Some Properties of Small Connected Quandles

June 12, 2013

1 Subquandles

The following is a list of subquandles of RIG quandles.

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# SubQuandles[n,i] is the set isomorphism classes of
# subquandles of C[n,i]. Trivial[k] represents a trivial
# quandle of order k. UID[k] represents a non-trivial and
# non-connected quandle of order k. It is possible that
# some trivial quandles of order > 4 are not listed.
# [m,j] represents the Rig quandle C[m,j].


SubQuandles[3,1]:= {}:
SubQuandles[4,1]:= {}:
SubQuandles[5,1]:= {}:
SubQuandles[5,2]:= {}:
SubQuandles[5,3]:= {}:
SubQuandles[6,1]:= {Trivial[2], [3, 1]}:
SubQuandles[6,2]:= {Trivial[2]}:
SubQuandles[7,1]:= {}:
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SubQuandles[8,1]:= {Trivial[2]}:
SubQuandles[8,2]:= {}:
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SubQuandles[9,1]:= {[3, 1]}:
SubQuandles[9,2]:= {[3, 1]}:
SubQuandles[9,3]:= {}:
SubQuandles[9,4]:= {[3, 1]}:
SubQuandles[9,5]:= {[3, 1]}:
SubQuandles[9,6]:= {[3, 1]}:
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SubQuandles[9,7]:= {}:
SubQuandles[9,8]:= {}:
SubQuandles[10,1]:= {Trivial[2], UID[4], [3, 1], [6, 1]}:
SubQuandles[11,1]:= {}:
SubQuandles[11,2]:= {}:
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SubQuandles[11,9]:= {}:
SubQuandles[12,1]:= {Trivial[2], UID[4], UID[6], [3, 1]}:
SubQuandles[12,2]:= {Trivial[2], UID[4]}:
SubQuandles[12,3]:= {Trivial[2]}:
SubQuandles[12,4]:= {[3, 1], [4, 1]}:
SubQuandles[12,5]:= {Trivial[2], UID[4], [6, 2]}:
SubQuandles[12,6]:= {Trivial[2], UID[4], [3, 1], [6, 1]}:
SubQuandles[12,7]:= {Trivial[2], UID[4]}:
SubQuandles[12,8]:= {Trivial[2], Trivial[3], Trivial[4], [3, 1], [6, 1]}:
SubQuandles[12,9]:= {Trivial[2], Trivial[3], Trivial[4], [6, 2]}:
SubQuandles[12,10]:= {Trivial[2], Trivial[3], [4, 1]}:
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SubQuandles[13,9]:= {}:
SubQuandles[13,10]:= {}:
SubQuandles[13,11]:= {}:
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SubQuandles[15,3]:= {[3, 1], [5, 2]}:
SubQuandles[15,4]:= {[3, 1], [5, 3]}:
SubQuandles[15,5]:= {[5, 1]}:
SubQuandles[15,6]:= {[3, 1], [5, 1]}:
SubQuandles[15,7]:= {Trivial[2], Trivial[3], UID[4], UID[6], UID[7], [3, 1], [6, 1], [10, 1]}:
SubQuandles[16,1]:= {[4, 1]}:
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SubQuandles[16,3]:= {}:
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SubQuandles[18,5]:= {Trivial[2], UID[6], [3, 1]}:
SubQuandles[18,6]:= {Trivial[2], UID[6], [3, 1]}:
SubQuandles[18,7]:= {Trivial[2], UID[6], [3, 1]}:
SubQuandles[18,8]:= {Trivial[2], UID[6], [3, 1], [9, 6]}:
SubQuandles[18,9]:= {Trivial[2], UID[6], [3, 1], [9, 5]}:
SubQuandles[18,10]:= {Trivial[2], UID[6], [3, 1], [9, 4]}:
SubQuandles[18,11]:= {Trivial[2], UID[4], UID[6], [3, 1], [9, 6]}:
SubQuandles[18,12]:= {Trivial[2], UID[4], UID[6], [3, 1], [6, 1], [9, 2]}:
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SubQuandles[19,8]:= {}:
SubQuandles[19,9]:= {}:
SubQuandles[19,10]:= {}:
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SubQuandles[19,15]:= {}:
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SubQuandles[19,17]:= {}:
SubQuandles[20,1]:= {Trivial[2], UID[8], [4, 1]}:
SubQuandles[20,2]:= {Trivial[2]}:
SubQuandles[20,3]:= {Trivial[2], UID[4], UID[6], UID[8], [3, 1], [12, 1]}:
SubQuandles[20,4]:= {[4, 1], [5, 1]}:
SubQuandles[20,5]:= {Trivial[2], Trivial[3], Trivial[4], [5, 1]}:
SubQuandles[20,6]:= {Trivial[2], Trivial[3], Trivial[4]}:
SubQuandles[20,7]:= {[4, 1], [5, 3]}:
SubQuandles[20,8]:= {[4, 1], [5, 2]}:
SubQuandles[20,9]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[7], UID[8], [3, 1
SubQuandles[20,10]:= {Trivial[2], Trivial[3], Trivial[4], UID[7], UID[8], [6, 2], [12, 9]}:
SubQuandles[21,1]:= {[3, 1], [7, 1]}:
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SubQuandles[21,4]:= {[3, 1], [7, 2]}:
SubQuandles[21,5]:= {[3, 1], [7, 3]}:
SubQuandles[21,6]:= {Trivial[2], Trivial[3], UID[4], UID[5], UID[9], [3, 1], [6, 1]}:
SubQuandles[21,7]:= {[7, 1]}:
SubQuandles[21,8]:= {[3, 1], [7, 1]}:
SubQuandles[21,9]:= {Trivial[2], Trivial[3], UID[4], UID[5], UID[6], UID[7], UID[9], UID[11],
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SubQuandles[23,9]:= {}:
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SubQuandles[23,17]:= {}:
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SubQuandles[23,18]:= {}:
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SubQuandles[23,20]:= {}:
SubQuandles[23,21]:= {}:
SubQuandles[24,1]:= {Trivial[2], UID[4], UID[6], UID[8], UID[12], [3, 1]}:
SubQuandles[24,2]:= {Trivial[2], Trivial[3], Trivial[4], UID[8]}:
SubQuandles[24,3]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], UID[12], [3,
SubQuandles[24,4]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], UID[12], [6, 2
SubQuandles[24,5]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], UID[12], [3,
SubQuandles[24,6]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], UID[12], [6, 2
SubQuandles[24,7]:= {Trivial[2], Trivial[3], Trivial[4]}:
SubQuandles[24,8]:= {Trivial[2], UID[6], [3, 1], [8, 1]}:
SubQuandles[24,9]:= {Trivial[2], Trivial[3]}:
SubQuandles[24,10]:= {Trivial[2], UID[4], UID[8], UID[12], [3, 1], [6, 1], [6, 2]}:
SubQuandles[24,11]:= {Trivial[2], UID[4], UID[8], UID[12], [3, 1], [6, 1], [6, 2]}:
SubQuandles[24,12]:= {Trivial[2], UID[4], UID[8]}:
SubQuandles[24,13]:= {Trivial[2], UID[4], UID[8]}:
SubQuandles[24,14]:= {Trivial[2], UID[4], UID[8]}:
SubQuandles[24,15]:= {Trivial[2], UID[4], UID[8]}:
SubQuandles[24,16]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[8], UID[
SubQuandles[24,17]:= {Trivial[2], UID[4], UID[6], UID[8], [3, 1], [12, 1]}:
SubQuandles[24,18]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8]}:
SubQuandles[24,19]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], [12, 2]}:
SubQuandles[24,20]:= {Trivial[2], Trivial[3], Trivial[4], Trivial[5], [8, 1]}:
SubQuandles[24,21]:= {Trivial[2], Trivial[3], UID[6]}:
SubQuandles[24,22]:= {Trivial[2], UID[8], [3, 1], [4, 1], [6, 1], [12, 4]}:
SubQuandles[24,23]:= {Trivial[2], UID[8], [4, 1], [6, 2]}:
SubQuandles[24,24]:= {[3, 1], [8, 2]}:
SubQuandles[24,25]:= {[3, 1], [8, 3]}:
SubQuandles[24,26]:= {Trivial[2], UID[4], UID[8], [12, 7]}:
SubQuandles[24,27]:= {Trivial[2], UID[4], UID[8], [3, 1], [6, 1], [12, 6]}:
SubQuandles[24,28]:= {Trivial[2], UID[4], UID[8], [6, 2], [12, 5]}:
SubQuandles[24,29]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], [6, 2], [12,
SubQuandles[24,30]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], [3, 1], [6,
SubQuandles[24,31]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], [6, 2], [12,
SubQuandles[24,32]:= {Trivial[2], UID[4], UID[8]}:
SubQuandles[24,33]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[6], UID[8], [12, 7]}:
SubQuandles[24,34]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[8], [3,
SubQuandles[24,35]:= {Trivial[2], UID[4], UID[6], UID[8], [6, 2], [12, 5]}:
SubQuandles[24,36]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[8], [6, 2
SubQuandles[24,37]:= {Trivial[2], UID[4], UID[6], UID[8], [3, 1], [6, 1], [12, 6]}:
SubQuandles[24,38]:= {Trivial[2], Trivial[3], Trivial[4], Trivial[5], [6, 2], [12, 9]}:
SubQuandles[24,39]:= {Trivial[2], Trivial[3], Trivial[4], Trivial[5], [3, 1], [6, 1], [12, 8]}
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SubQuandles[24,40]:= {Trivial[2], UID[4], UID[6], UID[8], [12, 7]}:
SubQuandles[24,41]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[8], [12,
SubQuandles[24,42]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[8], [6, 1
SubQuandles[25,1]:= {[5, 1]}:
SubQuandles[25,2]:= {[5, 1]}:
SubQuandles[25,3]:= {}:
SubQuandles[25,4]:= {[5, 2]}:
SubQuandles[25,5]:= {[5, 3]}:
SubQuandles[25,6]:= {[5, 2]}:
SubQuandles[25,7]:= {[5, 3]}:
SubQuandles[25,8]:= {[5, 2], [5, 3]}:
SubQuandles[25,9]:= {[5, 1], [5, 3]}:
SubQuandles[25,10]:= {[5, 1], [5, 2]}:
SubQuandles[25,11]:= {}:
SubQuandles[25,12]:= {}:
SubQuandles[25,13]:= {}:
SubQuandles[25,14]:= {[5, 1]}:
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SubQuandles[25,19]:= {}:
SubQuandles[25,20]:= {}:
SubQuandles[25,21]:= {[5, 3]}:
SubQuandles[25,22]:= {[5, 2]}:
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SubQuandles[25,24]:= {[5, 2]}:
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SubQuandles[25,30]:= {[5, 3]}:
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SubQuandles[25,33]:= {}:
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SubQuandles[27,3]:= {[3, 1], [9, 1]}:
SubQuandles[27,4]:= {[3, 1], [9, 1], [9, 2]}:
SubQuandles[27,5]:= {[3, 1], [9, 3]}:
SubQuandles[27,6]:= {Trivial[2], Trivial[3]}:
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SubQuandles[27,7]:= {[3, 1], [9, 1], [9, 4], [9, 5], [9, 6]}:
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SubQuandles[27,17]:= {[3, 1], [9, 1], [9, 4], [9, 5], [9, 6]}:
SubQuandles[27,18]:= {[3, 1], [9, 1]}:
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SubQuandles[28,4]:= {[4, 1], [7, 2]}:
SubQuandles[28,5]:= {[4, 1]}:
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SubQuandles[28,7]:= {[4, 1], [7, 4]}:
SubQuandles[28,8]:= {[4, 1], [7, 5]}:
SubQuandles[28,9]:= {[4, 1], [7, 1]}:
SubQuandles[28,10]:= {Trivial[2], UID[4], [3, 1], [7, 1]}:
SubQuandles[28,11]:= {[4, 1], [7, 2]}:
SubQuandles[28,12]:= {[4, 1], [7, 3]}:
SubQuandles[28,13]:= {Trivial[2], Trivial[3], Trivial[4], UID[4], UID[5], UID[6], UID[7], UID[8
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SubQuandles[29,4]:= {}:
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SubQuandles[29,6]:= {}:
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SubQuandles[29,12]:= {}:
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```

