A Weighted Moving Average Process for Forecasting “Economics and Environment”

By

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Basic Definition

The Difference Filter is defined as

$$(1-B)^d$$

Where

$$B^j x_t = x_{t-j}$$

- We can achieve stationarity after taking appropriate number of differencing.

The General ARIMA Model

The General ARIMA(*p*,*d*,*q*) is defined by

$$\phi_p(B)(1-B)^d x_t = \theta_q(B)\varepsilon_t$$

where $\phi_p(B) = (1-\phi_1 B-\phi_2 B^2-\ldots-\phi_p B^p)$

$$\theta_q(B) = (1-\theta_1 B-\theta_2 B^2-\ldots-\theta_q B^q)$$

and

$\{x_t\}$ is the realized time series

ϕ_p, θ_q are the coefficients

ε_t is the white noise

Model Development

- Check stationarity of the time series by determining the order of differencing *d*, where $d = 0,1,2,\ldots$
- Apply each *d* to KPSS test, until we achieve stationarity
- Deciding the order *m* of the process, for our case, we let $m = 5$
- After (*d*, *m*) being selected, listing all possible set of (*p*, *q*) for $p + q \le m$
- For each set of (*p*, *q*), estimating the parameters of each model, that is, $\phi_1, \phi_2, \ldots, \phi_p, \theta_1, \theta_2, \ldots, \theta_q$
- Compute the AIC for each model, and choose the smallest AIC

Analytical Structure for Our Proposed Model

- The *k*-th Moving Average of a series $\{x_t\}$ is defined by

$$y_t = \frac{1}{k}\sum_{j=0}^{k-1} x_{t-k+1+j}$$

where $t = k, k+1, \ldots, n$

- Apply the Classical Approach for series $\{y_t\}$
- Use the following back-shift operator to obtain the original series

$$\hat{x}_t = k\,\hat{y}_t - x_{t-1} - x_{t-2} - \ldots - x_{t-k+1}$$

Daily Closing Price for Stock XYZ

Classical Model

According to the procedure of the classical approach, we have the ARIMA(1,1,2) model as follows:

