Carbon Dioxide in the Atmosphere: CO_2

- Parametric Analysis
- Differential Equations

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Data

- The data used in the study consist of several data sets. The data of primary interest is atmospheric carbon dioxide in the air recorded at several sites located at various latitudes in the open water illustrated below in Figure 1. Data gathered and maintained by Scripps Institution of Oceanography. Monthly values are expressed in parts per million (ppm).

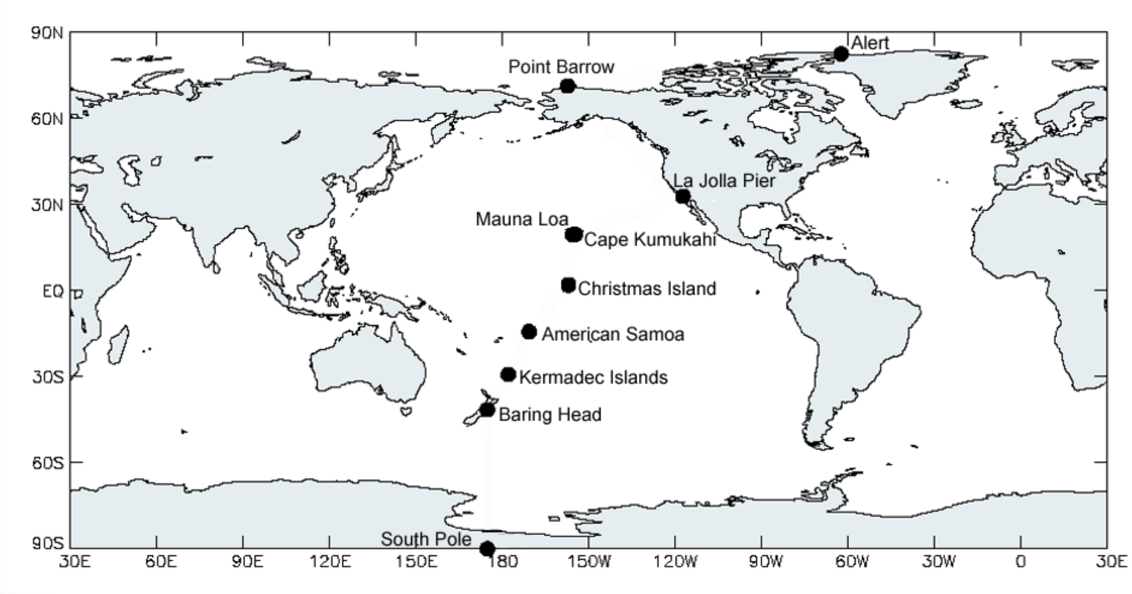


Figure 1: Map of monitoring sites for carbon dioxide in the atmosphere.

