Knotting Mathematics and Art : International Conference in Low Dimensional Topology and Mathematical Art

November 1 – 4, 2007
University of South Florida, Tampa, Florida, U.S.A.

Abstract (Nagle Lecture, Opening Talk)

John H. Conway (Princeton University)
From Topology to Symmetry
(This talk will be the *Nagle Lecture, http://www.math.usf.edu/Nagle/index.html*, scheduled on November 1, 7:30–8:30PM, also regarded as an opening lecture of the conference.)

It is well known that the symmetries of repeating patterns in the Euclidean plane belong to just one of 17 groups. Less well-known are the corresponding enumerations for frieze patterns (7 types) and patterns on the sphere (7 particular groups + 7 infinite series) and the hyperbolic plane, which supports a multiple infinity of groups. I shall describe some of these, and perhaps say something about the corresponding enumerations in 3 and more dimensions.

Abstract (Plenary Talks)

(In alphabetical order of the author's last name)

Thomas Banchoff (Brown University)
The Fourth Dimension and Salvador Dali

Salvador Dali was fascinated by geometric objects in the fourth dimension, including hypercubes and catastrophe surfaces. Where did he get his inspirations and how did he represent four-dimensional phenomena in his paintings? This talk will describe contacts with the artist over a ten-year period starting in 1975 as well as some new information about his sources.

J. Scott Carter (University of South Alabama), Tony Robbin (Artist and author)
Collaboration across the the two cultures

Starting from a mathematically simple idea — that the sum of cubes can be written as the square of a triangle — we begin investigating the aesthetics of the product of two isosoles right triangles from our respective disciplines. The investigation is simultaneuosly enhanced and obstructed by computer systems and electronic communications.

In this talk we will demonstrate the mathematical encryption of the figures, a 3 dimensional animation, some 2-dimensional projections, and rough sketches that indicate meaning of the figures. We also will show slides of some of Mr. Robbin's recent works. In the process, we will also discuss the importance of communication between us and how this on-going work has influenced our own points of view of our work.

Brent Collins (Artist), Carlo Séquin (University of California, Berkeley)

Analysis and synthesis of intuitively conceived geometrical art

The paper revisits some of the high points of a fruitful 12-year collaboration between Brent Collins, a wood sculptor who has created abstract geometrical art for several decades, and Carlo H. Séquin, a computer scientist who generates artistic geometric structures with computers and rapid prototyping machines. The contact between these two individuals was initiated by the 1992 special issue of Leonardo: "The Visual Mind." For the first two years all interactions took place over the phone. The flood of ideas generated during those weekly phone conversations prompted Squin to program a virtual visualization tool, called "Sculpture Generator I," to test out the emerging new ideas. Later the same tool created blue-prints for Collins, allowing him to carve more complex art objects than he would have been able to design himself with just ruler and compasses. Subsequently, Séquin captured the essence of different inspirational pieces carved intuitively by Collins, and created a modular sculpture design environment within the Berkeley SLIDE rendering program. In a recent collaboration, a first 6-foot diameter bronze sculpture has emerged from that program as a re-designed scaled-up version of Collins' "Pax Mundi" (1997). "Pax Mundi II" (2007) is located in the H&R Block headquarters in Kansas City.

Charles Frohman (University of Iowa)

The growth rate of the quantum hyperbolic invariants of Baseilhac and Benedetti

In the mid 1990's Kashaev developed topological invariants of a link in a triangulated manifold using the quantum dilogarithm. He then constructed a tangle functor for computing the invariants when the link is presented as a $(1,1)$-tangle in $\mathbb{R}^3$. He conjectured that the invariants grew exponentially in their level for hyperbolic knots where the growth rate is proportional to the hyperbolic volume of the complete hyperbolic structure on the complement of the knot. Later Murakami and Yokota realized that these invariants are evaluations of the colored Jones polynomial of the knot.

Through a careful analysis of the behavior of the quantum dilogarithm of Kashaev, Baseilhac and Benedetti were able to construct invariants of a manifold with torus boundary equipped with a representation of its fundamental group into $PSL(2,\mathbb{C})$. They call these the quantum hyperbolic invariants. The invariants depend on a branching, charge and flattening up to equivalence. We will look at how these choices effect the growth rate of the invariants presenting both numerical data for the figure eight knot and the knot 5_2 along with rigorous explorations of the phenomenah predicted by experiment.

