

Prediction Models for Carbon Dioxide Emissions and the Atmosphere

By

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Abstract

The object of the present study is to develop statistical models for predicting the carbon dioxide emissions and the atmosphere in the United States. We used monthly emissions data from 1981 to 2003 that was collected by the Carbon Dioxide Information Analysis Center. For the carbon dioxide in the atmosphere, we used the data that was collected in Mauna Loa from 1965 to 2004 by the Scripps Institution of Oceanography. The developed statistical models take into consideration trends and seasonal effects. The quality of the prediction process is illustrated using the actual data.

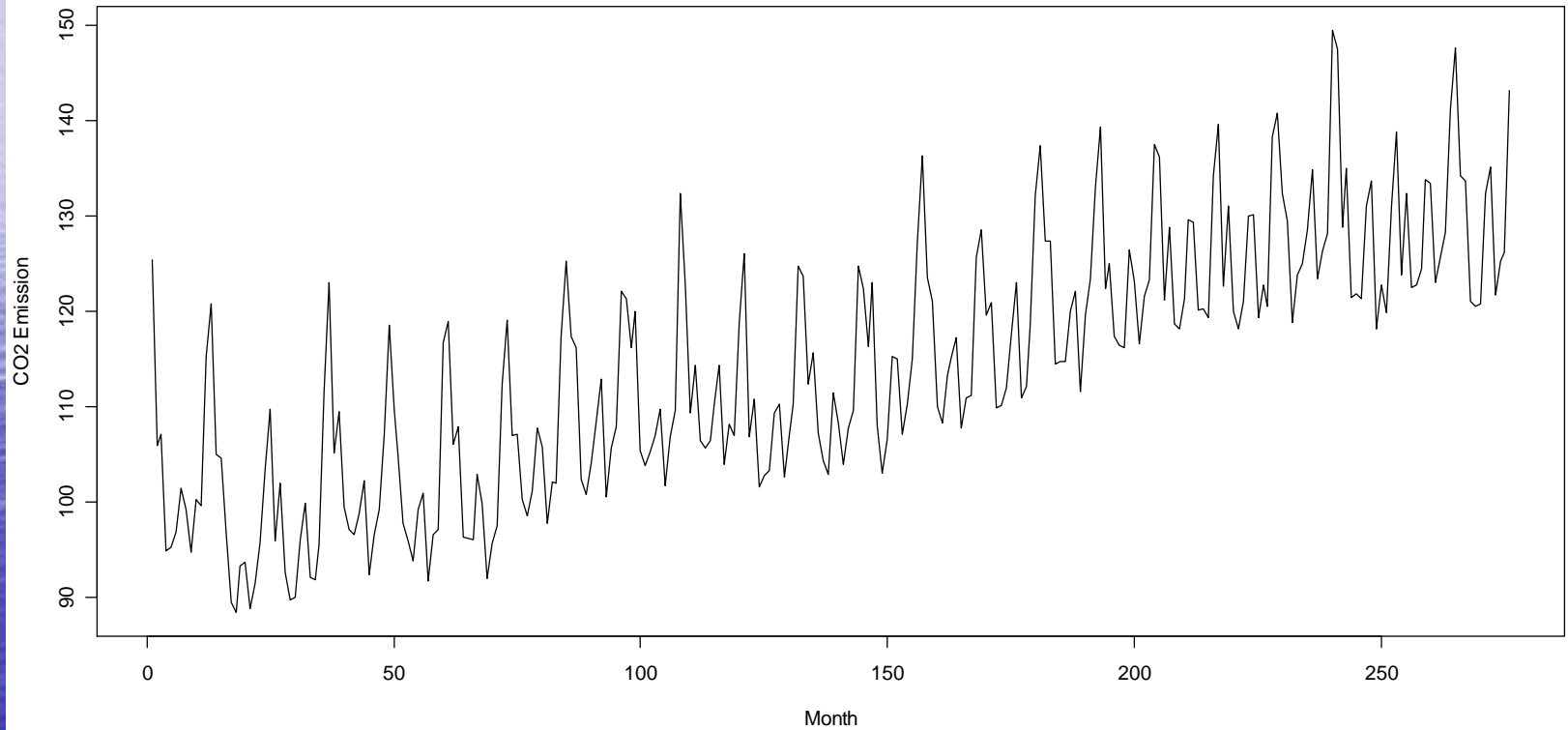
Outline

- The Data
- The Multiplicative ARIMA Model
- Brief Summary of Our Procedure
- Evaluation Criteria & Evaluation
- Conclusion
- References

The CO₂ Emission Data

- By Carbon Dioxide Information Analysis Center (CDIAC)
- Time: 1981 to 2003
- For detailed information, see (United States Environmental Protection Agency (EPA), 2004; Marland et al., 2003)