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SubQuandles[29,19]:= {}:
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SubQuandles[29,24]:= {}:
SubQuandles[29,25]:= {}:
SubQuandles[29,26]:= {}:
SubQuandles[29,27]:= {}:
SubQuandles[30,1]:= {Trivial[2], UID[4], UID[6], UID[10], [3, 1], [5, 1]}:
SubQuandles[30,2]:= {Trivial[2], UID[10], [3, 1], [5, 1], [6, 1], [15, 1]}:
SubQuandles[30,3]:= {Trivial[2], UID[10], [5, 2], [5, 3], [6, 2]}:
SubQuandles[30,4]:= {Trivial[2], Trivial[3], UID[3], UID[4], UID[5], UID[6], UID[9], UID[12], U
SubQuandles[30,5]:= {Trivial[2], UID[3], UID[4], UID[6], UID[10], [3, 1], [5, 1]}:
SubQuandles[30,6]:= {Trivial[2], UID[10], [5, 1], [6, 2]}:
SubQuandles[30,7]:= {Trivial[2], UID[10], [3, 1], [5, 2], [6, 1], [15, 3]}:
SubQuandles[30,8]:= {Trivial[2], UID[10], [3, 1], [5, 3], [6, 1], [15, 4]}:
SubQuandles[30,9]:= {Trivial[2], UID[10], [5, 2], [6, 2]}:
SubQuandles[30,10]:= {Trivial[2], UID[10], [5, 3], [6, 2]}:
SubQuandles[30,11]:= {Trivial[2], UID[4], UID[6], UID[12], [3, 1], [6, 1], [9, 2], [10, 1], [18
SubQuandles[30,12]:= {Trivial[2], UID[10], [5, 1], [15, 5]}:
SubQuandles[30,13]:= {Trivial[2], UID[10], [3, 1], [5, 1], [6, 1], [15, 6]}:
SubQuandles[30,14]:= {Trivial[2], UID[10], [5, 1]}:
SubQuandles[30,15]:= {Trivial[2], UID[10], [5, 1], [6, 2]}:
SubQuandles[30,16]:= {Trivial[2], UID[4], UID[6], UID[8], UID[12], UID[14], [3, 1], [12, 1], [1
SubQuandles[30,17]:= {Trivial[2], UID[6], UID[10], [5, 3], [6, 2]}:
SubQuandles[30,18]:= {Trivial[2], UID[6], UID[10], [5, 2], [6, 2]}:
SubQuandles[30,19]:= {Trivial[2], UID[6], UID[10], [5, 1], [15, 5]}:
SubQuandles[30,20]:= {Trivial[2], UID[6], UID[10], [3, 1], [5, 1], [6, 1], [15, 6]}:
SubQuandles[30,21]:= {Trivial[2], UID[4], UID[6], UID[10], UID[12], [3, 1], [9, 6], [18, 11]}:
SubQuandles[30,22]:= {Trivial[2], UID[4], UID[6], UID[10], UID[12], [3, 1], [6, 1], [9, 2], [10
SubQuandles[30,23]:= {Trivial[2], Trivial[3], Trivial[4], Trivial[5], UID[7], UID[8], UID[12],
SubQuandles[30,24]:= {Trivial[2], Trivial[3], Trivial[4], Trivial[5], UID[4], UID[5], UID[6], U
SubQuandles[31,1]:= {}:
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SubQuandles[31,4]:= {}:
SubQuandles[31,5]:= {}:
SubQuandles[31,6]:= {}:
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SubQuandles[31,28]:= {}:
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SubQuandles[32,2]:= {Trivial[2], UID[8], [4, 1], [8, 1]}:
SubQuandles[32,3]:= {Trivial[2], UID[8], [4, 1], [8, 1]}:
SubQuandles[32,4]:= {Trivial[2]}:
SubQuandles[32,5]:= {Trivial[2], UID[8], [4, 1]}:
SubQuandles[32,6]:= {Trivial[2], UID[8], [4, 1]}:
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SubQuandles[32,8]:= {Trivial[2], UID[8], [4, 1]}:
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SubQuandles[32,15]:= {}:
SubQuandles[32,16]:= {[4, 1], [8, 3]}:
SubQuandles[32,17]:= {[4, 1], [8, 2]}:
SubQuandles[33,1]:= {[3, 1], [11, 1]}:
SubQuandles[33,2]:= {[3, 1], [11, 7]}:
SubQuandles[33,3]:= {[3, 1], [11, 8]}:
```

```
SubQuandles[33,4]:= {[3, 1], [11, 9]}:
SubQuandles[33,5]:= {[3, 1], [11, 6]}:
SubQuandles[33,6]:= {[3, 1], [11, 5]}:
SubQuandles[33,7]:= {[3, 1], [11, 4]}:
SubQuandles[33,8]:= {[3, 1], [11, 3]}:
SubQuandles[33,9]:= {[3, 1], [11, 2]}:
SubQuandles[33,10]:= {[3, 1], [11, 1]}:
SubQuandles[33,11]:= {[11, 1]}:
SubQuandles[35,1]:= {[5, 1], [7, 1]}:
SubQuandles[35,2]:= {[5, 2], [7, 1]}:
SubQuandles[35,3]:= {[5, 3], [7, 1]}:
SubQuandles[35,4]:= {[5, 1], [7, 5]}:
SubQuandles[35,5]:= {[5, 1], [7, 4]}:
SubQuandles[35,6]:= {[5, 1], [7, 3]}:
SubQuandles[35,7]:= {[5, 1], [7, 2]}:
SubQuandles[35,8]:= {[5, 2], [7, 4]}:
SubQuandles[35,9]:= {[5, 2], [7, 5]}:
SubQuandles[35,10]:= {[5, 3], [7, 4]}:
SubQuandles[35,11]:= {[5, 3], [7, 5]}:
SubQuandles[35,12]:= {[5, 3], [7, 3]}:
SubQuandles[35,13]:= {[5, 3], [7, 2]}:
SubQuandles[35,14]:= {[5, 2], [7, 3]}:
SubQuandles[35,15]:= {[5, 2], [7, 2]}:
```

2 Product quandles

The table below lists the product quandles among rig quandles.

```
Notation:
PROD[n,i] = 0 if C[n,i] is not a product,
else if C[n,i] = C[n1,i1] x C[n2,i2] x... then PROD[n,i] =[[n1,i1],[n2,i2],...].

PROD[9,2]:=[[3,1],[3,1]]:

PROD[12,4]:=[[3,1],[4,1]]:

PROD[15,1]:=[[3,1],[5,1]]:
PROD[15,3]:=[[3,1],[5,2]]:
PROD[15,4]:=[[3,1],[5,3]]:

PROD[16,2]:=[[4,1],[4,1]]:
```

```
PROD[18,1]:=[[3,1],[6,1]]:
PROD[18,4]:=[[3,1],[6,2]]:

PROD[20,4]:=[[4,1],[5,1]]:
PROD[20,8]:=[[4,1],[5,2]]:
PROD[20,7]:=[[4,1],[5,3]]:

PROD[21,1]:=[[3,1],[7,1]]:
PROD[21,4]:=[[3,1],[7,2]]:
PROD[21,5]:=[[3,1],[7,3]]:
PROD[21,2]:=[[3,1],[7,4]]:
PROD[21,3]:=[[3,1],[7,5]]:

PROD[24,8]:=[[3,1],[8,1]]:
PROD[24,24]:=[[3,1],[8,2]]:
PROD[24,25]:=[[3,1],[8,3]]:
PROD[24,22]:=[[4,1],[6,1]]:
PROD[24,23]:=[[4,1],[6,2]]:

PROD[25,2]:=[[5,1],[5,1]]:
PROD[25,10]:=[[5,1],[5,2]]:
PROD[25,9]:=[[5,1],[5,3]]:
PROD[25,10]:=[[5,2],[5,1]]:
PROD[25,6]:=[[5,2],[5,2]]:
PROD[25,8]:=[[5,2],[5,3]]:
PROD[25,9]:=[[5,3],[5,1]]:
PROD[25,8]:=[[5,3],[5,2]]:
PROD[25,7]:=[[5,3],[5,3]]:

PROD[27,4]:=[[3,1],[9,1]]:
PROD[27,5]:=[[3,1],[9,3]]:
PROD[27,21]:=[[3,1],[9,4]]:
PROD[27,20]:=[[3,1],[9,5]]:
PROD[27,23]:=[[3,1],[9,6]]:
PROD[27,30]:=[[3,1],[9,7]]:
PROD[27,29]:=[[3,1],[9,8]]:
PROD[27,2]:=[[3,1],[3,1],[3,1]]:

PROD[28,9]:=[[4,1],[7,1]]:
PROD[28,1]:=[[4,1],[7,2]]:
PROD[28,2]:=[[4,1],[7,3]]:
PROD[28,7]:=[[4,1],[7,4]]:
```

```
PROD[28,8]:=[[4,1],[7,5]]:

PROD[30,11]:=[[3,1],[10,1]]:
PROD[30,2]:=[[5,1],[6,1]]:
PROD[30,6]:=[[5,1],[6,2]]:
PROD[30,7]:=[[5,2],[6,1]]:
PROD[30,9]:=[[5,2],[6,2]]:
PROD[30,8]:=[[5,3],[6,1]]:
PROD[30,10]:=[[5,3],[6,2]]:

PROD[32,2]:=[[4,1],[8,1]]:
PROD[32,17]:=[[4,1],[8,2]]:
PROD[32,16]:=[[4,1],[8,3]]:

PROD[33,1] =  [[3, 1], [11, 1]].
PROD[33,2] =  [[3, 1], [11, 7]].
PROD[33,3] =  [[3, 1], [11, 8]].
PROD[33,4] =  [[3, 1], [11, 9]].
PROD[33,5] =  [[3, 1], [11, 6]].
PROD[33,6] =  [[3, 1], [11, 5]].
PROD[33,7] =  [[3, 1], [11, 4]].
PROD[33,8] =  [[3, 1], [11, 3]].
PROD[33,9] =  [[3, 1], [11, 2]].