Seiichi Kamada (Hiroshima University)

On braid description of surface-links in 4-space

The Alexander and Markov theorems state that links in 3-space are in one-to-one correspondence to braids modulo the equivalence relation generated by conjugation and stabilization. A similar result is valid for surface-links in 4-space. Surface-links in 4-space are in one-to-one correspondence to 2-dimensional braids modulo the equivalence relation generated by braid ambient isotopy, conjugation and stabilization. An alternative version of this theorem is also introduced; generalizing the notion of stabilization, we can remove braid ambient isotopy from the generating moves.

Mikhail Khovanov (Columbia University)

Categorification in examples

We'll review the informal principles and intuition of categorification, illustrating them on the way with a few examples and explaining the relation between categorification and geometrization.

Yasutaka Nakanishi (Kobe University)

Local moves and Gordian complexes

(This is a joint work with Yoshiyuki Ohyama.) We consider local moves, a crossing-change, a Delta-move, C_n-moves, a pass-move, and a sharp-move. It is well-known that these are strongly related to the Conway polynomial. We will consider to what degree the relationship is strong.

Ivars Peterson (Mathematical Association of America)

A Journey into Mathematical Art

From Fibonacci numbers and the digits of pi to tetrahedra, fractals, Moebius strips, and minimal surfaces, mathematics has inspired a wide variety of artists. Many people are familiar with the work of M. C. Escher and aware of the intertwining of math and art during the Renaissance, but the realm of mathematical art is far wider and more diverse than most people realize. An illustrated survey of contemporary math-related art illuminates these rich interactions.

Abstract (General Talks)

(In alphabetical order of the author's last name)

Hirotaka Akiyoshi (Osaka City University Advanced Mathematical Institute)

Ford domains of punctured torus groups

In the 1970s, T. Jorgensen characterized in his famous unfinished paper the combinatorial structures of the Ford domains of quasifuchsian punctured torus groups, and introduced an invariant, the side parameter, for such groups. The side parameter is extended to an invariant on the closure of the quasifuchsian space of the punctured torus. In this talk, we prove that the extended side parameter is actually a parameter for the closure of the quasifuchsian space, and that it has the same continuity as the end invariant map defined by using the conformal structures or the ending laminations for the ends of the Kleinian manifolds for the groups.

Kheira Ameur

Quandles with nontrivial higher homology groups

In this talk we prove the nontriviality of homology groups of all odd dimensions for some families of Alexander quandles. The dimension 2 and 3 are proved using colorings and shadow colorings of torus knots that evaluate nontrivially by a polynomial cocycle. Higher dimensional polynomial cocycles along with cycles constructed by purely algebraic methods are used to prove the higher dimensional cases.

Angela Angeleska, Nataša Jonoska, Masahico Saito (University of South Florida)

DNA Rearrangements through assembly graphs

Motivated by DNA rearrangements that appear in some species of ciliates, we define assembly graphs as graphs with rigid vertices of valency 4 and two vertices of valency 1. DNA recombination can be modeled by smoothing of 4-valent vertices. Successful recombinations are studied combinatorially through smoothings in assembly graphs.

Jeff Boerner (University of Iowa)

Khovanov homology of links embedded in i-bundles

M. Asaeda, J. Przytycki and A. Sikora introduced a theory for links embedded in i-bundles. A new way to view their theory is introduced for i-bundles over orientable surfaces. The elements of the chain groups are surfaces rather than diagrams which gives the calculation of the theory a much more topological feel. This is done by borrowing ideas from D. Bar-Natan's work involving Khovanov Homology for tangles and cobordisms.

Carmen Caprau (California State University, Fresno)

The universal $sl(2)$-link cohomology via webs and foams

Khovanov has explained how rank two Frobenius systems lead to link homology theories and showed that there is a universal one corresponding to a certain Frobenius algebra structure defined on $Z[X,h,t]/(X^2-hX-t)$. We will introduce a geometric approach to the universal $sl(2)$-link cohomology theory that corresponds to $Z[i][X,h,t]/(X^2-hX-t)$, using webs and singular cobordisms between them, called foams. This theory categorifies the unnormalized Jones polynomial and is properly functorial under link cobordisms. In particular, it yields an invariant for a surface-knot or -link.

