Physica Scripta

PAPER

On lump and solitonic wave structures for the (3+1)-dimensional nonlinear evolution model

To cite this article: Reem K Alhefthi *et al* 2024 *Phys. Scr.* **99** 055223

IOP Publishing Phys. Scr. 99 (2024) 055223 https://doi.org/10.1088/1402-4896/ad368f

Physica Scripta

RECEIVED
21 January 2024

REVISED
11 March 2024

ACCEPTED FOR PUBLICATION
21 March 2024

PUBLISHED
5 April 2024

PAPER

On lump and solitonic wave structures for the (3+1)-dimensional nonlinear evolution model

Reem K Alhefthi[1], **Kalim U Tariq**[2,*], **Wen-Xiu Ma**[3,4,5,6,*] and **Fozia Mehboob**[2]

[1] Department of Mathematics, College of Sciences, King Saud University, PO Box 2455, Riyadh 11451, Saudi Arabia
[2] Department of Mathematics, Mirpur University of Science and Technology, Mirpur-10250 (AJK), Pakistan
[3] Department of Mathematics, Zhejiang Normal University, Jinhua 321004, Zhejiang, People's Republic of China
[4] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[5] Department of Mathematics and Statistics, University of South Florida Tampa, FL 33620, United States of America
[6] Department of Mathematical Sciences, North-West University, Mafikeng Campus, Mmabatho 2735, South Africa
[*] Authors to whom any correspondence should be addressed.

E-mail: kalimulhaq@must.edu.pk, mawx@cas.usf.edu and wma3@usf.edu

Keywords: nonlinear dynamics, the Hirota bilinear method, analytical wave solutions, lump wave solutions, periodic wave solutions

Abstract

In order to understand many complex situations in wave propagation, such as heat transfer, fluid dynamics, optical fibers, electrodynamics, physics, chemistry, biology, condensed matter physics, ocean engineering, and many other branches of nonlinear science, the majority of natural processes are routinely modelled and analysed using nonlinear evolution equations. In this study, the (3+1)-dimensional nonlinear evolution equation is investigated analytically. Initially, the Hirota bilinear approach is used to develop the bilinear version of the higher dimensional nonlinear model. Consequently, we are able to design periodic wave soliton solutions, lump wave and single-kink soliton solutions, and collisions between lumps and periodic waves. Later on, the unified method is applied to develop several new travelling wave solutions for the governing model substantially. Furthermore, numerous exact solutions are analyzed graphically to explore many fascinating nonlinear dynamical structures with the aid of 3D, contour, and 2D visualizations. A variety of higher dimensional nonlinear evolution models can also be investigated by employing present approaches arising in many fields of contemporary science and technology.

1. Introduction

Nonlinear evolution equations (NLEEs) involve numerous applications in diverse sectors such as hydrodynamics, optical fibres, chaos theory, ocean engineering, solitary wave theory, turbulence theory, and many more. Many nonlinear mathematical and physical events are known to heavily depend on the development of exact solutions and the investigation of accessible properties for nonlinear dynamical models. In numerous fields, NLEEs are frequently utilised to illustrate certain events including, optics [1], fluid mechanics [2], condensed matter physics [3], fluid dynamics [4], plasma physics [5] and nonlinear optics [6].

The study of dynamical structures of such models is the subject of intense research because of their crucial role in explaining many important phenomena in a wide range of real-world problems, such as those in thermodynamics, chemical physics, rheology, electrochemistry, chemical physics, quantum mechanics, and nonlinear dispersive models [7–9]. An understanding of various qualitative and quantitative features of nonlinear scientific processes is greatly aided by the travelling wave solutions. Their nonlocality is among these qualitative attributes, which indicates that a system status depends not just on its current position but also on all of its prior histories [10–14].

The lump solutions were systematically created by Ablowitz and Satsuma in 1978 using the Hirota bilinear technique [15, 16], while Ma developed the interaction solutions in 2015, including the lump-kink and the lump-soliton solutions [17] in comparison to the lump wave solutions [18, 19]. Furthermore, the rogue waves are a significant feature among the complicated interaction behaviours that have been identified based on interaction solutions for numerous eminent nonlinear dynamical models. Some studies suggest that rogue waves originate from the collision of a lump wave and a two-soliton. The lump wave solution is a unique sort of rational localised wave solution that algebraically decays to the background wave in the direction of space [20, 21].

The (3+1)-dimensional nonlinear evolution model reads

$$u_{yt} - u_{xxxy} - 3(u_x u_y)_x - 3u_{xx} - 3u_{zz} = 0, \tag{1}$$

which is originally proposed in 2016 by L. N. Gao *et al* [22], Yu-Hang Yin *et al* later developed a bilinear Bäcklund transformation that consists of four equations and six free parameters [23], while T. A. Sulaiman *et al* motivated to design and establish some lump collision phenomena [24], the interaction solutions to the dimensionally reduced equation have also been examined [25], along with the resonant behaviour of multiple wave solutions. In this paper, the Hirota bilinear approach is utilized to investigate the nature of the higher dimensional evolution model analytically. In order to develop a thorough analysis of lump waves and their collisions with periodic waves by identifying the bilinear form of the fractional model. We have successfully examined the collisions between lump waves and periodic wave soliton solutions, as well as the interactions between lump waves and single and double-kink soliton solutions. Additionally, the unified method [26–28] is implemented to achieve some new travelling wave structures for the governing model.

The research framework is structured as follows: section 2, defines the governing model and its bilinear form while section 3, contains the mathematical analysis. Section 4 demonstrates the graphical visualizations for the observed wave structures. At the end summary of this paper are illustrated.

2. Mathematical analysis

Consider the following transformation [29] to develop the bilinear form of the equation (1)

$$\omega(x, y, z, t) = 2\ln[F(x, y, z, t)]_x, \tag{2}$$

where $F(x, y, z, t)$ is the unknown wave function that is to be calculated for the solutions of equation (1). The bilinear form is shown below

$$(D_t D_y - D_x^3 D_y - 3D_x^2 + 3D_z^2)F.F = 0,$$

while the following bilinear operator are related [30],

$$D_x^m D_t^n (f.g) = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^m \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^n \times f(x, t).g(x', t')|x = x', t = t'.$$

For a result, we attain

$$\begin{aligned} &2FF_{yt} - 2F_t F_y + 6F_x F_{xxy} - 6F_{xx} F_{xy} + 2F_{xxx} F_y \\ &-2FF_{xxxy} - 3(FF_{xx} - F_x^2) + 3(FF_{zz} - F_z^2) = 0, \end{aligned} \tag{3}$$

obviously, if F satisfies (1), then $\omega = 2[lnF]x$ directs interest approach to the given equation (1).

2.1. Lump waves

In this section, we examine the operator F provided by

$$F = \psi_1^2 + \psi_2^2 + \varpi, \tag{4}$$

where

$$\psi_1 = a_1 x + a_2 y + a_3 z + a_4 t,\ \ \psi_2 = b_1 x + b_2 y + b_3 z + b_4 t,$$

and $a_i,\ b_i,\ \varpi$'s are the constants to be determined. Putting equation (4) into equation (3), we have a collection of results with different variables. Using a computational tool like Mathematica to resolve them, we reach the conclusions described below.

Case 1:

$$a_1 = -a_3 - ib_3,\quad b_1 = -2i(a_3 + ib_3),\quad b_2 = ia_2,$$
$$b_4 = -\frac{i(-15ia_3 b_3 - 6a_3^2 + a_2 a_4 + 6b_3^2)}{a_2},$$

where $a_2, a_3, a_4, b_3, \varpi$ are free variables. Using all known values, equation (4) becomes

$$\begin{aligned} F = \varpi &+ (x(-a_3 - ib_3) + a_4 t + a_2 y + a_3 z)^2 \\ &+ \left(-\frac{it(-15ia_3 b_3 - 6a_3^2 + a_2 a_4 + 6b_3^2)}{a_2} - 2ix(a_3 + ib_3) + ia_2 y + b_3 z\right)^2. \end{aligned} \tag{5}$$

Inserting (5) into (2), Finally calculate the $\omega_1(x, t, y, z)$ of (1).