CO_2 by Station

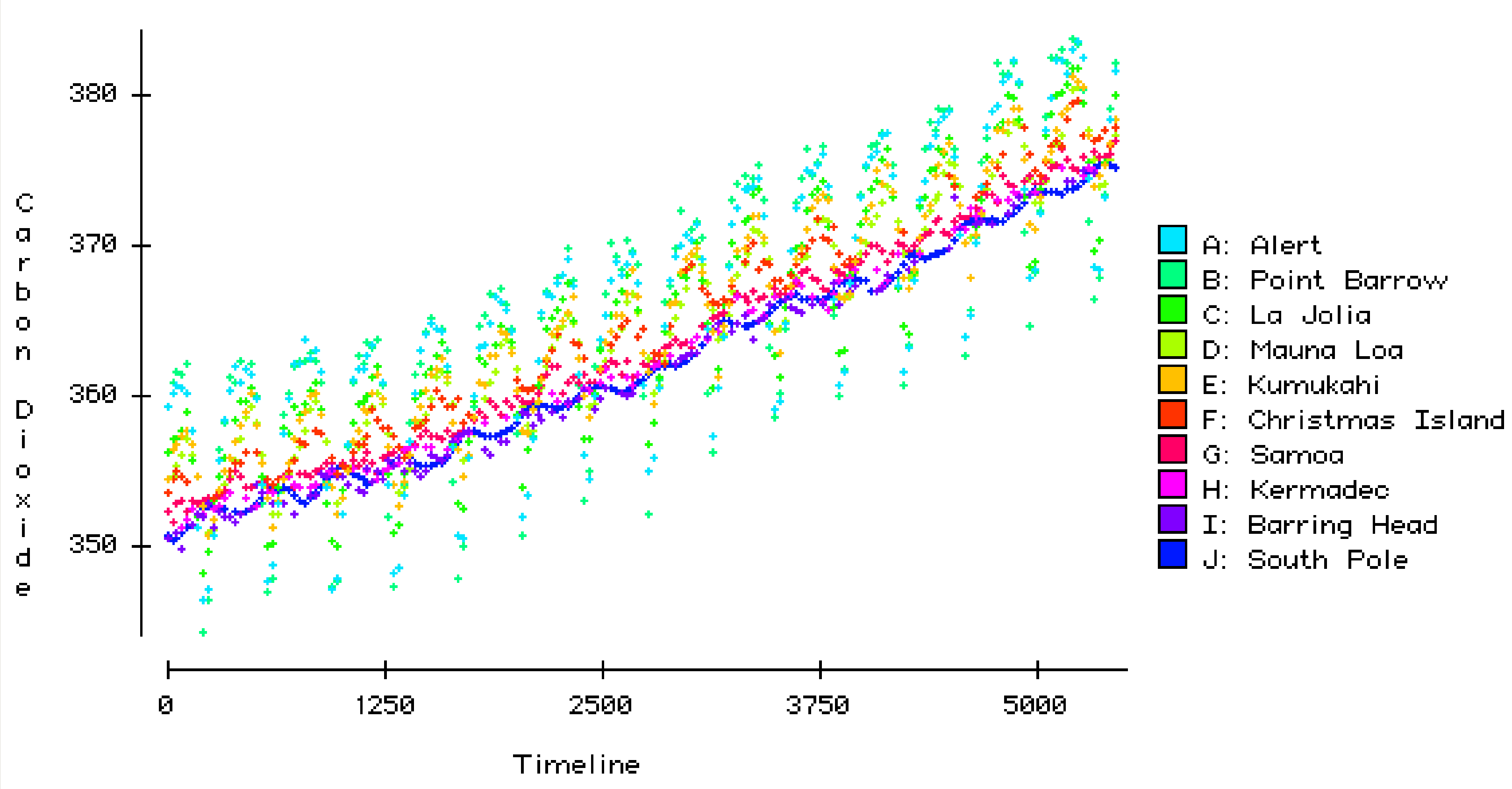


Figure 2: Line graph of atmospheric carbon dioxide (ppm) by location

CO_2 by Station

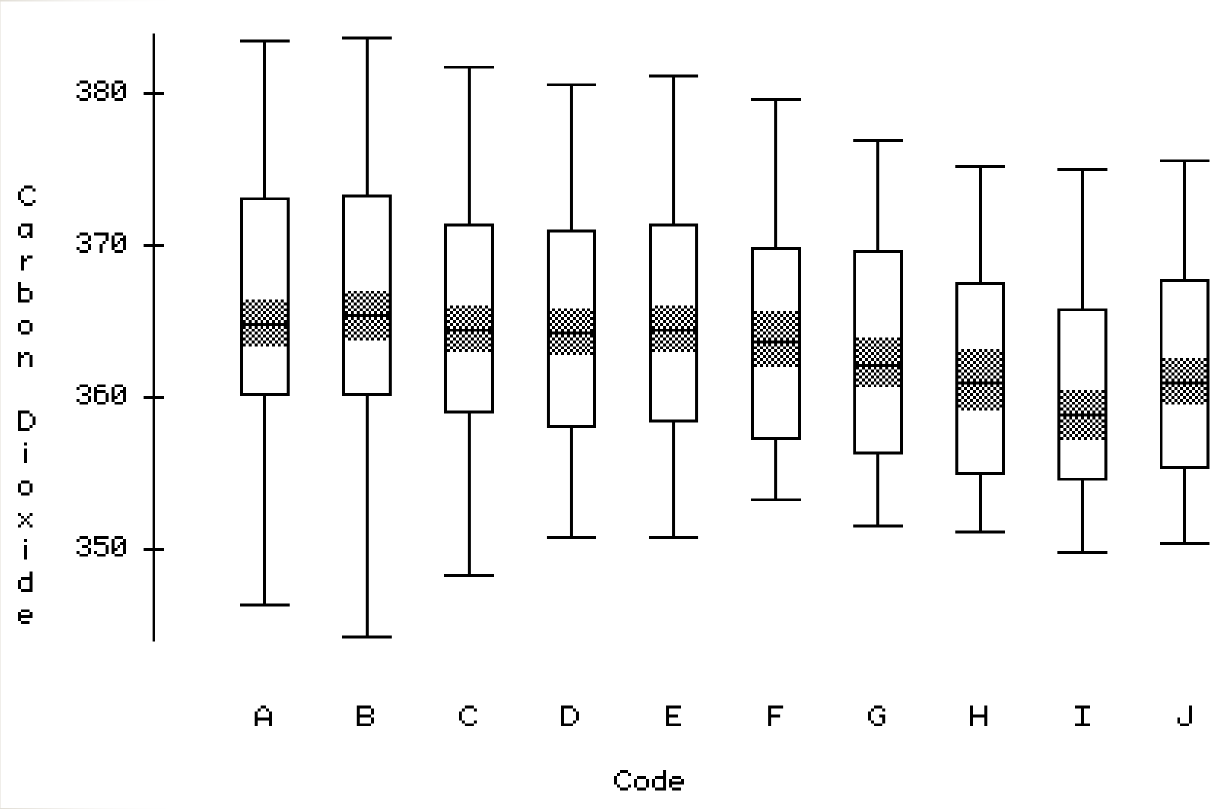
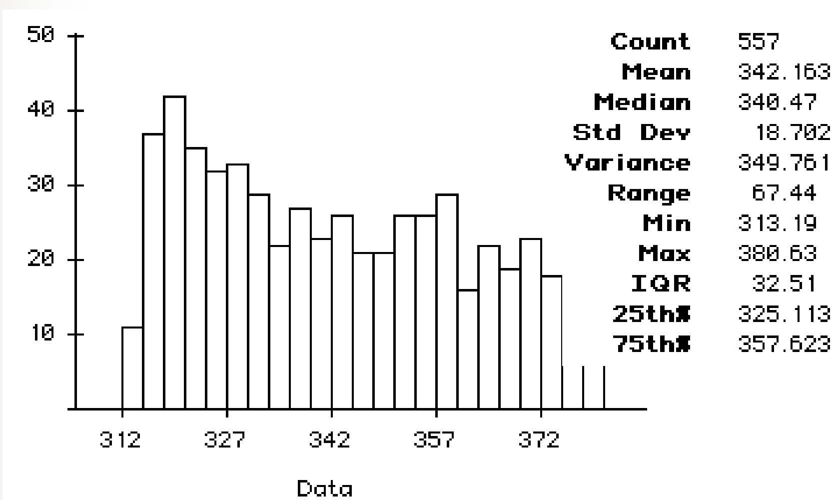


Figure 3: Box plots of carbon dioxide in the atmosphere by location

Parametric Analysis

- Figures 4 and 5 illustrate that the data's distribution not best characterized by the **normal probability distribution.** There is no symmetry and there is a heavy tail, which indicates the Frechet probability distribution; however, the strong peak best characterized by the Weibull probability distribution.