Abhijit Champanerkar (University of South Alabama)

Graphs on surfaces and Khovanov homology

Oriented ribbon graphs (dessins d'enfant) are graphs embedded in oriented surfaces. A quasi-tree of a ribbon graph is a spanning subgraph with one face, which is described by an ordered chord diagram. We show that for any link diagram L, there is an associated ribbon graph whose quasi-trees correspond bijectively to spanning trees of the graph obtained by checkerboard coloring L. This correspondence preserves the bigrading used for the spanning tree model of Khovanov homology, whose Euler characteristic is the Jones polynomial of L. Thus, Khovanov homology can be expressed in terms of ribbon graphs, with generators given by ordered chord diagrams. This is joint work with Ilya Kofman and Neal Stoltzfus.

Alissa S. Crans (Loyola Marymount University)

Musical actions of dihedral groups

Can you hear the dihedral group? Or its centralizer in a symmetric group? In the same way it is possible to see group structure in a crystal, it is also possible to hear group structure in music. We will investigate two ways that the dihedral group of order 24 acts on the set of major and minor chords and illustrate both geometrically and algebraically how these two actions are dual. Both actions and their duality have been used to analyze works of music as diverse as that of Beethoven and the Beatles.

Brian Curtin (University of South Florida)

Spin Leonard pairs and representations of the braid group

Let V denote a vector space of finite positive dimension. A pair A, A^* of linear operators on V is said to be a Leonard pair on V whenever for each $C \in \{A, A^*\}$, there exists a basis of V with respect to which the matrix representing C is diagonal and the matrix representing the other member of the pair is irreducible tridiagonal. A Leonard pair A, A^* on V is said to be a spin Leonard pair whenever there exist invertible linear operators U, U^* on V such that $UA = AU$, $U^*A^* = A^*U^*$, and $UA^*U^{-1} = (U^*)^{-1}AU^*$. We characterize the spin Leonard pairs in terms of certain polynomials in the terminating branch of the Askey-scheme, and we describe a finite-dimensional representation of the braid group B_3 constructed from each spin Leonard pair.

Oliver Dasbach (Louisiana State University)

The Jones polynomial and graphs on surfaces

Turaev constructed to each knot diagram an orientable surface on which the knot projects alternatingly. The induced checkerboard graphs are thus two graphs embedded on the Turaev surface that are dual to each other. We show that the Bollobas-Riordan-Tutte polynomial of either of these checkerboard graphs specialices to the Jones polynomial of the knot. We will give some applications of this approach.

Heather Dye (Mckendree University)

Virtual braids homotopic to the identity braid

Two virtual braids are homotopic when one braid can be transformed into the other by a sequence of virtual and classical Reidemeister moves and self crossing changes. In this talk, I will describe the set of pure virtual braids that are homotopic to the identity braid. This talk will use representations of virtual links and results from classical homotopy theory.

John Etnyre (Georgia Institute of Technology)

Fibered knots and the Bennequin bound

In the early 80's Bennequin proved that the self-linking number of a transverse knot in the standard contact structure on S^3 was bounded above by minus the Euler characteristic of any Seifert surface for the knot. It turns out there is a elegant interaction between the optimality of the Bennequin inequality for fibered knots and Giroux's work on the relation between open books and contact structures. In this talk I will explain this interaction and give a precise characterization of when the Bennequin bound is optimal for fibered knots.

Alex Feingold (State University of New York at Binghamton)

Mathematics as Art: Links and Knots in space and on surfaces

I will describe some of my experiences making knots and links out of wood, metal and stone. I have made such objects by carving wood or stone, by bending metal, and by casting bronze using the lost wax method. Mathematics offers many ideas and inspirations for the artist, challenging the audience to understand very abstract ideas in real solid forms. A mathematical knot is a continuous image of a circle which may be embedded in space or on a surface. A sculptured knot must have some thickness, so its cross-section can have an interesting geometry, giving a global structure that can, for example, illustrate the concept of a nontrivial fiber bundle. I have also found that some of my metal sculptures can be displayed in such a way that they can freely rotate, balanced on a pointed rod, or hanging from a wire, and when struck by a hammer they ring like a bell, earning the name of kinetic sound sculpture.

Kenichi Fujiwara (Tokyo Institute of Technology)

Re-construction of the Casson-Walker invariant for some rational homology spheres

Let M be an integral homology 3-sphere. Then, we can present M by surgery on a framed link with framings +/- 1 and linking numbers 0. Moreover, two such link-surgery presentations yield homeomorphic manifolds if and only if they are related by a sequence of refined Kirby calculus due to K. Habiro.

