A Temperature Forecasting Model for the Continental United States

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A Temperature Forecasting Model

Two major entities that play a major role in understanding Global Warming is temperature and Carbon Dioxide. The purpose of the present study is to utilize historical temperature in the Continental United States from 1895 to 2007 to develop a forecasting process to estimate future average monthly temperatures.
In addition, we shall study through our modeling if there is a difference in the two methods that are being used to collect and massage the temperatures in the Continental United States.

A Temperature Forecasting Model

The Version 1 data set consists of monthly mean temperature and precipitation for all 344 climate divisions in the contiguous U. S.
from January 1895 to June 2007.
The data is adjusted for time of observation bias, however, no other adjustments are made for inhomogeneities. These inhomogeneities include changes in instrumentation, observer, and observation practices, station and instrumentation moves,
and changes in station composition resulting
from stations closing and opening over time within
a division.

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The Version 2 data set was first become available in July 2007, and it consists of data from a network of 1219 stations in the contiguous United States that were defined by scientists at the Global Change Research Program of the U. S.
Department of Energy at National Climate Data Center.

A Temperature Forecasting Model

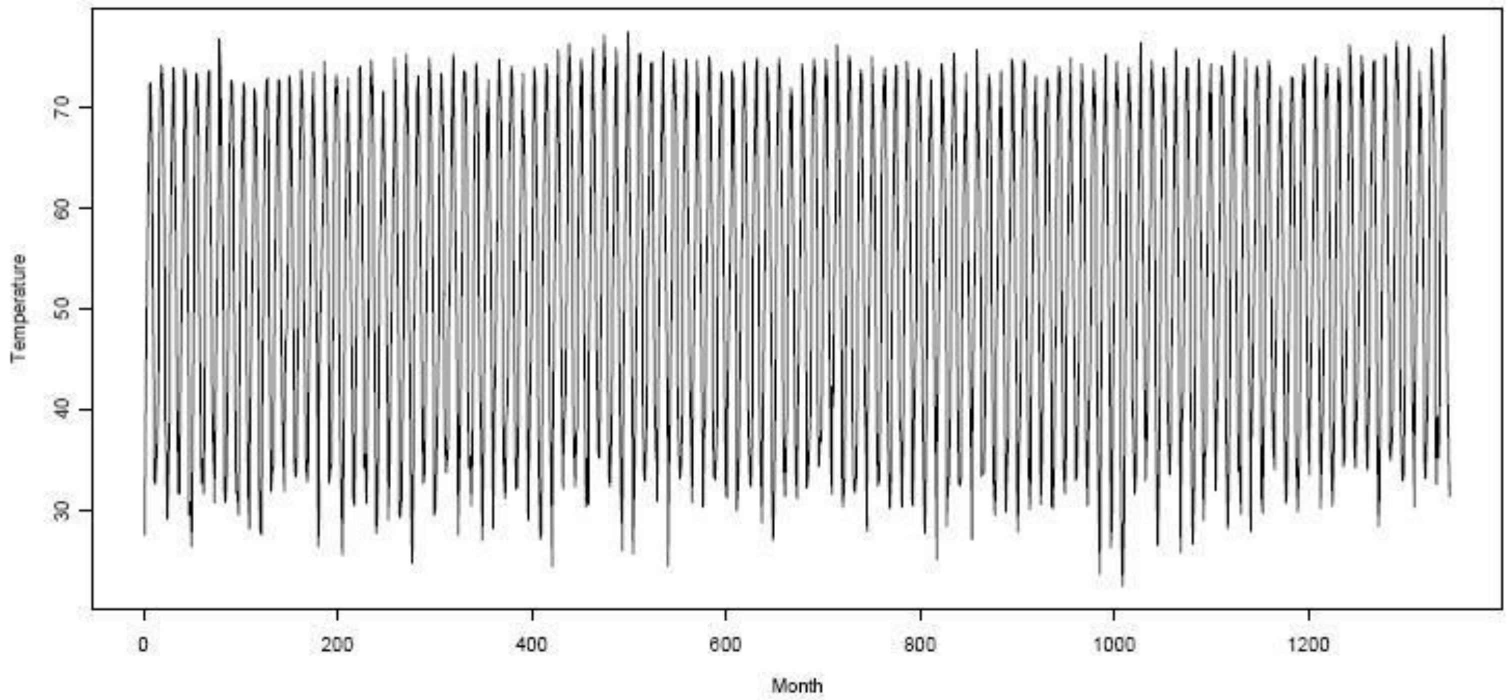

Figure 1.1 Time Series Plot for Monthly Temperature from the Continental United States 1895-2007 (Version 1 Dataset)