Case 2:

$$a_4 = -\frac{3(-2a_1b_1b_2 + a_2b_1^2 - a_2b_3^2 + 2a_3b_2b_3 + a_2a_3^2 - a_1^2a_2)}{2(a_2^2 + b_2^2)},$$
$$b_4 = \frac{3(a_1^2(-b_2) + 2a_2a_1b_1 + a_3^2b_2 - 2a_2a_3b_3 - b_2b_3^2 + b_1^2b_2)}{2(a_2^2 + b_2^2)},$$

where $a_1, a_2, a_3, b_1, b_2, b_3, \varpi$, are free variables. Utilizing the suggested values equation (4) becomes

$$F = \left(-\frac{3t(-2a_1b_1b_2 + a_2b_1^2 - a_2b_3^2 + 2a_3b_2b_3 + a_2a_3^2 - a_1^2a_2)}{2(a_2^2 + b_2^2)} + a_1x + a_2y + a_3z\right)^2 + \varpi + \left(\frac{3t(a_1^2(-b_2) + 2a_2a_1b_1 + a_3^2b_2 - 2a_2a_3b_3 - b_2b_3^2 + b_1^2b_2)}{2(a_2^2 + b_2^2)} + b_1x + b_2y + b_3z\right)^2. \quad (6)$$

Using equation (6) along with equation (2), Finally get the answer $\omega_2(x, t, y, z)$ of (1).

Case 3:

$$a_2 = 0, \quad a_4 = -\frac{3(a_3b_3 - a_1b_1)}{b_2}, \quad b_4 = \frac{3(-a_1^2 + a_3^2 + b_1^2 - b_3^2)}{2b_2},$$

where a_1,a_3, b_1, b_2, b_3, ϖ, are free variables.

Utilizing the suggested values equation (4) gives

$$F = \left(-\frac{3t(a_3b_3 - a_1b_1)}{b_2} + a_1x + a_3z\right)^2 + \varpi + \left(\frac{3t(-a_1^2 + a_3^2 + b_1^2 - b_3^2)}{2b_2} + b_1x + b_2y + b_3z\right)^2. \quad (7)$$

Adding (7) combine to (2), the determine the result $\omega_3(x, t, y, z)$ of (1).

Case 4:

$$a_1 = a_3 - ib_3, \quad b_1 = -2i(a_3 - ib_3), \quad b_2 = -ia_2,$$
$$b_4 = \frac{i(15ia_3b_3 - 6a_3^2 + a_2a_4 + 6b_3^2)}{a_2},$$

where a_2,a_3, a_4, b_3, ϖ, are free variables.

Using calculated results, equation (4) gives

$$F = \varpi + (x(a_3 - ib_3) + a_4t + a_2y + a_3z)^2 + \left(\frac{it(15ia_3b_3 - 6a_3^2 + a_2a_4 + 6b_3^2)}{a_2} - 2ix(a_3 - ib_3) - ia_2y + b_3z\right)^2. \quad (8)$$

Utilizing (8) together in (2), they determine the result $\omega_4(x, t, y, z)$ of (1).

2.2. Collision among lump wave and stripe soliton

Consider the operator of the form

$$F = \lambda_0 + \lambda_1\psi_1^2 + \lambda_2\psi_2^2 + \lambda_3e^{\psi_3}, \quad (9)$$

where

$$\psi_1 = a_0 + a_1x + a_2y + a_3z + a_4t, \quad \psi_2 = b_0 + b_1x + b_2y + b_3z + b_4t,$$
$$\psi_3 = c_0 + c_1x + c_2y + c_3z + c_4t,$$

here a_i, b_i, c_i's are the unknown integers. These definitions for ψ_1, ψ_2, and ψ_3 are still applicable throughout this article.

Using equation (9) into equation (3), the following results are attained after solving this system using a computer programme like Mathematica.

Case 1:

$$c_4 = \frac{2c_2c_1^3 + 3c_1^2 - 3c_3^2}{2c_2}, \quad \lambda_1 = 0, \quad \lambda_2 = 0,$$

where c_0, c_1, c_2, c_3, λ_0, λ_3 are free variables.

Therefore, applying these conditions, equation (9) becomes

$$F = \lambda_3 e\left(\frac{(2c_2c_1^3 + 3c_1^2 - 3c_3^2)t}{2c_2} + c_1x + c_2y + c_3z + c_0\right) + \lambda_0. \tag{10}$$

Using (10) along with (2), Finally find the solution.

$$\omega_5(x, y, z, t) = 2*\frac{c_1\lambda_3 e\left(\frac{(2c_2c_1^3 + 3c_1^2 - 3c_3^2)t}{2c_2} + c_1x + c_2y + c_3z + c_0\right)}{\lambda_3 e\left(\frac{(2c_2c_1^3 + 3c_1^2 - 3c_3^2)t}{2c_2} + c_1x + c_2y + c_3z + c_0\right) + \lambda_0}. \tag{11}$$

Case 2:

$$a_2 = -\frac{ib_2\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\quad b_1 = -\frac{ia_3\sqrt{\lambda_1}}{\sqrt{\lambda_2}},\quad b_3 = -\frac{ia_1\sqrt{\lambda_1}}{\sqrt{\lambda_2}},\quad b_4 = -\frac{ia_4\sqrt{\lambda_1}}{\sqrt{\lambda_2}},\quad \lambda_3 = 0,$$

while $a_0, a_1, a_3, a_4, b_0, b_2, \lambda_0, \lambda_1$ are free variables.

Therefore, applying these conditions, equation (9) becomes

$$F = \lambda_0 + \lambda_1\left(a_4t + a_1x + a_3z + a_0 - \frac{ib_2\sqrt{\lambda_2}\,y}{\sqrt{\lambda_1}}\right)^2 + \lambda_2\left(-\frac{ia_4\sqrt{\lambda_1}\,t}{\sqrt{\lambda_2}} - \frac{ia_3\sqrt{\lambda_1}\,x}{\sqrt{\lambda_2}} - \frac{ia_1\sqrt{\lambda_1}\,z}{\sqrt{\lambda_2}} + b_2y + b_0\right)^2. \tag{12}$$

Using (12) together with (2), We find the solution of the form:

$$\omega_6(x, y, z, t) = \frac{2\left(2a_1\lambda_1(a_4t + a_1x + a_3z + a_0 - a_2) - 2ia_3\sqrt{\lambda_1}\sqrt{\lambda_2}\left(-\frac{ia_1\sqrt{\lambda_1}z}{\sqrt{\lambda_2}} + b_2y + b_0 - b_3 - b_4\right)\right)}{\lambda_2\left(-\frac{ia_1\sqrt{\lambda_1}z}{\sqrt{\lambda_2}} + b_2y + b_0 - b_3 - b_4\right)^2 + \lambda_1(a_4t + a_1x + a_3z + a_0 - a_2)^2 + \lambda_0}. \tag{13}$$

where

$$a_2 = -\frac{ib_2\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\ b_3 = -\frac{ia_1\sqrt{\lambda_1}}{\sqrt{\lambda_2}},\ b_4 = -\frac{ia_4\sqrt{\lambda_1}}{\sqrt{\lambda_2}}.$$

Case 3:

$$a_2 = -\frac{ib_2\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\quad a_3 = 0,\quad b_1 = 0,\quad b_3 = -\frac{ia_1\sqrt{\lambda_1}}{\sqrt{\lambda_2}};\ b_4 = -\frac{ia_4\sqrt{\lambda_1}}{\sqrt{\lambda_2}},$$

where $a_0, a_1, a_4, a_3, b_0, b_2, \lambda_0, \lambda_1$ are free variables.

Using all of the following known values, equation (9) becomes

$$F = \lambda_0 + \lambda_1\left(a_4t + a_1x + a_0 - \frac{ib_2\sqrt{\lambda_2}\,y}{\sqrt{\lambda_1}}\right)^2 + \lambda_2\left(-\frac{ia_4\sqrt{\lambda_1}\,t}{\sqrt{\lambda_2}} - \frac{ia_1\sqrt{\lambda_1}\,z}{\sqrt{\lambda_2}} + b_2y + b_0\right)^2. \tag{14}$$

Inserting (14) into (2), they determine the result $\omega_7(x, t, y, z)$ of (1).