$$(1-.9631B)(1-B)x_t = (1-1.0531B+.0581B^2)\varepsilon_t$$

After we expanding the model, we have

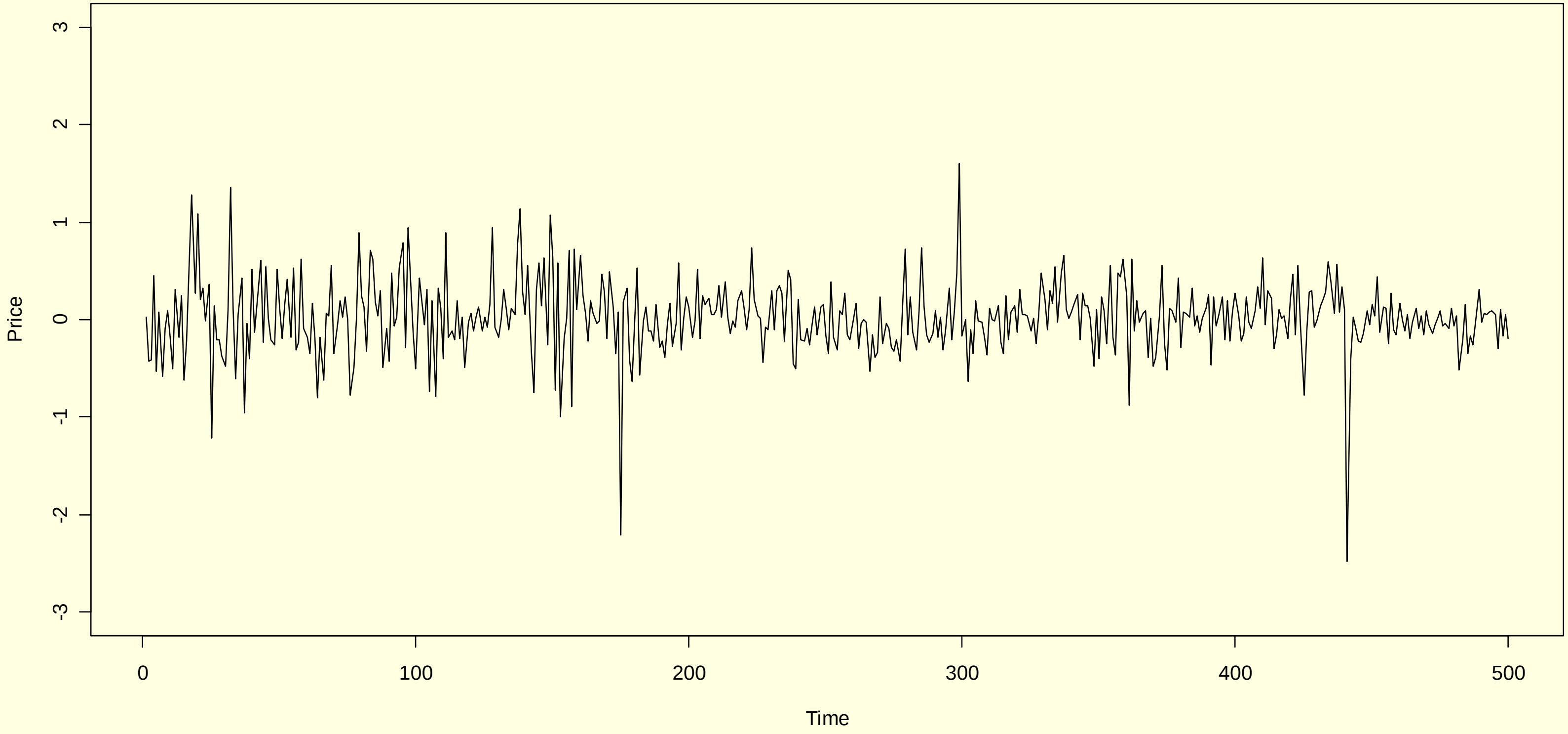
$$x_t = 1.9631x_{t-1} - .9631x_{t-2} + \varepsilon_t - 1.0531\varepsilon_{t-1} + .0581\varepsilon_{t-2}$$

By letting $\varepsilon_t = 0$, we have the one day ahead forecasting time series as