A Temperature Forecasting Model

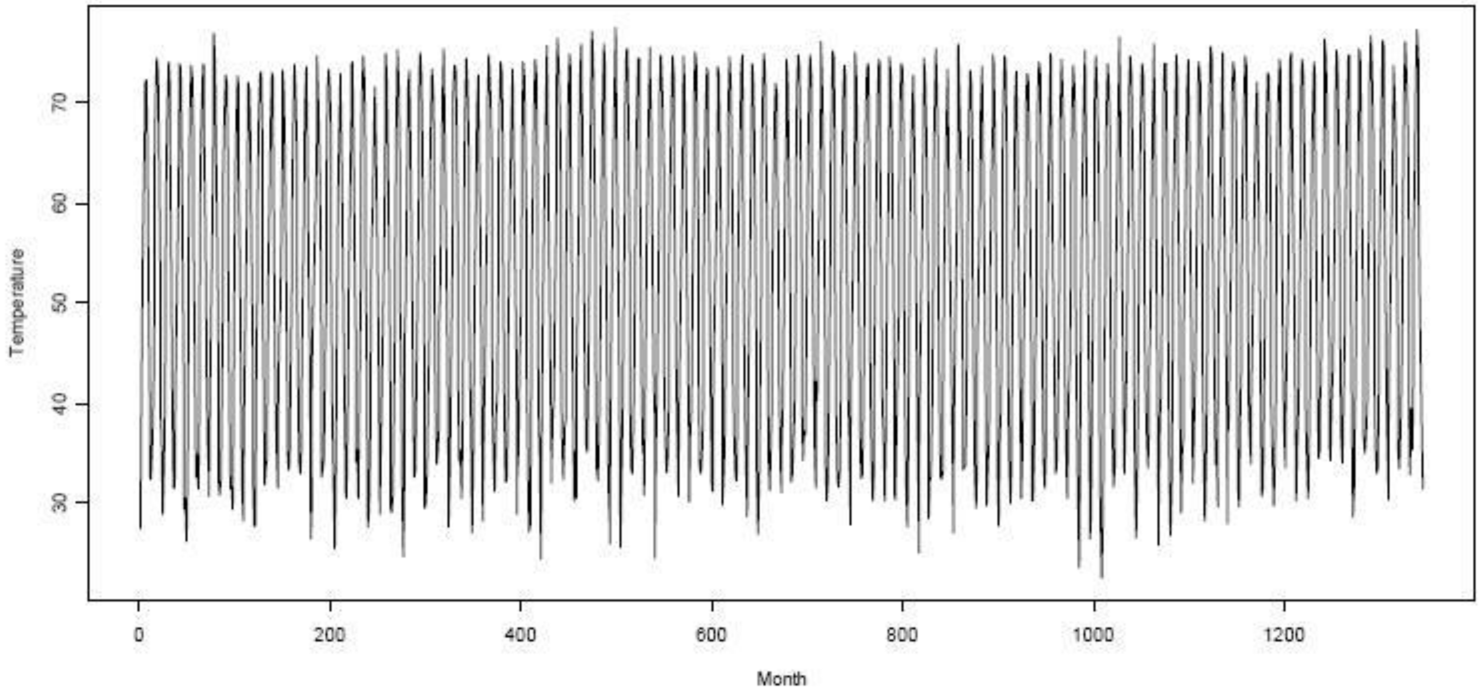

Figure 2.2 Time Series Plot for Monthly Temperature from the Continental United States 1895-2007 (Version 2 Dataset)

ANALYTICAL PROCEDURE

The multiplicative seasonal autoregressive integrated moving average, ARIMA model is defined by

$$\Phi_p(B^s)\phi_p(1-B)^d(1-B^s)^D x_t = \theta_q(B)\Gamma_Q(B^s)\varepsilon_t \qquad (2.1)$$

p is the order of the autoregressive process
d is the order of regular differencing
q is the order of the moving average process
P is the order of the seasonal autoregressive process
D is the order of the seasonal differencing
Q is the order of the seasonal moving average process
s refers to the seasonal period

ANALYTICAL PROCEDURE

$\text{ARIMA}(p,d,q)\times(P,D,Q)_s$, and $\phi_p(B),\theta_q(B),\Phi_P(B^s),\Gamma_q(B^s)$

$$\phi_p(B)=(1-\phi_1B-\phi_2B^2-...-\phi_pB^p)$$

$$\theta_q(B)=(1-\theta_1B-\theta_2B^2-...-\theta_qB^q)$$

$$\Phi_P(B^s)=1-\Phi_1B^s-\Phi_2B^{2s}-...-\Phi_PB^{Ps}$$

$$\Gamma_Q(B^s)=1-\Gamma_1B^s-\Gamma_2B^{2s}-...-\Gamma_QB^{Qs}.$$

ARIMA(p,d,q) x (P, D, Q)

Below we summarize the model identifying procedure:

- Determine the seasonal period s.
- Check for stationarity of the given time series $\{x_t\}$ by determining the order of differencing d, where $d = 0,1,2,...$ according to KPSS test, until we achieve stationarity.
- Deciding the order m of the process, for our case, we let $m = 5$ where $p + q + P + Q = m$.
- After (d, m) being selected, listing all possible configurations of (p, q, P, Q) for $p + q + P + Q \leq m$.
- For each set of (p, q, P, Q), estimates the parameters for each model, that is, $\phi_1, \phi_2, ..., \phi_p, \theta_1, \theta_2, ..., \theta_q, \Phi_1, \Phi_2, ..., \Phi_P, \Gamma_1, \Gamma_2, ..., \Gamma_Q$.
- Compute the AIC for each model, and choose the one with smallest AIC.
- After (p, d, q, P, Q) is selected, we determine the seasonal differencing filter by selecting the smaller AIC between the model with $D = 0$ and $D = 1$.
- Our final model will have identified the order of (p, d, q, P, D, Q).