PROD[35,1] =  [[5, 1], [7, 1]].
PROD[35,2] =  [[5, 2], [7, 1]].
PROD[35,3] =  [[5, 3], [7, 1]].
PROD[35,4] =  [[5, 1], [7, 5]].
PROD[35,5] =  [[5, 1], [7, 4]].
PROD[35,6] =  [[5, 1], [7, 3]].
PROD[35,7] =  [[5, 1], [7, 2]].
PROD[35,8] =  [[5, 2], [7, 4]].
PROD[35,9] =  [[5, 2], [7, 5]].
PROD[35,10] =  [[5, 3], [7, 4]].
PROD[35,11] =  [[5, 3], [7, 5]].
PROD[35,12] =  [[5, 3], [7, 3]].
PROD[35,13] =  [[5, 3], [7, 2]].
PROD[35,14] =  [[5, 2], [7, 3]].
PROD[35,15] =  [[5, 2], [7, 2]].
```

3 Dual quandles

Let $(Q, *)$ be a quandle. From definition, for any $y, z \in Q$, there is a unique $x \in Q$ such that $x * y = z$. Write $x = z \bar{*} y$. Then $(Q', \bar{*})$ is a quandle, called the dual of $(Q, *)$.

```
Notation:
DualQ[n,i] = [n,j] means that the dual of C[n,i] is C[n,j].

DualQ[3,1]:=[3,1]:
DualQ[4,1]:=[4,1]:
DualQ[5,1]:=[5,1]:
DualQ[5,2]:=[5,3]:
DualQ[5,3]:=[5,2]:
DualQ[6,1]:=[6,1]:
DualQ[6,2]:=[6,2]:
DualQ[7,1]:=[7,1]:
DualQ[7,2]:=[7,3]:
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DualQ[11,1]:=[11,1]:
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DualQ[11,8]:=[11,7]:
DualQ[12,1]:=[12,1]:
DualQ[12,2]:=[12,2]:
DualQ[12,3]:=[12,3]:
```

```
DualQ[12,4]:=[12,4]:
DualQ[12,5]:=[12,5]:
DualQ[12,6]:=[12,6]:
DualQ[12,7]:=[12,7]:
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DualQ[17,4]:=[17,6]:
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```
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```
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DualQ[27,62]:=[27,65]:
DualQ[27,65]:=[27,62]:
DualQ[27,63]:=[27,64]:
```

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DualQ[29,20]:=[29,17]:
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DualQ[29,22]:=[29,23]:
DualQ[29,23]:=[29,22]:
DualQ[29,27]:=[29,27]:
DualQ[30,1]:=[30,1]:
DualQ[30,2]:=[30,2]:
```

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DualQ[30,17]:=[30,18]:
DualQ[30,18]:=[30,17]:
DualQ[30,19]:=[30,19]:
DualQ[30,20]:=[30,20]:
DualQ[30,21]:=[30,21]:
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DualQ[31,13]:=[31,19]:
DualQ[31,19]:=[31,13]:
DualQ[31,14]:=[31,28]:
```

```
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DualQ[35,5]:=[35,4]:
DualQ[35,6]:=[35,7]:
DualQ[35,7]:=[35,6]:
```

```
DualQ[35,8]:=[35,11]:
DualQ[35,11]:=[35,8]:
DualQ[35,9]:=[35,10]:
DualQ[35,10]:=[35,9]:
DualQ[35,12]:=[35,15]:
DualQ[35,15]:=[35,12]:
DualQ[35,13]:=[35,14]:
DualQ[35,14]:=[35,13]:
```

A quandle is called *self-dual* if it is isomorphic to its dual. The following list gives some descriptions of the self-dual rig quandles.

```
C[3,1] = (Z_3)[t]/(t+1):

C[4,1] = (Z_2)[t]/(t^2+t+1):

C[5,1] = (Z_5)[t]/(t+1):

C[6,1] =  not Alexander,  Kei    , not Latin, faithful
C[6,2] =  not Alexander,  not Kei, not Latin, faithful

C[7,1] = (Z_7)[t]/(t+1):

C[8,1] =  not Alexander,  not Kei, not Latin, not faithful

C[9,1] = (Z_9)[t]/(t+1):
C[9,2] = (Z_3)[t]/(t+1) x (Z_3)[t]/(t+1) :
C[9,3] = (Z_3)[t]/(t^2+1):
C[9,6] = (Z_3)[t]/(t^2+1+2*t):

C[10,1] =  not Alexander,  Kei    , not Latin, faithful

C[11,1] = (Z_11)[t]/(t+1):

C[12,1] =  not Alexander,  Kei    , not Latin, not faithful
C[12,2] =  not Alexander,  not Kei, not Latin, not faithful
C[12,3] =  not Alexander,  not Kei, not Latin, faithful
C[12,4] = (Z_3)[t]/(t+1) x (Z_2)[t]/(t^2+t+1) :
C[12,5] =  not Alexander,  not Kei, not Latin, faithful
C[12,6] =  not Alexander,  Kei    , not Latin, faithful
C[12,7] =  not Alexander,  not Kei, not Latin, faithful
C[12,8] =  not Alexander,  Kei    , not Latin, faithful
C[12,9] =  not Alexander,  not Kei, not Latin, faithful
C[12,10] =  not Alexander,  not Kei, not Latin, faithful
```

```
C[13,11] = (Z_13)[t]/(t+1):

C[15,1] = (Z_15)[t]/(t+1):
C[15,2] =  not Alexander,  Kei    , not Latin, faithful
C[15,5] =  not Alexander,  not Kei, Latin    , faithful
C[15,6] =  not Alexander,  Kei    , Latin    , faithful
C[15,7] =  not Alexander,  Kei    , not Latin, faithful

C[16,1] = (Z_4)[t]/(t^2+t+1):
C[16,2] = (Z_2)[t]/(t^2+t+1) x (Z_2)[t]/(t^2+t+1) :
C[16,3] = (Z_2)[t]/(t^4+t^3+t^2+t+1):
C[16,4] = (Z_2)[t]/(t^4+t^2+1):
C[16,7] = (Z_4)[t]/(t^2+3*t+1):

C[17,15] = (Z_17)[t]/(t+1):

C[18,1] = C[3, 1] x C[6, 1] not Alexander,  Kei    , not Latin, faithful
C[18,2] =  not Alexander,  Kei    , not Latin, faithful
C[18,3] =  not Alexander,  not Kei, not Latin, faithful
C[18,4] = C[3, 1] x C[6, 2] not Alexander,  not Kei, not Latin, faithful
C[18,5] =  not Alexander,  not Kei, not Latin, faithful
C[18,8] =  not Alexander,  not Kei, not Latin, faithful
C[18,11] =  not Alexander,  not Kei, not Latin, faithful
C[18,12] =  not Alexander,  Kei    , not Latin, faithful

C[19,17] = (Z_19)[t]/(t+1):

C[20,1] =  not Alexander,  not Kei, not Latin, faithful
C[20,2] =  not Alexander,  not Kei, not Latin, faithful
C[20,3] =  not Alexander,  Kei    , not Latin, not faithful
C[20,4] = (Z_2)[t]/(t^2+t+1) x (Z_5)[t]/(t+1) :
C[20,5] =  not Alexander,  Kei    , not Latin, faithful
C[20,6] =  not Alexander,  not Kei, not Latin, faithful
C[20,9] =  not Alexander,  Kei    , not Latin, faithful
C[20,10] =  not Alexander,  not Kei, not Latin, faithful

C[21,1] = (Z_21)[t]/(t+1):
C[21,6] =  not Alexander,  Kei    , not Latin, faithful
C[21,7] =  not Alexander,  not Kei, Latin    , faithful
C[21,8] =  not Alexander,  Kei    , Latin    , faithful
C[21,9] =  not Alexander,  Kei    , not Latin, faithful
```

```
C[23,21] = (Z_23)[t]/(t+1):

C[24,1] =  not Alexander,  Kei    , not Latin, not faithful
C[24,2] =  not Alexander,  not Kei, not Latin, not faithful
C[24,3] =  not Alexander,  Kei    , not Latin, not faithful
C[24,4] =  not Alexander,  not Kei, not Latin, not faithful
C[24,5] =  not Alexander,  Kei    , not Latin, not faithful
C[24,6] =  not Alexander,  not Kei, not Latin, not faithful
C[24,7] =  not Alexander,  not Kei, not Latin, not faithful
C[24,8] = C[3, 1] x C[8, 1] not Alexander,  not Kei, not Latin, not faithful
C[24,9] =  not Alexander,  not Kei, not Latin, faithful
C[24,10] =  not Alexander,  not Kei, not Latin, faithful
C[24,11] =  not Alexander,  not Kei, not Latin, faithful
C[24,12] =  not Alexander,  not Kei, not Latin, faithful
C[24,13] =  not Alexander,  not Kei, not Latin, faithful
C[24,14] =  not Alexander,  not Kei, not Latin, not faithful
C[24,15] =  not Alexander,  not Kei, not Latin, not faithful
C[24,16] =  not Alexander,  Kei    , not Latin, not faithful
C[24,17] =  not Alexander,  Kei    , not Latin, not faithful
C[24,18] =  not Alexander,  not Kei, not Latin, not faithful
C[24,19] =  not Alexander,  not Kei, not Latin, not faithful
C[24,20] =  not Alexander,  not Kei, not Latin, not faithful
C[24,21] =  not Alexander,  not Kei, not Latin, faithful
C[24,22] = C[4, 1] x C[6, 1] not Alexander,  not Kei, not Latin, faithful
C[24,23] = C[4, 1] x C[6, 2] not Alexander,  not Kei, not Latin, faithful
C[24,26] =  not Alexander,  not Kei, not Latin, faithful
C[24,27] =  not Alexander,  Kei    , not Latin, faithful
C[24,28] =  not Alexander,  not Kei, not Latin, faithful
C[24,29] =  not Alexander,  not Kei, not Latin, faithful
C[24,30] =  not Alexander,  Kei    , not Latin, faithful
C[24,31] =  not Alexander,  not Kei, not Latin, faithful
C[24,32] =  not Alexander,  not Kei, not Latin, faithful
C[24,33] =  not Alexander,  not Kei, not Latin, faithful
C[24,34] =  not Alexander,  Kei    , not Latin, faithful
C[24,35] =  not Alexander,  not Kei, not Latin, faithful
C[24,36] =  not Alexander,  not Kei, not Latin, faithful
C[24,37] =  not Alexander,  Kei    , not Latin, faithful
C[24,38] =  not Alexander,  not Kei, not Latin, faithful
C[24,39] =  not Alexander,  Kei    , not Latin, faithful
C[24,40] =  not Alexander,  not Kei, not Latin, faithful
C[24,41] =  not Alexander,  not Kei, not Latin, faithful
C[24,42] =  not Alexander,  not Kei, not Latin, faithful
```

