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A study of lump-type and interaction solutions to a (3+1)-dimensional Jimbo–Miwa-like equation

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ABSTRACT

Using generalized bilinear equations, we construct a (3+1)-dimensional Jimbo–Miwa-like equation which possesses the same bilinear type as the standard (3+1)-dimensional Jimbo–Miwa equation. Classes of lump-type solutions and interaction solutions between lump-type and kink solutions to the resulting Jimbo–Miwa-like equation are generated through Maple symbolic computation. We discuss the conditions guaranteeing analyticity and positiveness of the solutions. By taking special choices of the involved parameters, 3D plots are presented to illustrate the dynamical features of the solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) have a variety of applications in many nonlinear sciences such as physics, applied mathematics, atmospheric and oceanic sciences [1,2]. Therefore, searching for exact solutions to NLEEs is a crucial concern for researches and scientists [3]. In recent years, rational solutions attracted a lot of attention. Among these rational solutions, lump solution is a kind of rational function solution localized in all directions in the space [4]. A new direct method, based on quadratic function, to obtain lump and lump-type solutions has been proposed [5]. Many nonlinear equations possess lump and lump-type solutions, for example, the BKP equation [4], the KPI equation [5], and others [6–10] using Hirota bilinear forms [11] or generalized bilinear forms [12]. Moreover, the interaction of lump solutions with other type of solutions such as kink solution [13,14], and resonance stripe solitons [15,16] has also attracted much attention. Tang et al. [17] explored the interaction between lump and a stripe of (2+1)-dimensional Ito equation and showed that the lump is swallowed by a stripe soliton. Fokas et al. [18] investigated the interaction of N-lumps with a line soliton for the Davey–Stewartson II equation. More interaction solutions between lumps and other kind of solutions have been analyzed in the literature [19,20].

The (3+1)-dimensional Jimbo–Miwa (JM) equation [21]

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1.1}$$

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is the second equation in the well known KP-hierarchy of integrable systems [21–24], which is used to describe certain interesting (3+1)-dimensional waves in physics. Although the JM Eq. (1.1) is non-integrable, exact solutions for the equation have been studied, particularly the Hirota bilinear method [25], the exponential-function method [26], the transformed rational function method [27], the three-wave method [28], the Wronskian method [29], and other methods were applied [30,31] to obtain solitons, periodic, complexiton, and traveling wave solutions.

Lump and lump-kink solutions for the JM Eq. (1.1) and two extended JM equations obtained by extending the last two linear terms u_{yt} and u_{xz} in Eq. (1.1) to $u_{xt} + u_{yt} + u_{zt}$ and $u_{xz} + u_{yz} + u_{zz}$ were considered in [32]. Classes of lump-type solutions for Eq. (1.1) were presented in [33,34]. Lump solutions to a reduced JM-like equation were investigated in [35].

This paper aims at seeking lump-type solutions and interaction solutions between lump-type and kink solutions for a JM-like equation in (3+1)-dimension through symbolic computations with Maple. The JM-like equation is generated from a generalized bilinear differential equation of JM type with $p = 3$. The organization of the paper is as follows: a JM-like equation is formulated in Section 2. In Section 3, we obtain lump-type solutions for the JM-like equation and analyze their dynamics. In Section 4, we present interaction solutions between lump-type and kink solutions. In the last section, conclusions and some remarks are given.

2. A (3+1)-dimensional Jimbo–Miwa-like equation

Via Hirota bilinear derivatives [11], Eq. (1.1) becomes the Hirota bilinear equation

$$\begin{aligned} B_{JM}(f) &:= (D_x^3 D_y + 2D_y D_t - 3D_x D_z) f.f \\ &= 2(ff_{xxxy} - f_y f_{xxx} + 3f_{xy} f_{xx} - 3f_x f_{xxy} + 2f_{ty} f - 2f_t f_y - 3f_{xz} f + 3f_x f_z) = 0, \end{aligned} \tag{2.1}$$

through the Cole–Hopf transformation $u = 2(\ln f)_x$.