Case 4:

$$a_2 = 0,\quad a_4 = -\frac{3(a_3b_3 - a_1b_1)}{b_2},\quad b_4 = -\frac{3(a_1^2\lambda_1 - a_3^2\lambda_1 - b_1^2\lambda_2 + b_3^2\lambda_2)}{2b_2\lambda_2},$$

where $a_0, a_1, a_3, b_0, b_1, b_2, b_3, \lambda_0, \lambda_1$ contain free variables.

Therefore, given each for the following known variables equation (9) becomes

$$F = \lambda_1\left(-\frac{3t(a_3b_3 - a_1b_1)}{b_2} + a_1x + a_3z + a_0\right)^2 + \lambda_0 + \lambda_2\left(-\frac{3t(a_1^2\lambda_1 - a_3^2\lambda_1 - b_1^2\lambda_2 + b_3^2\lambda_2)}{2b_2\lambda_2} + b_1x + b_2y + b_3z + b_0\right)^2. \tag{15}$$

using (15) along with (2), they determine the result $\omega_8(x, t, y, z)$ of (1).

2.3. Collisions between lump wave and double strip soliton

Consider the following steps

$$F = \lambda_0 + \lambda_1\psi_1^2 + \lambda_2\psi_2^2 + \lambda_3\cosh(\psi_3). \tag{16}$$

where

$$\psi_1 = a_0 + a_1x + a_2y + a_3z + a_4t,\ \ \psi_2 = b_0 + b_1x + b_2y + b_3z + b_4t,$$
$$\psi_3 = c_0 + c_1x + c_2y + c_3z + c_4t,$$

and $a_i,\ b_i,\ c_i$'s are the unknown integers.

Inserting (16) into (3), We come up with a collection of equations involving different variables. With the use of a computer programme like Mathematica, we are able to solve this system and obtain the given results.

Case 1:

$$c_4 = -\frac{-8c_2c_1^3 - 3c_1^2 + 3c_3^2}{2c_2},\quad \lambda_0 = 0,\quad \lambda_1 = 0,\quad \lambda_2 = 0,$$

where $c_0, c_1, c_2, c_3, \lambda_3$ are free variables.

In basis of these factors, equation (16) becomes

$$F = \lambda_3\cosh\left(-\frac{(-8c_2c_1^3 - 3c_1^2 + 3c_3^2)t}{2c_2} + c_1x + c_2y + c_3z + c_0\right). \tag{17}$$

using (17) together with (2), they find the solution.

$$\omega_9(x, y, z, t) = 2\left[c_1\tanh\left(-\frac{(-8c_2c_1^3 - 3c_1^2 + 3c_3^2)t}{2c_2} + c_1x + c_2y + c_3z + c_0\right)\right]. \tag{18}$$

Case 2:

$$a_1 = -\frac{ib_3\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\quad a_3 = -\frac{ib_1\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\quad a_4 = \frac{ib_4\sqrt{\lambda_2}}{\sqrt{\lambda_1}},\quad b_2 = \frac{ia_2\sqrt{\lambda_1}}{\sqrt{\lambda_2}},\quad \lambda_0 = 0,$$

where $a_0, a_2, a_4, b_0, b_1, b_3, b_4, \lambda_1, \lambda_2, \lambda_3$ are free variables. In basis of these factors, equation (16) becomes

$$F = \lambda_1\left(a_2y + a_0 + \frac{ib_4\sqrt{\lambda_2}\,t}{\sqrt{\lambda_1}} - \frac{ib_3\sqrt{\lambda_2}\,x}{\sqrt{\lambda_1}} - \frac{ib_1\sqrt{\lambda_2}\,z}{\sqrt{\lambda_1}}\right)^2 + \lambda_3\cosh(\psi_3) + \lambda_2\left(\frac{ia_2\sqrt{\lambda_1}\,y}{\sqrt{\lambda_2}} + b_4t + b_1x + b_3z + b_0\right)^2. \tag{19}$$

the (19) combined by (2), we find the solution $\omega_{10}(x, y, z, t)$ of (1).

Case 3:

$$a_4 = -\frac{3(a_3^2\lambda_1 - a_1^2\lambda_1 + b_1^2\lambda_2 - b_3^2\lambda_2)}{2a_2\lambda_1},\quad b_2 = 0,\quad b_4 = -\frac{3(a_3b_3 - a_1b_1)}{a_2},\quad \lambda_0 = 0,$$

where $a_0, a_1, a_2, a_3, b_0, b_1, b_3, \lambda_1, \lambda_2, \lambda_3$, are free variables.

In basis of these factors, equation (16) becomes

$$F = \lambda_2\left(-\frac{3t(a_3b_3 - a_1b_1)}{a_2} + b_1x + b_3z + b_0\right)^2 + \lambda_1\left(-\frac{3t(a_3^2\lambda_1 - a_1^2\lambda_1 + b_1^2\lambda_2 - b_3^2\lambda_2)}{2a_2\lambda_1} + a_1x + a_2y + a_3z + a_0\right)^2 + \lambda_3\cosh(\psi_3). \tag{20}$$

using (20) along with (2), we acquire the solution $\omega_{11}(x, y, z, t)$ of (1).

2.4. Collision between lump and periodic waves

The following section investigates the function F, which is defined as

$$F = \lambda_0 + \lambda_1\psi_1^2 + \lambda_2\psi_2^2 + \lambda_3\cos(\psi_3), \tag{21}$$

where

$$\psi_1 = a_0 + a_1x + a_2y + a_3z + a_4t,\ \ \psi_2 = b_0 + b_1x + b_2y + b_3z + b_4t,$$
$$\psi_3 = c_0 + c_1x + c_2y + c_3z + c_4t,$$

and $a_i,\ b_i,\ c_i$'s are the unknown integers.

Inserting (21) into (3), we obtain a collection of solutions with different given constants. After solving this system with a computer application like Mathematica they get to the corresponding results.

Case 1:

$$c_4 = \frac{-8c_2 c_1^3 + 3c_1^2 - 3c_3^2}{2c_2}, \quad \lambda_0 = 0, \quad \lambda_1 = 0, \quad \lambda_2 = 0,$$

where $c_0, c_1, c_2, c_3, \lambda_0, \lambda_3$, are free variables.

Therefore, using all of the given variables, equation (21) obtains

$$F = \lambda_3 \cos\left(\frac{(-8c_2 c_1^3 + 3c_1^2 - 3c_3^2)t}{2c_2} + c_1 x + c_2 y + c_3 z + c_0\right). \tag{22}$$

using all of these results together with (22) and then using (2), Finally get the solution.

$$\omega_{12}(x, t, y, z) = 2(-c_1)\tan\left(\frac{(-8c_2 c_1^3 + 3c_1^2 - 3c_3^2)t}{2c_2} + c_1 x + c_2 y + c_3 z + c_0\right). \tag{23}$$

Case 2:

$$a_4 = -\frac{3(a_3^2\lambda_1 - a_1^2\lambda_1 + b_1^2\lambda_2 - b_3^2\lambda_2)}{2a_2\lambda_1}, \quad b_2 = 0, \quad b_4 = -\frac{3(a_3 b_3 - a_1 b_1)}{a_2}, \quad \lambda_0 = 0,$$

where $a_0, a_1, a_2, a_3, b_0, b_1, b_3, \lambda_1, \lambda_2, \lambda_3$, are free variables.

Therefore, using all of the given variables, equation (21) obtains

$$\begin{aligned} F = {} & \lambda_2\left(-\frac{3t(a_3 b_3 - a_1 b_1)}{a_2} + b_1 x + b_3 z + b_0\right)^2 \\ & + \lambda_1\left(-\frac{3t(a_3^2\lambda_1 - a_1^2\lambda_1 + b_1^2\lambda_2 - b_3^2\lambda_2)}{2a_2\lambda_1} + a_1 x + a_2 y + a_3 z + a_0\right)^2 \\ & + \lambda_3 + \cos(c_4 t + c_1 x + c_2 y + c_3 z + c_0). \end{aligned} \tag{24}$$

using all of these results together with (24) and then using (2), we find the result.

$$\omega_{13}(x, y, z, t) = \frac{2\left(2b_1\lambda_2\left(-\frac{3t(a_3 b_3 - a_1 b_1)}{a_2} + b_1 x + b_3 z + b_0\right) + 2a_1\lambda_1\psi_1 - c_1\lambda_3\sin(\psi_3)\right)}{\lambda_2\left(-\frac{3t(a_3 b_3 - a_1 b_1)}{a_2} + b_1 x + b_3 z + b_0\right)^2 + \lambda_1\psi_1^2 + \lambda_3\cos(\psi_3)}. \tag{25}$$

where

$$\begin{aligned} & a_4 = -\frac{3(a_3^2\lambda_1 - a_1^2\lambda_1 + b_1^2\lambda_2 - b_3^2\lambda_2)}{2a_2\lambda_1}, \quad \psi_1 = a_4 t + a_1 x + a_2 y + a_3 z + a_0, \\ & \psi_3 = c_4 t + c_1 x + c_2 y + c_3 z + c_0. \end{aligned}$$

Case 3:

$$a_1 = 0, \quad a_3 = -\frac{ib_1\sqrt{\lambda_2}}{\sqrt{\lambda_1}}, \quad a_4 = \frac{ib_4\sqrt{\lambda_2}}{\sqrt{\lambda_1}}, \quad b_2 = \frac{ia_2\sqrt{\lambda_1}}{\sqrt{\lambda_2}}, \quad b_3 = 0, \quad \lambda_0 = 0,$$

where $a_0, a_2, b_0, b_1, b_4, \lambda_1, \lambda_2, \lambda_3$, are free variables.

