = N_1 w^2(\eta) + N_2 w(\eta) \end{cases}, \tag{4.1.1}$$

wherein, $N_1 = -3$, $N_2 = m + 6$. At this instant, the Hamiltonian function could be extracted for the dynamical system in Eq. (4.1.1),

$$H_1(w,\ T) = \frac{T^2}{2} - \frac{N_1 w^3}{3} - \frac{N_2 w^2}{2} = h_1,$$

where, $\mathbb{h}_1$ signify the Hamiltonian constant. To compute the equilibrium points of Eq. (4.1.1), we solve the following dynamical system,

$$\begin{cases} T = 0 \\ N_1 w^2 + N_2 w = 0 \end{cases}.$$

After a successful computation, one can originate the equilibrium points of the above dynamical system are (0, 0), $\left(-\frac{N_2}{N_1},\ 0\right)$. The dynamical system in Eq. (4.1.1) has Jacobian

$$\mathfrak{D}_1(w,\ T) = \begin{vmatrix} 0 & 1 \\ 2N_1 w + N_2 & 0 \end{vmatrix} = -2N_1 w - N_2.$$

Planar dynamical systems theory exposes us that,

1. The equilibrium standpoint $(w,\ T)$ represents a saddle point, while $\mathfrak{D}_1(w,\ T) < 0$;
2. The equilibrium standpoint $(w,\ T)$ denotes a center point, while $\mathfrak{D}_1(w,\ T) > 0$;
3. The equilibrium standpoint $(w,\ T)$ signifies a cuspid point, while $\mathfrak{D}_1(w,\ T) = 0$.

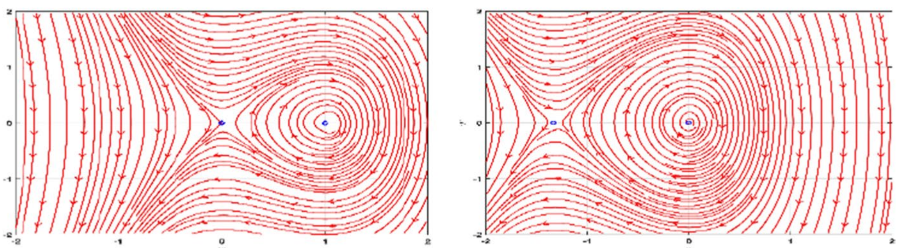
Instantly, the possible outcomes that can be attained after altering the pertinent parameters as follows.

$$\text{Situation 1}:\ N_1 < 0 \text{ and } N_2 > 0$$

For suitable value of the parameter $m = -3$, we notice that the equilibrium standpoint $(0,\ 0)$ denotes a saddle point and the equilibrium standpoint $(1,\ 0)$ denotes a center point that is presented in Fig. 1a.

$$\text{Situation 2}:\ N_1 < 0 \text{ and } N_2 < 0$$

We scrutinize that the equilibrium standpoint $(0,\ 0)$ denotes for a center point, as well as the equilibrium standpoint $(-1.3333,\ 0)$ denotes for saddle point together with the value of the parameter $m = -10$, that have already shown in Fig. 1b.



(a) (b)

Fig. 1 Bifurcation analysis of the recommended system with diverse situations for N_1 and N_2 along with numerous values of the parameters

4.2 Bifurcation Analysis of the Jimbo-Miwa Equation

This subdivision is organized for bifurcation analysis of the indicated equation. Here, we operate.

a modification $w' = U$ to the Eq. (3.2.3) and then utilize the Galilean transformation to the equation modified equation which provide us a dynamical system,

$$\begin{cases} \frac{dU(\eta)}{d\eta} = S \\ \frac{dS(\eta)}{d\eta} = N_3 U^2(\eta) + N_4 U(\eta) \end{cases}, \tag{4.2.1}$$

where, $N_3 = -3$, $N_4 = 2m + 3$. At this moment, the Hamiltonian function could be obtained for the dynamical system in Eq. (4.2.1),

$$H_2(U, S) = \frac{S^2}{2} - \frac{N_3 U^3}{3} - \frac{N_4 U^2}{2} = h_2,$$

here, $\mathbb{h}_2$ means the Hamiltonian constant. To calculate the equilibrium points of Eq. (4.2.1), we resolve the succeeding dynamical system,

$$\begin{cases} S = 0 \\ N_3 U^2 + N_4 U = 0 \end{cases}.$$

After a successful computation, we have the equilibrium points of the above dynamical system are (0, 0), $\left(-\frac{N_4}{N_3}, 0\right)$. The dynamical system in Eq. (4.2.1) carries out Jacobian

$$\mathfrak{D}_2(U, S) = \begin{vmatrix} 0 & 1 \\ 2N_3 w + N_4 & 0 \end{vmatrix} = -2N_3 w - N_4.$$