Based on a prime number p, a kind of generalized bilinear differential operators is introduced as follows [12]

$$\begin{aligned} (D_{p,x}^m D_{p,t}^n) f.f &= \left(\partial_x + \alpha_p \partial_{x'}\right)^m \left(\partial_t + \alpha_p \partial_{t'}\right)^n f(x,t) f(x',t')|_{x'=x,t'=t} \\ &= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^{(i)}} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^{(j)}} f(x,t) f(x',t')|_{x'=x,t'=t} \\ &= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i} \binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} f(x,t)}{\partial x^{m-i} t^{n-j}} \frac{\partial^{i+j} f(x,t)}{\partial x^i t^j} \end{aligned} \tag{2.2}$$

where $m, n \geq 0$, $\alpha_p^s = (-1)^{r_p(s)}$, if $s \equiv r_p(s) \bmod p$.

Taking $p = 3$, we have

$$\alpha_3 = -1, \qquad \alpha_3^2 = 1, \qquad \alpha_3^3 = 1, \qquad \alpha_3^4 = -1, \qquad \alpha_3^5 = 1, \qquad \alpha_3^6 = 1, \ldots$$

which leads to

$$\begin{aligned} D_{3,y} D_{3,t} f.f &= 2f_{yt} f - 2f_y f_t \\ D_{3,x} D_{3,z} f.f &= 2f_{xz} f - 2f_x f_z \\ D_{3,x}^3 D_{3,y} f.f &= 6f_{xx} f_{xy} \end{aligned}$$

So we can generalize the bilinear JM Eq. (2.1) into

$$\begin{aligned} GB_{JM}(f) &:= (D_{3,x}^3 D_{3,y} + 2D_{3,y} D_{3,t} - 3D_{3,x} D_{3,z}) f.f \\ &= 2(3f_{xx} f_{xy} + 2f_{yt} f - 2f_y f_t - 3f_{xz} f + 3f_x f_z) = 0. \end{aligned} \tag{2.3}$$

By the link between f and u

$$u = 2(\ln f)_x \tag{2.4}$$

the generalized bilinear JM Eq. (2.3) is transformed to

$$GP_{JM}(u) := \frac{9}{8} u^2 u_x v + \frac{3}{8} u^3 u_y + \frac{3}{4} u v u_{xx} + \frac{3}{4} u_x^2 v + \frac{3}{4} u^2 u_{xy} + \frac{9}{4} u u_x u_y + \frac{3}{2} u_{xx} u_y + \frac{3}{2} u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0 \tag{2.5}$$

where $u_y = v_x$. The transformation (2.4) is a characteristic one in establishing Bell polynomial theories of integral equations [36,37] and can transform generalized bilinear equations to nonlinear equations. The actual relation between the JM-like Eq. (2.5) and the generalized bilinear Eq. (2.3) reads

$$GP_{JM}(u) = \left[\frac{GB_{JM}(f)}{f^2} \right]_x \tag{2.6}$$

The (3+1)-dimensional JM-like Eq. (2.5) has more terms and higher nonlinearity than the standard JM Eq. (1.1), and if f solves the generalized bilinear Eq. (2.3), then $u = 2(\ln f)_x$ will solve the JM-like Eq. (2.5).

3. Lump-type solutions to the Jimbo–Miwa-like equation

In order to find lump-type solutions for Eq. (2.5), we search for positive quadratic function solutions to the generalized bilinear Eq. (2.3) by expressing f as

$$f = g^2 + h^2 + a_{11}, \quad g = a_1x + a_2y + a_3z + a_4t + a_5, \quad h = a_6x + a_7y + a_8z + a_9t + a_{10} \tag{3.1}$$

where $a_i\,(1 \le i \le 11)$ are real constants to be determined. Substituting Eq. (3.1) into Eq. (2.3) and equating all the coefficients of different polynomials of x, y, z, t to zero, we obtain a set of algebraic equations in the parameters $a_i\,(1 \le i \le 11)$. Taking advantage of the computer algebra system Maple, we solve the set of algebraic equations and get the following sets of solutions

Case 1:

$$a_2 = -\frac{a_6a_7}{a_1}, \quad a_3 = -\frac{2}{3}\frac{a_4a_6a_7}{a_1^2}, \quad a_8 = \frac{2}{3}\frac{a_4a_7}{a_1}, \quad a_9 = \frac{a_4a_6}{a_1}, \quad a_i = a_i(i = 1, 4, 5, 6, 7, 10, 11), \tag{3.2}$$

which needs to satisfy the conditions $a_1 \neq 0$, and $a_{11} > 0$ to ensure the well defined and the positiveness of f, respectively.