```
C[25,1] = (Z_25)[t]/(t+1):
C[25,2] = (Z_5)[t]/(t+1) x (Z_5)[t]/(t+1) :
C[25,3] = (Z_5)[t]/(t^2+t+1):
C[25,8] = (Z_5)[t]/(t^2+1):
C[25,11] = (Z_5)[t]/(t^2+4*t+1):
C[25,14] = (Z_5)[t]/(t^2+1+2*t):

C[27,1] =  not Alexander,  Kei    , not Latin, not faithful
C[27,2] = (Z_3)[t]/(t+1) x (Z_3)[t]/(t+1) x (Z_3)[t]/(t+1):
C[27,3] = (Z_27)[t]/(t+1):
C[27,4] = (Z_3)[t]/(t+1) x (Z_9)[t]/(t+1) :
C[27,5] = (Z_3)[t]/(t^3+t^2+t+1):
C[27,6] =  not Alexander,  not Kei, not Latin, not faithful
C[27,7] =  not Alexander,  not Kei, Latin    , faithful
C[27,8] =  not Alexander,  Kei    , Latin    , faithful
C[27,10] =  not Alexander,  Kei    , Latin    , faithful
C[27,12] =  not Alexander,  not Kei, Latin    , faithful
C[27,13] =  not Alexander,  Kei    , Latin    , faithful
C[27,14] =  not Alexander,  not Kei, not Latin, not faithful
C[27,15] =  not Alexander,  Kei    , Latin    , faithful
C[27,16] =  not Alexander,  not Kei, Latin    , faithful
C[27,17] =      Alexander,  not Kei, Latin    , faithful
C[27,22] = (Z_9)[t]/(t^2+1+2*t, 3+3*t) :
C[27,23] = (Z_3)[t]/(t+1) x (Z_3)[t]/(t^2+1+2*t) :
C[27,24] = (Z_3)[t]/(t^3+1):
C[27,25] = (Z_9)[t]/(1+8*t+t^2, 3+3*t) :
C[27,26] = (Z_9)[t]/(1+5*t+t^2, 3+3*t) :
C[27,55] =  not Alexander,  not Kei, Latin    , faithful

C[28,9] = (Z_2)[t]/(t^2+t+1) x (Z_7)[t]/(t+1) :
C[28,10] =  not Alexander,  Kei    , not Latin, faithful
C[28,13] =  not Alexander,  Kei    , not Latin, faithful

C[29,27] = (Z_29)[t]/(t+1):

C[30,1] =  not Alexander,  Kei    , not Latin, not faithful
C[30,2] = C[5, 1] x C[6, 1] not Alexander,  Kei    , not Latin, faithful
C[30,3] =  not Alexander,  not Kei, not Latin, faithful
C[30,4] =  not Alexander,  Kei    , not Latin, not faithful
C[30,5] =  not Alexander,  Kei    , not Latin, not faithful
C[30,6] = C[5, 1] x C[6, 2] not Alexander,  not Kei, not Latin, faithful
C[30,11] = C[3, 1] x C[10, 1] not Alexander,  Kei    , not Latin, faithful
C[30,12] =  not Alexander,  not Kei, not Latin, faithful
```

```
C[30,13] =  not Alexander,  Kei    , not Latin, faithful
C[30,14] =  not Alexander,  not Kei, not Latin, faithful
C[30,15] =  not Alexander,  not Kei, not Latin, faithful
C[30,16] =  not Alexander,  Kei    , not Latin, not faithful
C[30,19] =  not Alexander,  not Kei, not Latin, faithful
C[30,20] =  not Alexander,  Kei    , not Latin, faithful
C[30,21] =  not Alexander,  not Kei, not Latin, faithful
C[30,22] =  not Alexander,  Kei    , not Latin, faithful
C[30,23] =  not Alexander,  not Kei, not Latin, faithful
C[30,24] =  not Alexander,  Kei    , not Latin, faithful

C[31,29] = (Z_31)[t]/(t+1):

C[32,1] =  not Alexander,  not Kei, not Latin, not faithful
C[32,2] = C[4, 1] x C[8, 1] not Alexander,  not Kei, not Latin, not faithful
C[32,3] =  not Alexander,  not Kei, not Latin, not faithful
C[32,4] =  not Alexander,  not Kei, not Latin, not faithful
C[32,5] =  not Alexander,  not Kei, not Latin, not faithful
C[32,6] =  not Alexander,  not Kei, not Latin, not faithful
C[32,9] =  not Alexander,  not Kei, not Latin, not faithful

C[33,1] = (Z_33)[t]/(t+1):
C[33,10] =  not Alexander,  Kei    , Latin    , faithful
C[33,11] =  not Alexander,  not Kei, Latin    , faithful

C[35,1] = (Z_35)[t]/(t+1):
```

4 Epimorphisms

```
 Here [n,i]  means C[n,i] and
Epis[n,i] is the set of all epimorphic images of C[n,i].  So if [m,j] is not in
Epis[n,i] this means that C[m,j] is not an epimorphic image of C[n,i].


(  1) Epis[3,1]:={} (simple quandle)
(  2) Epis[4,1]:={} (simple quandle)
(  3) Epis[5,1]:={} (simple quandle)
(  4) Epis[5,2]:={} (simple quandle)
(  5) Epis[5,3]:={} (simple quandle)
(  6) Epis[6,1] = {[3, 1]}
(  7) Epis[6,2] = {[3, 1]}
(  8) Epis[7,1]:={} (simple quandle)
```

```
(  9) Epis[7,2]:={} (simple quandle)
( 10) Epis[7,3]:={} (simple quandle)
( 11) Epis[7,4]:={} (simple quandle)
( 12) Epis[7,5]:={} (simple quandle)
( 13) Epis[8,1] = {[4, 1]}
( 14) Epis[8,2]:={} (simple quandle)
( 15) Epis[8,3]:={} (simple quandle)
( 16) Epis[9,1] = {[3, 1]}
( 17) Epis[9,2] = {[3, 1]}
( 18) Epis[9,3]:={} (simple quandle)
( 19) Epis[9,4] = {[3, 1]}
( 20) Epis[9,5] = {[3, 1]}
( 21) Epis[9,6] = {[3, 1]}
( 22) Epis[9,7]:={} (simple quandle)
( 23) Epis[9,8]:={} (simple quandle)
( 24) Epis[10,1]:={} (simple quandle)
( 25) Epis[11,1]:={} (simple quandle)
( 26) Epis[11,2]:={} (simple quandle)
( 27) Epis[11,3]:={} (simple quandle)
( 28) Epis[11,4]:={} (simple quandle)
( 29) Epis[11,5]:={} (simple quandle)
( 30) Epis[11,6]:={} (simple quandle)
( 31) Epis[11,7]:={} (simple quandle)
( 32) Epis[11,8]:={} (simple quandle)
( 33) Epis[11,9]:={} (simple quandle)
( 34) Epis[12,1] = {[3, 1], [6, 1]}
( 35) Epis[12,2] = {[3, 1], [6, 2]}
( 36) Epis[12,3]:={} (simple quandle)
( 37) Epis[12,4] = {[3, 1], [4, 1]}
( 38) Epis[12,5] = {[3, 1], [6, 1]}
( 39) Epis[12,6] = {[3, 1], [6, 1]}
( 40) Epis[12,7] = {[3, 1], [6, 2]}
( 41) Epis[12,8] = {[3, 1], [6, 1]}
( 42) Epis[12,9] = {[3, 1], [6, 1], [6, 2]}
( 43) Epis[12,10] = {[4, 1]}
( 44) Epis[13,1]:={} (simple quandle)
( 45) Epis[13,2]:={} (simple quandle)
( 46) Epis[13,3]:={} (simple quandle)
( 47) Epis[13,4]:={} (simple quandle)
( 48) Epis[13,5]:={} (simple quandle)
( 49) Epis[13,6]:={} (simple quandle)
( 50) Epis[13,7]:={} (simple quandle)
( 51) Epis[13,8]:={} (simple quandle)
```