Planar dynamical systems theory reveals us that,

1. The equilibrium standpoint (U, S) represents a saddle point, while $\mathfrak{D}_2(U, S) < 0$;
2. The equilibrium standpoint (U, S) denotes a center point, while $\mathfrak{D}_2(U, S) > 0$;
3. The equilibrium standpoint (U, S) signifies a cuspid point, while $\mathfrak{D}_2(U, S) = 0$.

Quickly, the possible consequences that can be achieved by selecting the appropriate parameters as follows.

$$\text{State 1}: \; N_3 < 0 \text{ and } N_4 > 0$$

For suitable value of the parameter $m = 1$, we see that the equilibrium standpoint (0, 0) signifies a saddle point and the equilibrium standpoint (1.6667, 0) signifies center point that is offered in Fig. 2a.

$$\text{State 2}: \; N_3 < 0 \text{ and } N_4 < 0$$

We inspect that the equilibrium standpoint (0, 0) means for a center point, as well as the equilibrium standpoint (−1, 0) denotes for a saddle point, that have already disclosed in Fig. 2b.

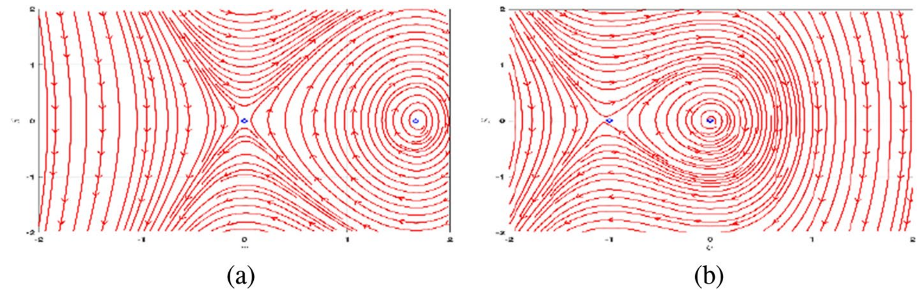


Fig. 2 Bifurcation analysis of the suggested system with various situations for $\boldsymbol{N}_3$ and $\boldsymbol{N}_4$ alongside frequent values of the parameters

5 Chaotic Phenomena

Tiny changes in input may occur in wildly varied outcomes in chaotic systems, whereby nonlinear dynamics convey apparently haphazard behavior. In this portion, we observe the chaotic nature of the (3 + 1)-dimensional Kadomtsev-Petviashvili equation and the (3 + 1) -dimensional Jimbo-Miwa equation.

5.1 Chaotic Behavior of the Kadomtsev-Petviashvili Equation

Here, we include a perturbed term in the dynamical system Eq. (4.1.1), to realize the chaotic nature of the declared equation. So, we have the succeeding dynamical system,

$$\begin{cases} \frac{dw(t)}{dt} = T \\ \frac{dT(t)}{dt} = N_1 w^2(t) + N_2 w(t) \ + \kappa_{\ 1} cos\left(\kappa_{\ 2} t\right) \end{cases}, \tag{5.1.1}$$

where, the amplitude and frequency of the dynamical system in Eq. (5.1.1) is specified by $\kappa_{\ 1}$ and $\kappa_{\ 2}$, respectively. Here and now, the chaotic nature of the suggested dynamical system is surveyed, we also explain the phase portraits of this dynamical system for copious values of the parameters, in Fig. 3a $m = -5,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = \pi\ /2$; 3b $m = -10,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = \pi\ /2$; 4a $m = -5,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = \pi$; 4b $m = -10,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = \pi$; 5a $m = -5,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = 2\pi$ and 5b $m = -10,\ \kappa_{\ 1} = 1, \kappa_{\ 2} = 2\pi$. In Figs. 3a and 5a, we find out complex dynamics,