Case 2:

$$a_2 = -\frac{a_6a_7}{a_1}, \quad a_3 = -\frac{a_6a_8}{a_1}, \quad a_4 = \frac{3}{2}\frac{a_1a_8}{a_7}, \quad a_9 = \frac{3}{2}\frac{a_6a_8}{a_7}, \quad a_{10} = \frac{a_5a_6}{a_1},$$
$$a_i = a_i(i = 1, 5, 6, 7, 8, 11), \tag{3.3}$$

which needs to satisfy the conditions $a_1a_7 \neq 0$, and $a_{11} > 0$ to make the corresponding solutions f well defined and positive, respectively.

Case 3:

$$a_3 = -\frac{2}{3}\frac{a_1a_2a_4 - a_1a_7a_9 + a_2a_6a_9 + a_4a_6a_7}{a_1^2 + a_6^2}, \quad a_8 = \frac{2}{3}\frac{a_1a_2a_9 + a_1a_4a_7 - a_2a_4a_6 + a_6a_7a_9}{(a_1^2 + a_6^2)},$$
$$a_{11} = -\frac{3}{2}\frac{(a_1a_2 + a_6a_7)(a_1^2 + a_6^2)^2}{(a_1a_9 - a_4a_6)(a_1a_7 - a_2a_6)}, \quad a_i = a_i(i = 1, 2, 4, 5, 6, 7, 9, 10), \tag{3.4}$$

which needs to satisfy the following conditions

i. $a_1^2 + a_6^2 \neq 0$, to make the corresponding solutions f well defined,
ii. $\frac{(a_1a_2+a_6a_7)}{(a_1a_9-a_4a_6)(a_1a_7-a_2a_6)} < 0$, to guarantee the positiveness of f,
iii. $(a_1a_9 - a_4a_6)(a_1a_7 - a_2a_6) \neq 0$, to ensure the localization of u in some directions in the space.

These cases of solutions for the parameters lead to three classes of quadratic function solutions $f_i\,(i = 1, 2, 3)$, defined by (3.1), to the generalized bilinear JM Eq. (2.3); and the resulting quadratic function solutions, in turn, yield three classes of lump-type solutions $u_i\,(i = 1, 2, 3)$, to the JM-like Eq. (2.5) through the transformation (2.4). All the rational function solutions $u_i \to 0$, $(i = 1, 2, 3)$, when the corresponding sum of squares $g^2 + h^2 \to \infty$. However, they do not approach zero in all directions in the space of x, y, z due to the character of (3+1)-dimensions in the resulting solutions, therefore, they are lump-type solutions but not lump solutions.

With the attention of analyzing the dynamic behavior of the solutions u_i, $(i = 1, 2, 3)$, we choose the following values for the parameters:

$$a_1 = 2, \quad a_4 = 1, \quad a_5 = -1, \quad a_6 = 2, \quad a_7 = 3, \quad a_{10} = 4, \quad a_{11} = 1,$$

for case 1, then we get the following lump-type solution:

$$u_1 = \frac{4(4x + 2t + 3)}{4x^2 + 9y^2 + z^2 + t^2 + 6yz + 4tx + 6x + 15y + 5z + 3t + 9}. \tag{3.5}$$

For case 2, we let

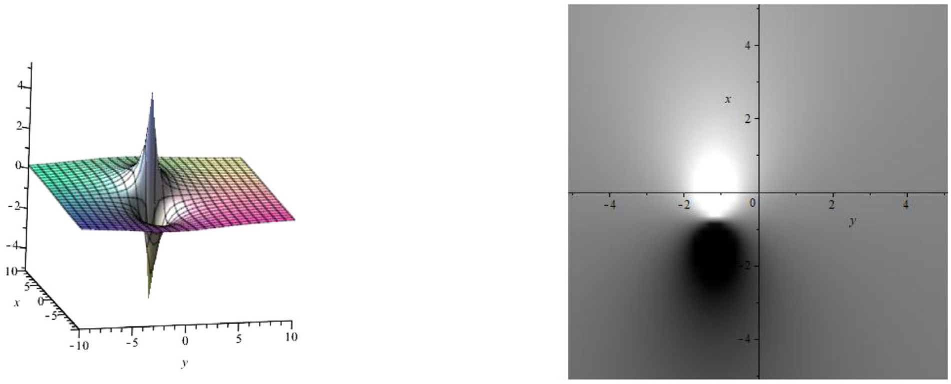
$$a_1 = 2, \quad a_5 = -1, \quad a_6 = 2, \quad a_7 = 3, \quad a_8 = 4, \quad a_{11} = 1,$$