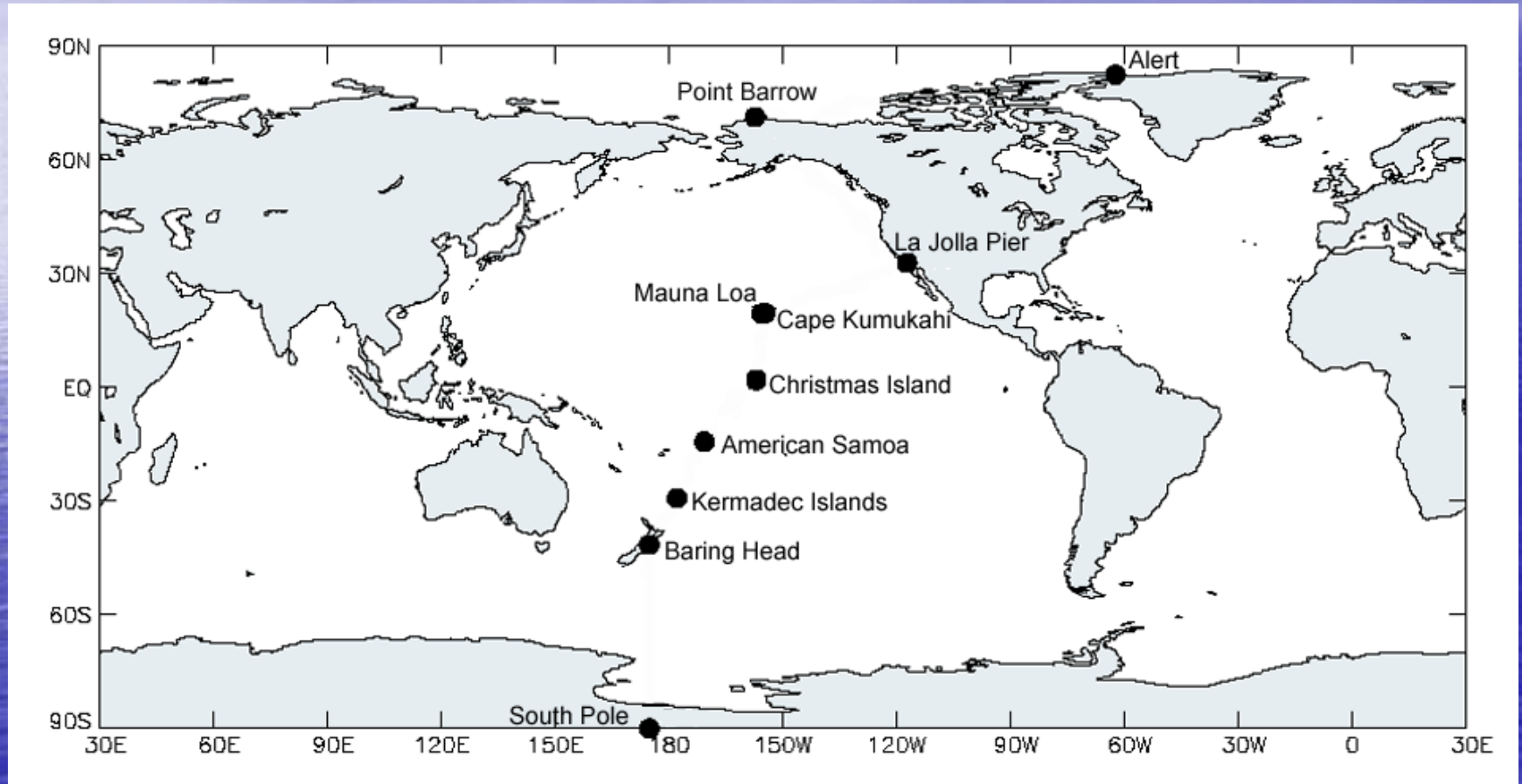
Time Series Plot on Monthly CO₂ Emissions 1981-2003



