1 798 943 05C69 05C35
Clark, W. Edwin (1-SFL);
Suen, Stephen [Suen, W.-C. Stephen] (1-SFL);
Dunning, Larry A. (1-BLGS-C)
Tight upper bounds for the domination numbers of graphs with given order and minimum degree. II. (English. English summary)
Electron. J. Combin. **7** (2000), *no.* 1, *Research Paper* 58, 19 *pp.* (*electronic*).

2000m:05168 05C69
Clark, W. Edwin (1-SFL);
Suen, Stephen [Suen, W.-C. Stephen] (1-SFL)
An inequality related to Vizing's conjecture. (English. English summary)
Electron. J. Combin. **7** (2000), *no.* 1, *Note* 4, 3 *pp.* (*electronic*).
Summary: "Let $\gamma(G)$ denote the domination number of a graph G and let $G\square H$ denote the Cartesian product of graphs G and H. We prove that $\gamma(G)\gamma(H) \leq 2\gamma(G\square H)$ for all simple graphs G and H."

2000k:94056 94B60
Clark, W. Edwin (1-SFL);
Suen, Stephen [Suen, W.-C. Stephen] (1-SFL)
On the probability that a t-subset of a finite vector space contains an r-subspace—with applications to short, light codewords in a BCH code. (English. English summary)
Proceedings of the Thirtieth Southeastern International Conference on Combinatorics, Graph Theory, and Computing (Boca Raton, FL, 1999).
Congr. Numer. **137** (1999), 139–159.
Summary: "Motivated by the problem of finding light (i.e., low weight) and short (i.e., low degree) codewords in narrow-sense, primitive BCH codes, we consider the problem of determining the probability that a random t-set of vectors in an n-dimensional vector space over GF(q) contains an r-dimensional subspace (or affine subspace). We find some bounds for this and similar probabilities and apply these techniques to estimate how short a minimum weight codeword can be in a narrow-sense BCH code."

{For the entire collection see MR 2000k:05005.}

2000a:26023 26E99 05C05

Clark, W. Edwin (1-SFL); **McColm, Gregory L.** (1-SFL);
Shekhtman, Boris (1-SFL)

An application of spanning trees to k-point separating families of functions. (English. English summary)

J. London Math. Soc. (2) **58** (1998), *no.* 2, 297–310.

Let R be the set of all real numbers. Let $\mathrm{LS}_k(\mathbf{R}^n,\mathbf{R})$ be the cardinality of a smallest k-point separating family of linear functions from $\mathbf{R}^n$ to R. For smooth functions the cardinality is denoted by $\mathrm{DS}_k(\mathbf{R}^n,\mathbf{R})$. The authors establish that if $n,k\geq 2$ then $\mathrm{LS}_k(\mathbf{R}^n,\mathbf{R})=\mathrm{DS}_k(\mathbf{R}^n,\mathbf{R})=n(k-1)$. The linear case result is also extended to a larger class of fields. Some graph-theoretical results are invoked in the proofs. *K. Chandrasekhara Rao* (Karaikudi)

99j:05103 05C35

Clark, W. Edwin (1-SFL); **Shekhtman, Boris** (1-SFL);
Suen, Stephen [Suen, W.-C. Stephen] (1-SFL);
Fisher, David C. (1-COD)

Upper bounds for the domination number of a graph. (English. English summary)

Proceedings of the Twenty-ninth Southeastern International Conference on Combinatorics, Graph Theory and Computing (Boca Raton, FL, 1998).

Congr. Numer. **132** (1998), 99–123.

Summary: "This paper presents several upper bounds based on the degree sequence of a graph G for the domination number $\gamma(G)$. Our strongest result shows that the domination number of a graph G with p vertices and minimum degree δ is at most $(1-S_\delta)p$ where $S_\delta=\prod_{k=1}^{\delta+1}(k/(k+1/\delta))$. This supersedes bounds of Arnautov that $\gamma(G)\leq(p/(\delta+1))H_{\delta+1}$ where $H_k=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{k}$. Slight improvements are obtained in the case of regular graphs. Miscellaneous extremal results are also obtained."

{For the entire collection see MR 99i:00022.}

99i:20080 20M14

Clark, W. E. (1-SFL); **Holland, W. C.** (1-BLGS);
Székely, G. J. (1-BLGS)

Decompositions in discrete semigroups. (English. English summary)

Studia Sci. Math. Hungar. **34** (1998), *no.* 1-3, 15–23.

Summary: "We prove that under some finiteness conditions in a (not necessarily commutative and not necessarily cancellative) semigroup every non-unit is a product of weakly irreducible elements. In commutative, finitely generated semigroups, every infinitely divisible element is idempotent. Without commutativity this is not true. An interesting open problem is to find necessary and sufficient conditions for this implication." *J. L. Chrislock* (Santa Cruz, CA)

98m:05096 05C35

Clark, W. E. (1-SFL); **Shekhtman, B.** (1-SFL)

On the domination number of certain analogues of Kneser graphs. (English. English summary)

Proceedings of the Twenty-eighth Southeastern International Conference on Combinatorics, Graph Theory and Computing (Boca Raton, FL, 1997).