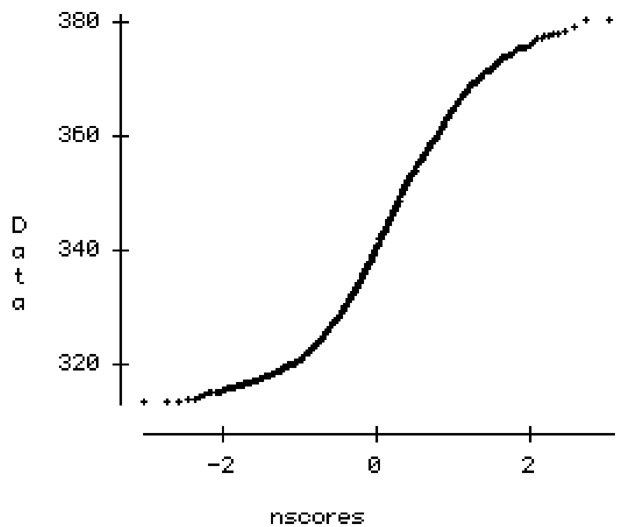


Figure 4: Histogram of CO_2 in the atmosphere measured in Hawaii.

Figure 5: Normal probability plot

Parametric Analysis

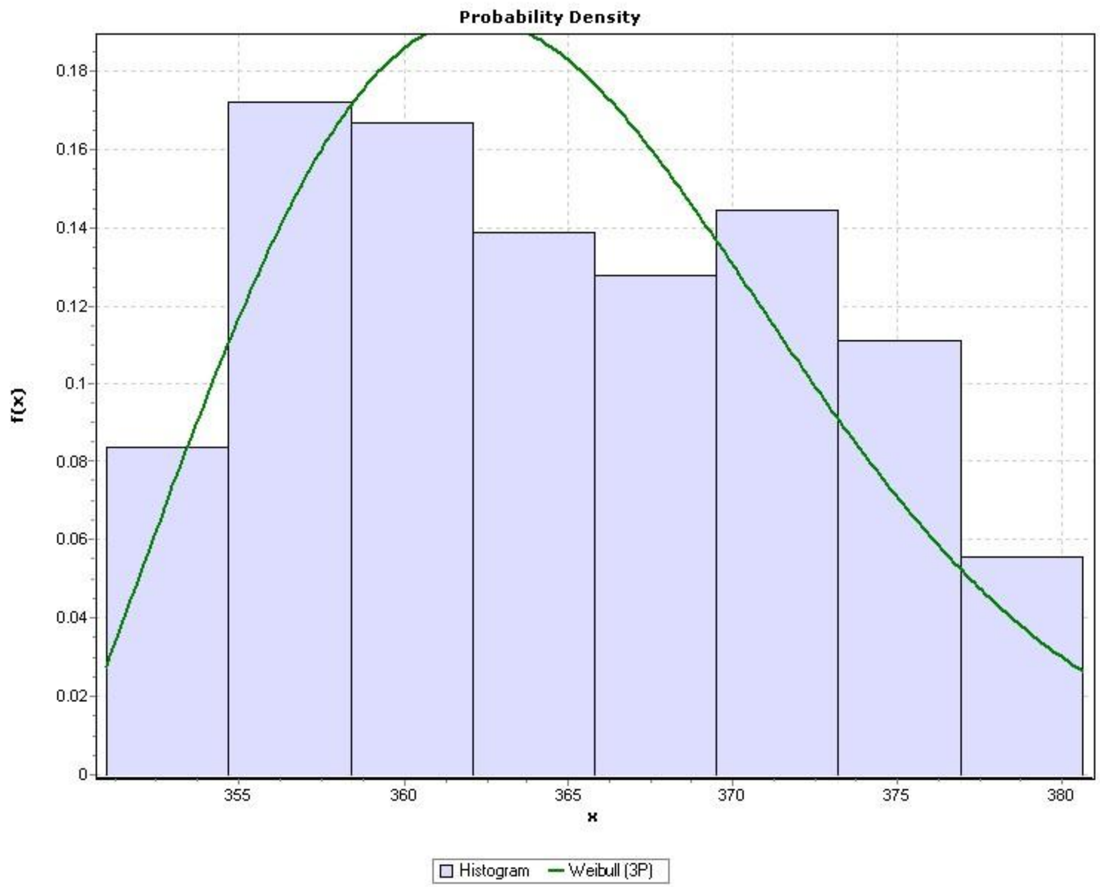


Figure 5: Three-parameter Weibull probability distribution fit to data from Hawaii