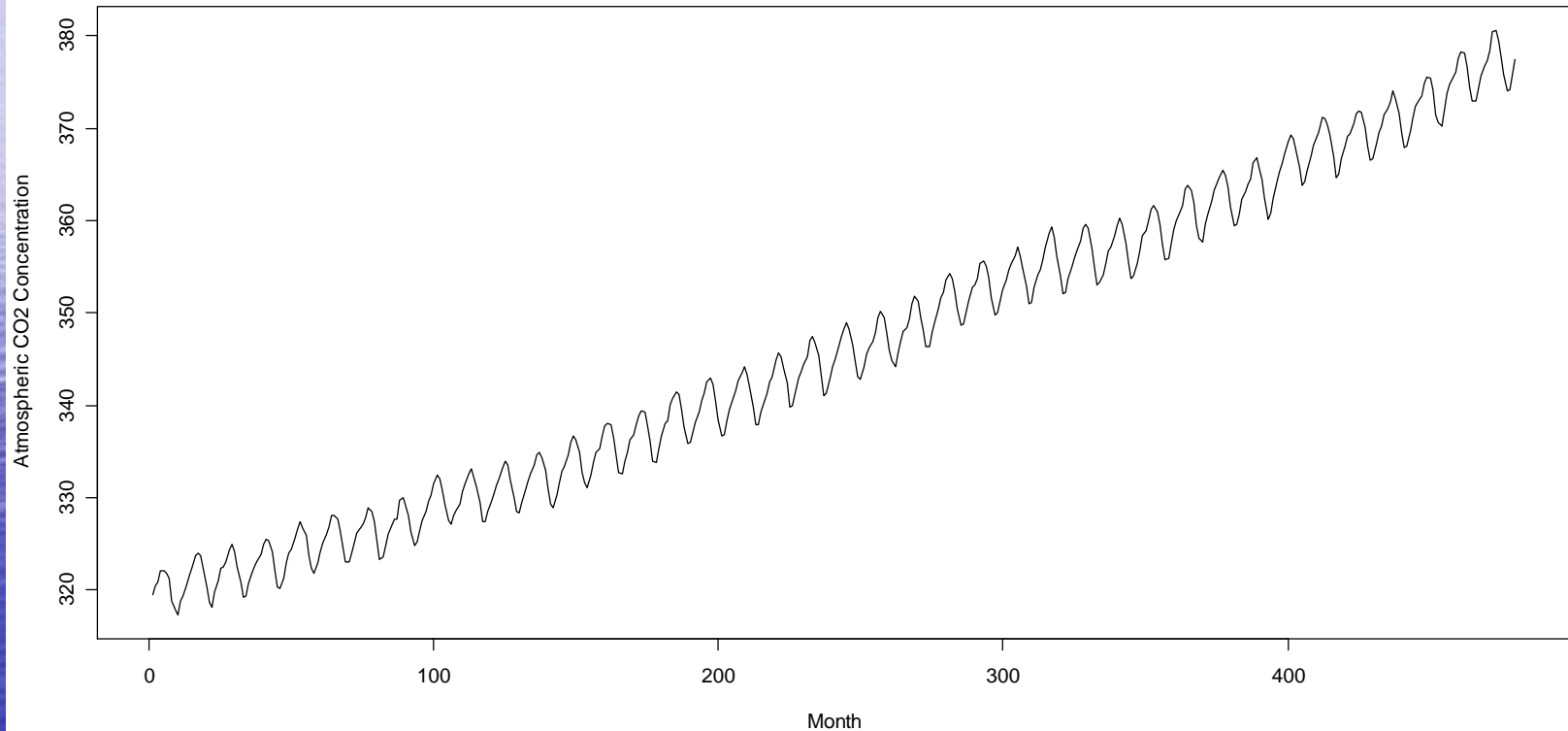
The Atmospheric CO₂ Data

- Carbon Dioxide Research Group, Scripps Institution of Oceanography, University of California
- Time: 1958 to 2004
- Location: Mauna Loa

Geographical Location of Mauna Loa



Time Series Plot for Monthly CO₂ in the Atmosphere 1965-2004



The Multiplicative ARIMA Model

ARIMA $(p, d, q) \times (P, D, Q)_s$ is defined by

$$\Phi_P(B^s)\phi_p(B)(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Gamma_Q(B^s)\varepsilon_t$$

where

$$\phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p)$$

$$\theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q)$$

$$\Phi_P(B^s) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \dots - \Phi_P B^{Ps}$$

$$\Gamma_Q(B^s) = 1 - \Gamma_1 B^s - \Gamma_2 B^{2s} - \dots - \Gamma_Q B^{Qs}$$

Brief Summary of Our Procedure on Developing the Subject Model-1

- Determine the seasonal period s
- Check for stationarity by determining the order of differencing d , where $d = 0, 1, 2, \dots$ according to KPSS test, until we achieve stationarity
- Deciding the order of the process, for our case, we let $m = 5$, where $p + q + P + Q = m$
- After (d, m) being selected, listing all possible configurations of (p, q, P, Q) for $p + q + P + Q \leq m$

Brief Summary of Our Procedure on Developing the Subject Model-2

- For each set of (p, q, P, Q) , estimates the parameters for each model
- Compute the AIC for each model, and choose the one with smallest AIC
- After (p, q, P, Q) is selected, we determine the seasonal differencing filter by selecting the smaller AIC between the model with $D = 0$ and $D = 1$
- Our final model will have identified the order of (p, d, q, P, D, Q)