In order to determine how good our proposed model is, we shall define several

ARIMA(p,d,q) x (P, D, Q)

In order to determine how good our proposed model is, we shall define several statistical criteria that we shall use to evaluate the subject forecasting model. The residuals of the model, $r_t = x_t - \hat{x}_t$, where x_t and $\hat{x}_t$ are the actual value and predicted value, respectively. Mean of the residuals, $\bar{r} = \frac{\sum_{t=1}^{n} r_t}{n}$. Variance of the residuals, $S_r^{\,2} = \frac{\sum_{t=1}^{n} (r_t - \bar{r})}{n-1}$. Standard deviation of the residuals, $S_r = \sqrt{S_r^{\,2}}$. Standard error of the residuals, $SE = S_r^{\,2} / \sqrt{n}$. Mean square error, $MSE = \frac{\sum_{t=1}^{n} r_t^{\,2}}{n}$.

DEVELOPMENT OF FORECASTING MODELS

The historical temperature data for the continental United States that we shall use are shown by Figure 1 and 2. A visual inspection does not show any obvious trends being present. Thus, we let the seasonal period $s = 12$. Following the step-by-step procedure we described above, we found that the model best characterizes the average monthly temperature of the Continental United States for both Version 1 and 2 is a $\text{ARIMA}(2,1,1)\times(1,1,1)_{12}$ process, analytical given by

$$(1-\Phi_1 B^{12})(1-\phi_1 B-\phi_2 B^2)(1-B)(1-B^{12})x_t = (1-\theta_1 B)(1-\Gamma_1 B^{12})\varepsilon_t \qquad (3.1)$$

Expanding both sides of the above ARIMA, we have

$$\begin{aligned}&[1-(1+\phi_1)B+(\phi_1-\phi_2)B^2+\phi_2 B^3-(1+\Phi_1)B^{12}+(1+\phi_1+\Phi_1+\phi_1\Phi_1)B^{13}\\&+(\phi_2+\phi_2\Phi_1-\phi_1-\phi_1\Phi_1)B^{14}-(\phi_2+\phi_2\Phi_1)B^{15}+\Phi_1 B^{24}-(\phi_1+\Phi_1)B^{25}\\&+(\phi_1\Phi_1-\phi_2\Phi_1)B^{26}+\phi_2\Phi_1 B^{27}]x_t=(1-\theta_1 B-\Gamma_1 B^{12}+\theta_1\Gamma_1 B^{13})\varepsilon_t\end{aligned}$$

Simplify it, we get

$$\begin{aligned}&x_t-(1+\phi_1)x_{t-1}+(\phi_1-\phi_2)x_{t-2}+\phi_2 x_{t-3}-(1+\Phi_1)x_{t-12}+(1+\phi_1+\Phi_1+\phi_1\Phi_1)x_{t-13}\\&+(\phi_2+\phi_2\Phi_1-\phi_1-\phi_1\Phi_1)x_{t-14}-(\phi_2+\phi_2\Phi_1)x_{t-15}+\Phi_1 x_{t-24}-(\phi_1+\Phi_1)x_{t-25}\\&+(\phi_1\Phi_1-\phi_2\Phi_1)x_{t-26}+\phi_2\Phi_1 x_{t-27}=\varepsilon_t-\theta_1\varepsilon_{t-1}-\Gamma_1\varepsilon_{t-12}+\theta_1\Gamma_1\varepsilon_{t-13}\end{aligned}$$

DEVELOPMENT OF FORECASTING MODELS

The one-step ahead forecasting model for Version 1 data is given by

$$\begin{aligned}\hat{x}_t = {} & 1.0941x_{t-1} - 0.057x_{t-2} - 0.0371x_{t-3} + 0.9954x_{t-12} - 1.0891x_{t-13} + \\ & 0.0567x_{t-14} + 0.0369x_{t-15} + 0.0046x_{t-24} + 0.0895x_{t-25} - 0.0004x_{t-26} + \\ & 0.00017x_{t-27} - 0.9861\varepsilon_{t-1} - 0.9742\Gamma_1\varepsilon_{t-12} + 0.9607\varepsilon_{t-13}\end{aligned} \qquad (3.2)$$

and the one-step ahead forecasting model for Version 2 data is given by

$$\begin{aligned}\hat{x}_t = {} & 1.0952x_{t-1} - 0.0556x_{t-2} - 0.0396x_{t-3} + 0.9964x_{t-12} - 0.9009x_{t-13} + \\ & 0.0554x_{t-14} + 0.0395x_{t-15} + 0.0036\Phi_1 x_{t-24} + 0.0916x_{t-25} + 0.0002x_{t-26} + \\ & 0.00014x_{t-27} - 0.9855\varepsilon_{t-1} - 0.9741\varepsilon_{t-12} + 0.9599\varepsilon_{t-13}\end{aligned} \qquad (3.3)$$

Note the closeness of the two forecasting models.

EVALUATION OF THE PROPOSED MODELS

We begin by forecasting for the last one hundred observations the monthly average temperature in the Continental United States for both Version 1 and 2, using the models given by expression 3.2 and 3.3. A graphical presentation of the results are presented below by Figure 4.1 and 4.2.

