Chirality of Assembly Graphs

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1 Introduction

This file is located at http://www.math.usf.edu/∼saito/DNAweb/chirality.pdf, and is one of the files posted at the web site http://www.math.usf.edu/∼saito/DNAweb/. The web site contains information and results on *assembly graphs*, that are mathematical models for DNA recombination processes. See http://www.math.usf.edu/∼saito/DNAweb/bachground.pdf for background materials, which will be referred to as [BG].

This file will be updated often during 2009–2013 to post our current results on chirality of assembly graphs.

2 Methods to detect chirality

Many assembly graphs are planar. For a relation between planarity and chirality, we have the following well known observation.

Lemma 2.1 *If a spatial graph lies on a plane, then is it achiral.*

Proof. It is easy to see that the 180° turn of the plane produces the mirror of the given graph. □

To detect planarilty, we use an analog of Kauffman's *odd writhe* [**?**] originally defined for knots. A concept similar to below can also be found in [**?**].

Definition 2.2 Let w be a double occurrence word. A letter j is said to be *odd* if the number of the letters between two copies of j, that are neither 1 nor the last letter of w, is an odd integer.

The number of odd letters is called the *odd index* of w and denoted by $I_{\text{odd}}(w)$. The same terms are defined corresponding assembly graphs, and the odd index of an assembly graph Γ is denoted by $\text{odd}(\Gamma)$.

Lemma 2.3 *If* $\text{odd}(\Gamma) > 0$*, then* Γ *is not planar.*

Proof. The proof is similar to that in [**?**]. □

Kauffman [**?**] gave two methods of constructing families of knots from a given spatial graph with rigid vertices. We review his methods.

[1] Knots in graphs.

[2] Replacements of rigid vertices by symmetric tangles.

We combine these methods with an analog of doubling that is used in knot theory. Here we define a doubling for diagrams of assembly graphs.

Definition 2.4 A *double* of a diagram of a spatial assembly graph Γ is a spatial assembly graph $D(\Gamma)$ obtained from Γ by taking parallel edges according to the blackboard framing, as indicated in Fig. **??**.

A 1-valent vertex is replaced by a hairpin curve, and a 4-valent vertex is replaced by four copies of it formed by four parallel strings as shown.

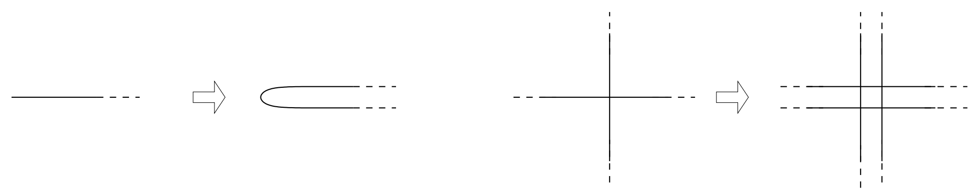

Figure 1: Doubling assembly graphs

Since the doubling depends on the blackboard framing, it is not an invariant of a given assembly graph. Considerations on writhe is important when this method is applied.

3 Chirality of assembly graphs in the table

We examine the assembly graphs in the table posted at http://www.math.usf.edu/∼saito/DNAweb/table.pdf.

3.1 Simple assembly graphs with two end points

- Lengths 1 and 2:

  In this case, all graphs are planar, hence all are achiral.

- Length 3:

  From Lemma **??**, there are two non-planar graphs: 121323 and 123231. Our first goal would be to see whether these are chiral.