```
( 52) Epis[13,9]:={} (simple quandle)
( 53) Epis[13,10]:={} (simple quandle)
( 54) Epis[13,11]:={} (simple quandle)
( 55) Epis[15,1] = {[3, 1], [5, 1]}
( 56) Epis[15,2]:={} (simple quandle)
( 57) Epis[15,3] = {[3, 1], [5, 2]}
( 58) Epis[15,4] = {[3, 1], [5, 3]}
( 59) Epis[15,5] = {[3, 1]}
( 60) Epis[15,6] = {[3, 1]}
( 61) Epis[15,7]:={} (simple quandle)
( 62) Epis[16,1] = {[4, 1]}
( 63) Epis[16,2] = {[4, 1]}
( 64) Epis[16,3]:={} (simple quandle)
( 65) Epis[16,4] = {[4, 1]}
( 66) Epis[16,5] = {[4, 1]}
( 67) Epis[16,6] = {[4, 1]}
( 68) Epis[16,7] = {[4, 1]}
( 69) Epis[16,8]:={} (simple quandle)
( 70) Epis[16,9]:={} (simple quandle)
( 71) Epis[17,1]:={} (simple quandle)
( 72) Epis[17,2]:={} (simple quandle)
( 73) Epis[17,3]:={} (simple quandle)
( 74) Epis[17,4]:={} (simple quandle)
( 75) Epis[17,5]:={} (simple quandle)
( 76) Epis[17,6]:={} (simple quandle)
( 77) Epis[17,7]:={} (simple quandle)
( 78) Epis[17,8]:={} (simple quandle)
( 79) Epis[17,9]:={} (simple quandle)
( 80) Epis[17,10]:={} (simple quandle)
( 81) Epis[17,11]:={} (simple quandle)
( 82) Epis[17,12]:={} (simple quandle)
( 83) Epis[17,13]:={} (simple quandle)
( 84) Epis[17,14]:={} (simple quandle)
( 85) Epis[17,15]:={} (simple quandle)
( 86) Epis[18,1] = {[3, 1], [6, 1], [9, 2]}
( 87) Epis[18,2] = {[3, 1], [6, 1], [9, 1]}
( 88) Epis[18,3] = {[3, 1], [6, 2], [9, 1]}
( 89) Epis[18,4] = {[3, 1], [6, 2], [9, 2]}
( 90) Epis[18,5] = {[3, 1], [6, 2], [9, 6]}
( 91) Epis[18,6] = {[3, 1], [6, 2], [9, 4]}
( 92) Epis[18,7] = {[3, 1], [6, 2], [9, 5]}
( 93) Epis[18,8] = {[3, 1], [6, 1], [9, 6]}
( 94) Epis[18,9] = {[3, 1], [6, 1], [9, 5]}
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( 95) Epis[18,10] = {[3, 1], [6, 1], [9, 4]}
( 96) Epis[18,11] = {[3, 1], [6, 1]}
( 97) Epis[18,12] = {[3, 1], [6, 1]}
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( 99) Epis[19,2]:={} (simple quandle)
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(101) Epis[19,4]:={} (simple quandle)
(102) Epis[19,5]:={} (simple quandle)
(103) Epis[19,6]:={} (simple quandle)
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(105) Epis[19,8]:={} (simple quandle)
(106) Epis[19,9]:={} (simple quandle)
(107) Epis[19,10]:={} (simple quandle)
(108) Epis[19,11]:={} (simple quandle)
(109) Epis[19,12]:={} (simple quandle)
(110) Epis[19,13]:={} (simple quandle)
(111) Epis[19,14]:={} (simple quandle)
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(113) Epis[19,16]:={} (simple quandle)
(114) Epis[19,17]:={} (simple quandle)
(115) Epis[20,1]:={} (simple quandle)
(116) Epis[20,2]:={} (simple quandle)
(117) Epis[20,3] = {[10, 1]}
(118) Epis[20,4] = {[4, 1], [5, 1]}
(119) Epis[20,5] = {[5, 1]}
(120) Epis[20,6] = {[5, 1]}
(121) Epis[20,7] = {[4, 1], [5, 3]}
(122) Epis[20,8] = {[4, 1], [5, 2]}
(123) Epis[20,9] = {[10, 1]}
(124) Epis[20,10] = {[10, 1]}
(125) Epis[21,1] = {[3, 1], [7, 1]}
(126) Epis[21,2] = {[3, 1], [7, 4]}
(127) Epis[21,3] = {[3, 1], [7, 5]}
(128) Epis[21,4] = {[3, 1], [7, 2]}
(129) Epis[21,5] = {[3, 1], [7, 3]}
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(131) Epis[21,7] = {[3, 1]}
(132) Epis[21,8] = {[3, 1]}
(133) Epis[21,9]:={} (simple quandle)
(134) Epis[23,1]:={} (simple quandle)
(135) Epis[23,2]:={} (simple quandle)
(136) Epis[23,3]:={} (simple quandle)
(137) Epis[23,4]:={} (simple quandle)
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(138) Epis[23,5]:={} (simple quandle)
(139) Epis[23,6]:={} (simple quandle)
(140) Epis[23,7]:={} (simple quandle)
(141) Epis[23,8]:={} (simple quandle)
(142) Epis[23,9]:={} (simple quandle)
(143) Epis[23,10]:={} (simple quandle)
(144) Epis[23,11]:={} (simple quandle)
(145) Epis[23,12]:={} (simple quandle)
(146) Epis[23,13]:={} (simple quandle)
(147) Epis[23,14]:={} (simple quandle)
(148) Epis[23,15]:={} (simple quandle)
(149) Epis[23,16]:={} (simple quandle)
(150) Epis[23,17]:={} (simple quandle)
(151) Epis[23,18]:={} (simple quandle)
(152) Epis[23,19]:={} (simple quandle)
(153) Epis[23,20]:={} (simple quandle)
(154) Epis[23,21]:={} (simple quandle)
(155) Epis[24,1] = {[3, 1], [6, 1], [12, 1]}
(156) Epis[24,2] = {[3, 1], [6, 2], [12, 2]}
(157) Epis[24,3] = {[3, 1], [6, 1], [12, 1], [12, 6]}
(158) Epis[24,4] = {[3, 1], [6, 1], [12, 1], [12, 5]}
(159) Epis[24,5] = {[3, 1], [6, 1], [12, 1], [12, 8]}
(160) Epis[24,6] = {[3, 1], [6, 1], [6, 2], [12, 1], [12, 9]}
(161) Epis[24,7] = {[12, 3]}
(162) Epis[24,8] = {[3, 1], [4, 1], [8, 1], [12, 4]}
(163) Epis[24,9]:={} (simple quandle)
(164) Epis[24,10] = {[3, 1], [6, 1], [12, 1]}
(165) Epis[24,11] = {[3, 1], [6, 1], [12, 1]}
(166) Epis[24,12] = {[3, 1], [6, 2], [12, 2]}
(167) Epis[24,13] = {[3, 1], [6, 2], [12, 2]}
(168) Epis[24,14] = {[3, 1], [6, 2], [12, 7]}
(169) Epis[24,15] = {[3, 1], [6, 2], [12, 7]}
(170) Epis[24,16] = {[3, 1], [6, 1], [12, 8]}
(171) Epis[24,17] = {[3, 1], [6, 1], [12, 8]}
(172) Epis[24,18] = {[3, 1], [6, 2], [12, 2], [12, 7]}
(173) Epis[24,19] = {[3, 1], [6, 1], [6, 2], [12, 2], [12, 9]}
(174) Epis[24,20] = {[8, 1], [12, 10]}
(175) Epis[24,21] = {[4, 1], [8, 1]}
(176) Epis[24,22] = {[3, 1], [4, 1], [6, 1], [12, 4]}
(177) Epis[24,23] = {[3, 1], [4, 1], [6, 2], [12, 4]}
(178) Epis[24,24] = {[3, 1], [8, 2]}
(179) Epis[24,25] = {[3, 1], [8, 3]}
(180) Epis[24,26] = {[3, 1], [6, 1], [12, 5]}