Standard Statistic Test for Goodness of Fit

Kolmogorov Smirnov			Anderson Darling			Chi-Squared		
Distribution	Statistic	Rank	Distribution	Statistic	Rank	Distribution	Statistic	Rank
Gen. Extreme Value	0.04067	1	Weibull (3P)	5.6362	1	Chi-Squared (2P)	60.799	1
Weibull (3P)	0.04229	2	Gen. Extreme Value	5.684	2	Gen. Extreme Value	61.298	2
Gamma (3P)	0.04256	3	Gamma (3P)	7.2995	3	Gamma (3P)	62.451	3
Lognormal (3P)	0.0461	4	Lognormal (3P)	7.739	4	Weibull (3P)	63.747	4
Chi-Squared (2P)	0.04795	5	Chi-Squared (2P)	7.7394	5	Inv. Gaussian (3P)	68.17	5

Figure 6: Goodness-of-Fit Test (all stations)

Three-Parameter Weibull

- The three-parameter Weibull best characterizes the probability distribution of the amount of carbon dioxide in the atmosphere where the cumulative three-parameter probability distribution is given by

$$F(x) = 1 - \exp\left\{ -\left(\frac{x-\gamma}{\beta} \right)^{\alpha} \right\}$$

where γ is the location parameter, β is the scale parameter and α is shape parameter; the support of this probability density function is

and n^{th} moment is given by $\mu = \beta\Gamma\left(1 + \frac{1}{\alpha}\right)$, where Γ is the Gamma function. The mean is

and the variance is $\sigma^2 = \beta^2\Gamma\left(1 + \frac{2}{\alpha}\right) - \mu^2$.

Estimation of the Parameters

Data Source	Parameter Estimates
All Stations	$\hat{\alpha} = 2.779, \hat{\beta} = 23.029, \hat{\gamma} = 343.7$
Hawaii	$\hat{\alpha} = 2.108, \hat{\beta} = 17.092, \hat{\gamma} = 349.6$

- Hence, for the stations overall, the cumulative probability distribution is given by

$$F(x) = 1 - \exp\left\{ -\left(\frac{x - 343.7}{23.029} \right)^{2.779} \right\}$$

and for the station located in Mauna Loa, Hawaii is given by

$$F(x) = 1 - \exp\left\{ -\left(\frac{x - 349.6}{17.092} \right)^{2.108} \right\}$$

Trend Analysis

- To determine if this probability distribution of carbon dioxide in the atmosphere dependents on time, consider the three-parameter Weibull probability distribution function by considering the mean yearly carbon dioxide in the atmosphere as a function of time in years given by $y = f(t) + \varepsilon$, where $f(t)$ is either a constant, linear, quadratic or exponential function.

- It was determine using standard statistical tests that the better fit function is as follows:

$$\hat{y} = 314.028 + 0.00224666t + 8.7475 \times 10^{-8} t^2$$

Profiling

- The cumulative conditional probability distribution is given by

$$F(x) = 1 - \exp\left\{-\left(\frac{x-\gamma_t}{\beta}\right)^{\alpha}\right\}$$

$$\hat{\mu}_t = 314.028 + 0.00224666t + 8.7475\times 10^{-8}t^2 \qquad \hat{\beta} = 17.092$$

$$\hat{\alpha} = 2.108 \qquad \Gamma\left(1+\frac{1}{2.108}\right) \approx 0.8857$$

$$\hat{\beta}_t = \frac{\hat{\mu}_t}{0.8857}$$

Hence, consider the cumulative probability distribution function given by

$$F(x) = 1 - \exp\left\{-\left(\frac{x-\left(354.5535 + 0.002537t + 9.87637\times 10^{-8}t^2\right)}{17.092}\right)^{2.108}\right\}$$

Profiling and Projections

- Projecting into the future 10 year (to 2017), at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 381.35 and 410.11 ppm, Figure 7. Projecting twenty years into the future (to 2027), at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 397.20 and 425.96 ppm. Projecting fifty years into the future (to 2057), at a 95% level of confidence, we have that the probable amount of carbon dioxide in the atmosphere will be between 460.56 and 489.32 ppm, Figure 8.