Evaluation Criteria

- We define the residuals as

$$r_t = x_t - \hat{x}_t$$

- Mean of the residuals

$$\bar{r} = \frac{\sum_{t=1}^n r_t}{n}$$

- Variance of the residuals

$$S_r^2 = \frac{\sum_{t=1}^n (r_t - \bar{r})^2}{n-1}$$

- Standard Deviation

$$S_r = \sqrt{S_r^2}$$

- Standard Error

$$SE = \frac{S_r}{\sqrt{n}}$$

- Mean Square Error

$$MSE = \frac{\sum_{t=1}^n r_t^2}{n}$$

The CO₂ Emissions Model

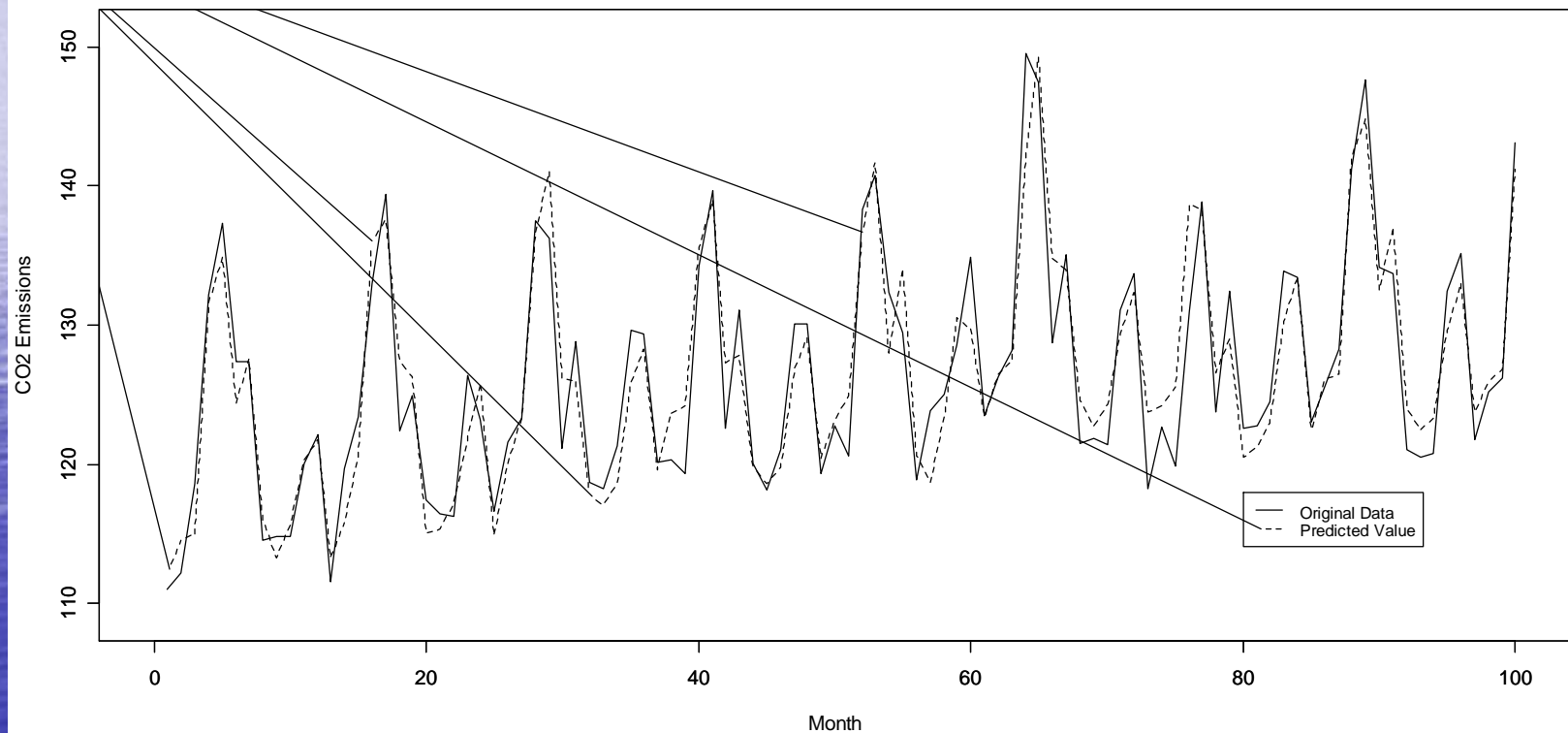
- ARIMA(1,1,2)(1,1,1)₁₂

$$(1 - \Phi_1 B^{12})(1 - \phi_1 B)(1 - B)(1 - B^{12})x_t = (1 - \theta_1 B - \theta_2 B^2)(1 - \Gamma_1 B^{12})\varepsilon_t$$

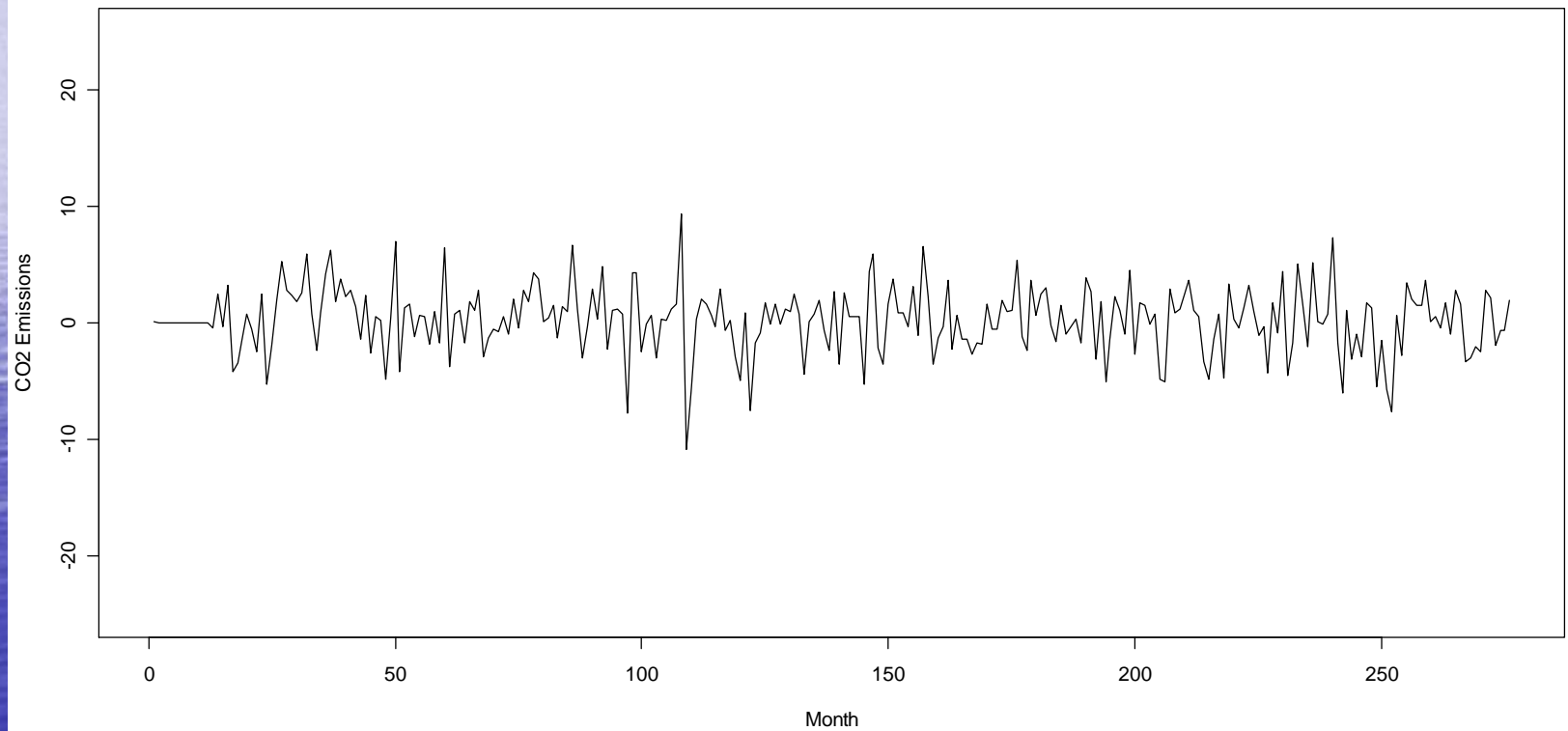
- After expand the model and put in the coefficients, we have

$$\begin{aligned}\hat{CO}_{2E} &= 1.5203x_{t-1} - 0.5203x_{t-2} + 1.0049x_{t-12} - 1.527749x_{t-13} + 0.5228495x_{t-14} \\ &\quad - 0.0049x_{t-24} + 0.007449x_{t-25} - 0.002549x_{t-26} - 0.9988\varepsilon_{t-1} + 0.1234\varepsilon_{t-2} \\ &\quad - 0.8523\varepsilon_{t-12} + 0.8512772\varepsilon_{t-13} - 0.10517\varepsilon_{t-14}\end{aligned}$$