Congr. Numer. **126** (1997), 175–181.

Summary: "Let G be a multigraph whose vertices are all k-dimensional subspaces of $\mathbf{R}_n$. Two vertices E_1 and E_2 are adjacent iff $E_1 \cap E_2^{\perp} = \{0\}$. We show that the domination number $\gamma(k,n)$ of this graph is no more than $k(n-k)+1$. This result complements the previous results in [W. E. Clark and B. Shekhtman, Proc. Amer. Math. Soc. 125 (1997), no. 1, 251–254; MR 97c:15003; Bull. Inst. Combin. Appl. 19 (1997), 83–92; MR 97g:05100], where similar estimates were proved for finite fields and for an algebraically closed field. The proof relies on the methods of integral geometry."

{For the entire collection see MR 98h:05005.}

98m:05095 05C35

Clark, W. Edwin (1-SFL); **Dunning, Larry A.** (1-BLGS)

Tight upper bounds for the domination numbers of graphs with given order and minimum degree. (English. English summary)

Electron. J. Combin. **4** (1997), *no.* 1, *Research Paper* 26, 25 *pp.* (*electronic*).

The authors are interested in determining the maximum possible domination number of a graph (not necessarily connected) with a given number of vertices, say n, and with a specified minimum degree. Using results of J. F. Fink et al. [Period. Math. Hungar. 16 (1985), no. 4, 287–293; MR 87e:05085], W. D. McCuaig and F. B. Shepherd [J. Graph Theory 13 (1989), no. 6, 749–762; MR 90i:05053] and B. A. Reed [Combin. Probab. Comput. 5 (1996), no. 3, 277–295; MR 97k:05160] which deal with minimum degree one, two and three respectively, they first determine the value exactly when the minimum degree is no more than three. They also consider the case when the minimum degree equals $n-k$ and n is sufficiently large relative to k. For n at most 14 (and for all but 6 values when n is no more than 16) they give the exact value as well. *Bert L. Hartnell* (3-STMR)

97j:05039 05C45 05A10

Clark, W. Edwin (1-SFL); **Ismail, Mourad E. H.** (1-SFL)

Binomial and Q-binomial coefficient inequalities related to the Hamiltonicity of the Kneser graphs and their Q-analogues. (English. English summary)

J. Combin. Theory Ser. A **76** (1996), *no.* 1, 83–98.

The Kneser graph $K(n,k)$ has as vertices all the k-subsets of a fixed n-set and has as edges those pairs that are disjoint. The reviewer and B.-L. Chen [J. Combin. Theory Ser. B 42 (1987), no. 3, 257–263; MR 89a:05100] proved that $K(n,k)$ is Hamiltonian if $\binom{n-1}{k-1}\le\binom{n-k}{k}$ when $n\ge 2k+1$ and $k\ge 1$. F. J. Zhang and X. F. Guo [Sichuan Daxue Xuebao 26 (1989), Special Issue, 90–94; MR 91c:05123] proved that $K(n,k)$ is Hamiltonian connected if the above inequality is strict. Chen and Lih defined $e(k)$ to be the minimum n such that the inequality holds for $n\ge 2k+1$ and proved that $e(k)\le k(k+1)/2$. The authors of the paper under review obtain a more precise approximation to $e(k)$. In addition they give an asymptotic formula for the solution of

$$k\Gamma(n)\Gamma(n-2k+1)=\Gamma^2(n-k+1)$$

for $n \geq 2k+1$, as $k \to \infty$, when n and k are not restricted to take integer values. The q-analogue of $K(n,k)$ is the graph $K_q(n,k)$ whose vertices are the k-subspaces of the n-dimensional vector space $V(n,q)$ over the finite field with q elements and whose edges are those pairs having the zero subspace as their intersection. The authors establish q-analogues of the foregoing results of Chen-Lih and Zhang-Guo. They prove that for $n \geq 2k$ and $k \geq 1$ the graph $K_q(n,k)$ is Hamiltonian if $\begin{bmatrix} n-1 \\ k-1 \end{bmatrix}_q \leq \begin{bmatrix} n-k \\ k \end{bmatrix}_q q^{k^2}$ and Hamiltonian connected if the inequality is strict. They then show that the inequality holds strictly if $n \geq 2k$, $k \geq 1$, and q is any prime power. Hence the graphs $K_q(n,k)$ are Hamiltonian and Hamiltonian connected. *Ko-Wei Lih* (RC-AST)

97g:05100 05C35 05C70

Clark, W. Edwin (1-SFL); **Shekhtman, Boris** (1-SFL)

Domination numbers of q-analogues of Kneser graphs. (English. English summary)

Bull. Inst. Combin. Appl. **19** (1997), 83–92.

Summary: "We investigate the domination number and total domination number of the graph $K_q(n,k)$ whose vertices are all the k-subspaces of an n-dimensional vector space over a field with q elements and whose edges are the pairs $\{U,W\}$ of vertices such that $U \cap W = \{0\}$. Bounds are obtained in general and exact results are obtained for $n \geq k^2+k-1$ and in other cases when q is sufficiently large relative to n and k. We also consider analogous bipartite graphs."