and we obtain the lump-type solution:

$$u_2 = \frac{16(2x + 4t - 1)}{8x^2 + 18y^2 + 32z^2 + 32t^2 + 48yz + 32tx - 8x - 16t + 3}. \tag{3.6}$$

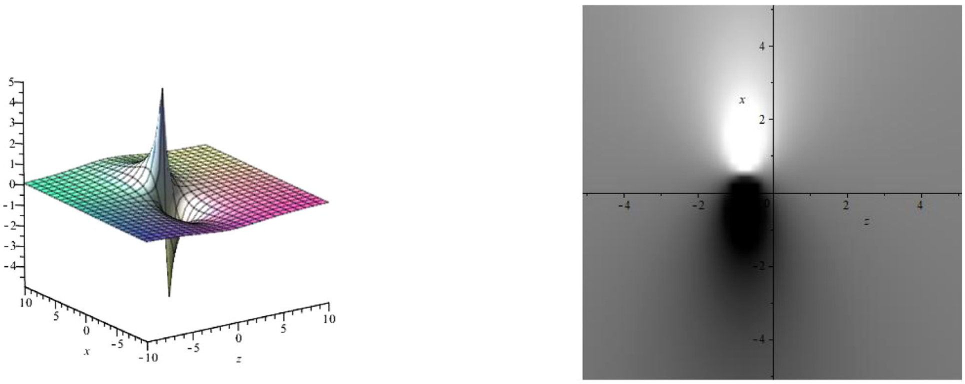
We choose

$$a_1 = 1, \quad a_2 = 0, \quad a_4 = 1, \quad a_5 = 2, \quad a_6 = -1, \quad a_7 = 1, \quad a_9 = 0, \quad a_{10} = 0,$$



(a) (b)

Fig. 1. Profiles of the lump-type solution (3.5) with $z = 1$ at $t = 0$: (a) 3D plot, (b) density plot.



(a) (b)

Fig. 2. Profiles of the lump-type solution (3.6) with $y = 1$ at $t = 0$: (a) 3D plot, (b) density plot.

for case 3, therefore we attain the lump-type solution:

$$u_3 = \frac{12(6x - 3y - 2z + 3t + 6)}{18x^2 + 9y^2 + 2z^2 + 9t^2 - 18xy - 12xz + 6yz + 18tx - 6tz + 36x - 12z + 36t + 90}. \tag{3.7}$$

Figs. 1–3 show the profiles of the lump-type solutions (3.5)–(3.7).

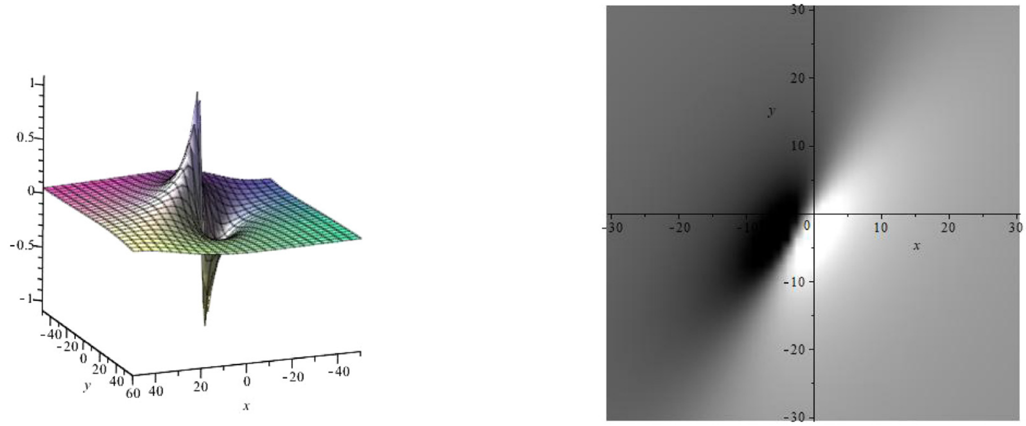
4. Interaction solutions between lump-type solutions and one stripe soliton

