

Polygonal Hamiltonian Paths of Assembly Graphs

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1 Introduction

This file is located at <http://www.math.usf.edu/~saito/DNAweb/table.pdf>, and is one of the files posted at the web site <http://www.math.usf.edu/~saito/DNAweb/>. The web site contains information and results on *assembly graphs*, that are mathematical models for DNA recombination processes. See <http://www.math.usf.edu/~saito/DNAweb/background.pdf> for background materials, which will be referred to as [BG].

This file will be updated often during 2009–2013 to post our current results on polygonal Hamiltonian paths of assembly graphs.

This document contains a definition of equivalence of polygonal paths, and study of equivalence classes of Hamiltonian polygonal paths for small assembly graphs in the table.

2 Equivalences of polygonal paths

Note that every polygonal path has three other variations by changing the edge at the two ends of the given path, by switching turns from right to left and vice versa. Thus it is natural to define these variations as equivalent. Otherwise we take isomorphisms of graphs to define equivalence as follows.

Definition 2.1 Let $\gamma_i = ((e_0^{(i)} \setminus v_0^{(i)}), v_1^{(i)}, e_1^{(i)}, \dots, e_m^{(i)}, v_m^{(i)}, (e_{m+1}^{(i)} \setminus v_{m+1}^{(i)}))$, for $i = 1, 2$, be two open polygonal paths in an assembly graph Γ .

The two paths γ_1 and γ_2 are called *end equivalent* if $v_j^{(1)} = v_j^{(2)}$ and $e_j^{(1)} = e_j^{(2)}$ for $j = 1, \dots, m$. In other words, they are end equivalent if they match except end arcs.

The two paths γ_1 and γ_2 are said *equivalent by graph isomorphism* if there is an isomorphism $f : \Gamma \rightarrow \Gamma$ of rigid vertex graphs such that $f(\gamma_1) = \gamma_2$.

The two paths are *equivalent* if they are equivalent under the equivalence relation generated by end equivalence and equivalence by graph isomorphism.

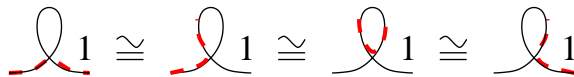


Figure 1: Polygonal path is unique for 1-letter word

Remark 2.2 For simple assembly graphs with two end points, there are at most two graph isomorphisms: the identity and the reversal. The reversal is an isomorphism that exchanges the end points.

If an assembly graph is palindromic, then there is a reversal. Otherwise there is no non-trivial isomorphism.

3 Simple assembly graphs with two end points

Since the path at the end vertex does not change the equivalence class of paths, we use a convention in diagrams that do not specify and indicate the portion of the path at the end. Specifically, if $\gamma = ((e_0 \setminus v_0), v_1, e_1, \dots, e_m, v_m, (e_{m+1}) \setminus v_{m+1})$ is a polygonal path, then we draw a path $(v_1, e_1, \dots, e_m, v_m)$ in figures. See Fig. 2 for this convention.

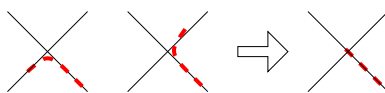


Figure 2: Abbreviating end edges

Lemma 3.1 *There is a unique polygonal Hamiltonian path for the assembly graph of length 1.*

Proof. All the cases are equivalent as depicted in Fig. 1. \square

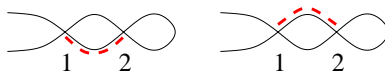


Figure 3: Polygonal paths for 2-letter word 1221

Lemma 3.2 *There is a unique class of Hamiltonian polygonal path for the assembly word 1221. There are two classes for 1212.*

Proof. For 1221, there are two edges from the vertex 1 to vertex 2. Two paths, each containing each of these two edges, are depicted in Fig. 3. All the others are end equivalent to these two. These two are equivalent by graph isomorphism. Hence we have a unique class. \square

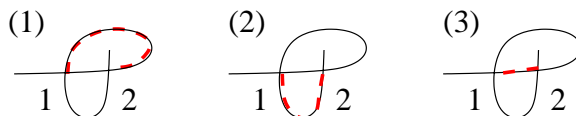


Figure 4: Polygonal paths for 2-letter word 1212

For 1212, there are three edges from the vertex 1 to vertex 2. The corresponding end representative paths are depicted in Fig. 4, (1) – (3). There is a graph isomorphism of this graph Γ reversing the order of 1 and 2, and the paths (2) and (3) in the figure are equivalent under this isomorphism. Hence there are two equivalence classes of paths represented by (1) and (2). \square

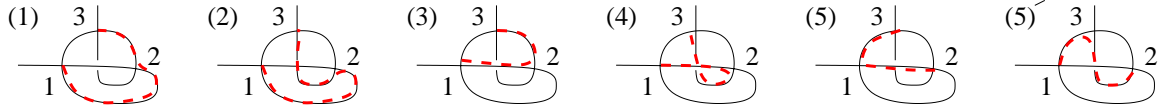


Figure 5: Polygonal paths for 3-letter word 121323

Lemma 3.3 *There are two classes of polygonal Hamiltonian paths for 121323.*

Proof. The word 121323 is palindromic, and there is a graph isomorphism switching the order of the word (transposition of 1 and 3). All paths starting from the vertex 3, under this isomorphism, are mapped to paths that start with 1, so that we list the paths starting 1 and 2. There are two edges from 1 to 2, and for each case, there are two edges from 2 to 3. The total four corresponding paths are depicted in Fig. 5. In (5) and (5)', paths starting from 2 are depicted. Since there is only one edge from 2 to 1 and one edge from 2 to 3, there is a unique path going $2 \rightarrow 1 \rightarrow 3$, and a unique path going $2 \rightarrow 3 \rightarrow 1$, respectively. These two paths are equivalent. Hence there are total five representatives. \square