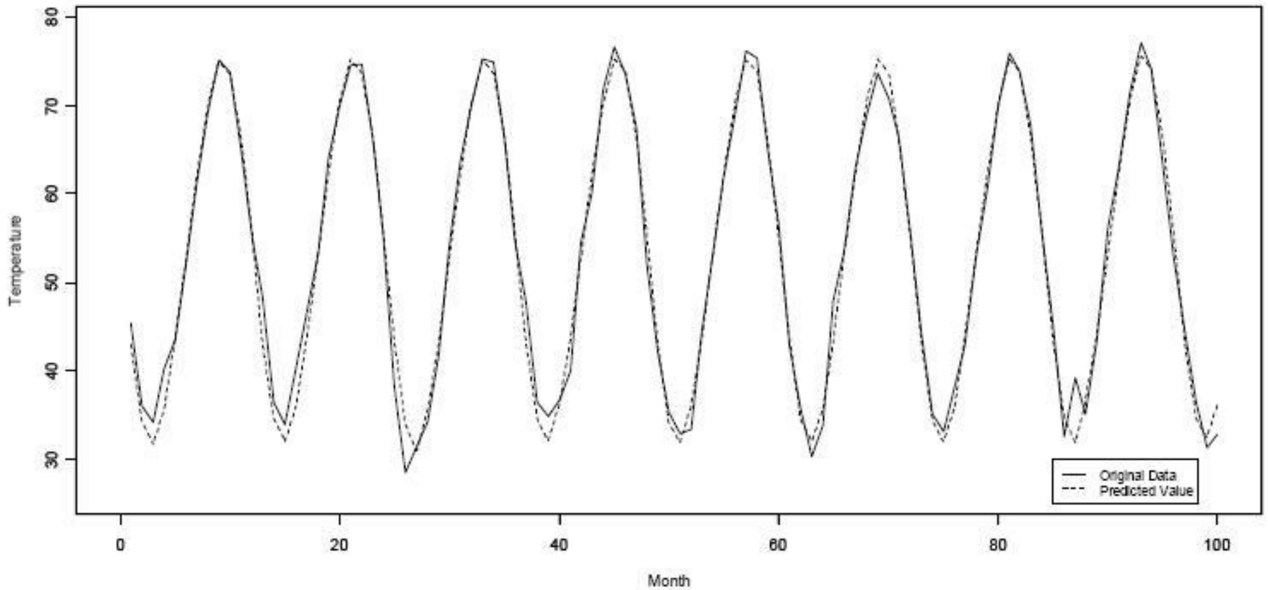

Figure 4.1 Monthly Temperature VS. Our Predicted Values for the Last 100 Observations (Version 1 Dataset)

EVALUATION OF THE PROPOSED MODELS

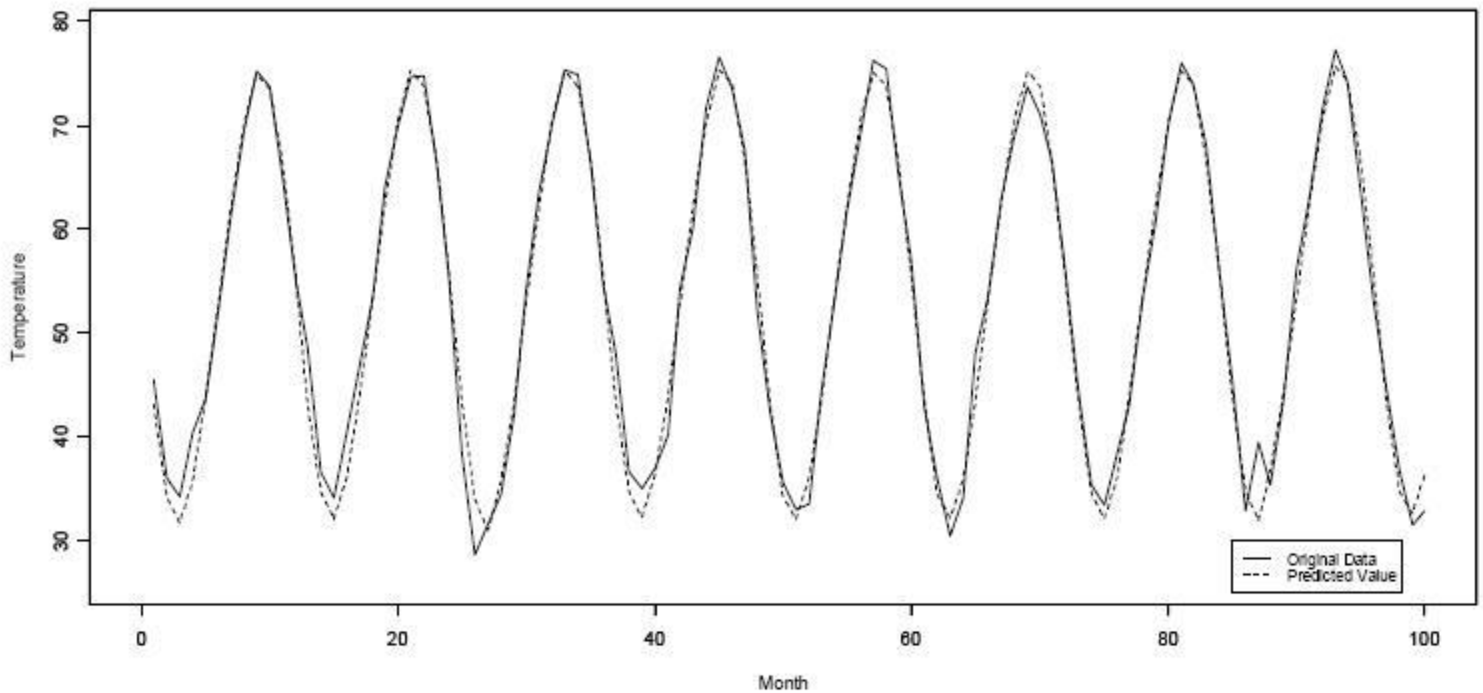

Figure 4.2 Monthly Temperature VS. Our Predicted Values for the Last 100 Observations (Version 2 Dataset)

Residual Plot (Version 1)

The residuals estimates, $r_t = x_t - \hat{x}_t$, for both forecasting process given by (3.2) and (3.3). The results are graphically presented below by Figure 4.3 and 4.4.