When M is a rational homology 3-sphere with its first homology group of prime order p=4n-1, the speaker proved a similar theorem. Here, we may assume that every link-surgery presentation consists of link-components with linking numbers 0 and framings +/- 1 except one p-framed component.

In this talk, we apply this theorem to re-construct the Casson-Walker invariant for those rational homology 3-spheres.

Daniela Genova (University of North Florida)

Topological properties of a biomolecular computing model

This is a summary of joint work with N. Jonoska on topological properties of forbidding-enforcing systems. DNA molecules can be conveniently represented as strings and actions of restriction enzymes as operations on strings. Hence, DNA computing models are often studied in the context of Formal Language Theory. Forbidding-enforcing systems (fe-systems) model molecular processes by defining classes of languages (fe-families) based on boundary conditions. Forbidding conditions disallow certain combinations of strings (molecules) and enforcing conditions imply the presence of new strings (molecules) if some combinations of strings are already present. We consider the topological space on the set of languages consisting of finite strings over a finite alphabet along with the standard language metric and show that this space is homeomorphic to the Cantor space. We specify the necessary and sufficient conditions under which the fe-families are open in that space. It follows that none of the Chomsky families of languages can be defined by a fe-system. Hence, fe-systems define completely new classes of languages.

Patrick Gilmer (Louisiana State University)

Congruence and quantum invariants

Let f be an integer greater than one. We study three progressively finer equivalence relations on closed 3-manifolds generated by Dehn surgery with denominator f: weak f-congruence, f-congruence, and strong f-congruence. If f is odd, weak f-congruence preserves the ring structure on cohomology with $\mathbb{Z}_f$-coefficients. We show that strong f-congruence coincides with a relation previously studied by Lackenby.

Lackenby showed that the quantum $SU(2)$ are well-behaved under this congruence. We strengthen this result and extend it to the $SO(3)$ quantum invariants. We also obtain some corresponding results for the coarser equivalence relations, and for quantum invariants associated to more general modular categories. We give finite lists of the only possible f for which there can be strong f-congruences between S^3, $\pm\Sigma(2,3,5)$ and $\pm\Sigma(2,3,7)$. Moreover we realize some of these strong congruences. We distinquish 0-framed surgery on the Whitehead link and $\#^2 S^1 \times S^2$ up to weak f-congruence for f an odd prime greater than three.

Chaim Goodman-Strauss (University of Arkansas)

Mathematical Illustration

Though illustration is not generally regarded as central to the mathematician's craft, many of the tools required are especially suited to those with an analytical outlook. On the occasion of the publication of "The Symmetries of Things" and the artistic focus of this conference, I would like to share tricks from the mathematical illustrator's toolbox.

George W. Hart (Stony Brook University)

The Truncated 120-Cell

The conference art exhibit will include a six-foot diameter construction of the Truncated 120-Cell. This model is an orthogonal projection, into three dimensions, of a four-dimensional uniform polytope first described a century ago. Several thousand small components are to be physically assembled as a "mathematics barn raising" by a group of volunteers. In such a group event, the participants discover the mathematical patterns and relationships of the structure in an immediate hands-on manner, thereby gaining a deeper appreciation of its beauty. My talk will highlight some of the mathematical ideas underlying the form and will illustrate the long history of three-dimensional models of four-dimensional polytopes.

Chuichiro Hayashi (Japan Women's University), Miwa Iwakura (Japan Women's University)

Q-fundamental surfaces in lens spaces

Fudamental surafces are in simple positions with respect to trianglations of 3-manifolds. For naturally triangulated (p, q)-lens spaces with $q = 1$ and 2, we obtain complete lists of Q-fundamental surfaces with respect to the Q-matching equations introduced by J. Tollefson. Among them, non-orientable ones give examples of vertex surfaces (with respect to the usual matching equations) which are Q-fundamental and not Q-vertex surfaces. For general coprime pairs of integers (p, q), we look into a certain class of Q-fundamental surfaces which we expect to contain a non-orientable surface with maximal Euler characteristic, and then make comparison with the result by Bredon and Wood.

Allison Henrich (Dartmouth College)

New Vassiliev Invariants for Virtual Knots

I will introduce a collection of new degree 1 Vassiliev invariants for virtual knots, including a universal Vassiliev invariant of degree 1. We will explore some applications and discuss which groups may be realized as values of these invariants.