Therefore, using all of the given variables, equation (21) obtains

$$\begin{aligned} F = {} & \lambda_1\left(a_2 y + a_0 + \frac{ib_4\sqrt{\lambda_2}\,t}{\sqrt{\lambda_1}} - \frac{ib_1\sqrt{\lambda_2}\,z}{\sqrt{\lambda_1}}\right)^2 \\ & + \lambda_3\cos(\psi_3) + \lambda_2\left(\frac{ia_2\sqrt{\lambda_1}\,y}{\sqrt{\lambda_2}} + b_4 t + b_1 x + b_0\right)^2. \end{aligned} \tag{26}$$

using all of these results together with (26) and then using (2), we find the result.

$$\omega_{14}(x, y, z, t) = \frac{2\left(-c_1\lambda_3\sin(\psi_3) + 2b_1\lambda_2\left(\frac{ia_2\sqrt{\lambda_1}\,y}{\sqrt{\lambda_2}} + b_4 t + b_1 x + b_0\right)\right)}{\lambda_2\left(\frac{ia_2\sqrt{\lambda_1}\,y}{\sqrt{\lambda_2}} + b_4 t + b_1 x + b_0\right)^2 + \lambda_1\left(a_2 y + a_0 + \frac{ib_4\sqrt{\lambda_2}\,t}{\sqrt{\lambda_1}} - \frac{ib_1\sqrt{\lambda_2}\,z}{\sqrt{\lambda_1}}\right)^2 + \lambda_3\cos(\psi_3)}. \tag{27}$$

2.5. Periodic solitons

The following section investigates the function F, which is defined as

$$F = \lambda_0 + \lambda_1 e^{-\psi_1} + \lambda_2 e^{\psi_1} + \lambda_3 \cos(\psi_2). \tag{28}$$

where

$$\psi_1 = a_0 + a_1 x + a_2 y + a_3 z + a_4 t,\ \ \psi_2 = b_0 + b_1 x + b_2 y + b_3 z + b_4 t.$$

and $a_i,\ b_i,\ c_i$'s are the unknown integers.

Inserting (28) into (3), we obtain a collection of solutions with different given constants. After solving this system with a computer application like Mathematica they get to the corresponding results.

Case 1:

$$a_4 = \frac{-6a_2 a_1 b_1^2 + 2a_2 a_1^3 + 3a_1^2 - 3a_3^2}{2a_2},\quad b_2 = 0,$$
$$b_3 = -b_1,\quad b_4 = -\frac{b_1(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 - 3a_3)}{a_2},\quad \lambda_0 = 0,$$

where $a_0, a_1, a_2, b_0, b_1, a_3, \lambda_1, \lambda_2, \lambda_3$ are free variables.

Therefore, using all of the given variables, equation (28) obtains

$$\begin{aligned} F = {} & \lambda_1\left(\frac{t(-6a_2 a_1 b_1^2 + 2a_2 a_1^3 + 3a_1^2 - 3a_3^2)}{2a_2} + a_1 x + a_2 y + a_3 z + a_0\right)^2 \\ & + \lambda_2\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 - 3a_3)}{a_2} + b_1 x - b_1 z + b_0\right)^2 \\ & + \lambda_3 \cos\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 - 3a_3)}{a_2} + b_1 x - b_1 z + b_0\right). \end{aligned} \tag{29}$$

using all of these results together with (29) and then using (2), we find the result.

$$\omega_{15}(x, y, z, t) = \frac{2\left(b_1\lambda_3\left(-\sin\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 - 3a_3)}{a_2} + b_1 x - b_1 z + b_0\right)\right) + 2a_1\lambda_1\psi_1 + 2b_1\lambda_2\psi_2\right)}{\lambda_3 \cos\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 - 3a_3)}{a_2} + b_1 x - b_1 z + b_0\right) + \lambda_1\psi_1^2 + \lambda_2\psi_2^2}. \tag{30}$$

where

$$a_4 = \frac{-6a_2 a_1 b_1^2 + 2a_2 a_1^3 + 3a_1^2 - 3a_3^2}{2a_2},\quad \psi_1 = a_4 t + a_1 x + a_2 y + a_3 z + a_0,$$
$$\psi_2 = b_4 t + b_1 x + b_2 y + b_3 z + b_0.$$

Case 2:

$$a_4 = \frac{-6a_2 a_1 b_1^2 + 2a_2 a_1^3 + 3a_1^2 - 3a_3^2}{2a_2},\quad b_2 = 0,$$
$$b_3 = b_1,\quad b_4 = -\frac{b_1(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 + 3a_3)}{a_2},\quad \lambda_0 = 0,$$

where $a_0, a_1, a_2, a_3, b_0, b_1, b_3, \lambda_1, \lambda_2, \lambda_3$ are free variables.

Therefore, using all of the given variables, equation (28) obtains

$$\begin{aligned} F = {} & \lambda_1\left(\frac{t(-6a_2 a_1 b_1^2 + 2a_2 a_1^3 + 3a_1^2 - 3a_3^2)}{2a_2} + a_1 x + a_2 y + a_3 z + a_0\right)^2 \\ & + \lambda_2\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 + 3a_3)}{a_2} + b_1 x + b_1 z + b_0\right)^2 \\ & + \lambda_3 \cos\left(-\frac{b_1 t(a_2 b_1^2 - 3a_2 a_1^2 - 3a_1 + 3a_3)}{a_2} + b_1 x + b_1 z + b_0\right). \end{aligned} \tag{31}$$

using all of these results together with (31) and then using (2), we find the result.

$$\omega_{16}(x, y, z, t) = \frac{2\left(2a_1\lambda_1\left(\frac{t(-6a_2a_1b_1^2+2a_2a_1^3+3a_1^2-3a_3^2)}{2a_2} + a_1x + a_2y + a_3z + a_0\right) + 2b_1\lambda_2\psi_2 - b_1\lambda_3\sin(\psi_2)\right)}{\lambda_1\left(\frac{t(-6a_2a_1b_1^2+2a_2a_1^3+3a_1^2-3a_3^2)}{2a_2} + a_1x + a_2y + a_3z + a_0\right)^2 + \lambda_2\psi_2^2 + \lambda_3\cos(\psi_2)}.$$

where

$$b_4 = -\frac{b_1(a_2b_1^2 - 3a_2a_1^2 - 3a_1 + 3a_3)}{a_2}, \quad \psi_2 = b_4t + b_1x + b_2y + b_3z + b_0.$$

2.6. Traveling wave solutions by the unified method

The aim of the section is to achieve some new traveling wave solutions for the (3+1)-dimensional nonlinear evolution model (1) by employing the unified technique [31]. Consider the following wave transformation

$$\omega(x, y, z, t) = u(\zeta), \quad \zeta = \tau x + \rho y + \frac{\sigma z^\alpha}{\alpha} - \frac{\mu t^\beta}{\beta}, \tag{32}$$

where τ and ρ is parameters, while σ, μ is the waves velocity. An ordinary differential equation is obtained using the transformation (32) in (1).