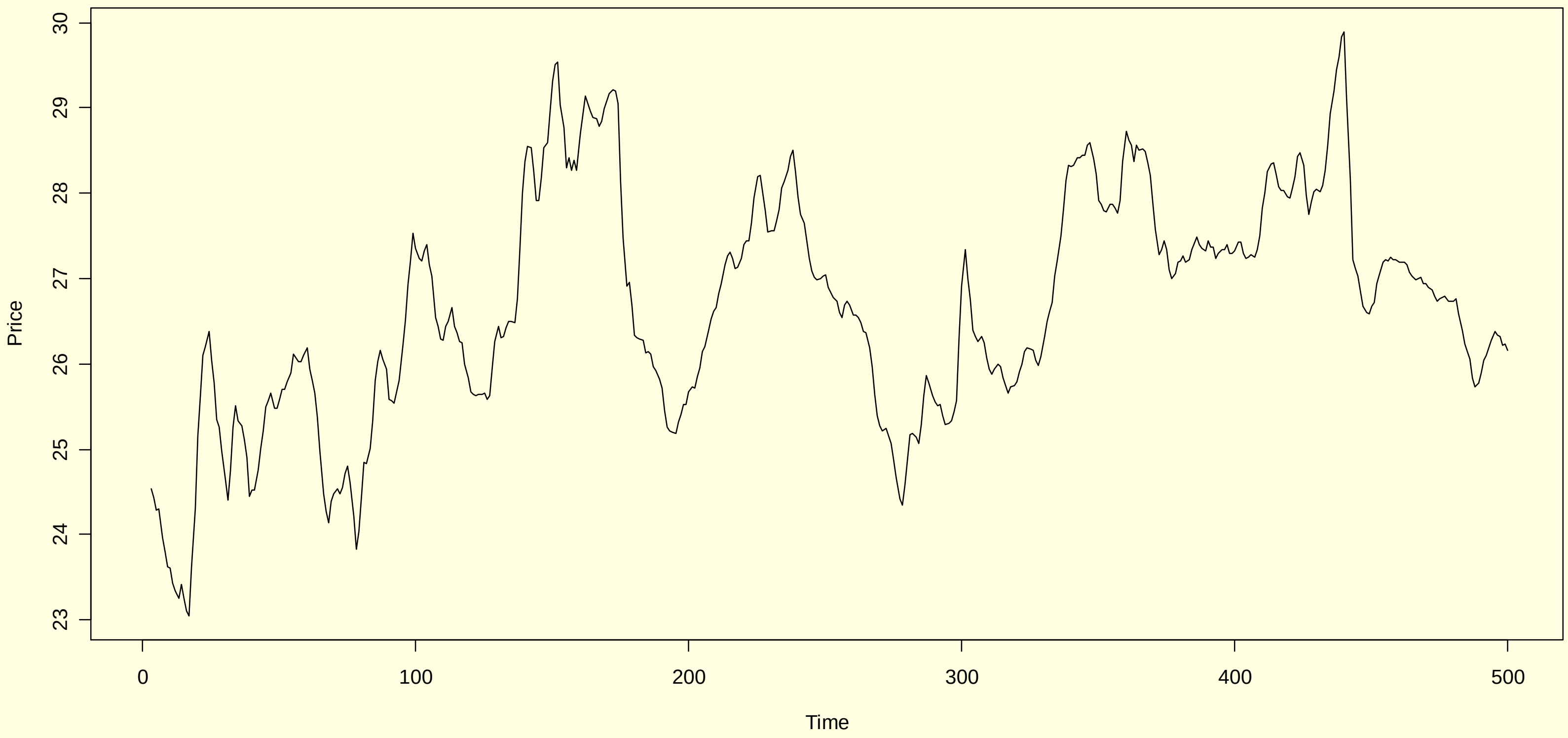
$$\hat{x}_t = 1.9631x_{t-1} - .9631x_{t-2} - 1.0531\varepsilon_{t-1} + .0581\varepsilon_{t-2}$$

Comparisons on Classical ARIMA Model VS. Original Time Series

Time Series Plot of the Residuals for Classical Model



Three Days Moving Average on Daily Closing Price of Stock XYZ



Our Proposed Model

Transform the series $\{x_t\}$ into $\{y_t\}$ by using

$$y_t = \frac{x_{t-2} + x_{t-1} + x_t}{3}$$

Following the classical approach, the best model on $\{y_t\}$ is ARIMA(2,1,3), that is

$$(1 - .8961B - .0605B^2)(1 - B)y_t = (1 + .0056B - .0056B^2 - B^3)\varepsilon_t$$