Kazuhiro Ichihara (Nara University of Education)

On the maximal number of exceptional surgeries

The well-known Hyperbolic Dehn surgery Theorem due to W.P. Thurston says that every hyperbolic knot admits only finitely many Dehn surgeries yielding non-hyperbolic manifolds, called exceptional surgeries. Concerning the maximal number of exceptional surgeries, C.McA Gordon conjectured that they are at most 10 for each knot, and the knot admitting 10 is only the figure-eight knot in the 3-sphere. In this talk, it will be shown that each hyperbolic knot in the 3-sphere admits at most 10 integral exceptional surgeries. Based on this, it will be shown that each hyperbolic alternating knot in the 3-sphere admits at most 10 exceptional surgeries.

Ayumu Inoue (Tokyo Institute of Technology)

Two kinds of knot quandles and their representations

For any dimensional knots, we can define two kinds of quandles, the Joyce's knot quandle and a conjugate quandle, called the reduced knot quandle, of a knot group. It is known that the reduced knot quandle is a quotient of the Joyce's knot quandle and any representation of a reduced knot quandle to a fixed quandle induces one of a Joyce's knot quandle to the quandle. In this talk, we show that the set of all representations of a reduced knot quandle to an Alexander quandle is calculated by the first homology group of the infinite cyclic covering space of the knot complement. Furthermore, in classical case, we show the set of all representations of a Joyce's knot quandle to an Alexander quandle coincides with the reduced knot quandle's one.

Atsushi Ishii (RIMS, Kyoto University), Kokoro Tanaka (Gakushuin University)

Khovanov homology for virtual links with two types of maps for Möbius cobordisms

Khovanov homology is a homology theory for (classical) links which is a categorification of the Jones polynomial. If we want to extend Khovanov homology to virtual links, Khovanov's construction does not immediately work and the main difficulty arising is the existence of Möbius cobordisms (bifurcations of type $1 \to 1$). Recently, V. O. Manturov succeeded in extending Khovanov homology to virtual links. In his construction, the zero map is assigned to each of the Möbius cobordisms because of the grading reasons.

In this talk, we construct a new extension of Khovanov homology to virtual links by taking suitable grading shifts and assigning one of two non-zero maps to each of the Möbius cobordisms. Our homology theory is a categorification of a one-variable specialization of the Miyazawa polynomial, which is known as a multi-variable generalization of the Jones-Kauffman polynomial.

Masahide Iwakiri (Osaka City University)

The lower bound of the w-indices of surface links via quandle cocycle invariants

The w-index of a surface link F is the minimal number of the triple points of surface braids representing F. In this talk, for a given 3-cocycle, we consider the minimal number of the w-indices of surface links whose quandle cocycle invariants associated with the 3-cocycle are non-trivial. As a consequence, for given a non-negative integer g, there are surface knots with genus g such that the w-index of them are 6.

Yasuyuki Miyazawa (Yamaguchi University)

Unoriented virtual links and a polynomial invariant

In this talk, we introduce a multi-variable polynomial invariant H for oriented virtual links derived from a virtual magnetic graph with oriented vertices, and then, we construct a multi-variable polynomial invarinat Y for unoriented virtual links as a certain weighted sum of the H-polynomials on virtual links associated with a given virtual link. We show some features of the Y-polynomial including an evaluation of the virtual crossing number of a virtual link.

Fumikazu Nagasato (JSPS research fellow, Tokyo Institute of Technology), Yoshikazu Yamaguchi (COE fellow, University of Tokyo)

On the geometry of certain slices of the character variety of knot groups

For a knot K in 3-sphere, we can consider representations of the knot group G_K into $SL(2,\mathbb{C})$. Their characters construct an algebraic set. This is so-called the $SL(2,\mathbb{C})$-character variety of G_K and denoted by $X(G_K)$.

In this talk, we introduce a slice (a subset) $S_0(K)$ of $X(G_K)$, which is closely related to the Casson-Lin invariant, the A-polynomial and abelian knot contact homology. This slice $S_0(K)$ has a structure of 2-fold branched covering. Then we show that the branched point set consists entirely of the characters of metabelian representations. We mainly explain these facts.

Teruo Nagase (Tokai University), Akiko Shima (Tokai University)

On surface braids represented by charts with two crossings

We show that if a surface braid is represented by a chart with two crossings, and if its closure is a disjoint union of 2-spheres, then its closure is a ribbon surface. We introduce 'tangles' in charts. We study tangles in a disk which does not contain any crossings and whose boundary intersects three consecutive labels.