97c:15003 15A03

Clark, W. Edwin (1-SFL); **Shekhtman, Boris** (1-SFL)

Covering by complements of subspaces. II. (English. English summary)

Proc. Amer. Math. Soc. **125** (1997), *no.* 1, 251–254.

Summary: "Let V be an n-dimensional vector space over an algebraically closed field K. Define $\gamma(k,n,K)$ to be the least positive integer t for which there exists a family $E_1, E_2, \cdots, E_t$ of k-dimensional subspaces of V such that every $(n-k)$-dimensional subspace F of V has at least one complement among the E_i. Using algebraic geometry we prove that $\gamma(k,n,K) = k(n-k)+1$."

{For Part I see the preceding review.}

B. N. Moyls (Vancouver, BC)

97c:15002 15A03

Clark, W. Edwin (1-SFL); **Shekhtman, Boris** (1-SFL)

Covering by complements of subspaces. (English. English summary)

Linear and Multilinear Algebra **40** (1995), *no.* 1, 1–13.

Summary: "Let V be an n-dimensional vector space over a field F. We attempt to determine the least positive integer $\gamma = \gamma(k, n, F)$ for which there exists a family $U_1, U_2, \cdots, U_\gamma$ of k-dimensional subspaces of V such that for every $(n-k)$-dimensional subspace W of V there is an $i \in \{1, 2, \cdots, \gamma\}$ satisfying $U_i \oplus W = V$. We find upper and lower bounds for $\gamma(k, n, F)$. In a few special cases we find exact values."

B. N. Moyls (Vancouver, BC)

96b:05129 05C70 05C35

Chang, Tony Yu (1-SFL); **Clark, W. Edwin** (1-SFL);
Hare, Eleanor O. (1-CLEM-C)

Domination numbers of complete grid graphs. I. (English. English summary)

Ars Combin. **38** (1994), 97–111.

This paper concerns the domination numbers $\gamma_{k,n}$ of the complete $k \times n$ grid graphs $(P_k \times P_n)$ for $1 \le k \le 10$ and $n \ge 1$. These numbers were previously established for $1 \le k \le 6$. Here, dominating sets are presented for $5 \le k \le 10$ and $n \ge 1$, giving new upper bounds for $\gamma_{k,n}$ for $7 \le k \le 10$ and large n. Evidence which indicates that these bounds may be the exact values of $\gamma_{k,n}$ is also discussed.

Christine M. Mynhardt (3-VCTR)

96a:11018 11B65 05A19 05A30 11A07 33D20

Clark, W. Edwin (1-SFL)

q-analogue of a binomial coefficient congruence. (English. English summary)

Internat. J. Math. Math. Sci. **18** (1995), *no.* 1, 197–200.

This short and sweet paper is devoted to proving that $\left[{na \atop nb}\right]_q \equiv \left[{a \atop b}\right]_{q^{n^2}} \bmod \Phi_n(q)^2$, where

$$\begin{bmatrix} A \\ B \end{bmatrix}_q = \begin{cases} 0 & \text{if } B < 0 \text{ or } B > A \\ \frac{(1-q^A)(1-q^{A-1})\cdots(1-q^{A-B+1})}{(1-q^B)(1-q^{B-1})\cdots(1-q)} & \text{otherwise,} \end{cases}$$

and $\Phi_n(q)$ is the nth cyclotomic polynomial in the variable q.

The result is the q-analogue of the known binomial coefficient congruence $\binom{pa}{pb} \equiv \binom{a}{b} \bmod p^2$ where p is a prime.

The proof relies on a natural extension of the q-analogue of the Chu-Vandermonde summation and the factorization of the Gaussian polynomial. *George E. Andrews* (1-PAS)

95k:52009 52A37

Clark, W. Edwin (1-SFL)

Separating sets with parallel classes of hyperplanes. (English. English summary)

Bull. Inst. Combin. Appl. **13** (1995), 65–82.

Let V be an n-dimensional vector space over a field F of order q. A hyperplane in V is a translation of an $(n-1)$-dimensional subspace W of V and the set of all translations of W is called a parallel class of hyperplanes. A parallel class H is said to separate a subset $S \subseteq V$ if each hyperplane in H meets S in at most one point. A family $\mathcal{P}$ of parallel classes of hyperplanes is said to be k-point separating if for every k-subset S of V there is a parallel class in $\mathcal{P}$ that separates S. In the paper it is proved that there is an integer $c(n,k)$ such that if $q \geq c(n,k)$ and $n,k \geq 2$ then any k-point separating family $\mathcal{P}$ contains at least $n(k-1)$ members. A partial result is also proved when $q < c(n,k)$. *Bela Uhrin* (H-AOS-C)

94g:15001 15A03 05C99

Clark, W. Edwin (1-SFL)

Bounds on a class of partial partitions of a vector space over GF(2): a graph theoretical approach. (English. English summary)

Linear and Multilinear Algebra **32** (1992), *no.* 3-4, 225–235.