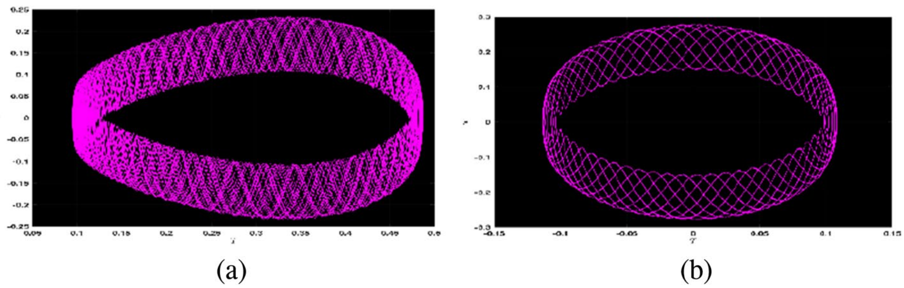


Fig. 3 Chaotic nature of the proposed dynamical system with $\boldsymbol{\kappa}_{\ 2} = \boldsymbol{\pi}\ /2$ with abundant values of the parameters

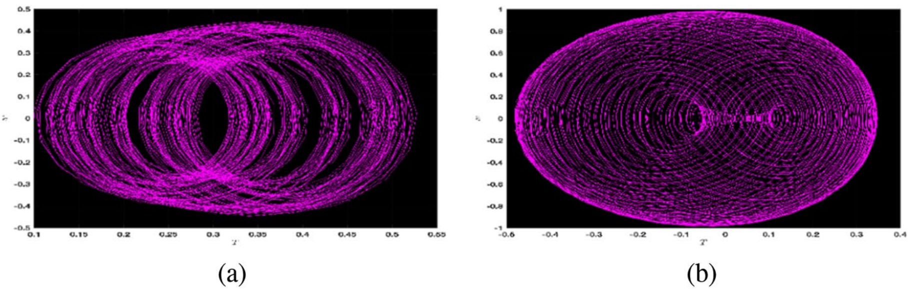


Fig. 4 Chaotic nature of the proposed dynamical system having $\kappa_2 = \pi$ and diverse values of the parameters

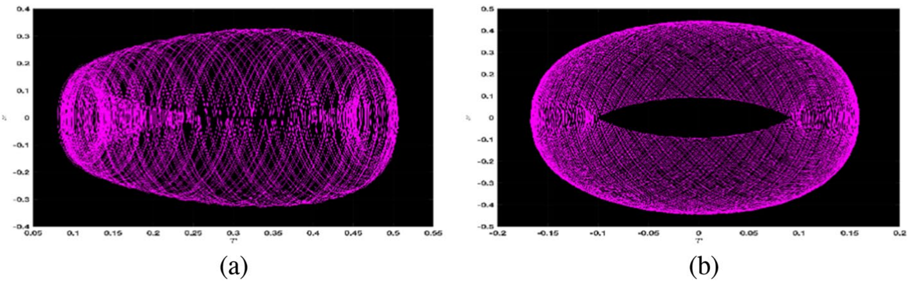


Fig. 5 Chaotic nature of the proposed dynamical system being $\kappa_2 = 2\pi$ as well as numerous values of the parameters

and also in Figs. 3b as well 5b have ringlet dynamics. In addition, periodic dynamics are exhibited in Fig. 4a, likewise, surprising periodic dynamics are also exhibited in Fig. 4b. This analysis supplies us the stronger concept of the system's insight which might be helpful in modern mathematics, physics, and contemporary engineering.

5.2 Chaotic Behavior of the Jimbo-Miwa Equation

In this portion, a perturbed term has added in the dynamical system mentioned in Eq. (4.2.1), so that one could achieve the chaotic nature of the announced equation,

$$\begin{cases} \frac{dU(t)}{dt} = S \\ \frac{dS(t)}{dt} = N_3 U^2(t) + N_4 U(t) + \kappa_3 cos\left(\kappa_4 t\right) \end{cases} . \quad (5.2.1)$$

In the above dynamical system, we have two specific constants one is κ_3 means the amplitude of this system and the other is κ_4 means the frequency of the earlier system. Numerous values of the parameters, in Fig. 6a $m = -5,\ \kappa_3 = 1, \kappa_4 = \pi/2$; 6b $m = -1,\ \kappa_3 = 1, \kappa_4 = \pi/2$; 7a $m = -5,\ \kappa_3 = 1, \kappa_4 = \pi$; 7b $m = -1,\ \kappa_3 = 1, \kappa_4 = \pi$; 8a $m = -5,\ \kappa_3 = 1, \kappa_4 = 2\pi$ and 8b $m = -1,\ \kappa_3 = 1, \kappa_4 = 2\pi$, support us to measure the chaotic nature of the recommended dynamical system and to present the phase portraits of the directly above system. From a close investigation, one could see the ringlet dynamics in Figs. 6a and 8a, alongside the complex dynamics in Figs. 6b and 8b. Furthermore, the periodic dynamics and surprising periodic are demonstrated in Fig. 7a and b, respectively. Our understanding of the