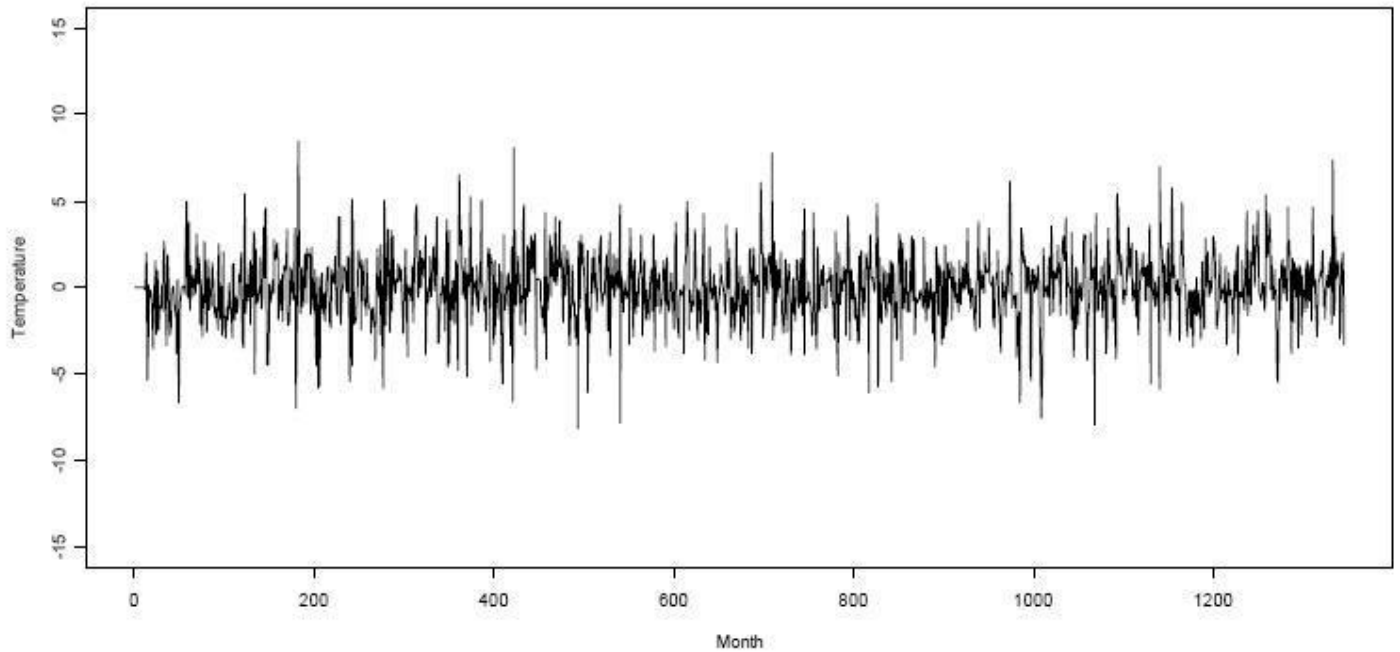

Figure 4.3 Residual Plot for Monthly Temperature on Continental United States 1895-2007 (Version 1 Dataset)

Residual Plot (Version 2)