$$(-\mu\rho + 3\sigma^2 - 3\tau^2)u''(\zeta) + 3\sigma u'(\zeta) - 6\rho\tau^2u'(\zeta)u''(\zeta) - \rho\tau^3u^{(4)}(\zeta) = 0, \tag{33}$$

by integrating and skipping integration constant, then get the solutions

$$(-\mu\rho + 3\sigma^2 - 3\tau^2)u'(\zeta) + 3\sigma u(\zeta) + \frac{1}{2}\rho\tau^2u'(\zeta)^2 - \rho\tau^3u^{(3)}(\zeta) = 0. \tag{34}$$

Assume the solution to equation (34) provided by

$$u(\zeta) = A_0 + \sum_{i=1}^{n}(A_i\theta^i(\zeta) + B_i\theta^{-i}(\zeta)), \tag{35}$$

here $A_i, B_i (0 \leqslant i \leqslant n)$ are parameters, and while $\theta(\zeta)$ satifies

$$\theta'(\zeta) = \theta^2(\zeta) + \Omega, \tag{36}$$

Group 1: When $\Omega < 0$,

$$\theta(\zeta) = \frac{\sqrt{\Omega(-(A^2 + B^2))} - A\sqrt{-\Omega}\cosh(2\sqrt{-\Omega}(\zeta + K_0))}{A\sinh(2\sqrt{-\Omega}(\zeta + K_0)) + B}, \tag{37}$$

$$\theta(\zeta) = \frac{-\sqrt{\Omega(-(A^2 + B^2))} - A\sqrt{-\Omega}\cosh(2\sqrt{-\Omega}(\zeta + K_0))}{A\sinh(2\sqrt{-\Omega}(\zeta + K_0)) + B}, \tag{38}$$

$$\theta(\zeta) = \frac{2A\sqrt{-\Omega}}{A + B + \cosh(2\sqrt{-\Omega}(\zeta + K_0) - \sinh(2\sqrt{-\Omega}(\zeta + K_0)))} + \sqrt{-\Omega}, \tag{39}$$

$$\theta(\zeta) = \frac{2A\sqrt{-\Omega}}{A + B + \cosh(2\sqrt{-\Omega}(\zeta + K_0) - \sinh(2\sqrt{-\Omega}(\zeta + K_0)))} - \sqrt{-\Omega}, \tag{40}$$

where K_0 is an integration constant.

Group 2: When $\Omega > 0$,

$$\theta(\zeta) = \frac{\sqrt{\Omega(A^2 - B^2)} - A\sqrt{\Omega}\cos(2\sqrt{\Omega}(\zeta + K_0))}{A\sin(2\sqrt{\Omega}(\zeta + K_0)) + B}, \tag{41}$$

$$\theta(\zeta) = \frac{-\sqrt{\Omega(A^2 - B^2)} - A\sqrt{\Omega}\cos(2\sqrt{\Omega}(\zeta + K_0))}{A\sin(2\sqrt{\Omega}(\zeta + K_0)) + B}, \tag{42}$$

$$\theta(\zeta) = -\frac{2\mathrm{Ai}\sqrt{\Omega}}{A - \mathrm{iSin}(2\sqrt{\Omega}(\zeta + K_0)) + \cos(2\sqrt{\Omega}(\zeta + K_0))} + i\sqrt{\Omega}, \tag{43}$$

$$\theta(\zeta) = -\frac{2\mathrm{Ai}\sqrt{\Omega}}{A - \mathrm{iSin}(2\sqrt{\Omega}(\zeta + K_0)) + \cos(2\sqrt{\Omega}(\zeta + K_0))} - i\sqrt{\Omega}. \tag{44}$$

Group 3: When $\Omega = 0$,

$$\theta(\zeta) = -\frac{1}{\zeta + K_0}, \tag{45}$$

by comparing $u^{(3)}(\zeta)$ and $u'(\zeta)^2$ in (34), then find $n = 1$. So, equation (35) can be explained using the following

$$u(\zeta) = A_0 + A_1\theta(\zeta) + \frac{B_1}{\theta(\zeta)}, \tag{46}$$

by using equation (46) including equation (36) to equation (34) and now balancing $\theta(\zeta)$ to zero, here attain a collection of results with various distinct parameters, They have a system of algebraic equations.

$$\begin{aligned}
&- A_1\mu\rho\Omega - 2A_1\rho\tau^3\Omega^2 + \frac{1}{2}A_1^2\rho\tau^2\Omega^2 + 3A_1\sigma^2\Omega + 3A_0\sigma - 3A_1\tau^2\Omega = 0,\\
&- A_1\mu\rho - 2A_1\rho\tau^3\Omega + A_1^2\rho\tau^2\Omega + 3A_1\sigma^2 - 3A_1\tau^2 = 0,\\
&- A_1B_1\rho\tau^2\Omega^2 + B_1\mu\rho\Omega + 2B_1\rho\tau^3\Omega^2 - 3B_1\sigma^2\Omega + 3B_1\tau^2\Omega = 0,\\
&- 2A_1B_1\rho\tau^2\Omega - A_1B_1\rho\tau^2 + B_1\mu\rho + 2B_1\rho\tau^3\Omega - 3B_1\sigma^2 + 3B_1\tau^2 = 0,\\
&6B_1\rho\tau^3\Omega^3 + 6B_1\rho\tau^3\Omega^2 + \frac{1}{2}B_1^2\rho\tau^2\Omega^2 + B_1^2\rho\tau^2\Omega + \frac{1}{2}B_1^2\rho\tau^2 = 0.
\end{aligned}$$

The following outcomes from resolving this solution

$$A_1 = -\frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}}{\sqrt{\rho}\tau}, \quad B_1 = 0,$$

$$\mu = \frac{\sqrt{6}\sqrt{A_0}\sqrt{\rho}\sqrt{\sigma}\tau + 2\rho\tau^3 + 3\sigma^2 - 3\tau^2}{\rho}.$$

The following solutions for the given model are derived using these parameters.

Group 1: When $\Omega < 0$,

$$\omega_{17}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}(\sqrt{\Omega(-A^2 - B^2)} - A\sqrt{-\Omega}\cosh(2\sqrt{-\Omega}(\zeta\mu + K_0)))}{\sqrt{\rho}\tau(A\sinh(2\sqrt{-\Omega}(\zeta\mu + K_0)) + B)}, \tag{47}$$

$$\omega_{18}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}(-\sqrt{\Omega(-A^2 - B^2)} - A\sqrt{-\Omega}\cosh(2\sqrt{-\Omega}(\zeta\mu + K_0)))}{\sqrt{\rho}\tau(A\sinh(2\sqrt{-\Omega}(\zeta\mu + K_0)) + B)}, \tag{48}$$

$$\omega_{19}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}\left(\frac{2A\sqrt{-\Omega}}{A + B + \cosh(\sinh(2\sqrt{-\Omega}(\zeta\mu + K_0)) - 2\sqrt{-\Omega}(\zeta\mu + K_0))} + \sqrt{-\Omega}\right)}{\sqrt{\rho}\tau}, \tag{49}$$

$$\omega_{20}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}\left(\frac{2A\sqrt{-\Omega}}{A + B + \cosh(\sinh(2\sqrt{-\Omega}(\zeta\mu + K_0)) - 2\sqrt{-\Omega}(\zeta\mu + K_0))} - \sqrt{-\Omega}\right)}{\sqrt{\rho}\tau}, \tag{50}$$

where

$$\mu = \frac{\sqrt{6}\sqrt{A_0}\sqrt{\rho}\sqrt{\sigma}\tau + 2\rho\tau^3 + 3\sigma^2 - 3\tau^2}{\rho}, \quad \zeta = -\frac{\mu t^\beta}{\beta} + \tau x + \rho y + \frac{\sigma z^\alpha}{\alpha}.$$