Takuji Nakamura (Osaka Electro-Communication University)

Pass-move and Conway polynomial

A pass-move is one of local moves for knots. It is known that a pass-move keeps the parity of the second coefficient of the Conway polynomial. So we see that the trefoil knot cannot be unknotted by a finite sequence of pass-moves. In this talk, we will discuss a realization problem of the Conway polynomial by a knot with respect to a pass-move. This is a joint work with Yasutaka Nakanishi (Kobe University).

Sam Nelson (Pomona College)

Quandles and linking number

As a complete invariant of classical links up to reflection, many link invariants should be derivable from the knot quandle. We will see a family of finite quandles whose counting invariants determine the linking number of a two-component link up to sign.

Maciej Niebrzydowski (University of Louisiana at Lafayette)

Quandle of trefoil knot as Dehn quandle of torus

In this talk, we will describe the isomorphism between the universal quandle of the trefoil knot (as defined by Joyce) and the Dehn quandle of the torus (as introduced by Zablow). This is a joint work with Jozef Przytycki.

Ryo Nikkuni (Kanazawa University)

Homotopy on spatial graphs and generalized Sato-Levine invariants

Edge-homotopy and vertex-homotopy are equivalence relations on spatial graphs which are generalizations of Milnor's link-homotopy. T. Fleming and the speaker introduced some edge (resp. vertex)-homotopy invariants of spatial graphs by applying the Sato-Levine invariant for the 2-component constituent algebraically split links. In this talk, we construct some edge (resp. vertex)-homotopy invariants of spatial graphs without any restriction of linking numbers of 2-component constituent links by applying the generalized Sato-Levine invariant.

Kanako Oshiro (Hiroshima University)

Trivial quandles and surface knot invariants

For any trivial quandle, the involutions are good involutions. Using a trivial quandle with a good involution and the quandle homology group of order 3, we can obtain several surface link invariants. In this talk, we explain that the invariants are equivalent to triple linking invariants and $\mathbb{Z}_2$-triple point invarients.

Makoto Ozawa (Komazawa University)

Essential state surfaces for knots and links

We introduce a canonical spanning surface obtained from a knot or link diagram depending on a given state, and give a sufficient condition for the surface to be essential. Using the essential surface, we can see the triviality and splittability of a knot or link from its diagrams.

Val Pinciu (Southern Connecticut State University)

Edge unfoldable polyhedra in mathematics and art

The question whether a convex polyhedron can be cut along edges and unfolded into a non-overlapping net goes back to the German renaissance painter Albrecht Durer. His book "The Painter's Manual" includes a description of many polyhedra, which he presented as surface unfoldings, what are now called nets. The first formal description of the problem is due to Shephard who in 1975 conjectured that every convex polyhedron can be cut along edges and unfolded into a non-overlapping net. While this problem is still open, we present several classes of polyhedra for which Shephard's conjecture is true. For general polyhedra we present a bound for the number of non-overlapping nets.

Jozef H. Przytycki (George Washington University)

History of knot theory: art and science

Our goal is to present the history of ideas which lead up to the development of modern knot theory. We argue, using our "case study" that mathematics is an actively developing field which has more questions than solutions and some of the questions are very elementary.

When did knot theory start? Was it in 1794 when the (future) famous mathematician Johann Carl Friedrich Gauss (1777-1855) copied figures of knots from an English book? Or before that, in 1771, when A.Vandermonde considered knots and braids as a part of Leibniz' analysis situs? Possibly drawings by Leonardo da Vinci and Durer should be taken into account.

We can go back in time even further to ancient Greece where surgeons were considering sling knots. Even earlier, we can enjoy ancient stamps and seals with knots and links as their motifs. One of the oldest example, I am aware of, is from the pre-Hellenic Greece. Excavations at Lerna by the American School of Classical Studies discovered two rich deposits of clay seal-impressions. The second deposit dated about 2200BC contains several impressions of knots and links.

Knot Theory has a long history but despite this, or maybe because of this, one still can find inspiring elementary open problems. These problems are not just interesting puzzles but they lead to very interesting theory (structure).

Tony Robbin (Artist, author)

Painting and higher-dimensional projective space

My talk will briefly review five periods of my artwork from 1970 to the present, and show how each period investigated a different aspect of four-dimensional geometry. Focusing on current work, I will discuss higher-dimensional projection and how this mode of representation squares with the geometry of special relativity.