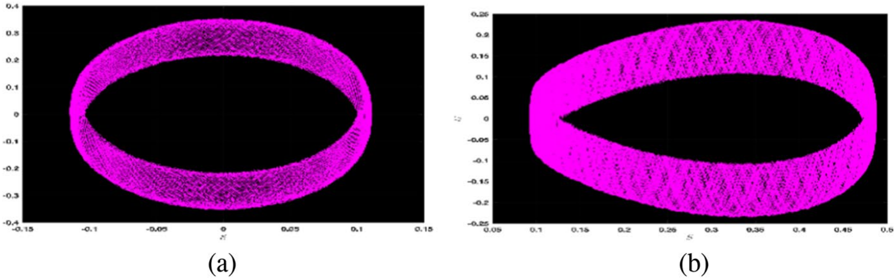


Fig. 6 Chaotic nature of the advised dynamical system with $\boldsymbol{\kappa}_4 = \boldsymbol{\pi}/2$ with copious values of the parameters

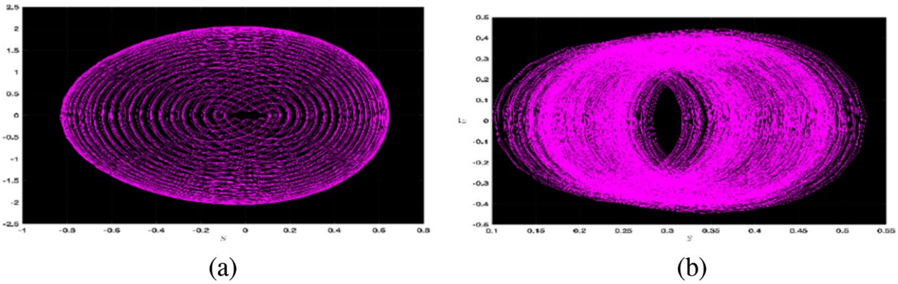


Fig. 7 Chaotic nature of the advised dynamical system having $\kappa_4 = \pi$ and varied values of the parameters

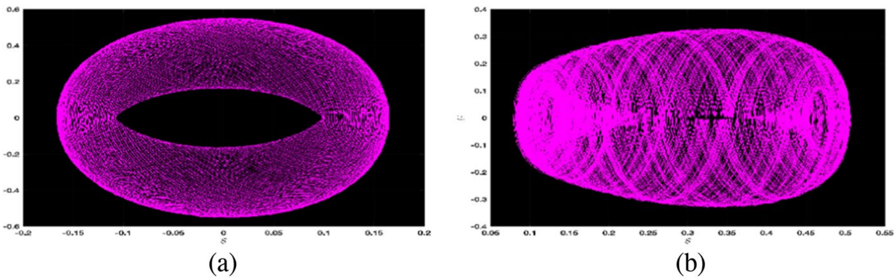


Fig. 8 Chaotic nature of the advised dynamical system being $\kappa_4 = 2\pi$ as well as several values of the parameters

system's insight has been strengthened as a result of this research, and it could be useful in modern engineering, physics, and mathematics.

6 Sensitivity Analysis

A technique that can compute the effect of one or more input variables on the output variables is known as sensitivity analysis. Sensitivity analysis of the $(3+1)$-dimensional Kadomtsev-Petviashvili equation and the $(3+1)$-dimensional Jimbo-Miwa equation is identified in this segment.

6.1 Sensitivity Analysis of the Kadomtsev-Petviashvili Equation

In this subsection, Runge–Kutta procedure is implemented to observe the sensitivity of the dynamical system designated in Eq. (4.1.1). Here, we apply the well-established Runge–Kutta technique to resolve the succeeding dynamical system to detect its sensitivity,

$$\begin{cases} \frac{dw(t)}{dt} = T \\ \frac{dT(t)}{dt} = N_1 w^2(t) + N_2 w(t) \end{cases} . \tag{6.1.1}$$

We set the specific value of the parameter $m = -10$. Initial condition of the above stated dynamical system have been afforded by,

a. *a*) $w(0) = 0.5$ and $T = 0$; *b*) $w(0) = 0$ and $T = 0.5$
b. *c*) $w(0) = 0.2$ and $T = 0$; *d*) $w(0) = 0$ and $T = 0.2$.

The consequence of the dynamical system after considering the different states of initial conditions is displayed in Fig. 9. The dynamics of class w and dynamics of class T are shown by blue curves and pink curves, respectively. From Fig. 9, one could easily notice that a little change in the initial condition affects a significant effect on the dynamical system.