A collection Q of linearly independent w-subsets of the n-dimensional vector space $V(n)$ over GF(2) is called a w-quilt if whenever X and Y are distinct elements of Q, then X is disjoint from the linear span of Y. The main results of the paper are the following: (1) If $w \geq 6$, then every w-quilt Q in $V(w+2)$ satisfies $|Q| \leq 21(2^{w-4}/w)+4$, where $|Q|$ denotes the cardinality of Q. (2) If $w \geq 12$, then every w-quilt Q in $V(w+2)$ satisfies

$$|Q| \leq (21 - \tfrac{1}{12}(1-(\tfrac{1}{4})^{t-3}))2^{w-4}/w + t + 1,$$

where $t = [(w - \log_2(w))/2]$. *K. Chandrasekhara Rao* (Karaikudi)

94b:05103 05C35

Chang, Tony Yu (1-SFL); **Clark, W. Edwin** (1-SFL)

The domination numbers of the $5 \times n$ and $6 \times n$ grid graphs. (English. English summary)

J. Graph Theory **17** (1993), *no.* 1, 81–107.

Let G be a graph with vertex set V. A subset D of V is a dominating set if for every vertex x not in D, there is a vertex of D adjacent to x. The order of the smallest dominating set is called the domination number, and denoted $\gamma(G)$. Motivated by the long-standing conjecture of Vizing that $\gamma(G \times H) \geq \gamma(G) \times \gamma(H)$, numerous authors have determined $\gamma(G \times H)$ for various G and H. The reviewer and L. F. Kinch [Ars Combin. 18 (1984), 33–44; MR 87a:05087] determined $\gamma(P_m \times P_n)$ exactly, for $m = 2, 3$ and 4 and all values of n. In this paper, the authors give an exact formula for $\gamma(P_m \times P_n)$ with $m = 5$ and 6, and all values of n. The formulas agree with those given by E. O. Hare ["Algorithms for grid and grid-like graphs", Ph.D. Thesis, Dept. Comput. Sci., Clemson Univ., Clemson, SC, 1989; per bibl.] for $n \leq 500$. *Michael Jacobson* (1-LSVL)

93h:11139 11T99 05B10 11B13

Clark, W. Edwin (1-SFL); **Pedersen, John** (1-SFL)

Sum-free sets in vector spaces over GF(2). (English. English summary)

J. Combin. Theory Ser. A **61** (1992), *no.* 2, 222–229.

A subset S of an abelian group is said to be sum-free if $a, b \in S$ implies $a + b \notin S$. Such a set is maximal sum-free (msf) if it is not a proper subset of another sum-free set. Sum-free sets have applications in areas such as Ramsey theory and coding.

If $V(n)$ denotes the vector space of dimension n over GF(2), it is shown that there are no msf sets S in $V(n)$ if $5 \cdot 2^{n-4} < |S| < 2^{n-1}$. For $n \geq 4$ there are msf sets having cardinalities $5 \cdot 2^{n-4}$ and $2^{n-s} + 2^{s+t} - 3 \cdot 2^t$ for $0 \leq t \leq n-4$ and $2 \leq S \leq [(n-t)/2]$. Also, if S is an msf set then $|S| \geq (\sqrt{2^{n+3} - 7} - 1)/2$. A computer search has found all msf sets if $n \leq 6$ and numerous examples for $7 \leq n \leq 10$. *W. A. Webb* (1-WAS)

93g:94012 94B05 94B15

Clark, W. Edwin (1-SFL); **Dunning, Larry A.** (1-BLGS-C)

Partial partitions of vector spaces arising from the construction of byte error control codes. (English. English summary)

Ars Combin. **33** (1992), 161–177.

Let $H = \{H_1, H_2, \cdots, H_m\}$ be a partial partition of the vector space V defined over $\mathrm{GF}(q)$ with the H_i pairwise disjoint w-subsets and let $\langle H_i\rangle$ denote the subspace of V spanned by H_i. Of concern are partitions that have one or more of the following properties: (i) Each H_i is linearly independent; (ii) $H_i \cap \langle H_j\rangle = \varnothing$ if $i \neq j$; (iii) $\langle H_i\rangle \cap \langle H_j\rangle = \{0\}$ if $i \neq j$; (iv) any two elements of $H_1 \cup H_2 \cup \cdots \cup H_m$ are linearly independent; (v) any three elements of $H_1 \cup H_2 \cup \cdots \cup H_m$ are linearly independent. This paper examines the following: (1) the relationship between byte-error-control codes and partial partitions with various subsets of the listed properties; (2) the largest partial partitions, for fixed r and w, having properties (i) and (iii); (3) the largest partition (over $\mathrm{GF}(2)$), called a quilt, for fixed r and w, having properties (i) and (ii). It also presents solutions to the following: (4) Find the largest partial partition, for fixed r and w, having property (i) only; and (5) find the largest partition, for fixed r and w, having property (i), such that every component H_i of H is contained in the set of columns of the parity-check matrix of a cyclic or an extended cyclic code. *J. J. Stiffler* (Marlborough, MA)

93g:05112 05C70

Clark, W. Edwin (1-SFL)

Matching subspaces with complements in finite vector spaces. (English. English summary)

Bull. Inst. Combin. Appl. **6** (1992), 33–38.