Yongwu Rong (George Washington University)

Categorifications for knots, plane graphs, and Feynman diagrams

It is well-known that knots, plane graphs, and Feynman diagrams (a.k.a. chord diagrams) are intimately related to each other. We investigate several categorifications associated to these objects. As a result, we obtain Khovanov type homologies that are related to the Alexander polynomial, the Penrose polynomial, and the transition polynomial respectively. Most of the work here is jointly with Kerry Luse.

Toshio Saito (Nara Women's University)

Meridionally destabilizing number and connected sum of knots

It is well-known that the tunnel number of a knot is not super-additive under connected sum if the exterior of a factor knot admits a minimal genus Heegaard splitting with meridional destabilization. Moreover, Morimoto has shown that the converse is also true if the factor knots are tunnel number one. This indicates that such a composite knot is tunnel number two if and only if a factor knot is a (1,1)-knot. On the other hand, if a composite knot admits a minimal genus Heegaard splitting with meridional destabilization, then what do we know about factor knots?

Based on this standpoint, we discuss a relationship between the meridionally destabilizing number of a composite knot and that of a factor knot.

Shin Satoh (Kobe University)

On tricolorable 2-knots of triple point number four

If a 2-knot has a non-trivial cocycle invariant associated with tricolorings, then the following are equivalent; (i) the triple point number is equal to four, and (ii) it is ribbon-concordant to the 2-twist-spun trefoil.

Radmila Sazdanovic (Graduate student GWU), Slavik Jablan (Mathematical Institute, Serbia)

Unlinking gap

Computing unlinking number is a very difficult and complex problem. Therefore we define BJ-unlinking number and BJ-unlinking gap which will be computable due to the algorithmic nature of their definition. According to the Bernhard-Jablan conjecture unknotting/unlinking number is the same as the BJ- unlinking number. We compute BJ-unlinking number for various families of knots and links for which the unlinking number is unknown and give experimental results for families of rational links with arbitrarily large BJ-unlinking gap and polyhedral links with constant non-trivial BJ-unlinking gap.

Nadrian C. Seeman (New York University)

DNA nanotechnology inherently leads to knots and catenanes

DNA nanotechnology entails the assembly of branched DNA components into larger objects and lattices. The double helical character of DNA molecules implies that these structures contain interlocked molecular strands that can produce knots and catenanes. A half-turn of right-handed B-DNA (about six nucleotide pairs) leads to a unit tangle. The sign can be reversed by using left-handed Z-DNA. Catenanes whose edges are linked like a cube and a truncated octahedron have been built. Borromean rings have aso been constructed. A variety of small knots have also been assembled. The limiting factor in DNA-based molecular topology is characterization of the product.

Carlo Séquin (University of California, Berkeley)

Knotty sculptures

This presentation explores the use of simple knots as constructivist building blocks for abstract geometrical sculptures. One approach places simple n-foil knots on the n-sided faces of a Platonic or Archimedean polyhedron. In the simplest case, four trefoil knots form an interlinked tetrahedral configuration. Following this principle, twelve pentafoil knots can form an almost spherical configuration of dodecahedral symmetry. Another investigation explores various generating principles for the construction of recursive knots. For instance, a simple crossing of two strands is replaced with a more complicated tangled version of two strands, and the process is then repeated recursively. Alternatively, a suitable projection of a knot is deformed, so that a scaled-down copy of the knot can be placed onto a sub-structure of the original knot. Then the details of the connections are suitably re-arranged to form a single knot. A third study looks for ways to densely pack 3D space with mutually interlinking knots. The simplest case is a union of three simple arrays of toroidal unknots in three mutually orthogonal directions. A more complicated web is obtained by interlacing Figure-8 knots.

Reiko Shinjo (Osaka City University)

An infinite sequence of non-conjugate braids whose closures result in the same knot

By the Classification Theorem of closed 3-braids proved by J. Birman and W. Menasco, for $n = 1, 2$ or 3 it is known that there are only finitely many mutually non-conjugate n-braids having the same closure. On the other hand, infinite sequences of non-conjugate 4-braids having the same closure are known. In this talk, for any $n \geq 4$ we give a knot of braid index n which has an infinite sequence of pairwise non-conjugate n-braids which close to the knot.

Alexander Shumakovitch (The George Washington University)
Finite-type invariants of divide knots

A divide is the image of a generic (relative) immersion of a 1-dimensional compact manifold into the standard unit disk D^2. To every divide P, one can associate a link $L(P)$ in the 3-sphere S^3 using a procedure developed by Norbert A'Campo in 1974. In this talk, we restrict ourselves to the case of P being an immersion of a closed interval. Then $L(P)$ is a knot, called a divide knot. Divide knots arise naturally in the study of real morsifications of isolated complex plane curve singularities.