In this section, we explore the interaction between a lump-type and a stripe of the (3+1)-dimensional JM-like Eq. (2.5) by adding an exponential function in the quadratic function solution (3.1)

$$\begin{aligned} &f = g^2 + h^2 + e^l + a_{16}, \quad g = a_1x + a_2y + a_3z + a_4t + a_5, \quad h = a_6x + a_7y + a_8z + a_9t + a_{10}, \\ &l = a_{11}x + a_{12}y + a_{13}z + a_{14}t + a_{15} \end{aligned} \tag{4.1}$$

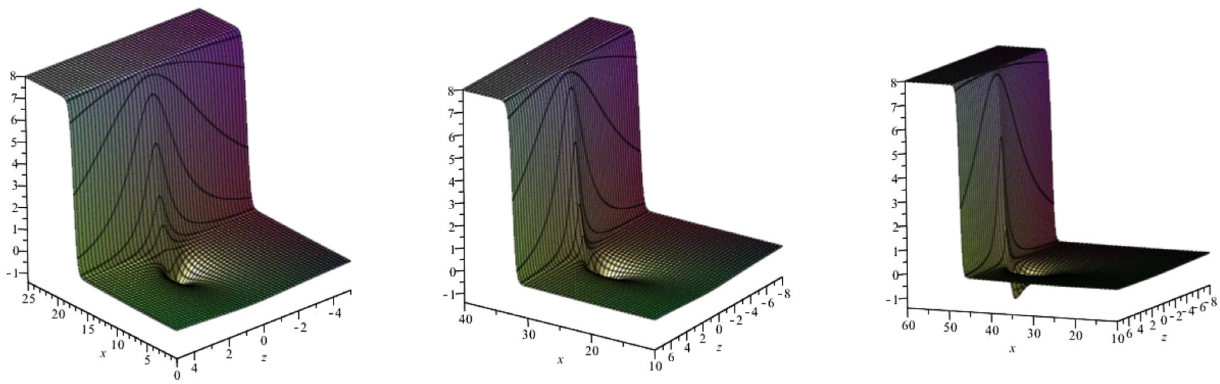
where $a_i (i = 1, \ldots, 16)$ are constants to be determined and $a_{16} > 0$. Symbolic computation via Maple on a direct substitution of Eq. (4.1) into Eq. (2.3), and by collecting all the coefficients about $x, y, z, t, e^{a_{11}x+a_{12}y+a_{13}z+a_{14}t+a_{15}}$, we reach the following set of constraining relations among the parameters

$$\begin{aligned} &a_1 = -\frac{a_6a_7}{a_2}, \quad a_3 = \frac{2}{3}\frac{a_2a_9}{a_6}, \quad a_4 = -\frac{a_7a_9}{a_2}, \quad a_8 = -\frac{2}{3}\frac{a_9a_7}{a_6}, \quad a_{12} = a_{13} = 0, \quad a_{14} = \frac{a_9a_{11}}{a_6}, \\ &a_i = a_i(i = 2, 5, 6, 7, 9, 10, 11, 15, 16), \end{aligned} \tag{4.2}$$



(a) (b)

Fig. 3. Profiles of the lump-type solution (3.7) with $z = 1$ at $t = 1$: (a) 3D plot, (b) density plot.