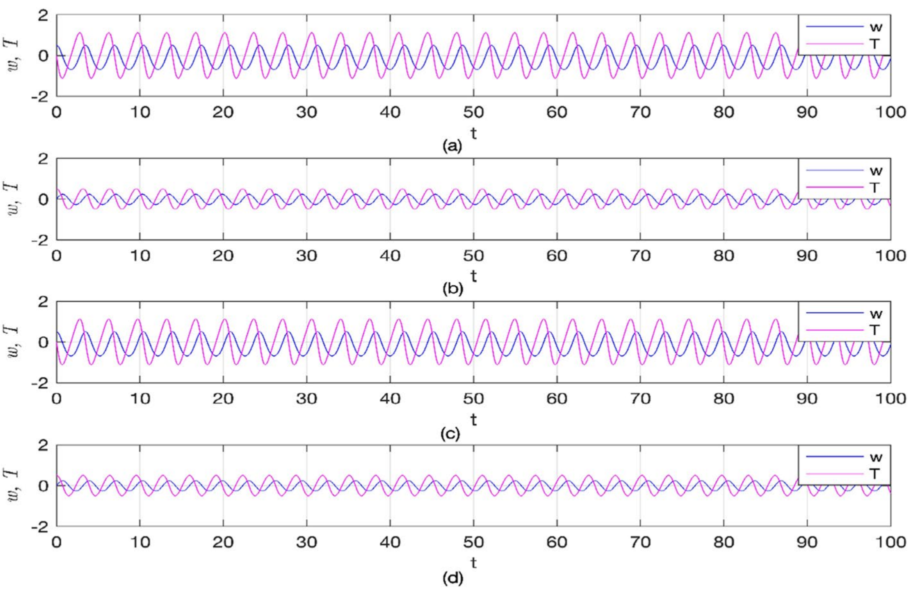


Fig. 9 Sensitivity analysis of the intended dynamical system along with some initial conditions and the value of the parameter $m = -10$

6.2 Sensitivity Analysis of the Jimbo-Miwa Equation

In this part, to study the sensitivity of the dynamical system in Eq. (4.2.1), we exploit the widely acknowledged Runge–Kutta scheme. So, the earlier mentioned process is operated to unravel the next dynamical system to perceive its sensitivity,

$$\begin{cases} \frac{dU(t)}{d\eta} = S \\ \frac{dS(t)}{dt} = N_3 U^2(t) + N_4 U(t) \end{cases}. \tag{6.2.1}$$

At this point, we consider the definite value of the parameters m, N_3, $N_4 = -3$. The system's initial situations are given below,

iii. $a)$ $U(0) = 0.5$ and $S = 0$; $b)$ $U(0) = 0$ and $S = 0.5$
iv. $c)$ $U(0) = 0.2$ and $S = 0$; $d)$ $U(0) = 0$ and $S = 0.2$.

The fluctuations of the system's nature for different initiation positions are shown in Fig. 10. Blue curves and pink curves characterize the dynamics of class U and the dynamics of class S, respectively. The presented figure in Fig. 10 confirms that a slide modification in the initial position impacts a huge change in the system's behavior.

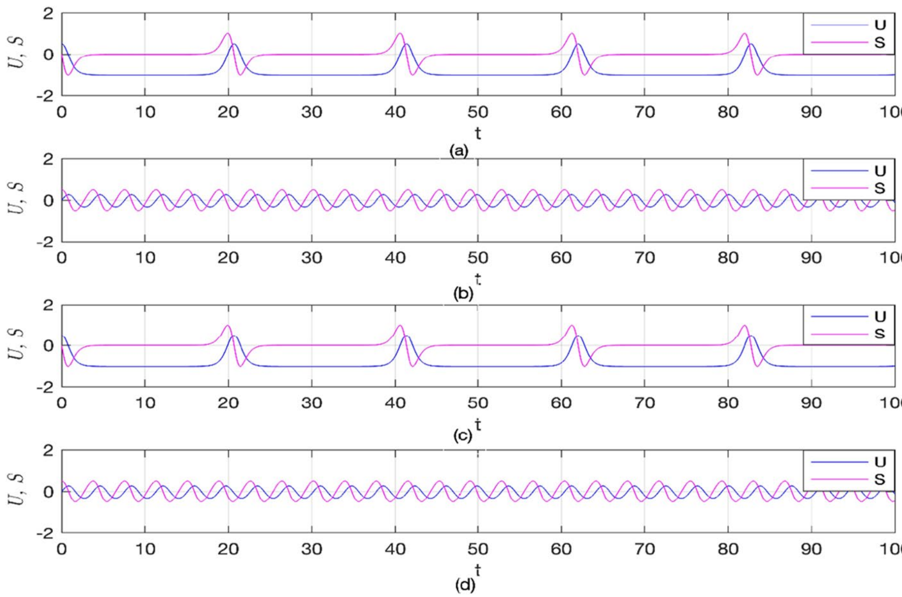
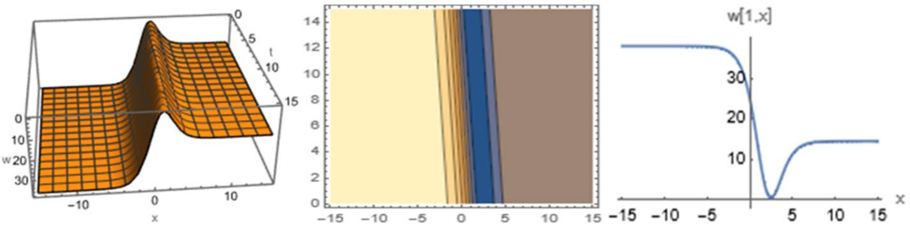