Monthly CO₂ Emissions VS. Forecast Values for the Last 100 Observations



Residuals Plot for CO2 Emissions



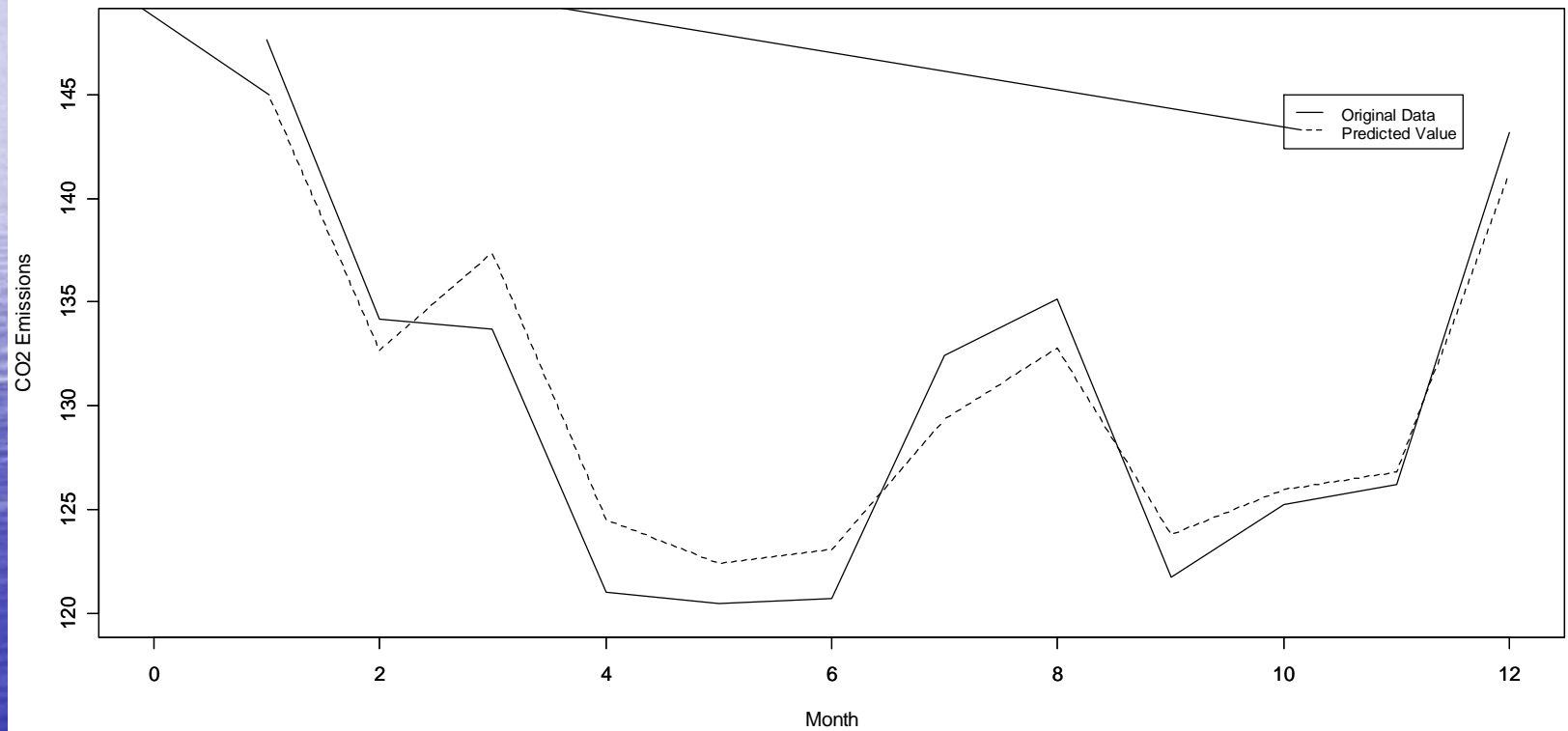
Basic Evaluation on CO₂ Emissions

\bar{r}	S_r^2	S_r	SE	MSE
0.2339641	8.055668	2.838251	0.1708426	8.08122

CO₂ Emissions Forecast

	Original Values	Forecast Values	Residuals
Jan-03	147.6298	145.2361	2.3937
Feb-03	134.1716	132.6554	1.5162
Mar-03	133.6979	137.3912	-3.6933
Apr-03	121.0047	124.5518	-3.5471
May-03	120.4789	122.4091	-1.9302
Jun-03	120.7394	123.101	-2.3616
Jul-03	132.4187	129.3481	3.0706
Aug-03	135.1314	132.787	2.3444
Sep-03	121.7753	123.8295	-2.0542
Oct-03	125.2487	125.9811	-0.7324
Nov-03	126.2127	126.812	-0.5993
Dec-03	143.1509	141.1834	1.9675