Summary: "Let $C(n,q)$ denote the graph whose vertices are the subspaces of the n-dimensional vector space V over $\mathrm{GF}(q)$ and whose edges are the pairs $\{X, Y\}$ where $X \oplus Y = V$. We show that $C(n,q)$ has a perfect matching if and only if q is odd or n is odd. If q and n are even and $q > 2$ we show there exist matchings which omit only one vertex."

92j:51016 51E21 05B25 51E22 94B05

Clark, W. Edwin (1-SFL)

Blocking sets in finite projective spaces and uneven binary codes.

Discrete Math. **94** (1991), *no.* 1, 65–68.

Summary: "A 1-blocking set in the projective space $\mathrm{PG}(m,2)$, $m \geq 2$, is a set B of points such that any $(m-1)$-flat meets B and no 1-flat is contained in B. A binary linear code is said to be uneven if it contains at least one codeword of odd weight. If B is a 1-blocking set in $\mathrm{PG}(r-1,2)$ and $\dim\langle B\rangle = r-1$, any matrix H whose columns are the vectors in B is a parity check matrix for an uneven binary code of length $n=|B|$, redundancy r, and minimum distance at least 4; conversely, if B is the set of columns of the parity check matrix of such a code then it is a 1-blocking set. Using this and results on uneven binary codes of minimum distance 4, we show that there exists a 1-blocking set of cardinality n if and only if $5 \leq n \leq 5\cdot 2^{m-3}$."

91f:94023 94B20

Clark, W. Edwin (1-SFL); **Dial, Gur** (BR-FSC)

Remarks on the Sharma-Kaushik metrics for error-correcting codes.

J. Combin. Inform. System Sci. **13** (1988), *no.* 3-4, 74–78.

Summary: "We show that the definition of a class of metrics introduced by B. D. Sharma and M. L. Kaushik may be simplified and we answer a question of Sharma concerning possible generalizations of this class of metrics."

91c:94028 94B05

Clark, W. E. (1-SFL); **Dunning, L. A.** (1-BLGS); **Rogers, D. G.**

Binary set functions and parity check matrices.

Discrete Math. **80** (1990), *no.* 3, 249–265.

The authors study binary linear (n,k,d) codes, where d is the minimum distance, $d \geq 4$ and $r=n-k$ is the redundancy, for which one can construct a parity check matrix in which all of the columns are of odd weight. This class of codes is put in correspondence with (δ,r)-parity standard systems, where $\delta = 2^{r-1}-n$, $r\geq 4$.

The authors prove that, for $r \geq 4$, the value $n(r) = 5\times 2^{r-4}+1$ is a lower bound for the values n such that every binary $(n,n-r,d\geq 4)$ code has a parity check matrix formed only of odd-weight columns. They also show that the value $\delta(r) = 3\times 2^{r-4}-1$ is an upper bound for the values δ such that every (δ,r)-parity system is standard.

This equivalence is applied to the study of binary codes with unequal error protection, and new constructions and bounds are found for those with minimum distance $d = 3, 4$.

REVISED *Josep Rifa* (E-BARA-I)

90f:94049 94B40 11T71

Clark, W. Edwin (1-SFL); **Lewis, Larry W.** (1-SFL)

Prime cyclic arithmetic codes and the distribution of power residues.

J. Number Theory **32** (1989), *no.* 2, 220–225.

The authors discuss the weight distribution of prime cyclic arithmetic codes, which are analogues of irreducible cyclic codes, for which this problem has been studied in detail. The paper deals with number theory, and the reader needs no background in coding theory.

Let r and n be integers > 1, $m = r^n - 1$, p a prime divisor of m, $a = m/p$, and Z_m the ring of integers mod m. A prime cyclic arithmetic code (with length n and base r) is an ideal of Z_m generated by a. Such a code can be taken equal to Z_p, and the following norm (or weight) is defined in Z_p: let $\langle r\rangle$ denote the subgroup of U_p, the group of nonzero elements in Z_p, generated by r; $\|x\|$ is the number of elements of the coset $\langle r\rangle x$ which lie in the interval $\mathcal{M}(p,r) = \{[p/(r+1)]+1, [p/(r+1)]+2, \cdots, [rp/(r+1)]\}$.

The authors study the function $\Delta(p,r) = \max\{|\mathcal{M}(p,r)|/d - \|x\|/x \in U_p\}$, where d is the index of $\langle r\rangle$ in U_p, and they give upper bounds on Δ, as well as conditions on r and p to have $\Delta(p,r) = 0$ (i.e., equidistant codes). Several examples of equidistant codes, obtained by computer search, are presented. *Antoine Lobstein* (Paris)

88h:94027 94B15

Clark, W. Edwin (1-SFL)

Cyclic codes over GF(q) with simple orbit structure.