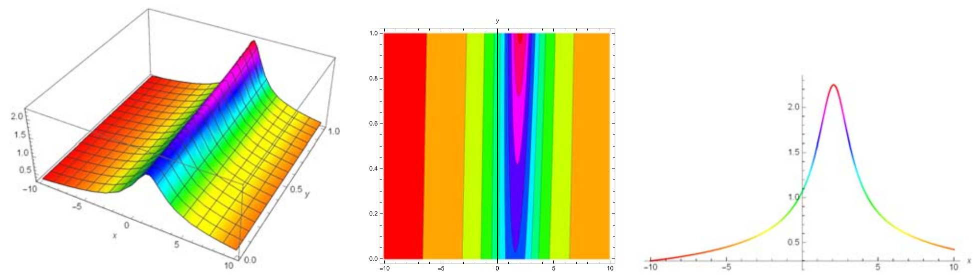
Group 2: When $\Omega > 0$,

$$\omega_{21}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}(\sqrt{\Omega(A^2 - B^2)} - A\sqrt{\Omega}\cos(2\sqrt{\Omega}(\zeta\mu + K_0)))}{\sqrt{\rho}\tau(A\sin(2\sqrt{\Omega}(\zeta\mu + K_0)) + B)}, \tag{51}$$

$$\omega_{22}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}(-\sqrt{\Omega(A^2 - B^2)} - A\sqrt{\Omega}\cos(2\sqrt{\Omega}(\zeta\mu + K_0)))}{\sqrt{\rho}\tau(A\sin(2\sqrt{\Omega}(\zeta\mu + K_0)) + B)}, \tag{52}$$

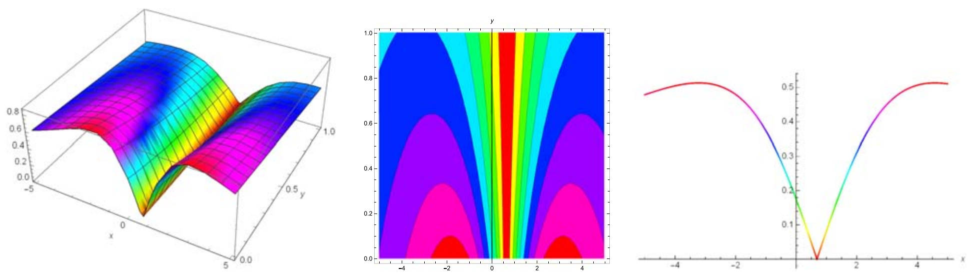
$$\omega_{23}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}\left(-\frac{2\mathrm{Ai}\sqrt{\Omega}}{A - \mathrm{iSin}(2\sqrt{\Omega}(\zeta\mu + K_0)) + \cos(2\sqrt{\Omega}(\zeta\mu + K_0))} + i\sqrt{\Omega}\right)}{\sqrt{\rho}\tau}, \tag{53}$$

$$\omega_{24}(x, y, z, t) = A_0 - \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}\left(-\frac{2\mathrm{Ai}\sqrt{\Omega}}{A - \mathrm{iSin}(2\sqrt{\Omega}(\zeta\mu + K_0)) + \cos(2\sqrt{\Omega}(\zeta\mu + K_0))} - i\sqrt{\Omega}\right)}{\sqrt{\rho}\tau}, \tag{54}$$



(a) (b) (c)

Figure 1. Approaches for the bell solution $|\omega_1(x, y, z, t)|$ with $a_3 = 1.22,\ a_4 = 2,\ t = 1,\ a_1 = 1.25,\ a_2 = 1.88,\ b_1 = 1.73,\ b_2 = 1.92,\ b_3 = 1.44,\ z = 1,\ \varpi = 2$.



(a) (b) (c)

Figure 2. Approaches for the bright solution $|\omega_2(x, y, z, t)|$ with $a_3 = 1.8,\ a_4 = 1,\ t = 1,\ a_1 = 0.6,\ a_2 = 1.6,\ b_1 = 1,\ b_2 = -1,\ b_3 = 1.3,\ z = 1,\ \varpi = 1$.

where

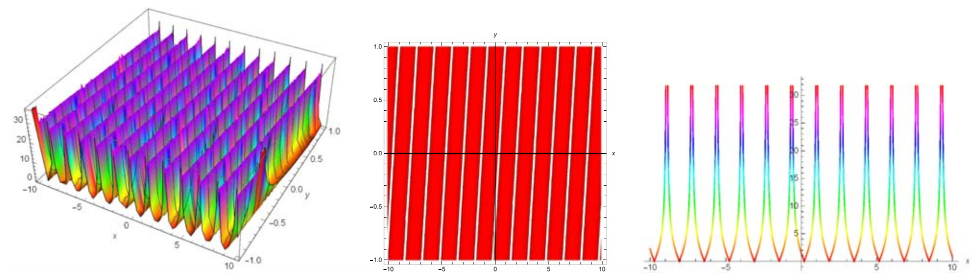
$$\mu = \frac{\sqrt{6}\sqrt{A_0}\sqrt{\rho}\sqrt{\sigma}\tau + 2\rho\tau^3 + 3\sigma^2 - 3\tau^2}{\rho}, \quad \zeta = -\frac{\mu t^{\beta}}{\beta} + \tau x + \rho y + \frac{\sigma z^{\alpha}}{\alpha}.$$

Group 3: When $\Omega = 0$,

$$\omega_{25}(x, y, z, t) = \frac{\sqrt{6}\sqrt{A_0}\sqrt{\sigma}}{\sqrt{\rho}\tau\left(-\frac{t^{\beta}(\sqrt{6}\sqrt{A_0}\sqrt{\rho}\sqrt{\sigma}\tau + 2\rho\tau^3 + 3\sigma^2 - 3\tau^2)}{\beta\rho} + K_0 + \tau x + \rho y + \frac{\sigma z^{\alpha}}{\alpha}\right)} + A_0. \tag{55}$$

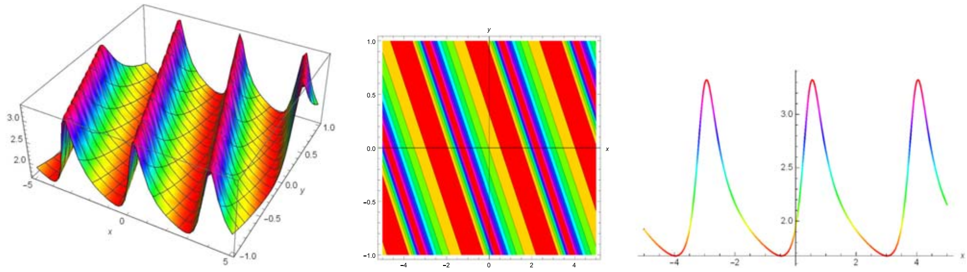
3. Discussion and results

In current section of the study, we present the different graphical behaviours of the (3+1) dimensional nonlinear evolution equation (1) using the advanced computational tool like Mathematica or Maple. Several distinct, bright, dark, periodic, and bell-shaped solitons appear for a certain range of values. Figure 1 tells graphs representations for $|\omega_1(x, y, z, t)|$ based on different parameter values of $a_3 = 1.22,\ a_4 = 2,\ t = 1,\ a_1 = 1.25,\ a_2 = 1.88,\ b_1 = 1.73,\ b_2 = 1.92,\ b_3 = 1.44,\ z = 1,\ \varpi = 2$ represent the bell shaped soliton solutions. Figure 2 tells graphs representations for $|\omega_2(x, y, z, t)|$ based on different parameter values of $a_3 = 1.8,\ a_4 = 1,\ t = 1,\ a_1 = 0.6,\ a_2 = 1.6,\ b_1 = 1,\ b_2 = -1,\ b_3 = 1.3,\ z = 1,\ \varpi = 1$ represent the bright shaped soliton solutions. Likewise, figure 3 displays the periodic soliton solutions utilizing different values of constant $|\omega_4(x, y, z, t)|$ with $c_0 = 2,\ c_1 = -1.9,\ c_2 = 1.1,\ c_3 = 0.7,\ z = 0.2,\ t = 1$. Also if we talk for the figure 4 express the periodic soliton solution for the constant values $b_0 = 1.7, b_1 = -0.99, b_3 = 0.1, a_0 = 1.22, a_1 = 0.43, a_2 = 1.2, a_3 = 0.11, t = 1, z = 1, \lambda_0 = -1.6, \lambda_1 = -0.88$. Figure 5 displays the singular bell shaped soliton solutions utilizing different values of constant $|\omega_{10}(x, y, z, t)|$ with $a_0 = 0.2, a_1 = -1.3, a_4 = 1, b_0 = 3, b_4 = 1, \lambda_2 = -2, \lambda_1 = 1, \lambda_0 = 2.1, z = 1, t = 1$. Likewise, figures 6, 7 for given parameter values represents the periodic solution $a_1 = 1.2, a_0 = 1.3, a_2 = 1.4, a_3 = -1.2, \lambda_1 = 1.5, \lambda_0 = -1.2, t = 1, z = 1.02$ and $\rho = 1.9,\ A = 1.3,\ B = 1.12,\ \Omega = 1.7,\ \sigma = 0.44,\ \tau = 1.3,\ z = 1,\ t = 1,\ \alpha = 0.2,\ \beta = 0.2,\ K_0 = 1,\ A_0 = 1, y = 1$. Similarly, figure 8 displays a periodic soliton solution for the constant values is presented $\rho = 1.9,\ A = 1.3,\ B = 1.12,\ \Omega = 1.7,\ \sigma = 0.44,\ \tau = 1.3,$