Forecasting Results



The Atmospheric CO₂ Model

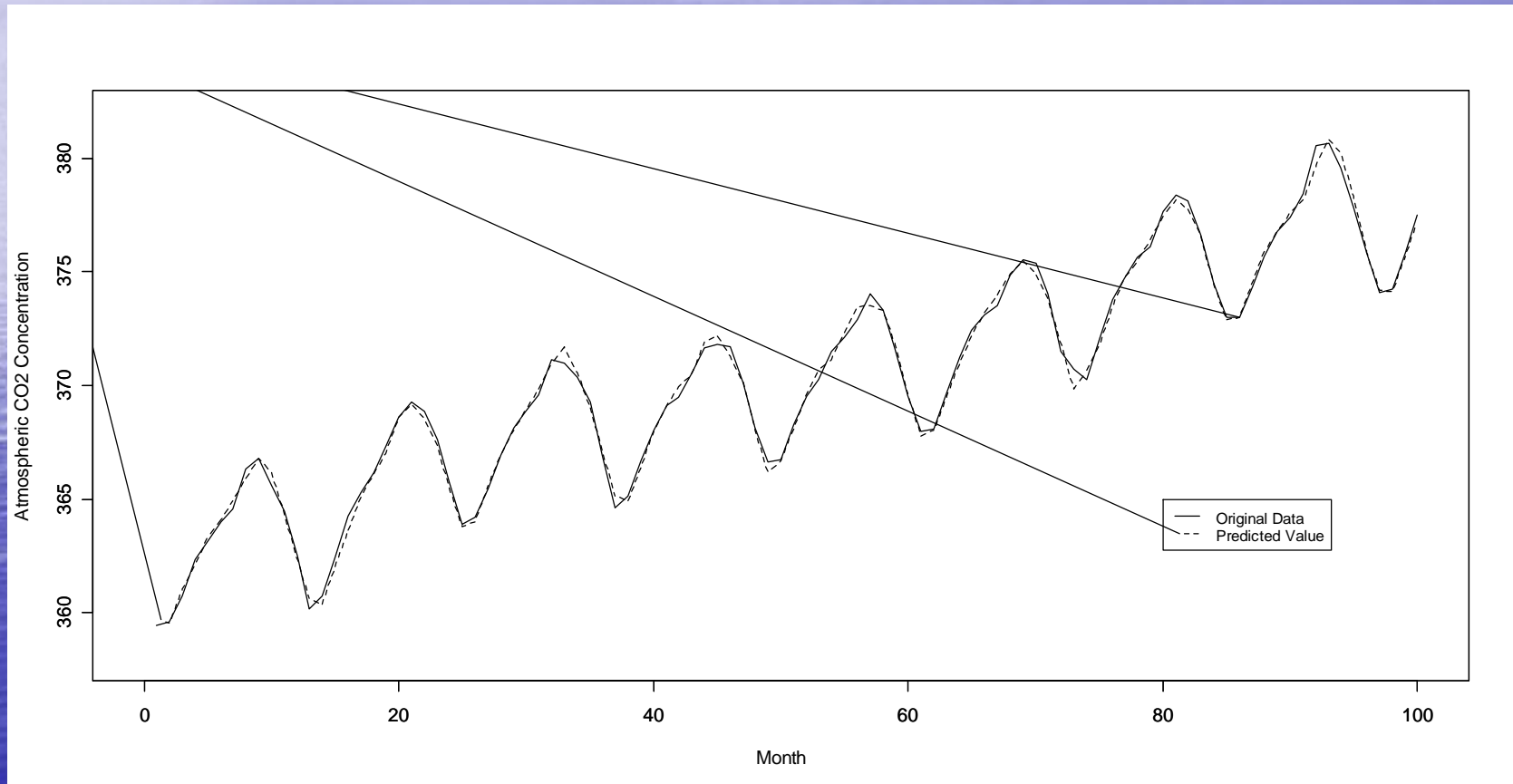
- ARIMA(2,1,0)(2,1,1)₁₂

$$(1 - \Phi_1 B^{12} - \Phi_2 B^{24})(1 - \phi_1 B - \phi_2 B^2)(1 - B)(1 - B^{12})x_t = (1 - \Gamma_1 B^{12})\varepsilon_t$$

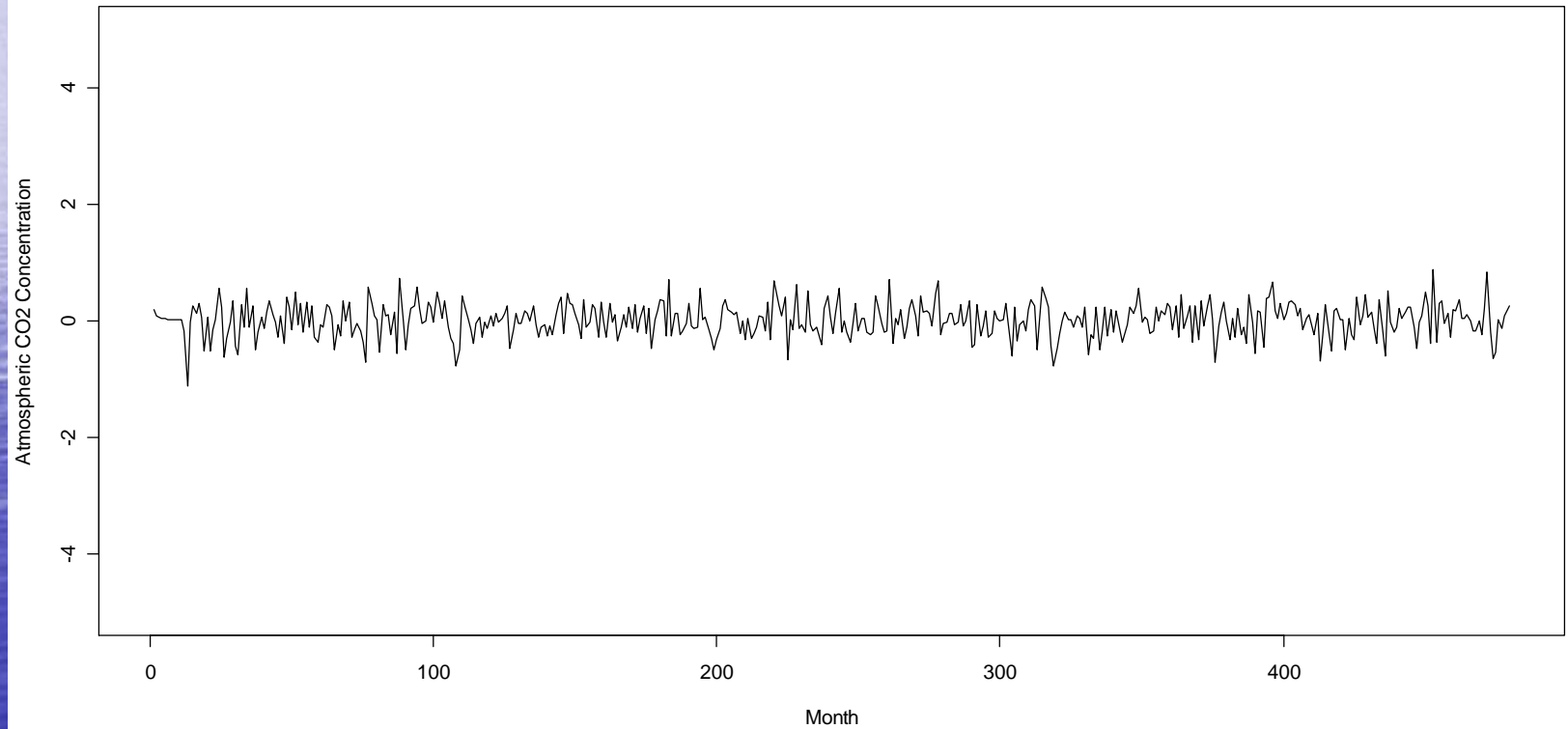
- After expand the model and put in the coefficients, we have