Discrete Math. **61** (1986), *no.* 2-3, 151–164.

Let C denote a cyclic (n,k) code over $F = \mathrm{GF}(q)$ having parity check polynomial $h(x)$ of degree k. The author assumes that n is the multiplicative order of x modulo $h(x)$; by $R = F[x]/(h(x))$ he denotes the parity check algebra of C. If R^* is the unit group of R and $\langle x\rangle$ is the subgroup of R^* generated by $x + (h(x))$, he calls $h(x)$ a simple orbit structure if $R^* = F^*\langle x\rangle$. He calls $h(x)$ local if R is a local ring. The author gives a description of when $h(x)$ is local and has simple orbit structure in terms of q and the polynomial $h(x)$. In addition, he shows how to compute the weight classes for a cyclic code having simple orbit structure. *Eugene Spiegel* (1-CT)

87c:94059 94B40

Clark, W. Edwin (1-SFL); **Liang, Joseph J.** (1-SFL)

Equidistant binary arithmetic codes.

IEEE Trans. Inform. Theory **32** (1986), *no.* 1, 106–108.

Author summary: "Let $C(B)$ denote the binary cyclic AN code with generator A, where $AB = 2^n - 1$. It is known that $C(B)$ is equidistant if B is a prime power p^k, where either 2 or -2 is primitive modulo B provided $p \equiv 1 \pmod 3$ if $k > 1$. It is conjectured that these are the only B such that $C(B)$ is equidistant. We have verified this for $B < 100\,000$. We establish several results that further limit the possibilities for counterexamples to the conjecture."

82k:39003 39B05

Clark, W. E.; Mukherjea, A.

Comments on a functional equation.

Real Anal. Exchange **6** (1980/81), *no.* 2, 192–199.

The authors study all solutions of (1) $f(x+s) = af(x)$, $f(x+t) = bf(x)$, where a, b, s, t are real numbers, $a > 0$, $b > 0$, and s/t is irrational. They note that they were motivated by wanting to know conditions on the equation $f(x+1) = 2f(x)$ to ensure that any solution equals 2^x a.e. They also observe that (1) with $a = 1 = b$ appears in a problem of W. Rudin [Real and complex analysis, second edition, McGraw-Hill, New York, 1974; MR 49#8783]. Their results: (a) If there is a solution of (1) which is positive at some point and bounded above on some interval, then $a^t = b^s$; (b) If $a^t = b^s$, then every measurable solution of (1) equals $ca^{x/s}$ a.e.

They also exhibit a solution of (1) which is not continuous a.e., but they are unable to determine whether their example is measurable. They assert that they have some generalizations to metric topological groups, and they provide one such result.

As would be expected, a Hamel basis for the reals figures prominently in their arguments. *R. A. Rosenbaum* (Middletown, Conn.)

81k:94037 94B40
Clark, W. Edwin; Liang, J. J. [Liang, Joseph J.]
On block irreducible forms over Euclidean domains.
Internat. J. Math. Math. Sci. **3** (1980), *no.* 1, 15–28.
For elements of a Euclidean ring R with a real valuation $v(R)$, the authors establish a new canonical form, which generalizes previously known forms, that is valid for integers with an arbitrary radix r and for Gaussian integers with radix $r = \pm 1 \pm i$. The authors call this more general canonical form with radix r of R a block irreducible form. They prove that this form is unique and minimal and thus allows us to determine the arithmetical weight of an element with radix r. They also prove a theorem on the existence in R of block irreducible forms and give a constructive algorithm for their determination.
V. A. Arakelov (Grozny)

56 #11508 94A10
Clark, W. Edwin; Liang, J. J.
Weak radix representation and cyclic codes over Euclidean domains.
Comm. Algebra **4** (1976), *no.* 11, 999–1028.
Authors' summary: "We show that, to some extent, the two theories cyclic arithmetic coding and cyclic polynomial coding can be subsumed under a 'unified' theory based on a more or less arbitrary Euclidean domain. In particular, we show that Hamming weight (for cyclic polynomial codes) and arithmetic weight (for cyclic arithmetic codes) are special cases of a more general notion of weight defined for Euclidean rings. Of fundamental importance in this development is a generalization of the notion of radix representation for integers which we call a weak radix representation. For several specific Euclidean rings we prove the existence of certain unique canonical weak radix representations for all ring elements. We show, for example, that every Gaussian integer has a unique representation of the form (1) $a_n r^n + \cdots + a_1 r + a_0$ where $r = i - 1$, each a_j is zero or a unit, and whenever $a_j \neq 0$, then $a_{j+1} = a_{j+2} = 0$. This is analogous to a known result of Reitwiesner that each rational integer has a unique representation of the form (1) where $r = 2$, each a_j is 0 or ± 1, and whenever $a_j \neq 0$, $a_{j+1} = 0$."

56 #2633 94A10

Clark, W. Edwin

Equidistant cyclic codes over GF(q)**.**

Discrete Math. **17** (1977), *no.* 2, 139–141.