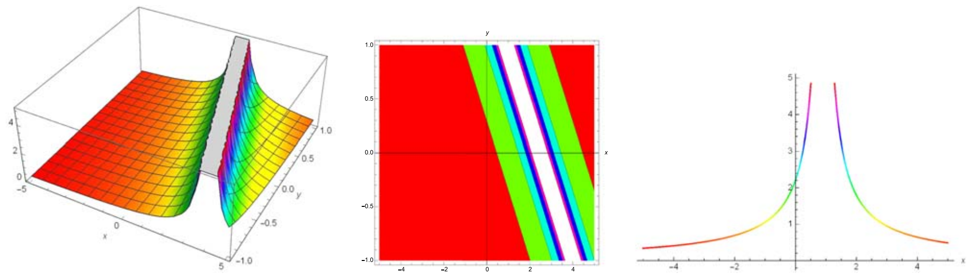
(a) (b) (c)

Figure 3. Approaches for the multi periodic solution $|\omega_4(x, y, z, t)|$ with $c_0 = 2,\ c_1 = -1.9,\ c_2 = 1.1,\ c_3 = 0.7,\ z = 0.2,\ t = 1.$



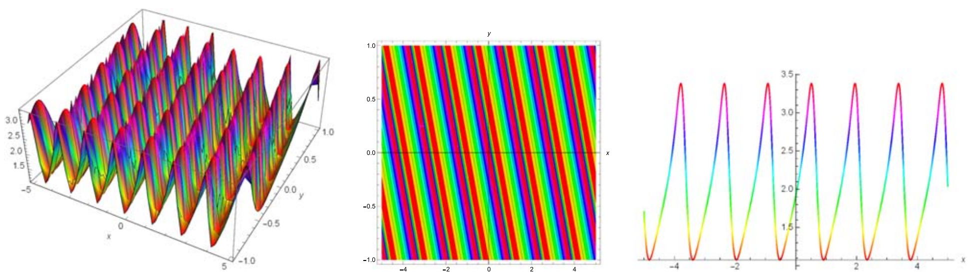
(a) (b) (c)

Figure 4. Approaches for the periodic solution $|\omega_6(x, y, z, t)|$ with $b_0 = 1.7, b_1 = -0.99, b_3 = 0.1, a_0 = 1.22, a_1 = 0.43, a_2 = 1.2, a_3 = 0.11, t = 1, z = 1, \lambda_0 = -1.6, \lambda_1 = -0.88.$



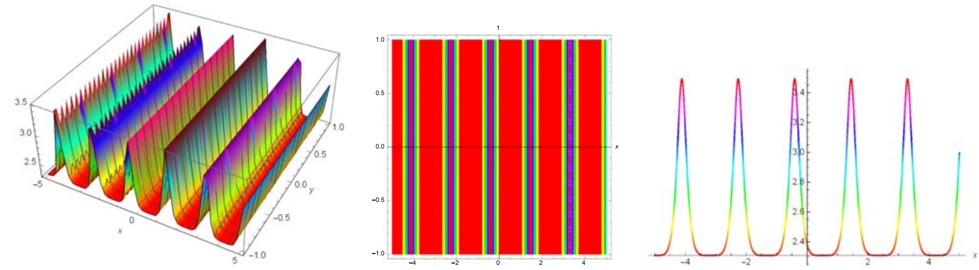
(a) (b) (c)

Figure 5. Approaches for the singular bell shaped solution $|\omega_{10}(x, y, z, t)|$ with $a_0 = 0.2, a_1 = -1.3, a_4 = 1, b_0 = 3, b_4 = 1, \lambda_2 = -2, \lambda_1 = 1, \lambda_0 = 2.1, z = 1, t = 1.$



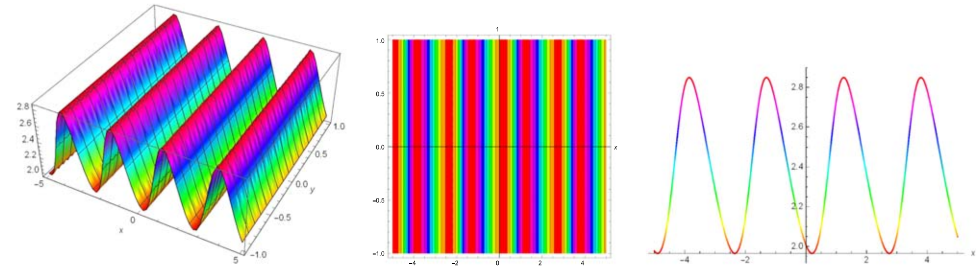
(a) (b) (c)

Figure 6. Approaches for the periodic solution $|\omega_8(x, y, z, t)|$ with a_1=1.2, $a_0 = 1.3, a_2 = 1.4, a_3 = -1.2, \lambda_1 = 1.5, \lambda_0 = -1.2, t = 1, z = 1.02.$



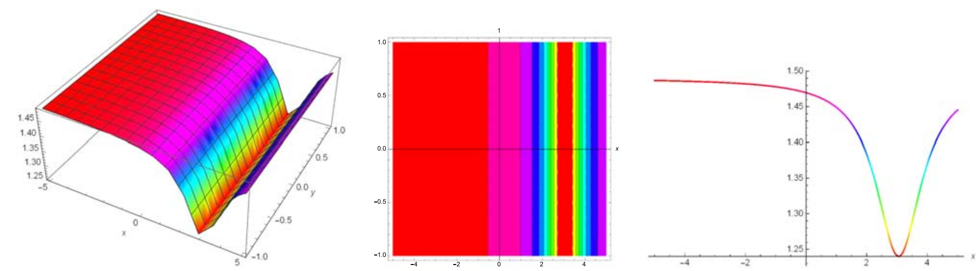
(a) (b) (c)

Figure 7. Approaches for the periodic solution $|\omega_{18}(x, y, z, t)|$ with $\rho = 1.9,\ A = 1.3,\ B = 1.12,\ \Omega = 1.7,\ \sigma = 0.44,\ \tau = 1.3,\ z = 1,$ $t = 1,\ \alpha = 0.2,\ \beta = 0.2,\ K_0 = 1,\ A_0 = 1, y = 1.$



(a) (b) (c)

Figure 8. Approaches for the periodic soliton solution $|\omega_{16}(x, y, z, t)|$ with $\rho = 1.2,\ A = 0.21,\ B = 0.6,\ \Omega = 0.9,\ \sigma = 0.3,\ \tau = 1.3,$ $z = 1,\ t = 1,\ \alpha = 0.15,\ \beta = 0.7,\ K_0 = 1,\ A_0 = 2,\ y = 1.$



(a) (b) (c)

Figure 9. Approaches for the dark type soliton solution $|\omega_{16}(x, y, z, t)|$ with $\rho = 1.98,\ A = 1.25,\ B = 1.51,\ \Omega = 1.68,\ \tau = 2,$ $\sigma = -0.2,\ z = 1,\ t = 1,\ \alpha = 2.11,\ \beta = 1,\ K_0 = 2,\ A_0 = 1.49,\ y = 1.$

$z = 1,\ t = 1,\ \alpha = 0.2,\ \beta = 0.2,\ K_0 = 1,\ A_0 = 1, y = 1$. At the end, figure 9 represents a dark type soliton solution for the constant values is presented $\rho = 1.98,\ A = 1.25,\ B = 1.51,\ \Omega = 1.68,\ \tau = 2,\ \sigma = -0.2,\ z = 1,\ t = 1,$ $\alpha = 2.11,\ \beta = 1,\ K_0 = 2,\ A_0 = 1.49,\ y = 1.$

The Two and three dimensional graphs of the computed results using various variable selections present more in-depth understanding of the dynamical wave structures. We have observed some periodic wave solutions representing a movement can be oscillatory or periodic, although oscillatory motion is limited to oscillating around an equilibrium point or between two states. Periodic motion is applicable to any movement that repeats over time. Another form of the developed wave structures are the singular wave forms representing the nature of the solution as the blow-up time draws near is extremely fascinating to imagine through different wave shapes. When the solution becomes unbounded in finite time, singularity takes on a simple form. Though its slope becomes infinite in finite time, we can claim that the wave has broken when the solution is still bounded. A point where the slope is vertical and the wave is said to have broken is eventually generated by the graph, which initially grows somewhat steeper as it propagates.