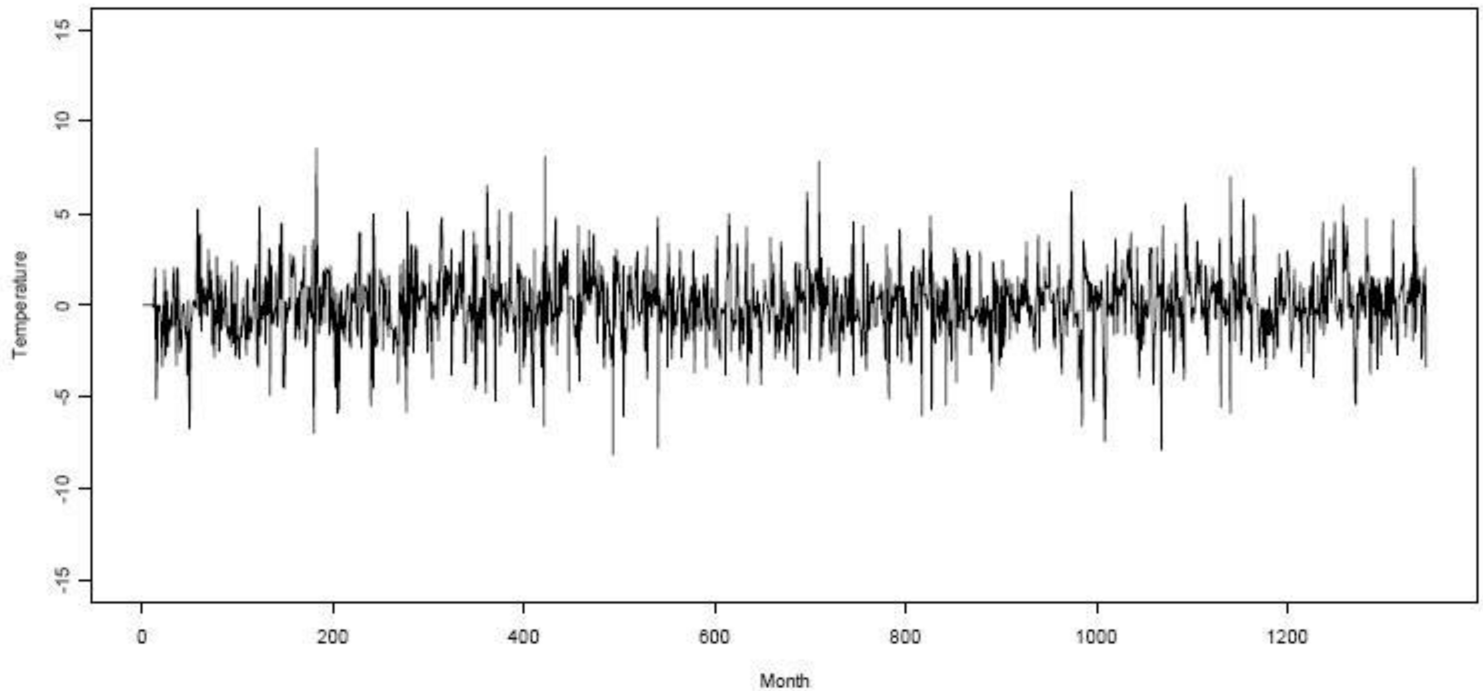

Figure 4.4 Residual Plot for Monthly Temperature on Continental United States 1895-2006 (Version 2 Dataset)

Basic Evaluation Statistics

The mean of the residuals, $\bar{r}$, the variance, S_r^2, the standard deviation, S_r, standard error, *SE*, and the mean square error, *MSE*. The results are presented below by Table 4.1 and 4.2, for Version 1 and Version 2 data, respectively.

Table 4.1 Basic Evaluation Statistics (Version 1 Dataset)

$\bar{r}$	S_r^2	S_r	*SE*	*MSE*
-0.008512476	4.331902	2.081322	0.05673052	4.328756

Table 4.2 Basic Evaluation Statistics (Version 2 Dataset)

$\bar{r}$	S_r^2	S_r	*SE*	*MSE*
-0.01310953	4.323726	2.079357	0.05667696	4.320685

We observe that all evaluation criteria support the quality of the proposed forecasting model. We can also conclude the similarity of the two models.

A Temperature Forecasting Model

we used the first 1334 observations $\{x_1, x_2, ..., x_{1334}\}$ to forecast $\hat{x}_{1335}$. Then we use the observations $\{x_1, x_2, ..., x_{1335}\}$ to forecast $\hat{x}_{1336}$, and continue this process until we obtain the forecasting values of the last 12 observations, that is, $\{\hat{x}_{1335}, \hat{x}_{1336}, ..., \hat{x}_{1346}\}$. Table 4.3, gives the actual, forecasting and residual data for the subject 12 months.

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Table 4.3 (Version 1 Dataset)

	Original Values	Forecast Values	Residuals
March 2006	43.31	44.0291	-0.7191
April 2006	56.03	53.1361	2.89395
May 2006	63.06	62.5318	0.52821
June 2006	71.44	70.6153	0.82467
July 2006	77.1	75.5855	1.51453
August 2006	74.1	74.2054	-0.1054
September 2006	63.69	66.6904	-3.0004
October 2006	52.97	55.4991	-2.5291
November 2006	44.68	43.2673	1.41275
December 2006	36.64	34.6357	2.00433
January 2007	31.39	32.58	-1.19
February 2007	32.86	36.2024	-3.3424

Figure 4.5 below gives a graphical presentation of the information presented in Table 4.3 for Version 1 observed time series.