We start the talk by briefly outlining the construction of divide knots and their main properties. We then describe explicit formulas for some finite-type invariants of divide knots and discuss various corollaries. The formulas are written in terms of Arnold's invariants of pieces of the divide.

Toshie Takata (Niigata University)
A complete set of relations for Ohtsuki's invariants of integral homology 3-spheres

We present a way to obtain a complete set of relations which characterize the set of perturbative invariants identifying the set of the logarithm of the LMO invariant for integral homology 3-spheres. Specially, we give a complete set of relations for Ohtsuki's invariants up to degree 6.

Toshifumi Tanaka (Osaka City University)
Maximal Thurston-Bennequin number of doubled knots

We give formulas for the maximal Thurston-Bennequin numbers for doubles of a positive torus knot and a two-bridge knot, by showing that the upper bound given by the Kauffman polynomial is sharp for doubles of positive knots and alternating knots. We also show that the Rasmussen invariant of any iterated untwisted positive double of a knot with non-negative maximal Thurston-Bennequin number equals two.

Reidun Twarock (University of York)
Hidden symmetries in virus architecture: art created by nature

It has long been known that viruses use icosahedral symmetry in the organisation of their protein shells that encapsulate and hence provide protection for the viral genome. In this talk I will show that symmetry is even more important for virus architecture than previously appreciated: The full three-dimensional structure of certain classes of viruses, including the thickness and shape of their protein containers and the organisation of the genomic material inside, is orchestrated by the monomial sets of certain infinite-dimensional groups. A classification of these groups leads to 41 blueprints that encode the essential structural features of these viruses and provide new insides in the mechanisms responsible for their assembly and evolution.

Yoshiaki Uchida (Yamagata University)
Many non-conjugate braids whose closures represent same knot via covering space

In this talk I will consider the Classification Theorem of closed braids. R. Shinjo give the proof of there exist an infinite sequence of pairwaise non-conjugate braid whose closure is same knot. She use Conway polynomial. I will give another proof by considering cyclic covering space. And I will mention the similar conjecture for 2-knot braid.

Hao Wu (George Washington University)

The Khovanov-Rozansky cohomology of Legendrian and transversal links

The study of Legendrian and transversal links is a central topic in 3-dimensional contact topology. I will review estimates of classical invariants of Legendrian and transversal links in the standard contact 3-space established using quantum link invariants. I will also explain how to use Wagner's Composition Product to give a proof of such an estimates from the Khovanov-Rozansky cohomology, which generalizes Jaeger's proof of the Morton-Franks-Williams inequality.

Ryosuke Yamamoto (Osaka City University Advanced Mathematical Institute)

On Alexander polynomials of fibered knots in 3-manifolds

As information about a fibered knot in a closed orientable 3-manifold, we focus on algebraic intersection numbers of essential arcs on the fiber surface with their images of the monodromy map of the fibered knot. We will see how Alexander polynomial of the fibered knot is calculated using this information and see that we may describe each coefficient of Conway polynomial of the knot by Pfaffians of some matrices of the algebraic intersection numbers.

Akira Yasuhara (Tokyo Gakugei University)

Self delta-equivalence for links whose Milnor's isotopy invariants vanish

For an n-component link L, the Milnor's isotopy invariant is defined for each multi-index $I = i_1 i_2 ... i_m$ $(i_j \in \{1, 2, ..., n\})$. Here m is called the length. Let $r(I)$ denote the maximum number of times that any index appears. It is known that Milnor invariants with $r = 1$ are link-homotopy invariant. N. Habegger and X. S. Lin showed that two string links are a link-homotopc if and only if their Milnor invariants with $r = 1$ coincide. This gives us that a link in S^3 is link-homotopic to a trivial link if and only if the all Milnor invariants of the link with $r = 1$ vanish. Although Milnor invariants with $r = 2$ are not link-homotopy invariants, T. Fleming and the speaker showed that Milnor invariants with $r \leq 2$ are self Δ-equivalence invariants. In this talk, we give a self Δ-equivalence classification of the set of n-component links in S^3 whose Milnor invariants with length $\leq 2n - 1$ and $r \leq 2$ vanish. As a corollary, we have that a link is self Δ-equivalent to a trivial link if and only if the all Milnor invariants of the link with $r \leq 2$ vanish.