4. Conclusion

In this paper, the (3+1)-dimensional nonlinear evolution model has been studied by applying the Hirota bilinear approach and the unified method. Based on the considered methodologies, we retrieved various fascinating forms of wave structures. The dynamic characteristics for the governing model are extensively visulized in the form of 3D, 2D and contour plots by using Mathematica 13. The observed solutions demonstared fascinating behaviours including the dark, bright, singular, periodic, bell-shaped, solutions and optical soliton solutions, for details see figures (1–9). This study is more valuable since the findings are more significant and reliable in describing various physical phenomena. These solutions demonstrate the physical behaviour of a variety of natural phenomena, including wave motion, fluid dynamics, and optical fiber properties. The applied strategies are thus shown to be desirable and helpful to deal with a number of other higher dimensional nonlinear evolution models that exist in hydrodynamic, plasma, mathematics, and other disciplines of engineering and science.

Acknowledgments

The authors would like to extend their sincere appreciation to the Researchers Supporting Project number (RSPD2024R802), King Saud University, Riyadh, Saudi Arabia.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study. The data that support the findings of this study are available upon reasonable request from the authors.

Declarations

Ethics approval

Not applicable.

Competing interests

The authors declare no conflict of interest.

Authors' contributions

1. Reem K. Alhefthi: Software, scientific computation, review and editing.
2. Kalim U. Tariq: Methodology, conceptualization, resource, validation.
3. Wen-Xiu Ma: Supervision, project administration.
4. Fozia Mehboob: Formal analysis and investigation, writing original draft, visualization.

Consent for publication

All the authors have agreed and given their consent for the publication of this research paper.

Funding

Not available.

ORCID iDs

Kalim U Tariq https://orcid.org/0000-0003-0300-6515

References

[1] Anderson D, Lisak M and Berntson A 2001 A variational approach to nonlinear evolution equations in optics *Pramana* **57** 917–36

[2] Hu L, Gao Y-T, Jia S-L, Su J-J and Deng G-F 2019 Solitons for the (2+1)-dimensional boiti–leon–manna-pempinelli equation for an irrotational incompressible fluid via the pfaffian technique *Mod. Phys. Lett.*B **33** 1950376

[3] He Z-M, Wen L, Wang Y-J, Chen G, Tan R-B, Dai C-Q and Zhang X-F 2019 Dynamics and pattern formation of ring dark solitons in a two-dimensional binary bose-einstein condensate with tunable interactions *Phys. Rev.* E **99** 062216

[4] Carillo S and Ragnisco O 2012 *Nonlinear Evolution Equations and Dynamical Systems* (Springer Science & Business Media)

[5] Abdikian A 2018 Cylindrical fast magnetosonic solitary waves in quantum degenerate electron-positron-ion plasma *Phys. Plasmas* **25** 022308

[6] Kumar D, Hosseini K and Samadani F 2017 The sine-gordon expansion method to look for the traveling wave solutions of the tzitzéica type equations in nonlinear optics *Optik* **149** 439–46

[7] Podlubny I 1998 *Fractional Differential Equations: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of their Solution and Some of their Applications* (Elsevier)

[8] Goufo E F D 2016 Application of the Caputo-Fabrizio fractional derivative without singular kernel to Korteweg-de Vries-Burgers equation *Mathematical Modelling and Analysis* **21** 188–98

[9] Benson D A, Wheatcraft S W and Meerschaert M M 2000 Application of a fractional advection-dispersion equation *Water Resour. Res.* **36** 1403–12

[10] Shermergor T 1966 On the use of fractional differentiation operators for the description of elastic-after effect properties of materials *J. Appl. Mech. Tech. Phys.* **7** 85–7

[11] Kilbas A 2006 *Theory and Applications of Fractional Differential Equations* **204** 1–523

[12] Zheng X, Zhang Z and Wang H 2020 Analysis of a nonlinear variable-order fractional stochastic differential equation *Appl. Math. Lett.* **107** 106461

[13] Benjamin T B, Bona J L and Mahony J J 1972 Model equations for long waves in nonlinear dispersive systems, Philosophical Transactions of the Royal Society of London *Series A, Mathematical and Physical Sciences* **272** 47–78

[14] Oustaloup A, Levron F, Mathieu B and Nanot F M 2000 Frequency-band complex noninteger differentiator: characterization and synthesis *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* **47** 25–39

[15] Dai C-Q and Wang Y-Y 2016 Spatiotemporal localizations in (3+1)-dimensional pt-symmetric and strongly nonlocal nonlinear media *Nonlinear Dyn.* **83** 2453–9

[16] Sun Y-L, Ma W-X, Yu J-P and Khalique C M 2019 Dynamics of lump solitary wave of kadomtsev-petviashvili-boussinesq-like equation *Comput. Math. Appl.* **78** 840–7

[17] Ma W-X 2015 Lump solutions to the kadomtsev-petviashvili equation *Phys. Lett.* A **379** 1975–8

[18] Liu N and Liu Y 2016 New multi-soliton solutions of a (3+1)-dimensional nonlinear evolution equation *Comput. Math. Appl.* **71** 1645–54

[19] Ma W-X 2023 Lump waves in a spatial symmetric nonlinear dispersive wave model in (2+1)-dimensions *Mathematics* **11** 4664

[20] Chen S-J, Lü X, Li M-G and Wang F 2021 Derivation and simulation of the m-lump solutions to two (2+1)-dimensional nonlinear equations *Phys. Scr.* **96** 095201

[21] Zhaqilao R 2013 waves and rational solutions of a (3+1)-dimensional nonlinear evolution equation *Phys. Lett.* A **377** 3021–6

[22] Gao L-N, Zhao X-Y, Zi Y-Y, Yu J and Lü X 2016 Resonant behavior of multiple wave solutions to a hirota bilinear equation *Comput. Math. Appl.* **72** 1225–9

[23] Yin Y-H, Lü X and Ma W-X 2022 Bäcklund transformation, exact solutions and diverse interaction phenomena to a (3+1)-dimensional nonlinear evolution equation *Nonlinear Dyn.* **108** 4181–94

[24] Sulaiman T A, Yusuf A, Abdeljabbar A and Alquran M 2021 Dynamics of lump collision phenomena to the (3+1)-dimensional nonlinear evolution equation *J. Geom. Phys.* **169** 104347

[25] Fang T and Wang Y-H 2018 Interaction solutions for a dimensionally reduced hirota bilinear equation *Comput. Math. Appl.* **76** 1476–85

[26] Shahen N H M, Rahman M *et al* 2022 Dispersive solitary wave structures with mi analysis to the unidirectional dgh equation via the unified method *Partial Differential Equations in Applied Mathematics* **6** 100444

[27] Raza N, Rafiq M H, Kaplan M, Kumar S and Chu Y-M 2021 The unified method for abundant soliton solutions of local time fractional nonlinear evolution equations *Results in Physics* **22** 103979

[28] Ullah M S, Abdeljabbar A, Roshid H-O and Ali M Z 2022 Application of the unified method to solve the biswas-arshed model *Results in Physics* **42** 105946

[29] Ma W-X 2022 Soliton solutions by means of hirota bilinear forms *Partial Differential Equations in Applied Mathematics* **5** 100220

[30] Ghanbari B 2021 Employing hirota's bilinear form to find novel lump waves solutions to an important nonlinear model in fluid mechanics *Results in Physics* **29** 104689

[31] Osman M, Korkmaz A, Rezazadeh H, Mirzazadeh M, Eslami M and Zhou Q 2018 The unified method for conformable time fractional schrö dinger equation with perturbation terms *Chin. J. Phys.* **56** 2500–6