$$\begin{aligned}\hat{CO}_{2A} = & 0.6887x_{t-1} + 0.1989x_{t-2} + 0.1124x_{t-3} + 1.0759x_{t-12} - 0.74097x_{t-13} - \\ & 0.213997x_{t-14} - 0.12093x_{t-15} - 0.0683x_{t-24} + 0.047038x_{t-25} + \\ & 0.013585x_{t-26} + 0.00768x_{t-27} - 0.0076x_{t-36} + 0.005234x_{t-37} + \\ & 0.0015116x_{t-38} + 0.00085x_{t-39} - 0.8787\varepsilon_{t-12}\end{aligned}$$

Monthly CO₂ in the Atmosphere VS. Forecast Values for the Last 100 Observations



Residuals Plot for Atmospheric CO₂



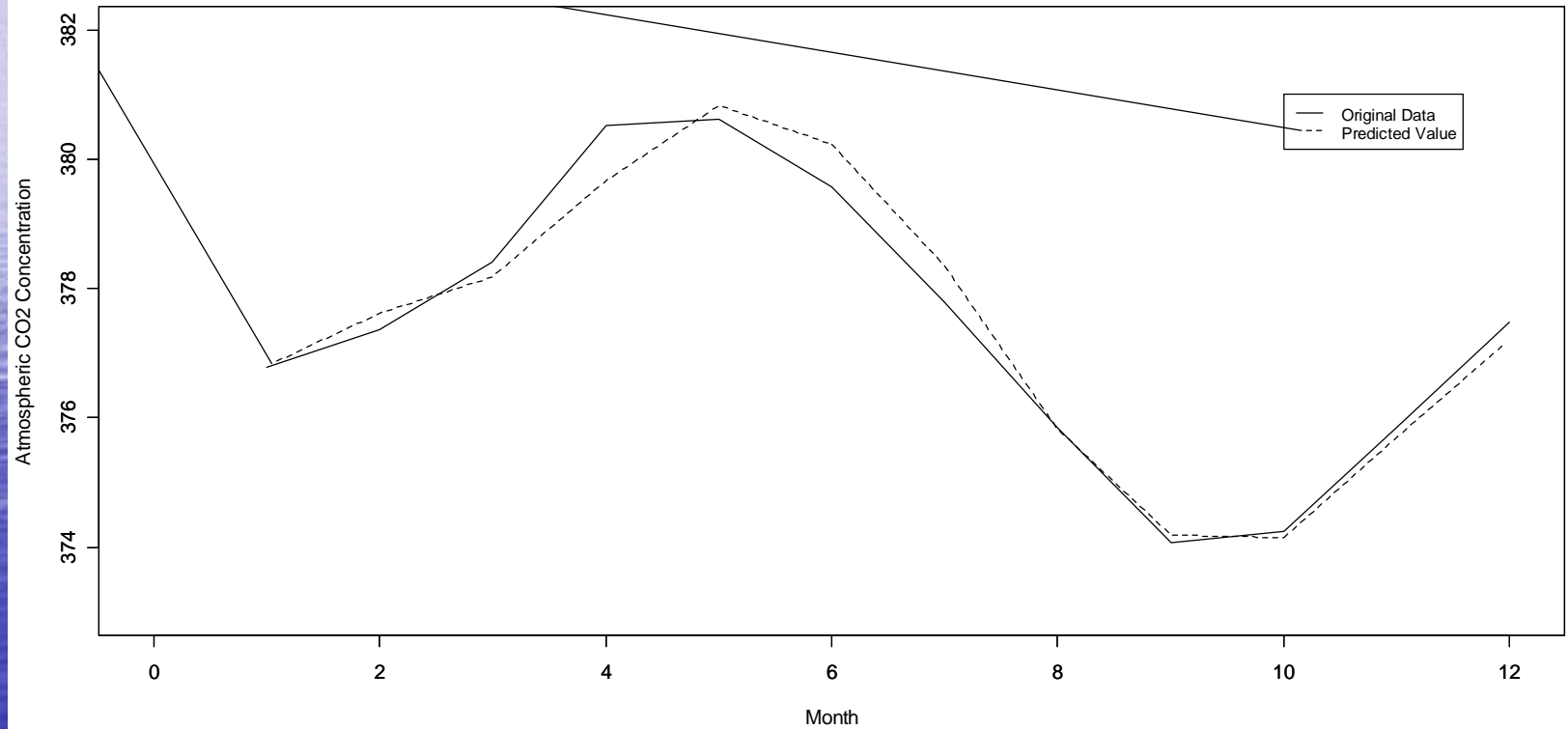
Basic Evaluation on Atmospheric CO₂

\bar{r}	S_r^2	S_r	SE	MSE
0.01140137	0.08460756	0.2908738	0.01327651	0.08456128

Atmospheric CO₂ Forecast

	Original Values	Forecast Values	Residuals
Jan-04	376.79	376.7963	-0.0063