Profiling: Ten Year Projections

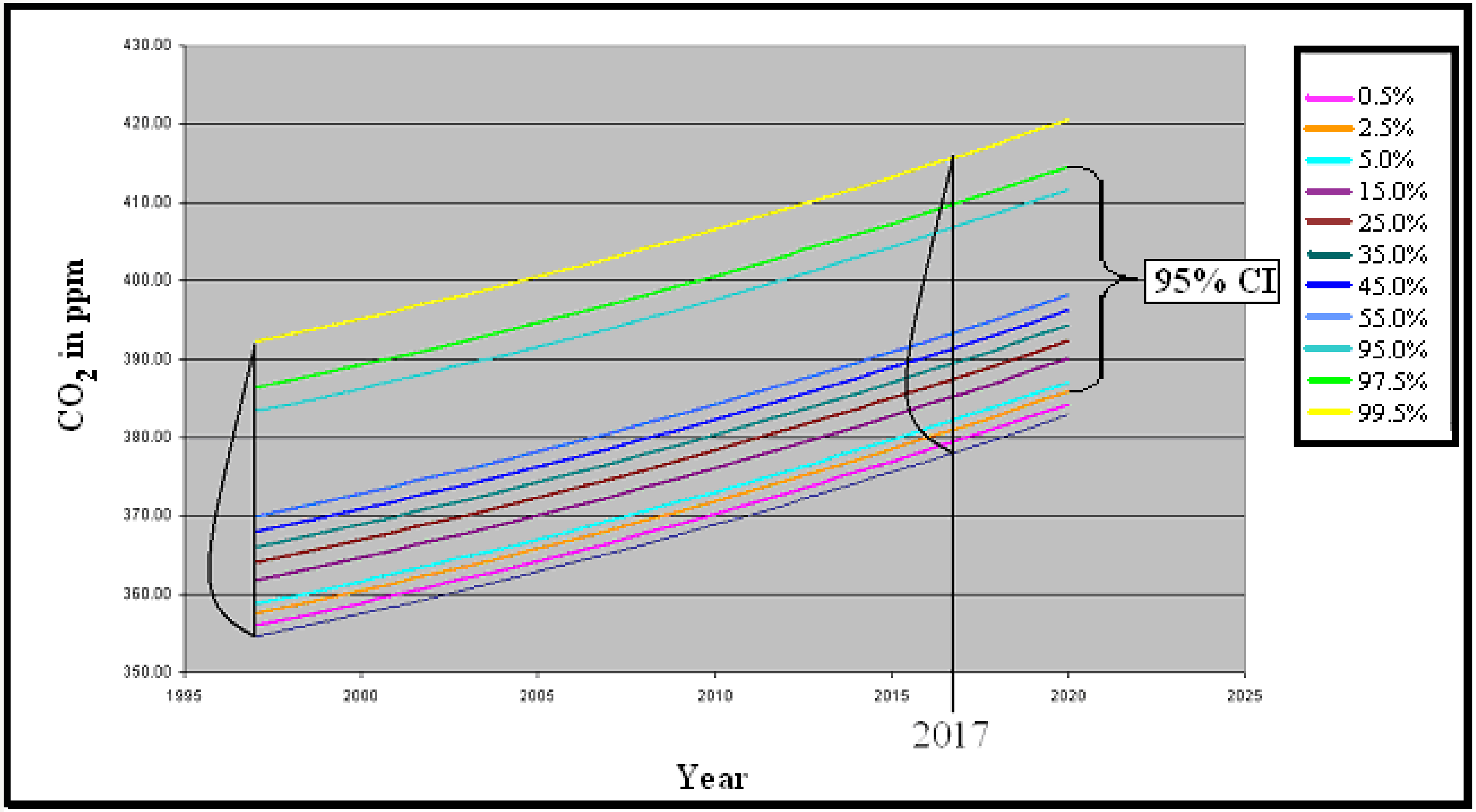


Figure 7: Projections through 2017

Profiling: Fifty Year Projections

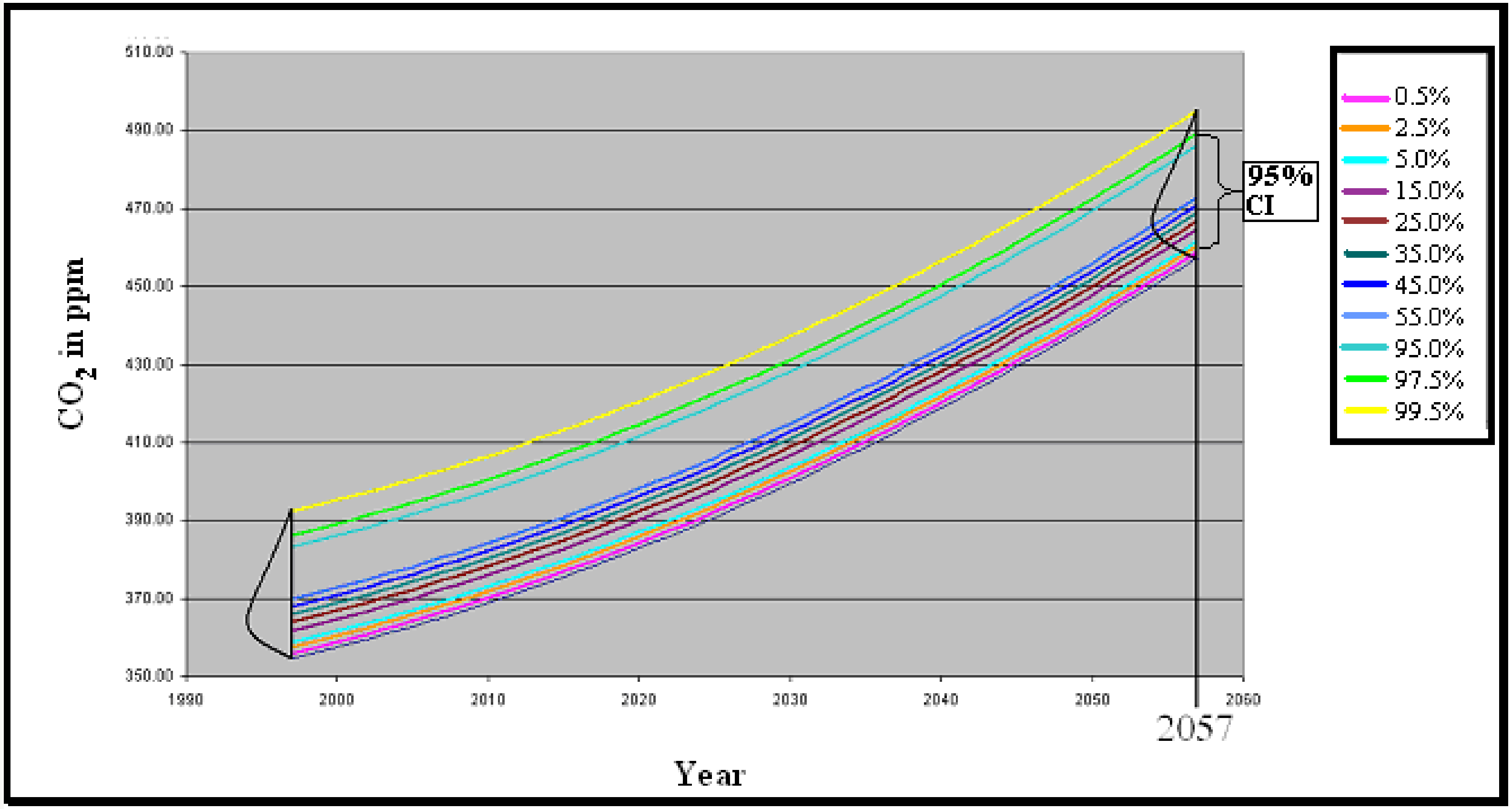


Figure 7: Projections through 2057

Confidence Intervals

	90% CI		95% CI		99% CI	
Year	Lower Limit	Upper Limit	Lower Limit	Upper Limit	Lower Limit	Upper Limit
2007	369.32	393.91	368.13	396.89	366.53	402.84
2008	370.52	395.11	369.33	398.09	367.73	404.04
2009	371.75	396.34	370.57	399.33	368.96	405.28
2010	373.01	397.60	371.82	400.58	370.22	406.53
2011	374.29	398.88	373.10	401.86	371.50	407.81
2012	375.60	400.18	374.41	403.17	372.81	409.12
2013	376.94	401.52	375.75	404.51	374.14	410.46
2014	378.30	402.88	377.11	405.87	375.50	411.82
2015	379.68	404.27	378.49	407.25	376.89	413.20
2016	381.09	405.68	379.91	408.67	378.30	414.62
2017	382.54	407.12	381.35	410.11	379.75	416.06
2018	384.00	408.59	382.81	411.57	381.21	417.52
2019	385.50	410.08	384.31	413.07	382.70	419.02
2020	387.01	411.60	385.82	414.58	384.22	420.53