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(181) Epis[24,27] = {[3, 1], [6, 1], [12, 6]}
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(183) Epis[24,29] = {[3, 1], [6, 1], [6, 2], [12, 5], [12, 6], [12, 9]}
(184) Epis[24,30] = {[3, 1], [6, 1], [12, 6], [12, 8]}
(185) Epis[24,31] = {[3, 1], [6, 1], [12, 5], [12, 8]}
(186) Epis[24,32] = {[3, 1], [6, 2], [12, 7]}
(187) Epis[24,33] = {[3, 1], [6, 1], [6, 2], [12, 7], [12, 9]}
(188) Epis[24,34] = {[3, 1], [6, 1], [12, 8]}
(189) Epis[24,35] = {[3, 1], [6, 1], [12, 8]}
(190) Epis[24,36] = {[3, 1], [6, 1], [12, 8]}
(191) Epis[24,37] = {[3, 1], [6, 1], [12, 8]}
(192) Epis[24,38] = {[6, 1], [6, 2], [12, 8], [12, 9]}
(193) Epis[24,39] = {[6, 1], [12, 8]}
(194) Epis[24,40] = {[3, 1], [6, 1], [6, 2], [12, 9]}
(195) Epis[24,41] = {[3, 1], [6, 1], [6, 2], [12, 9]}
(196) Epis[24,42] = {[3, 1], [6, 1], [6, 2], [12, 9]}
(197) Epis[25,1] = {[5, 1]}
(198) Epis[25,2] = {[5, 1]}
(199) Epis[25,3]:={} (simple quandle)
(200) Epis[25,4] = {[5, 2]}
(201) Epis[25,5] = {[5, 3]}
(202) Epis[25,6] = {[5, 2]}
(203) Epis[25,7] = {[5, 3]}
(204) Epis[25,8] = {[5, 2], [5, 3]}
(205) Epis[25,9] = {[5, 1], [5, 3]}
(206) Epis[25,10] = {[5, 1], [5, 2]}
(207) Epis[25,11]:={} (simple quandle)
(208) Epis[25,12]:={} (simple quandle)
(209) Epis[25,13]:={} (simple quandle)
(210) Epis[25,14] = {[5, 1]}
(211) Epis[25,15] = {[5, 1]}
(212) Epis[25,16] = {[5, 1]}
(213) Epis[25,17] = {[5, 1]}
(214) Epis[25,18] = {[5, 1]}
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(216) Epis[25,20]:={} (simple quandle)
(217) Epis[25,21] = {[5, 3]}
(218) Epis[25,22] = {[5, 2]}
(219) Epis[25,23] = {[5, 2]}
(220) Epis[25,24] = {[5, 2]}
(221) Epis[25,25] = {[5, 3]}
(222) Epis[25,26] = {[5, 2]}
(223) Epis[25,27] = {[5, 3]}
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(224) Epis[25,28] = {[5, 3]}
(225) Epis[25,29] = {[5, 2]}
(226) Epis[25,30] = {[5, 3]}
(227) Epis[25,31]:={} (simple quandle)
(228) Epis[25,32]:={} (simple quandle)
(229) Epis[25,33]:={} (simple quandle)
(230) Epis[25,34]:={} (simple quandle)
(231) Epis[27,1] = {[3, 1], [9, 2]}
(232) Epis[27,2] = {[3, 1], [9, 2]}
(233) Epis[27,3] = {[3, 1], [9, 1]}
(234) Epis[27,4] = {[3, 1], [9, 1], [9, 2]}
(235) Epis[27,5] = {[3, 1], [9, 3]}
(236) Epis[27,6] = {[9, 3]}
(237) Epis[27,7] = {[3, 1], [9, 2]}
(238) Epis[27,8] = {[3, 1], [9, 2]}
(239) Epis[27,9] = {[3, 1], [9, 2]}
(240) Epis[27,10] = {[3, 1], [9, 2]}
(241) Epis[27,11] = {[3, 1], [9, 2]}
(242) Epis[27,12] = {[3, 1], [9, 2]}
(243) Epis[27,13] = {[3, 1], [9, 2]}
(244) Epis[27,14] = {[3, 1], [9, 6]}
(245) Epis[27,15] = {[3, 1], [9, 2]}
(246) Epis[27,16] = {[3, 1], [9, 2]}
(247) Epis[27,17] = {[3, 1], [9, 2]}
(248) Epis[27,18] = {[3, 1], [9, 1]}
(249) Epis[27,19] = {[3, 1], [9, 1]}
(250) Epis[27,20] = {[3, 1], [9, 2], [9, 5]}
(251) Epis[27,21] = {[3, 1], [9, 2], [9, 4]}
(252) Epis[27,22] = {[3, 1], [9, 1], [9, 4], [9, 5], [9, 6]}
(253) Epis[27,23] = {[3, 1], [9, 2], [9, 6]}
(254) Epis[27,24] = {[3, 1], [9, 6]}
(255) Epis[27,25] = {[3, 1], [9, 6]}
(256) Epis[27,26] = {[3, 1], [9, 6]}
(257) Epis[27,27] = {[9, 8]}
(258) Epis[27,28] = {[9, 7]}
(259) Epis[27,29] = {[3, 1], [9, 8]}
(260) Epis[27,30] = {[3, 1], [9, 7]}
(261) Epis[27,31]:={} (simple quandle)
(262) Epis[27,32]:={} (simple quandle)
(263) Epis[27,33]:={} (simple quandle)
(264) Epis[27,34]:={} (simple quandle)
(265) Epis[27,35] = {[3, 1], [9, 2]}
(266) Epis[27,36] = {[3, 1], [9, 2]}
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(267) Epis[27,37] = {[3, 1], [9, 6]}
(268) Epis[27,38] = {[3, 1], [9, 6]}
(269) Epis[27,39] = {[3, 1], [9, 6]}
(270) Epis[27,40] = {[3, 1], [9, 6]}
(271) Epis[27,41] = {[3, 1], [9, 2]}
(272) Epis[27,42] = {[3, 1], [9, 2]}
(273) Epis[27,43] = {[3, 1], [9, 2]}
(274) Epis[27,44] = {[3, 1], [9, 2]}
(275) Epis[27,45] = {[3, 1], [9, 2]}
(276) Epis[27,46] = {[3, 1], [9, 2]}
(277) Epis[27,47] = {[3, 1], [9, 4]}
(278) Epis[27,48] = {[3, 1], [9, 5]}
(279) Epis[27,49] = {[3, 1], [9, 4]}
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(283) Epis[27,53] = {[3, 1], [9, 6]}
(284) Epis[27,54] = {[3, 1], [9, 6]}
(285) Epis[27,55] = {[3, 1], [9, 6]}
(286) Epis[27,56] = {[3, 1], [9, 2]}
(287) Epis[27,57] = {[3, 1], [9, 2]}
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(290) Epis[27,60] = {[3, 1], [9, 6]}
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(293) Epis[27,63]:={} (simple quandle)
(294) Epis[27,64]:={} (simple quandle)
(295) Epis[27,65]:={} (simple quandle)
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(297) Epis[28,2] = {[4, 1], [7, 3]}
(298) Epis[28,3] = {[7, 3]}
(299) Epis[28,4] = {[7, 2]}
(300) Epis[28,5] = {[7, 3]}
(301) Epis[28,6] = {[7, 2]}
(302) Epis[28,7] = {[4, 1], [7, 4]}
(303) Epis[28,8] = {[4, 1], [7, 5]}
(304) Epis[28,9] = {[4, 1], [7, 1]}
(305) Epis[28,10]:={} (simple quandle)
(306) Epis[28,11] = {[4, 1]}
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(308) Epis[28,13]:={} (simple quandle)
(309) Epis[29,1]:={} (simple quandle)
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(320) Epis[29,12]:={} (simple quandle)
(321) Epis[29,13]:={} (simple quandle)
(322) Epis[29,14]:={} (simple quandle)
(323) Epis[29,15]:={} (simple quandle)
(324) Epis[29,16]:={} (simple quandle)
(325) Epis[29,17]:={} (simple quandle)
(326) Epis[29,18]:={} (simple quandle)
(327) Epis[29,19]:={} (simple quandle)
(328) Epis[29,20]:={} (simple quandle)
(329) Epis[29,21]:={} (simple quandle)
(330) Epis[29,22]:={} (simple quandle)
(331) Epis[29,23]:={} (simple quandle)
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(333) Epis[29,25]:={} (simple quandle)
(334) Epis[29,26]:={} (simple quandle)
(335) Epis[29,27]:={} (simple quandle)
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(339) Epis[30,4] = {[10, 1]}
(340) Epis[30,5] = {[15, 2]}
(341) Epis[30,6] = {[3, 1], [5, 1], [6, 2], [15, 1]}
(342) Epis[30,7] = {[3, 1], [5, 2], [6, 1], [15, 3]}
(343) Epis[30,8] = {[3, 1], [5, 3], [6, 1], [15, 4]}
(344) Epis[30,9] = {[3, 1], [5, 2], [6, 2], [15, 3]}
(345) Epis[30,10] = {[3, 1], [5, 3], [6, 2], [15, 4]}
(346) Epis[30,11] = {[3, 1], [10, 1]}
(347) Epis[30,12] = {[3, 1], [6, 1], [15, 5]}
(348) Epis[30,13] = {[3, 1], [6, 1], [15, 6]}
(349) Epis[30,14] = {[3, 1], [6, 2], [15, 5]}
(350) Epis[30,15] = {[3, 1], [6, 2], [15, 6]}
(351) Epis[30,16] = {[15, 7]}
(352) Epis[30,17] = {[3, 1], [6, 2]}
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(353) Epis[30,18] = {[3, 1], [6, 2]}
(354) Epis[30,19] = {[3, 1], [6, 1]}
(355) Epis[30,20] = {[3, 1], [6, 1]}
(356) Epis[30,21] = {[10, 1]}
(357) Epis[30,22] = {[10, 1]}
(358) Epis[30,23] = {[15, 7]}
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(367) Epis[31,8]:={} (simple quandle)
(368) Epis[31,9]:={} (simple quandle)
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(374) Epis[31,15]:={} (simple quandle)
(375) Epis[31,16]:={} (simple quandle)
(376) Epis[31,17]:={} (simple quandle)
(377) Epis[31,18]:={} (simple quandle)
(378) Epis[31,19]:={} (simple quandle)
(379) Epis[31,20]:={} (simple quandle)
(380) Epis[31,21]:={} (simple quandle)
(381) Epis[31,22]:={} (simple quandle)
(382) Epis[31,23]:={} (simple quandle)
(383) Epis[31,24]:={} (simple quandle)
(384) Epis[31,25]:={} (simple quandle)
(385) Epis[31,26]:={} (simple quandle)
(386) Epis[31,27]:={} (simple quandle)
(387) Epis[31,28]:={} (simple quandle)
(388) Epis[31,29]:={} (simple quandle)
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(391) Epis[32,3] = {[4, 1], [16, 2]}
(392) Epis[32,4] = {[16, 3]}
(393) Epis[32,5] = {[4, 1], [8, 1], [16, 4]}
(394) Epis[32,6] = {[4, 1], [16, 4]}
(395) Epis[32,7] = {[4, 1], [8, 1], [16, 5]}
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(396) Epis[32,8] = {[4, 1], [8, 1], [16, 6]}
(397) Epis[32,9] = {[4, 1], [8, 1], [16, 7]}
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(399) Epis[32,11]:={} (simple quandle)
(400) Epis[32,12]:={} (simple quandle)
(401) Epis[32,13]:={} (simple quandle)
(402) Epis[32,14]:={} (simple quandle)
(403) Epis[32,15]:={} (simple quandle)
(404) Epis[32,16] = {[4, 1], [8, 3]}
(405) Epis[32,17] = {[4, 1], [8, 2]}
(406) Epis[33,1] = {[3, 1], [11, 1]}
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(409) Epis[33,4] = {[3, 1], [11, 9]}
(410) Epis[33,5] = {[3, 1], [11, 6]}
(411) Epis[33,6] = {[3, 1], [11, 5]}
(412) Epis[33,7] = {[3, 1], [11, 4]}
(413) Epis[33,8] = {[3, 1], [11, 3]}
(414) Epis[33,9] = {[3, 1], [11, 2]}
(415) Epis[33,10] = {[3, 1]}
(416) Epis[33,11] = {[3, 1]}
(417) Epis[35,1] = {[5, 1], [7, 1]}
(418) Epis[35,2] = {[5, 2], [7, 1]}
(419) Epis[35,3] = {[5, 3], [7, 1]}
(420) Epis[35,4] = {[5, 1], [7, 5]}
(421) Epis[35,5] = {[5, 1], [7, 4]}
(422) Epis[35,6] = {[5, 1], [7, 3]}
(423) Epis[35,7] = {[5, 1], [7, 2]}
(424) Epis[35,8] = {[5, 2], [7, 4]}
(425) Epis[35,9] = {[5, 2], [7, 5]}
(426) Epis[35,10] = {[5, 3], [7, 4]}
(427) Epis[35,11] = {[5, 3], [7, 5]}
(428) Epis[35,12] = {[5, 3], [7, 3]}
(429) Epis[35,13] = {[5, 3], [7, 2]}
(430) Epis[35,14] = {[5, 2], [7, 3]}
(431) Epis[35,15] = {[5, 2], [7, 2]}
```

5 Minimal generators

The following list contains
(1) A list of sets Gen$[n, i]$ which is a smallest set of generators for $C[n, i]$.
(2) A set Any2 which is the set of all $[n, i]$ such that any two elements of $C[n, i]$ will generate $C[n, i]$.

Note that if $[n, i]$ is in Any2 then there are no non-trivial subquandles of $C[n, i]$.

```
printf("Gen[n,i] is a smallest set of generators for C[n,i]. \n");

Gen[3,1]:={1, 2}:
Gen[4,1]:={1, 2}:
Gen[5,1]:={1, 2}:
Gen[5,2]:={1, 2}:
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Gen[12,5]:={1, 3, 5}:
Gen[12,6]:={1, 3, 5}:
Gen[12,7]:={1, 5}:
```

```
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```

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```

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```

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```

```
Gen[35,12]:={1, 8}:
Gen[35,13]:={1, 8}:
Gen[35,14]:={1, 8}:
Gen[35,15]:={1, 8}:

printf("Any2 is the set of all [n,i] such that
any two elements of C[n,i] will generate C[n,i]. \n");

Any2:={[3, 1], [4, 1], [5, 1], [5, 2], [5, 3], [7, 1], [7, 2], [7, 3], [7, 4], [7, 5],
[8, 2], [8, 3], [9, 3], [9, 7], [9, 8], [11, 1], [11, 2], [11, 3], [11, 4], [11, 5],
[11, 6], [11, 7], [11, 8], [11, 9], [13, 1], [13, 2], [13, 3], [13, 4], [13, 5],
[13, 6], [13, 7], [13, 8], [13, 9], [13, 10], [13, 11], [16, 3], [16, 8], [16, 9],
[17, 1], [17, 2], [17, 3], [17, 4], [17, 5], [17, 6], [17, 7], [17, 8], [17, 9],
[17, 10], [17, 11], [17, 12], [17, 13], [17, 14], [17, 15], [19, 1], [19, 2],
[19, 3], [19, 4], [19, 5], [19, 6], [19, 7], [19, 8], [19, 9], [19, 10], [19, 11],
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[29, 6], [29, 7], [29, 8], [29, 9], [29, 10], [29, 11], [29, 12], [29, 13], [29, 14],
[29, 15], [29, 16], [29, 17], [29, 18], [29, 19], [29, 20], [29, 21], [29, 22],
[29, 23], [29, 24], [29, 25], [29, 26], [29, 27], [31, 1], [31, 2], [31, 3],
[31, 4], [31, 5], [31, 6], [31, 7], [31, 8], [31, 9], [31, 10], [31, 11], [31, 12],
[31, 13], [31, 14], [31, 15], [31, 16], [31, 17], [31, 18], [31, 19], [31, 20],
[31, 21], [31, 22], [31, 23], [31, 24], [31, 25], [31, 26], [31, 27], [31, 28],
[31, 29], [32, 10], [32, 11], [32, 12], [32, 13], [32, 14], [32, 15]}:
```

6 Abelian extensions

A function $\phi : X \times X \to A$ for an abelian group A is called *quandle* 2*-cocycle* if it satisfies

$$\phi(x, y) - \phi(x, z) + \phi(x * y, z) - \phi(x * z, y * z) = 0$$

and $\phi(x, x) = 0$ for any $x, y, z \in X$. the website cited at the beginning. For a quandle 2-cocycle ϕ, $X \times A$ becomes a quandle by $(x, a) * (y, b) = (x * y, a + \phi(x, y))$ for $x, y \in X$, $a, b \in A$, and it is called an *abelian extension* of X by A.