Fig. 10 Sensitivity analysis of the projected dynamical system with various initial conditions together with the values of the parameters m, N_3, $N_4 = -3$.

7 Physical Complexion and Explanation of the Graph

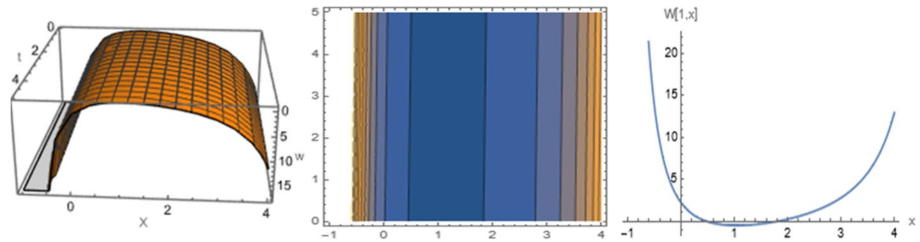
A graph is typically an illustrative depiction of data or quantities that have been organized systematically. It is indispensable to put together and demonstrate the information in a way that facilitates clarity for interpretation. The implementation of plots in several fields of research enhances analysis as well. Using different approaches, many scholars explored the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional Jimbo-Miwa equation. However, the mechanical features of the aforesaid equations are presented here in this unique analysis by applying a novel procedure, the $\left(\frac{G'}{G'+G+A}\right)$-expansion approach. For the NLPDEs subject to consideration, we share the traveling wave along with the soliton profile in 3D, 2D, and contour forms using proper values that remain constant. The mechanisms associated with the complex nonlinear physical phenomenon can be explained by the provided solutions. This finding assists researchers in a deeper understanding of the most fascinating characteristics of NLPDEs that explain the several marvels in nonlinear science and technology. This section involves the visual limning of the discovered closed-form soliton solutions to the stated previous equations.

At first, we arrange the solution of Eq. (3.1.6) in three formats, 3D with $x \in [-15, 15], t \in [0, 15]$, contour with $x \in [-15, 15], t \in [0, 15]$and 2D with $x \in [-15, 15]$and then achieve kink shape soliton that are crystalized in Fig. 11 for the parameters $y = 1, z = 2, R = \sqrt{5}, T = 1, B_1 = 0.01, B_2 = -0.0001, H = R^2 - 4T = 1, m = -0.1$ and $t = 1$. After then, we sketch the solution of Eq. (3.1.8) as well in three different arrangements, 3D using $x \in [-1, 4], t \in [0, 5]$, contour using $x \in [-1, 4], t \in [0, 5]$ and 2D using $x \in [-1, 4]$and have bell shape soliton, that is shown in Fig. 12 together with the parameters $y = 1, z = 2, R = \sqrt{3}, T = 1, B_1 = 0.001, B_2 = -0.0001, H = R^2 - 4T = -1, m = 0.1$ and $t = 1$. The bell shape soliton solution has been displayed of Eq. (3.1.10) in Fig. 13 in which 3D within $x \in [-10, 10], t \in [0, 10]$, contour within $x \in [-10, 10], t \in [0, 10]$and 2D within $x \in [-10, 10]$ including the parameters $y = 1, z = 2, R = \sqrt{5}, T = 1, B_1 = 0.01, B_2 = -0.0001, H = R^2 - 4T = 1, m = -0.1$ and $t = 2$. Besides these, Fig. 14 presents the singular periodic shaped soliton solution of Eq. (3.1.12), wherein 3D with $x \in [-15, 15], t \in [0, 15]$, contour with $x \in [-15, 15], t \in [0, 15]$and 2D with $x \in [-15, 15]$ in addition the parameters stand $y = 1, z = 2, R = \sqrt{3}, T = 1, B_1 = 0.001, B_2 = -0.0001, H = R^2 - 4T = -1, m = -0.001$ and $t = 1$.