(a) $y = 1, t = 5$ (b) $y = 2, t = 10$ (c) $y = 3, t = 15$

Fig. 4. Profiles of the interaction solution (4.5) with different values of y, t.

with $a_2a_6 \neq 0$. Then the exact interaction solution of u is expressed as follows

$$u = 2(\ln f)_x = \frac{2f_x}{f} = \frac{4a_1g + 4a_6h + 2a_{11}e^l}{f}, \tag{4.3}$$

where

$$f = g^2 + h^2 + e^l + a_{16}, \qquad g = -\frac{a_6a_7}{a_2}x + a_2y + \frac{2}{3}\frac{a_2a_9}{a_6}z - \frac{a_7a_9}{a_2}t + a_5,$$
$$h = a_6x + a_7y - \frac{2}{3}\frac{a_9a_7}{a_6}z + a_9t + a_{10}, \qquad l = a_{11}x + \frac{a_9a_{11}}{a_6}t + a_{15},$$

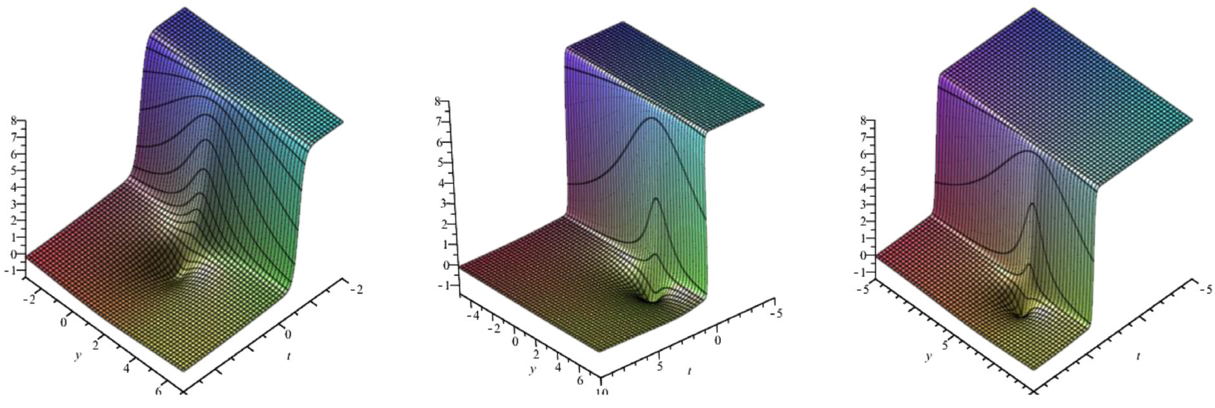
and $a_2, a_5, a_6, a_7, a_9, a_{10}, a_{11}, a_{15}$ and a_{16} are arbitrary real constants.

To illustrate the interaction phenomena between a lump-type solution and a stripe solution, we take special choices for the parameters as:

$$a_2 = 2, \quad a_5 = 1, \quad a_6 = -1, \quad a_7 = 2, \quad a_9 = 3, \quad a_{10} = 0, \quad a_{11} = 4, \quad a_{15} = -2, \quad a_{16} = 4. \tag{4.4}$$

Then the mixed lump-type stripe solution to Eq. (2.5) reads:

$$u = -\frac{4(-6t + 2x + 1 + 2e^{-12t+4x-2})}{18t^2 - 12tx + 2x^2 + 8y^2 - 32yz + 32z^2 - 6t + 2x + 4y - 8z + 5 + e^{-12t+4x-2}}. \tag{4.5}$$



(a) $x = 0, z = 1$ (b) $x = 3, z = 2$ (c) $x = 15, z = 4$

Fig. 5. Profiles of the interaction solution (4.5) with different values of x, z.

Figs. 4, and 5 show the asymptotic behaviors of solution (4.5). They exhibit the interaction between the lump-type soliton and the strip soliton.

5. Conclusions

In this paper, we studied the lump-type solutions and their dynamics for the nonlinear JM-like Eq. (2.5) making use of the generalized bilinear form and with the aid of Maple. Depending on the obtained lump-type solutions, we analyzed the interaction solutions between lump-type and kink solutions for the JM-like Eq. (2.5). In addition, we investigated the analyticity and localization conditions of the resulting solutions. A few particular classes of the derived solutions with specific values of the parameters were computed and plotted in Maple. These results are significant to understand the propagation processes for nonlinear waves in fluid mechanics. It would be an interesting topic to search for positive polynomial solutions to generalized bilinear or tri-linear differential equations . This kind of polynomial solutions will present lump or lump-type solutions to the corresponding nonlinear equations through $u = c(\ln f)_x$ or $u = c(\ln f)_{xx}$, where c is a constant related to the multi-linearity of the associated multi-linear equations.

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