It is known that the quandle cocycle invariant using a 2-cocycle mentioned above measures the failure of lifting a coloring by a quandle X to a coloring by Y, where Y is an abelian cocycle extension of X. In particular, if $\mathrm{Col}_X(K) > \mathrm{Col}_Y(K)$, then the cocycle invariant is non-trivial, and the cocycle is not a coboundary, so that $H_2(X, A) \neq 0$.

The list below shows the numbers of non-trivial colorings of the first 10 knots in the table by abelian extensions of quandles, for abelian extensions that satisfy the condition $\mathrm{Col}_X(K) > \mathrm{Col}_Y(K)$. Hence the list below gives two facts: (1) the cocycle invariant is non-trivial for knots, and (2) $H_2(X, A) \neq 0$.

```
Notations:
In the table below

         ColorVector[n,i] = [ v[1], v[2], ...., v[10] ]

means that v[i] is the number of colorings of knot[i] by quandle C[n,i].

Comments by M.S. are marked by %.


ColorVector[ 8, 1] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[ 4, 1] = [ 12,  12,   0,   0,   0,   0,   0,   0,  12,  12, ]
% Cocy inv non-trivial (known).

ColorVector[12, 2] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[ 6, 2] = [ 24,   0,   0,   0,  24,   0,   0,   0,   0,   0, ]
% Cocy inv non-trivial (known).

ColorVector[24, 2] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[ 6, 2] = [ 24,   0,   0,   0,  24,   0,   0,   0,   0,   0, ]
% Cocy inv non-trivial (known).

ColorVector[24, 7] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 3] = [ 60,   0,  60,  60,   0,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial, this may not be known.

ColorVector[24, 8] = [ 48,   0,   0,   0,  48,   0,   0,   0,   0,   0, ]
ColorVector[12, 4] = [132,  36,   0,   0,  24,   0,   0,   0,  36,  36, ] % Alex
% Cocy inv non-trivial, but  C[12,4] has a subquandle C[4,1] that has non-trivial cocy inv.

ColorVector[24,15] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 7] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial, this may not be known.

ColorVector[24,18] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 7] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-triviall, this may not be known.

ColorVector[24,19] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
```

```
ColorVector[12, 9] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial, this may not be known.

ColorVector[24,20] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12,10] = [108, 108,   0,   0,   0,   0,   0,   0, 108, 108, ] % Non-Alex
% Cocy inv non-trivial, this may not be known.

ColorVector[27, 6] = [  0,   0,   0,   0,   0,   0, 216,   0,   0,   0, ]
ColorVector[ 9, 3] = [  0,  72,   0,  72,   0,   0,  72,   0,   0,   0, ] % Alex
% Cocy inv non-trivial, C[9,3]=(Z_3)[t]/(t^2+1) known?

ColorVector[32, 1] = [ 96,  96,   0,   0,   0,   0,   0,   0,  96,  96, ]
ColorVector[16, 1] = [ 48, 240,   0,   0,   0,   0,   0,   0, 240, 240, ] % Alex
% Cocy inv non-trivial, C[16,1] = (Z_4)[t]/(t^2+t+1)  known?

ColorVector[32, 2] = [ 96,  96,   0,   0,   0,   0,   0,   0,  96,  96, ]
ColorVector[16, 2] = [240, 240,   0,   0,   0,   0,   0,   0, 240, 240, ] % Alex
% Cocy inv non-trivial, C[16,2]=C[4,1]XC[4,1], known.

ColorVector[32, 4] = [  0,   0, 160,   0,   0, 160, 160,   0,   0,   0, ]
ColorVector[16, 3] = [  0,   0, 240,   0,   0, 240, 240,   0,   0,   0, ] % Alex
% Cocy inv non-trivial, (Z_2)[t]/(t^4+t^3+t^2+t+1)  known?

ColorVector[32, 9] = [ 96,  96,   0,   0,   0,   0,   0,   0,  96,  96, ]
ColorVector[16, 7] = [240,  48,   0,   0,   0,   0,   0,   0,  48,  48, ] % Alex
First Exception: ColorVector[32,9][1] = 96 and ColorVector[16,7][1] = 240 for knot 3_1.
% Cocy inv non-trivial, (Z_4)[t]/(t^2+3*t+1) known?


Cocycles up to 6 elements quandles are known.
Many cocycles of Alexander quandles are known.
The following are non-Alexander extensions that give
non-trivial cocycle invariants, and this result may be new.

ColorVector[24, 7] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 3] = [ 60,   0,  60,  60,   0,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial

ColorVector[24,15] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 7] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial

ColorVector[24,18] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
```

```
ColorVector[12, 7] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial

ColorVector[24,19] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12, 9] = [ 96,   0,   0,   0,  96,   0,   0,   0,   0,   0, ] % Non-Alex
% Cocy inv non-trivial

ColorVector[24,20] = [  0,   0,   0,   0,   0,   0,   0,   0,   0,   0, ]
ColorVector[12,10] = [108, 108,   0,   0,   0,   0,   0,   0, 108, 108, ] % Non-Alex
% Cocy inv non-trivial
```

7 Knot colorings

For two quandles Q_1 and Q_2, and a set $\mathcal{K}$ of knots, we denote

- $Q_1 \approx_{\mathcal{K}} Q_2$ if $\langle Q_1 \mid K \rangle = \langle Q_2 \mid K \rangle$ for all $K \in \mathcal{K}$,
- $Q_1 \sim_{\mathcal{K}} Q_2$ if it holds that $\langle Q_1 \mid K \rangle \neq 0$ if and only if $\langle Q_2 \mid K \rangle \neq 0$ for all $K \in \mathcal{K}$.

Let $\mathcal{K}$ be 2977 knots in the table up to 12 crossings.
The equivalence classes of RIG quandles by $\approx_{\mathcal{K}}$ are listed below.

```
(1) { C[5,2],C[5,3] }
(2) { C[6,1],C[6,2] }
(3) { C[7,2],C[7,3] }
(4) { C[7,4],C[7,5] }
(5) { C[8,2],C[8,3] }
(6) { C[9,4],C[9,5] }
(7) { C[9,7],C[9,8] }
(8) { C[11,2],C[11,5] }
(9) { C[11,3],C[11,4] }
(10) { C[11,6],C[11,9] }
(11) { C[11,7],C[11,8] }
(12) { C[12,5],C[12,6] }
(13) { C[12,8],C[12,9] }
(14) { C[13,1],C[13,6] }
(15) { C[13,2],C[13,8] }
(16) { C[13,3],C[13,9] }
(17) { C[13,4],C[13,7] }
(18) { C[13,5],C[13,10] }
(19) { C[15,3],C[15,4] }
(20) { C[16,5],C[16,6] }
```

```
(21) { C[16,8],C[16,9] }
(22) { C[17,1],C[17,8] }
(23) { C[17,2],C[17,5] }
(24) { C[17,3],C[17,12] }
(25) { C[17,4],C[17,6] }
(26) { C[17,7],C[17,14] }
(27) { C[17,9],C[17,11] }
(28) { C[17,10],C[17,13] }
(29) { C[18,1],C[18,4] }
(30) { C[18,2],C[18,3] }
(31) { C[18,5],C[18,8] }
(32) { C[18,6],C[18,10] }
(33) { C[18,7],C[18,9] }
(34) { C[19,1],C[19,9] }
(35) { C[19,2],C[19,12] }
(36) { C[19,3],C[19,4] }
(37) { C[19,5],C[19,15] }
(38) { C[19,6],C[19,10] }
(39) { C[19,7],C[19,11] }
(40) { C[19,8],C[19,16] }
(41) { C[19,13],C[19,14] }
(42) { C[20,5],C[20,6] }
(43) { C[20,7],C[20,8] }
(44) { C[20,9],C[20,10] }
(45) { C[21,2],C[21,3] }
(46) { C[21,4],C[21,5] }
(47) { C[23,1],C[23,11] }
(48) { C[23,2],C[23,7] }
(49) { C[23,3],C[23,5] }
(50) { C[23,4],C[23,13] }
(51) { C[23,6],C[23,9] }
(52) { C[23,8],C[23,17] }
(53) { C[23,10],C[23,20] }
(54) { C[23,12],C[23,15] }
(55) { C[23,14],C[23,19] }
(56) { C[23,16],C[23,18] }
(57) { C[24,3],C[24,4] }
(58) { C[24,5],C[24,6],C[24,16],C[24,17] }
(59) { C[24,15],C[24,18] }
(60) { C[24,22],C[24,23] }
(61) { C[24,24],C[24,25] }
(62) { C[24,27],C[24,28] }
(63) { C[24,29],C[24,30],C[24,31] }
```

```
(64) { C[24,34],C[24,36] }
(65) { C[24,35],C[24,37] }
(66) { C[24,38],C[24,39] }
(67) { C[24,41],C[24,42] }
(68) { C[25,4],C[25,5] }
(69) { C[25,6],C[25,7],C[25,8] }
(70) { C[25,9],C[25,10] }
(71) { C[25,12],C[25,13] }
(72) { C[25,15],C[25,16] }
(73) { C[25,17],C[25,18] }
(74) { C[25,19],C[25,20] }
(75) { C[25,21],C[25,22] }
(76) { C[25,23],C[25,28] }
(77) { C[25,24],C[25,27] }
(78) { C[25,25],C[25,26] }
(79) { C[25,29],C[25,30] }
(80) { C[25,31],C[25,34] }
(81) { C[25,32],C[25,33] }
(82) { C[27,9],C[27,11] }
(83) { C[27,17],C[27,22] }
(84) { C[27,18],C[27,19] }
(85) { C[27,20],C[27,21] }
(86) { C[27,29],C[27,30] }
(87) { C[27,31],C[27,33] }
(88) { C[27,32],C[27,34] }
(89) { C[27,35],C[27,36] }
(90) { C[27,37],C[27,38] }
(91) { C[27,41],C[27,42] }
(92) { C[27,43],C[27,46] }
(93) { C[27,44],C[27,45] }
(94) { C[27,47],C[27,50] }
(95) { C[27,48],C[27,49] }
(96) { C[27,51],C[27,52] }
(97) { C[27,56],C[27,57] }
(98) { C[27,58],C[27,59] }
(99) { C[27,62],C[27,65] }
(100) { C[27,63],C[27,64] }
(101) { C[28,1],C[28,2] }
(102) { C[28,7],C[28,8] }
(103) { C[28,11],C[28,12] }
(104) { C[29,1],C[29,14] }
(105) { C[29,2],C[29,9] }
(106) { C[29,3],C[29,21] }
```

```
(107) { C[29,4],C[29,5] }
(108) { C[29,6],C[29,24] }
(109) { C[29,7],C[29,10] }
(110) { C[29,8],C[29,12] }
(111) { C[29,11],C[29,16] }
(112) { C[29,13],C[29,26] }
(113) { C[29,15],C[29,19] }
(114) { C[29,17],C[29,20] }
(115) { C[29,18],C[29,25] }
(116) { C[29,22],C[29,23] }
(117) { C[30,2],C[30,6] }
(118) { C[30,7],C[30,8],C[30,9],C[30,10] }
(119) { C[30,12],C[30,14] }
(120) { C[30,13],C[30,15] }
(121) { C[30,17],C[30,18] }
(122) { C[30,23],C[30,24] }
(123) { C[31,1],C[31,15] }
(124) { C[31,2],C[31,20] }
(125) { C[31,3],C[31,7] }
(126) { C[31,4],C[31,24] }
(127) { C[31,5],C[31,25] }
(128) { C[31,6],C[31,8] }
(129) { C[31,9],C[31,27] }
(130) { C[31,10],C[31,16] }
(131) { C[31,11],C[31,12] }
(132) { C[31,13],C[31,19] }
(133) { C[31,14],C[31,28] }
(134) { C[31,17],C[31,18] }
(135) { C[31,21],C[31,23] }
(136) { C[31,22],C[31,26] }
(137) { C[32,5],C[32,6] }
(138) { C[32,7],C[32,8] }
(139) { C[32,10],C[32,15] }
(140) { C[32,11],C[32,13] }
(141) { C[32,12],C[32,14] }
(142) { C[32,16],C[32,17] }
(143) { C[33,2],C[33,3] }
(144) { C[33,4],C[33,5] }
(145) { C[33,6],C[33,9] }
(146) { C[33,7],C[33,8] }
(147) { C[35,2],C[35,3] }
(148) { C[35,4],C[35,5] }
(149) { C[35,6],C[35,7] }
```