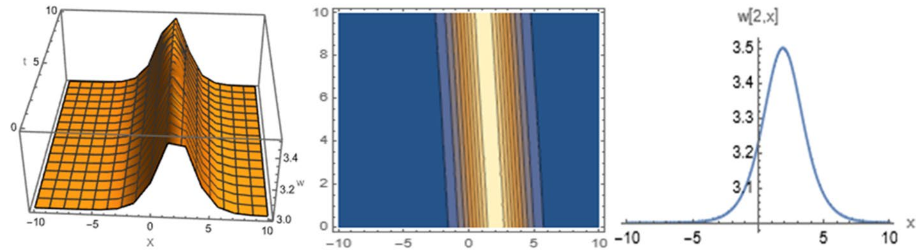
(a) (b) (c)

Fig. 11 The kink-shaped soliton solution of Eq. (3.1.6) presented in (**a**), 3D structure **b** provided the contour shape, and **c** display the 2D figure



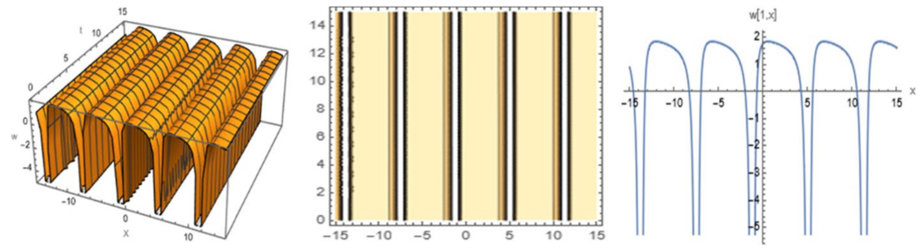
(a) (b) (c)

Fig. 12 The parabolic-shaped soliton solution of Eq. (3.1.8) furnished in (**a**), 3D structure **b** delivered the contour shape, and **c** assigned the 2D figure



(a) (b) (c)

Fig. 13 The compressed bell-shaped soliton solution of Eq. (3.1.10) presented in (**a**), 3D structure, **b** supplied the contour shape, and **c** offered the 2D figure



(a) (b) (c)

Fig. 14 Singular periodic shaped solution of Eq. (3.1.12) handed in (**a**), 3D structure, **b** presented the contour shape, and **c** allocated the 2D

The kink shape soliton solution has been demonstrated of Eq. (3.2.6) in Fig. 15 in which 3D within $x \in [-10, 10]$, $t \in [0, 10]$, contour within $x \in [-10, 10]$, $t \in [0, 10]$and 2D within $x \in [-10, 10]$ including the parameters $y = 2,\ z = 1,\ c_0 = 0.1,\ R = \sqrt{5},\ T = 1,\ B_1 = 0.0001,\ B_2 = -0.0001,\ H = R^2 - 4T = 1,\ m = -0.1$ and $t = 1$. Lastly, Fig. 16 depicts the kink shaped soliton solution of Eq. (3.2.8), wherein 3D with $x, t \in [0, 5]$, contour with $x, t \in [0, 5]$and 2D with $x \in [0, 5]$ in adding the parameters stand

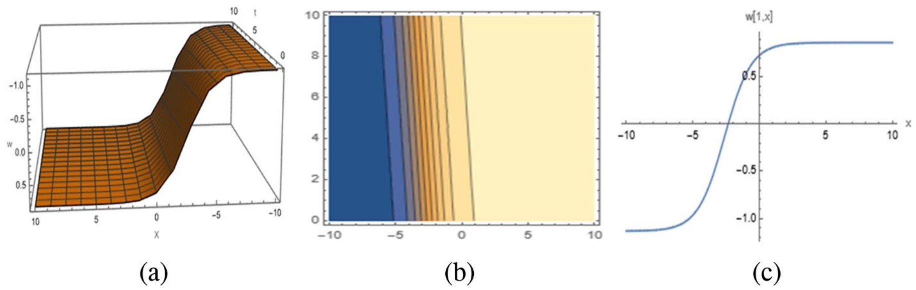


Fig. 15 The one sided kink-shaped soliton solution of Eq. (3.2.6) offered in (**a**), 3D structure, **b** organized the contour shape, and **c** assigned the 2D