2021	388.56	413.15	387.37	416.13	385.77	422.08
2022	390.13	414.72	388.94	417.70	387.34	423.65
2023	391.73	416.32	390.54	419.30	388.94	425.25
2024	393.35	417.94	392.16	420.92	390.56	426.87
2025	395.01	419.59	393.82	422.58	392.22	428.53
2026	396.68	421.27	395.49	424.25	393.89	430.20
2027	398.39	422.97	397.20	425.96	395.60	431.91

Differential Equation of CO_2

$$\frac{d(CO_2)}{dt} = f(E, D, R, S, O, P, A, B)$$

$$CO_2 = \int f(E, D, R, S, O, P, A, B)$$

E is fossil fuel Combustion, which is a function of the following: Gas fuel, Liquid fuel, Solid fuel, Gas flares, Cement production

D is Deforestation and Destruction of biomass and soil carbon, which is a function of the following: Deforestation, Destruction of biomass, Destruction of soil carbons, R is terrestrial plant Respiration

S is Soil respiration from soils and decomposers such as bacteria, fungi, and animals, which is a function of the following: Respiration from soils, Respiration from decomposers

O is the flux from Oceans to atmosphere

P is terrestrial Photosynthesis

A is the flux from Atmosphere to oceans

B is the Burial of organic carbon and limestone carbon in sediments and soils, which is a function of the following: Burial of organic carbon, Burial of limestone carbon

Developed Sub Models

$$E = -593503 + 2.0629 e^{\left(\frac{t}{1200}\right)}$$

$$D = 1073.05 + 0.0325t$$

$$S = \begin{cases} -0.044\left(1995 + \frac{t}{12}\right)^3 + 263.6\left(1995 + \frac{t}{12}\right)^2 \\ -52577\left(1995 + \frac{t}{12}\right) + 3 \times 10^8 \end{cases}$$

$$O - A = 42.814 - 4.533t + 0.29t^2$$

Developed Model

$$CO_2 = \begin{cases} k_E \left\{-593503t + 2.4755\times 10^9 e^{\left(\frac{t}{1200}\right)}\right\} \\ + k_D \left(10730.5t + 0.01625t^2\right) \\ + k_S \left\{ \begin{array}{l} -0.132\left(1995 + \frac{t}{12}\right)^4 + 1054.4\left(1995 + \frac{t}{12}\right)^3 \\ -315462\left(1995 + \frac{t}{12}\right)^2 + 3\times 10^8 t \end{array} \right\} \\ + k_{A-O} \left\{42.814t - 4.2665t^2 + 0.0967t^3\right\} \\ - k_P \int P dt - k_B \int B dt \end{cases}$$