Author's summary: "It is proved that a cyclic (n,k) code over GF(q) is equidistant if and only if its parity check polynomial is irreducible and has exponent $e=(q^k-1)/a$ where a divides $q-1$ and $(a,k)=1$. The length n may be any multiple of e. The proof of this theorem also shows that if a cyclic (n,k) code over GF(q) is not a repetition of a shorter code and the average weight of its nonzero code words is integral, then its parity check polynomial is irreducible over GF(q) with exponent $n=(q^k-1)/a$ where a divides $q-1$."

54 #2764 18H40 20M25 55J10

Clark, W. Edwin

Cohomology of semigroups via topology–with an application to semigroup algebras.

Comm. Algebra **4** (1976), *no.* 10, 979–997.

Let S be a semigroup with 0. Let $B(S)$ denote the chain complex whose nth component is the free abelian group generated by the set of all n-tuples $(s_1,\cdots,s_n)$, where $s_i\in S$ and $s_1s_2\cdots s_n\neq 0$. The boundary operator is defined as it is for the group case. Although the usual cohomology groups of a semigroup with zero are trivial, that need not be the case for the various homology and cohomology groups derived from this complex.

For a field F let F^* denote the multiplicative group of nonzero elements. Each cocycle $z\in Z^2(S,F^*)$ gives rise to an F-algebra $zF[S]$ with basis $S-\{0\}$ and multiplication given by $s*t\equiv z(s,t)st$ if $st\neq 0$ and 0 otherwise. This algebra is referred to as the twisted semigroup algebra of S over F determined by the cocycle z. If two cocycles z and w differ by a coboundary then $zK[S]$ and $wK[S]$ are functorially isomorphic. Other relationships between cohomology groups of S and twisted semigroup algebras are derived in the paper.

By a matrix units semigroup S is meant a subsemigroup of $\{e_{ij}\colon 1\leq i,j\leq n\}\cup\{0\}$ which contains 0 and all $e_i=e_{ii}$ (where $e_{ip}e_{pj}=e_{ij}$ and $e_{ip}e_{qj}=0$ for $p\neq q$). Such semigroups are in one-to-one correspondence with the quasi-orderings on the set $\{1,\cdots,n\}$. The order is defined by $i\leq j$ if $e_{ij}\in S$. Two "oriented simplicial complexes" are obtained from this quasi-ordered set which have the same cohomology groups as S for $n\geq 2$. Conversely, every simplicial complex can be associated with some matrix units semigroup which has the same cohomology. The paper closes with some examples and remarks.

Jimmie D. Lawson

52 #16850 94A10

Clark, W. Edwin; Liang, J. J.

On arithmetic weight for a general radix representation of integers.

IEEE Trans. Information Theory **IT-19** (1973), 823–826.

The expression $N=\sum_i b_i r^i$ is said to be a modified radix-r form for the positive integer N, where r is an arbitrary integer greater than 1, if the coefficients b_i are integers satisfying the condition $-r<b_i<r$. It is said to be a generalized nonadjacent form (GNAF) if the coefficients b_i satisfy the following two conditions: (1) $|b_i+b_{i+1}|<r$ for all i, and (2) $|b_i|<|b_{i+1}|$, if $b_i b_{i+1}<0$, where $|b|$ denotes the absolute value of b. A GNAF is proved to be unique, and the arithmetic weight of an integer is shown to be equal to the number of nonzero terms in the form. Two algorithms are presented for the computation of this form. If $r=2$, the GNAF coincides with the well-known modified binary nonadjacent form. The results are of interest in the theory of arithmetic codes. *G. M. Tenengolc*

52 #5847 22A15

Clark, W. E.; Mukherjea, A.; Tserpes, N. A.

Is topologically simple simple?

Semigroup Forum **11** (1975), *no.* 1, 90–93.

A topological semigroup is simple if it contains no proper ideal and topologically simple if it contains no proper closed ideal. Motivated by a conjecture of H. L. Chow [Amer. Math. Monthly 82 (1975), 155–156; Zbl 302 #22007] that a topologically simple subsemigroup of a compact semigroup must be simple, the authors first give a counterexample to the conjecture but show that it holds for locally compact subsemigroups. This follows from the more general result that, in a locally compact completely simple semigroup with compact maximal groups, each locally compact subsemigroup is completely simple. *K. N. Sigmon*

50 #12433 94A10

Clark, W. Edwin; Liang, J. J.

On modular weight and cyclic nonadjacent forms for arithmetic codes.

IEEE Trans. Information Theory **IT-20** (1974), 767–770.