and the final analytical form of the model is

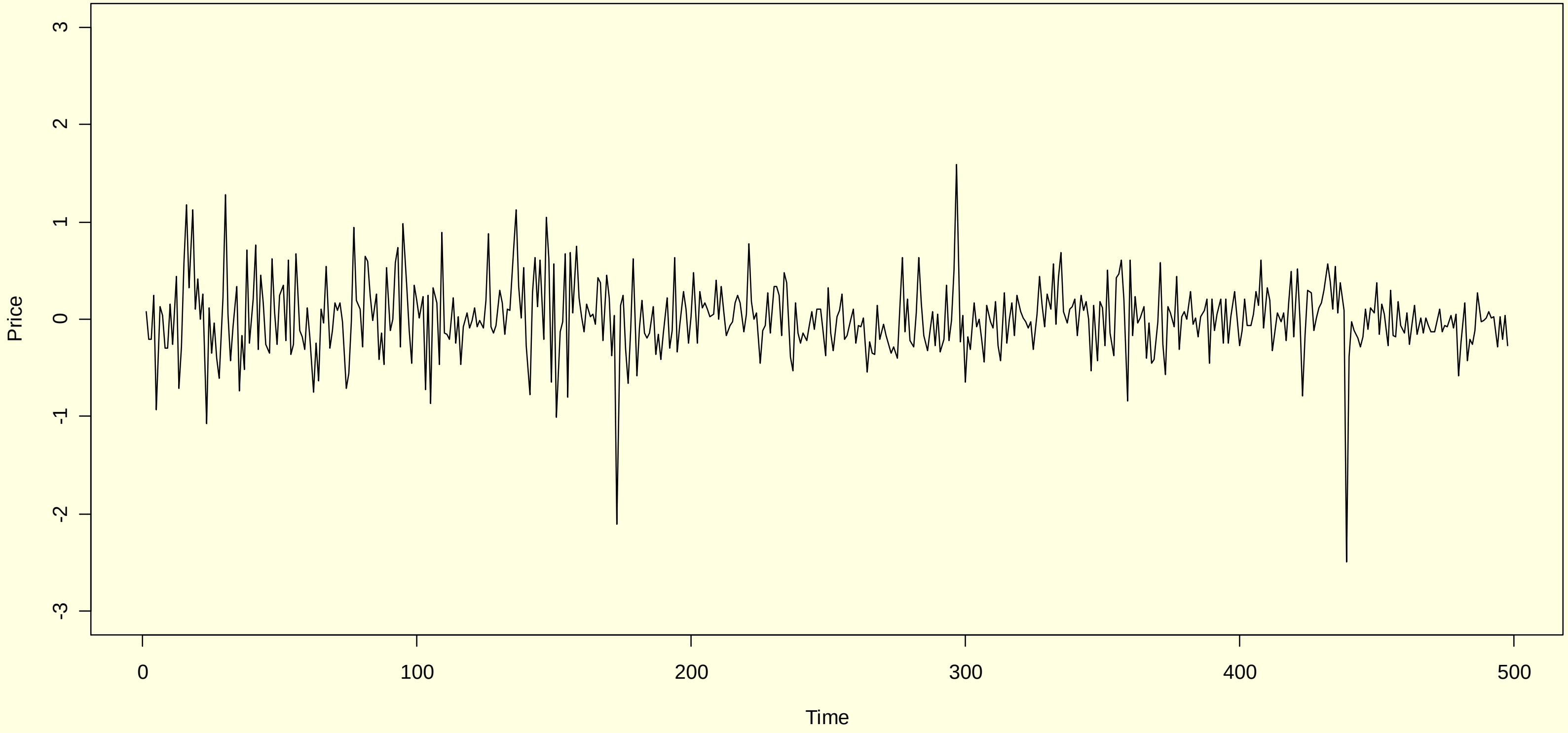
$$\hat{y}_t = 1.8961 y_{t-1} - .8356 y_{t-2} - .0605 y_{t-3} + .0056\varepsilon_{t-1} - .0056\varepsilon_{t-2} - \varepsilon_{t-3}$$