Monthly Temperature VS. our Predicted Values

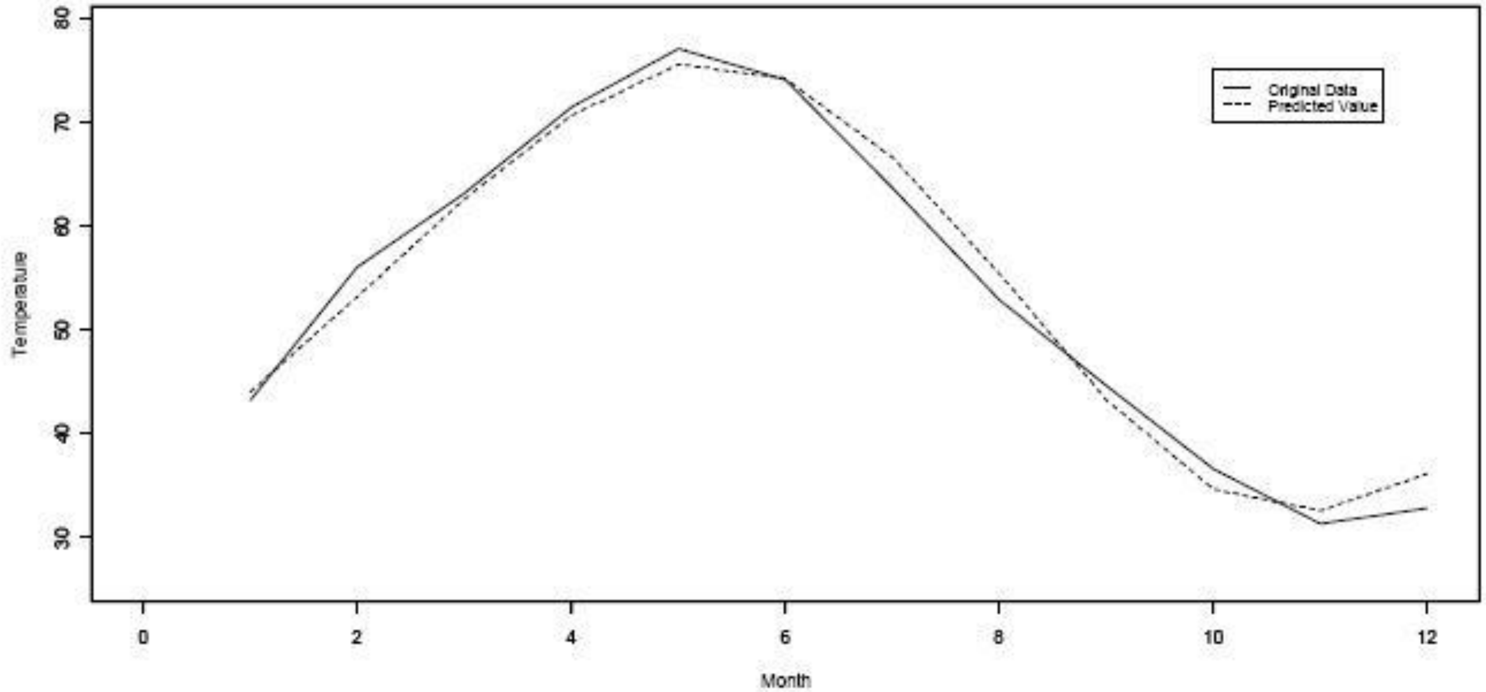


Figure 4.5. Monthly Temperature VS. Our Predicted Values for the Last 12 Observations (Version 1 Dataset)

Calculated the estimates values

Similarly, for Version 2 of the data set, we have calculated the estimates presented by Table 4.4.

Table 4.4

	Original Values	Forecast Values	Residuals
March 2006	43.45	44.1812	-0.7312
April 2006	56.12	53.2506	2.86942
May 2006	63.12	62.6351	0.48486
June 2006	71.55	70.7152	0.83478
July 2006	77.22	75.6947	1.52532
August 2006	74.19	74.3167	-0.1267
September 2006	63.86	66.8069	-2.9469
October 2006	53.13	55.6137	-2.4837
November 2006	44.58	43.3947	1.18529
December 2006	36.79	34.7224	2.06761
January 2007	31.46	32.6854	-1.2254
February 2007	32.86	36.3025	-3.4425

A graphical presentation of the results given in Table 4.4 are given below by Figure 4.6.