(150) { C[35,8],C[35,9],C[35,10],C[35,11] }
(151) { C[35,12],C[35,13],C[35,14],C[35,15] }

The list below are equivalence classes for $\sim_{\mathcal{K}}$.

(1) { C[3,1],C[6,1],C[6,2],C[9,1],C[9,2],C[9,4],C[9,5],C[9,6],C[10,1],
C[12,1],C[12,5],C[12,6],C[12,8],C[12,9],C[15,7],C[18,1],C[18,2],C[18,3],
C[18,4],C[18,5],C[18,6],C[18,7],C[18,8],C[18,9],C[18,10],C[18,11],C[18,12],
C[20,3],C[20,9],C[20,10],C[21,9],C[24,1],C[24,3],C[24,4],C[24,5]
,C[24,6],C[24,10],C[24,11],C[24,16],C[24,17],C[24,27],C[24,28],C[24,29],C[24,30],
C[24,31],C[24,34],C[24,35],C[24,36],C[24,37],C[24,38],C[24,39],
C[24,40],C[24,41],C[24,42],C[27,1],C[27,2],C[27,3],C[27,4],C[27,7],C[27,8],C[27,9],C[27,10],
C[27,11],C[27,12],C[27,13],C[27,14],C[27,15],C[27,16],C[27,17],C[27,18],C[27,19],C[27,20],
C[27,21],C[27,22],C[27,23],C[27,24],C[27,25],C[27,26],C[27,35],
C[27,36],C[27,37],C[27,38],C[27,39],
C[27,40],C[27,41],C[27,42],C[27,43],C[27,44],C[27,45],C[27,46],C[27,47],C[27,48],C[27,49],
C[27,50],C[27,51],C[27,52],C[27,53],C[27,54],C[27,55],C[27,56],C[27,57],C[27,58],C[27,59],
C[27,60],C[27,61],C[28,13],C[30,4],C[30,11],C[30,16],C[30,21],C[30,22],C[30,23],C[30,24] }
(2) { C[4,1],C[12,10],C[16,1],C[16,2],C[16,4],C[16,5],C[16,6],C[16,7],
C[32,1],C[32,2],C[32,3],C[32,5],C[32,6],C[32,7],C[32,8],C[32,9] }
(3) { C[5,1],C[20,5],C[20,6],C[25,1],C[25,2],C[25,14],C[25,15],C[25,16],C[25,17],C[25,18] }
(4) { C[5,2],C[5,3],C[25,4],C[25,5],C[25,6],C[25,7],C[25,8],C[25,21],
C[25,22],C[25,23],C[25,24],
C[25,25],C[25,26],C[25,27],C[25,28],C[25,29],C[25,30] }
(5) { C[7,2],C[7,3] }
(6) { C[7,4],C[7,5] }
(7) { C[8,1],C[24,20] }
(8) { C[8,2],C[8,3] }
(9) { C[9,7],C[9,8] }
(10) { C[11,2],C[11,5] }
(11) { C[11,3],C[11,4] }
(12) { C[11,6],C[11,9] }
(13) { C[11,7],C[11,8] }
(14) { C[12,2],C[24,19] }
(15) { C[12,4],C[24,22],C[24,23] }
(16) { C[12,7],C[24,14],C[24,32],C[24,33] }
(17) { C[13,1],C[13,6] }
(18) { C[13,2],C[13,8] }
(19) { C[13,3],C[13,9] }
(20) { C[13,4],C[13,7] }
(21) { C[13,5],C[13,10] }
(22) { C[15,1],C[15,6],C[30,2],C[30,6],C[30,13],C[30,15],C[30,20] }

```
(23) { C[15,2],C[30,1] }
(24) { C[15,3],C[15,4],C[30,7],C[30,8],C[30,9],C[30,10],C[30,17],C[30,18] }
(25) { C[15,5],C[30,12],C[30,14] }
(26) { C[16,3],C[32,4] }
(27) { C[16,8],C[16,9] }
(28) { C[17,1],C[17,8] }
(29) { C[17,2],C[17,5] }
(30) { C[17,3],C[17,12] }
(31) { C[17,4],C[17,6] }
(32) { C[17,7],C[17,14] }
(33) { C[17,9],C[17,11] }
(34) { C[17,10],C[17,13] }
(35) { C[19,1],C[19,9] }
(36) { C[19,2],C[19,12] }
(37) { C[19,3],C[19,4] }
(38) { C[19,5],C[19,15] }
(39) { C[19,6],C[19,10] }
(40) { C[19,7],C[19,11],C[31,5],C[31,25] }
(41) { C[19,8],C[19,16] }
(42) { C[19,13],C[19,14] }
(43) { C[20,7],C[20,8] }
(44) { C[21,1],C[21,8] }
(45) { C[21,2],C[21,3] }
(46) { C[21,4],C[21,5] }
(47) { C[23,1],C[23,11] }
(48) { C[23,2],C[23,7] }
(49) { C[23,3],C[23,5] }
(50) { C[23,4],C[23,13] }
(51) { C[23,6],C[23,9] }
(52) { C[23,8],C[23,17] }
(53) { C[23,10],C[23,20] }
(54) { C[23,12],C[23,15] }
(55) { C[23,14],C[23,19] }
(56) { C[23,16],C[23,18] }
(57) { C[24,15],C[24,18] }
(58) { C[24,24],C[24,25] }
(59) { C[25,9],C[25,10] }
(60) { C[25,12],C[25,13] }
(61) { C[25,19],C[25,20] }
(62) { C[25,31],C[25,34] }
(63) { C[25,32],C[25,33] }
(64) { C[27,27],C[27,28],C[27,29],C[27,30] }
(65) { C[27,31],C[27,33] }
```

```
(66) { C[27,32],C[27,34] }
(67) { C[27,62],C[27,65] }
(68) { C[27,63],C[27,64] }
(69) { C[28,1],C[28,2],C[28,3],C[28,4],C[28,11],C[28,12] }
(70) { C[28,7],C[28,8] }
(71) { C[29,1],C[29,14] }
(72) { C[29,2],C[29,9] }
(73) { C[29,3],C[29,21] }
(74) { C[29,4],C[29,5] }
(75) { C[29,6],C[29,24] }
(76) { C[29,7],C[29,10] }
(77) { C[29,8],C[29,12] }
(78) { C[29,11],C[29,16] }
(79) { C[29,13],C[29,26] }
(80) { C[29,15],C[29,19] }
(81) { C[29,17],C[29,20] }
(82) { C[29,18],C[29,25] }
(83) { C[29,22],C[29,23] }
(84) { C[31,1],C[31,15] }
(85) { C[31,2],C[31,20] }
(86) { C[31,3],C[31,7] }
(87) { C[31,4],C[31,24] }
(88) { C[31,6],C[31,8] }
(89) { C[31,9],C[31,27] }
(90) { C[31,10],C[31,16] }
(91) { C[31,11],C[31,12] }
(92) { C[31,13],C[31,19] }
(93) { C[31,14],C[31,28] }
(94) { C[31,17],C[31,18] }
(95) { C[31,21],C[31,23] }
(96) { C[31,22],C[31,26] }
(97) { C[32,10],C[32,15] }
(98) { C[32,11],C[32,13] }
(99) { C[32,12],C[32,14] }
(100) { C[32,16],C[32,17] }
(101) { C[33,1],C[33,10] }
(102) { C[33,2],C[33,3] }
(103) { C[33,4],C[33,5] }
(104) { C[33,6],C[33,9] }
(105) { C[33,7],C[33,8] }
(106) { C[35,2],C[35,3] }
(107) { C[35,4],C[35,5] }
(108) { C[35,6],C[35,7] }
```

```
(109) { C[35,8],C[35,9],C[35,10],C[35,11] }
(110) { C[35,12],C[35,13],C[35,14],C[35,15] }
```