Feb-04	377.37	377.609	-0.239
Mar-04	378.41	378.1837	0.2263
Apr-04	380.52	379.6653	0.8547
May-04	380.63	380.8268	-0.1968
Jun-04	379.57	380.2339	-0.6639
Jul-04	377.79	378.3489	-0.5589
Aug-04	375.86	375.837	0.023
Sep-04	374.06	374.1871	-0.1271
Oct-04	374.24	374.1482	0.0918
Nov-04	375.86	375.6897	0.1703
Dec-04	377.48	377.2186	0.2614

Forecasting Results



Conclusion

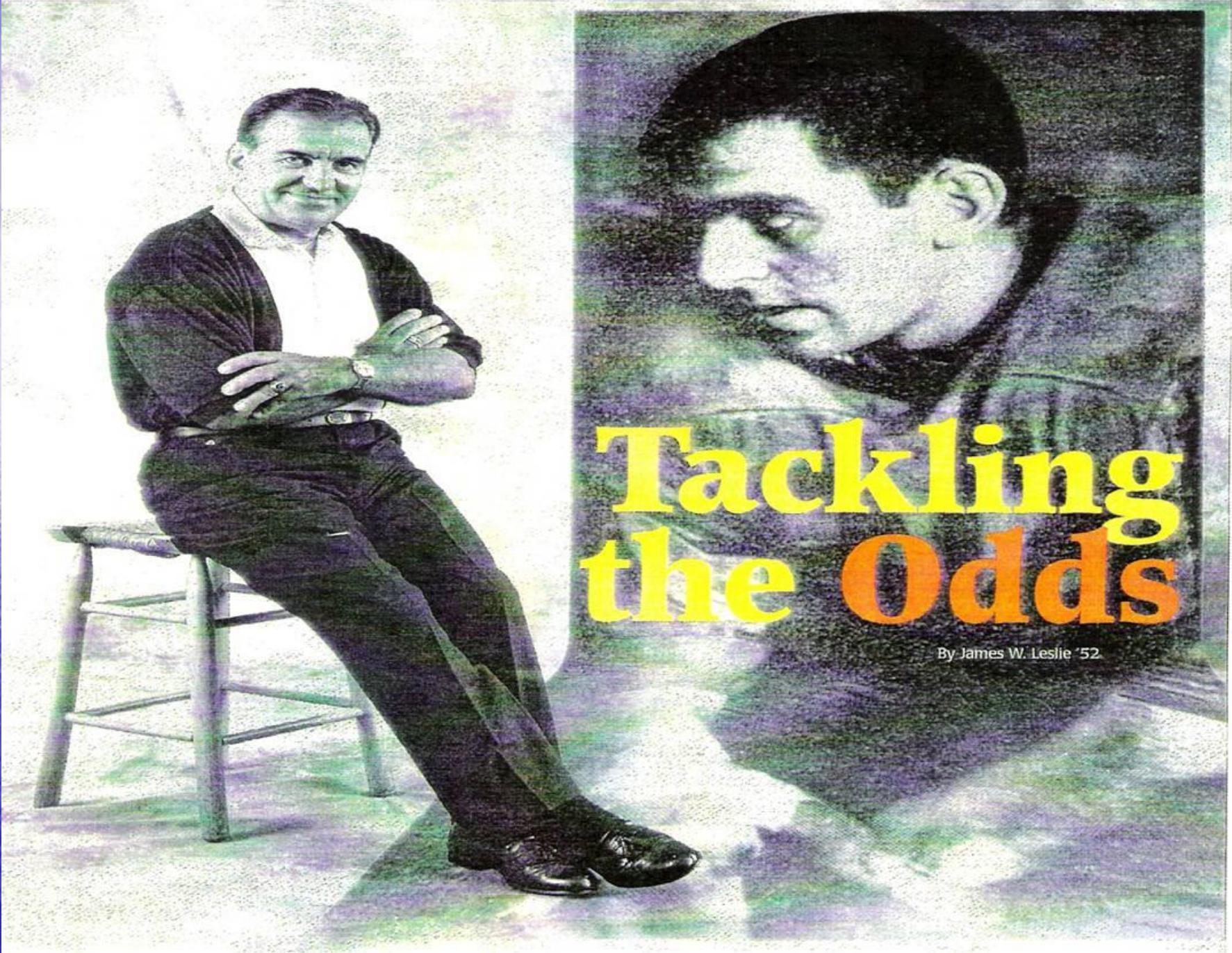
- Model on CO₂ Emissions
- Model on CO₂ in the Atmosphere
- Basic Evaluations on Both Models Are Good
- Both Models Perform Well Without Knowing the Future Information

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Tackling the Odds

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Thank You !