Monthly Temperature VS. our Predicted Values

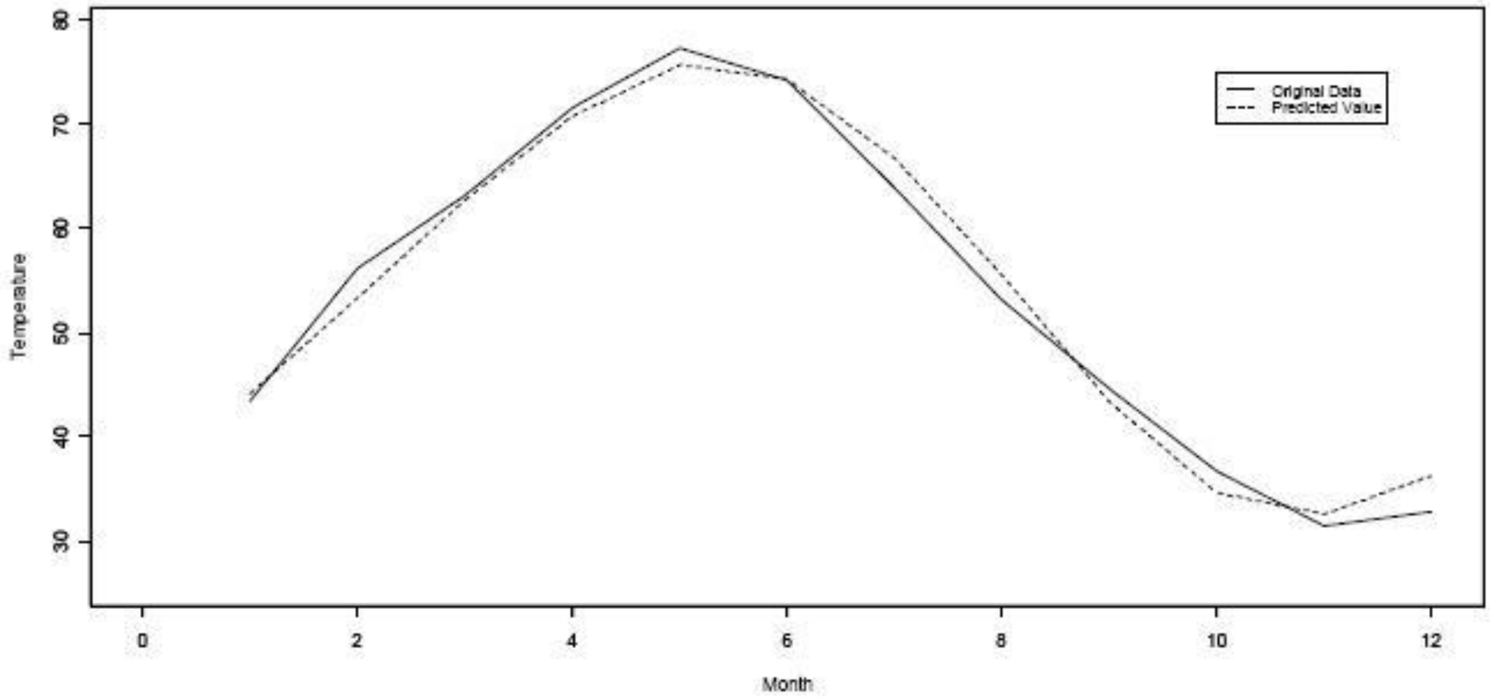
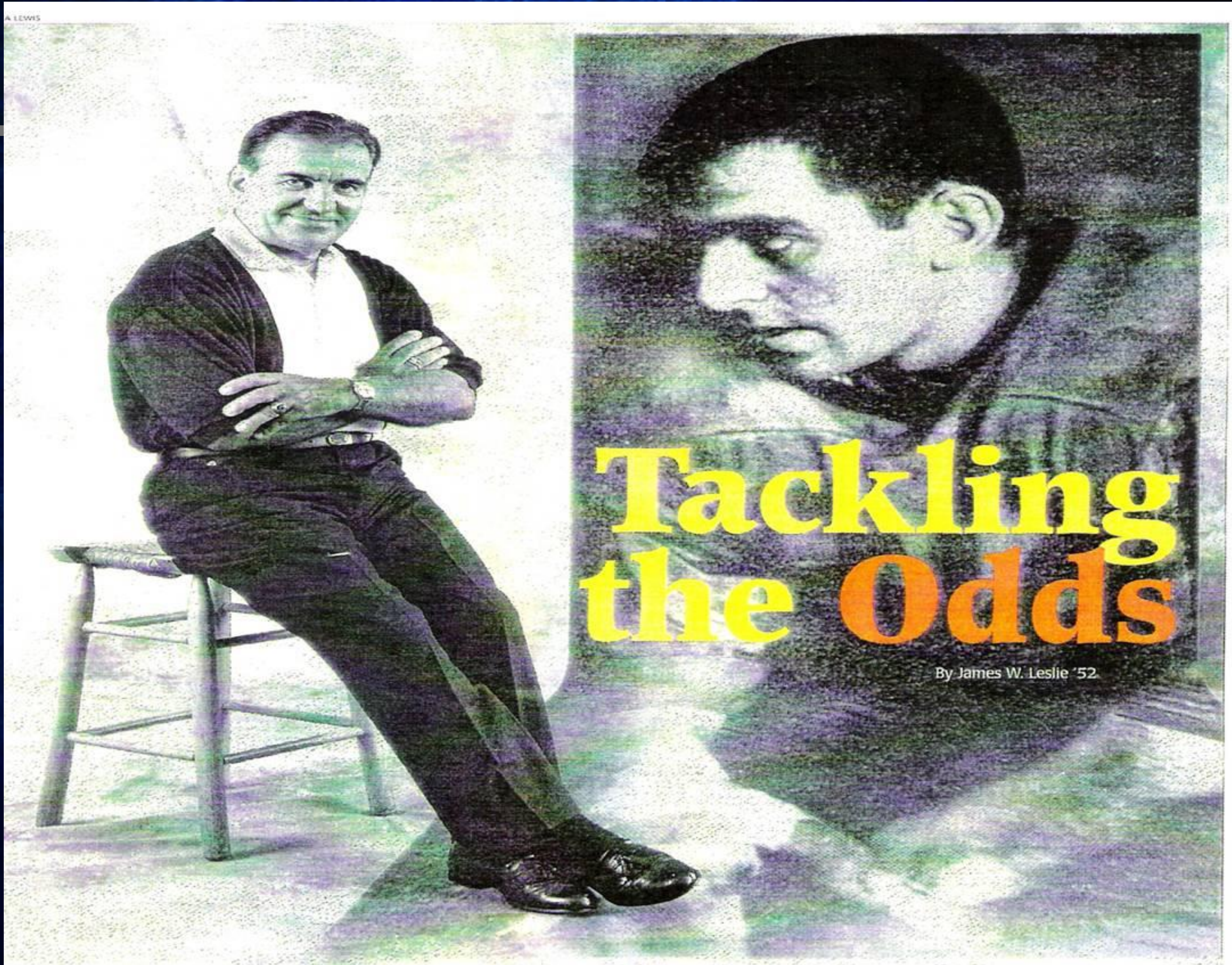


Figure 4.6. Monthly Temperature VS. Our Predicted Values for the Last 12 Observations (Version 2 Dataset)

CONCLUSIONS

We have developed two seasonal autoregressive integrated moving average models to forecast the monthly average temperature in the Continental United States using historical data for 1895-2007. The two models are based on two different methods, USCD and USHCN, that are been used to create the two temperature basis. The two developed models were evaluated and it was shown that the processes give good forecast values. In addition we can conclude that both Version 1 and 2 give really similar results and thus, both methods are not necessary.


A LEWIS

Tackling the Odds

By James W. Leslie '52

Thank You !

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