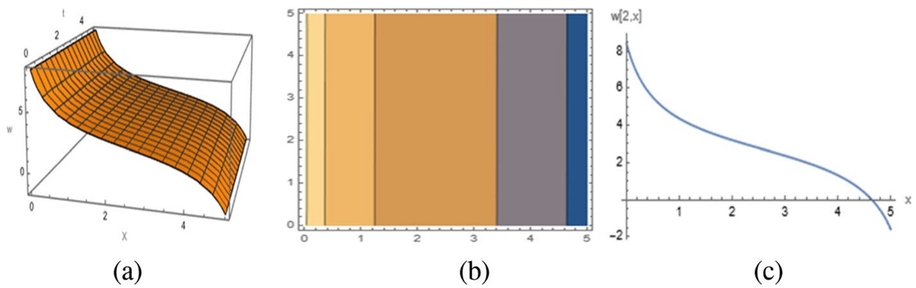


Fig. 16 The flat kink-shaped soliton solution of Eq. (3.2.8) handed in (**a**), 3D structure, **b** posed the contour shape, and **c** allotted the 2D

$y = 2,\ z = 1,\ c_0 = 0.1,\ \ R = \sqrt{3},\ T = 1,\ B_1 = 1,\ B_2 = 2,\ H = R^2 - 4T = -1,\ m = 0.001$ and $t = 2$.

We are able to emulate and display accurate physical behavior with the help of the 3D, 2D, and contour plots of the soliton solutions of NLPDEs, which are arisen in modern science and technology. Additionally, we recognize from each figure that the upcoming approach to the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional Jimbo-Miwa equation evaluation will be much more precise and fruitful. The outcomes of the above-mentioned model are pretty newly exposed and have not been established in the past literature, as we have lately exhibited.

8 Conclusion

Closed-form dynamic solitary solutions to the NLPDEs are used to investigate the most revolutionary occurrences in mathematical physics and contemporary engineering. In a nutshell, the $\left(\frac{G'}{G'+G+A}\right)$-expansion technique has made it possible for us to effectively analyze and collect innovative combined with generic closed-form soliton solutions for both the (3 + 1)-dimensional KP equation and the (3 + 1)-dimensional Jimbo-Miwa equation that possess multiple inherent applications in engineering technology and modern mathematics. There are noticeable differences between our produced outcomes and the ones that are currently available. Most physical systems, especially, plasma physics, solid state physics, fiber optics, water engineering, oceanography, dust acoustic waves, and shallow water waves with weakly nonlinear restoring forces could be thoroughly explained by the (3 + 1) -dimensional KP model. The most common model without the Painleve property resembles the (3 + 1)-dimensional JM equation that addresses precise attractive higher-dimensional waves in marine engineering, ocean sciences, fluid mechanics, mathematical modeling of infectious disease, circuit analysis, and computational neuroscience. A brief but insightful analysis concluded that our more recent and comprehensive results may be significant and beneficial for tangible applications as well, even if they weren't made available in other publications yet. Bifurcation analysis alongside phase portrait, chaotic analysis, and sensitivity analysis have been applied by now with the aim to figure out the nature of the outcomes. After performing these analyses, our research has successfully confirmed that our system seems sensitive and additionally, the systems have both a saddle point and a center point in their state of equilibrium. An incredible wave shape associated with nonlinear natural phenomena has been generated by the fantastic physical features of the gathered solutions. The visual representations of the computed exact soliton solutions having rational functions and trigonometric functions in the kink-shaped, parabolic-shaped soliton, singular periodic-shaped solution, one-sided kink-shaped soliton, and flat kink-shaped soliton have been depicted in contour, $2D$, and $3D$ diagram for a specific collection of constant quantities. From the outcomes of the present experiment, the advised approach is operative, profitable, valuable, trustworthy, simpler, and more rapid to detect multiple NLPDEs of mathematical physics to identify the novel exact soliton solutions.

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Data Availability No datasets were generated or analysed during the current study.

Declarations

Ethical Approval I hereby declare that this manuscript is the result of my independent creation under the reviewers' comments. Except for the quoted contents, this manuscript does not contain any research achievements that have been published or written by other individuals or groups.

Competing Interests The authors declare no competing interests.

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