Apply the following back-shift operator to obtain the forecasts for $\{x_t\}$

$$\hat{x}_t = 3\hat{y}_t - x_{t-1} - x_{t-2}$$

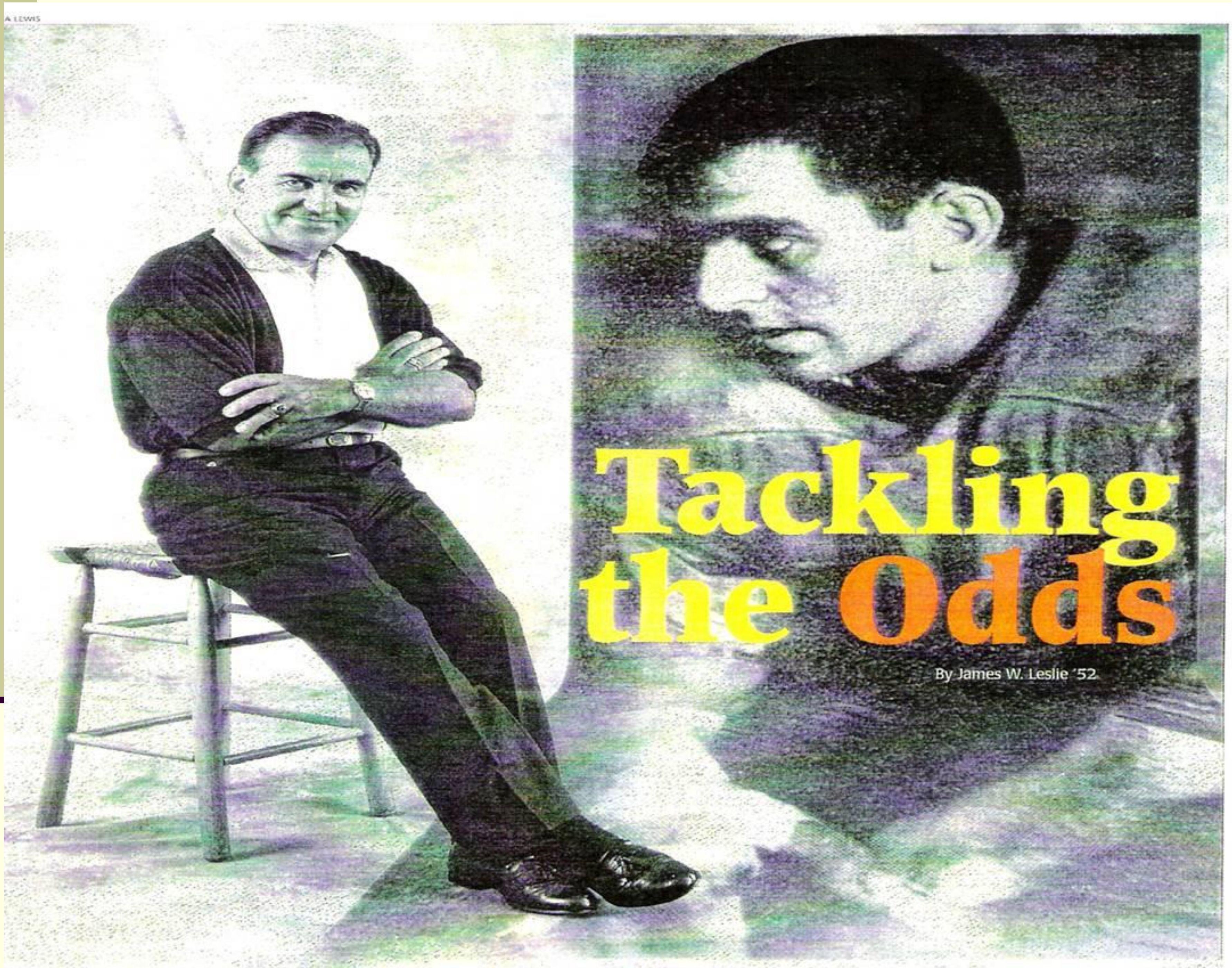
Comparisons on Our Proposed Model VS. Original Time Series

Time Series Plot for Residuals for Our Proposed Model



Comparison on Classical Approach Vs. Our Proposed model

	$\bar{r}$	S_r	$S_r/\sqrt{n}$	S_r^2
Classical	0.02209169	0.1445187	0.3801562	0.0170011
Proposed	0.01016814	0.1437259	0.3791119	0.01698841


Tackling
the Odds
By James W. Leslie '52

Thank You !