The modular weight of an integer a, $0 \le a < m$, used previously by T. R. N. Rao and O. N. Garcia [same Trans. IT-17 (1971), 85–91], was defined to be $w_m{}^*(a) = \min\{w(a), w(m-a)\}$, where $w(\cdot)$ is the arithmetic weight with radix r, r an integer not less than 2. In the present paper this is modified to $w_m(a) = \min\{w(x); x \equiv a \bmod m\}$ and it is shown that for $m = r^n$, $r^n - 1$ or $r^n + 1$, $w_m{}^*(a) = w_m(a)$, while if $m = r^n - 1$ or $r^n + 1$ then $w_m(ra) = w_m(a)$. For some values of m, $w_m{}^*$ may fail to be a metric, a defect which w_m does not have. The notion of a modular cyclic generalized nonadjacent form for $m = r^n - 1$ is introduced. An algorithm is given for computing the modular cyclic GNAF for any integer $N < r^n - 1$ from its radix r representation. The number of nonzero digits in the modular cyclic GNAF of an integer N, $0 \le N < m$, $m = r^n - 1$ is shown to be its modular weight. The use of residues in computing modular weight for $m = r^n - 1$ is discussed. *Ian Blake*

49 #2679 12C05 13B25

Clark, W. Edwin; Liang, Joseph J.

Enumeration of finite commutative chain rings.

J. Algebra **27** (1973), 445–453.

A commutative chain ring is a commutative ring with identity, ideals of which are linearly ordered by inclusion. For example, $\mathbf{Z}_q[x]/(f(x))$ is a finite commutative chain ring, where $q = p^n$ is a prime power, $f(x)$ is monic of degree r (say) and irreducible modulo p. This ring, which appears in an early work of W. Krull [Math. Ann. 92 (1924), 183–213, Jbuch 50, 72], is uniquely determined by p, n and r, and is denoted by $\mathrm{GR}(p^n, r)$. Every finite commutative chain ring R is easily seen to be of the form $S[\theta]$, where S is a coefficient ring and θ is a generator of the unique maximal (principal) ideal N of R. It is shown that $S \cong \mathrm{GR}(p^n, r)$, θ satisfies an Eisenstein polynomial equation $\theta^k = p(\sum_{i=0}^{k-1} a_i\theta^i)$ and $p^{n-1}\theta^t = 0$, where p^n is the characteristic of R, p^r is the order of R/N, k is the largest integer ($\le$ the index of nilpotency m of N) such that $p \in N^k$ and $t = m - (n-1)k$. The authors' main aim is to find the number of isomorphism classes of rings R with given invariants p, n, r, k and t. When $n = 1$, there is one such class $R = S[\theta]$ with $S \cong \mathrm{GF}(p^r)$ $(= \mathrm{GR}(p, r))$ and $\theta^k = 0$.

Assume that $n > 1$. R is called a pure chain ring when θ can be so chosen that the polynomial equation is of the form $\theta^k = pu$ with u a unit of S. The main result is that there are at least $\sum_{c|d} \varphi(c)/\tau(c)$ isomorphism classes of pure chain rings, where $d = (k, p^r - 1)$, φ is the Euler φ-function and $\tau(c)$ is the order of p in the group of units of the ring $\mathbf{Z}_c$. Moreover, if $(p, k) = 1$ then all chain rings are pure and this lower bound is attained. A theorem of Krull [op. cit.] for the case in which R/N is algebraically closed is extended to give necessary and sufficient conditions for one isomorphism class corresponding to given invariants in the general case. *D. Kirby*

48 #11200 16A44

Clark, W. Edwin; Drake, David A.

Finite chain rings.

Abh. Math. Sem. Univ. Hamburg **39** (1973), 147–153.

A ring with 1 is called a chain ring if its lattice of left ideals forms a chain, and it is known that a finite chain ring is a local uniserial ring and the lattice of its right ideals forms a chain, as well. L. A. Skornjakov [In memoriam: N. G. Cebotarev (Russian), pp. 75–88, Izdat. Kazan. Univ., Kazan, 1964; MR 34#190] showed that a finite chain ring is a homomorphic image of a ring of the form $\langle x, y\rangle/I$, where $\langle x, y\rangle$ is the ring of polynomials in the non-commuting indeterminates x, y over Z_{p^n} and I is an ideal generated by four elements of specified type. In the paper under review, the authors give a more revealing description of finite chain rings: Let R be a finite chain ring of characteristic p^n. Then R contains a unique (up to isomorphism) coefficient ring S which is isomorphic to a Galois ring $\mathrm{GR}(p^n, r)$ [cf. the first author, Proc. Amer. Math. Soc. 33 (1972), 25–28; MR 45#3481]. Let m be the nilpotency index of the radical of R. Then there exists an element $a \in R$, and positive integers $t \le k$ with $m = (n-1)k + t$, such that (1) ${}_SR = S \oplus Sa \oplus \cdots \oplus Sa^{k-1}$; (2) $a^k = pu$ with a unit $u \in R$; (3) $Sa^i \cong S$ $(1 \le i \le t-1)$ and $Sa^i \cong Sp$ $(t \le i \le k-1)$. Moreover, specializing to the commutative case, they show that a finite commutative ring is a chain ring if and only if it is a homomorphic image of an Eisenstein extension of a Galois ring. *H. Tominaga*

47 #210 13D15

Clark, W. Edwin; Bergman, George M.

The automorphism class group of the category of rings.

J. Algebra **24** (1973), 80–99.

Ausgangspunkt dieser Arbeit bildet die kategorietheoretische Charakterisierung von Ringeigenschaften. Dies ist nicht immer moglich, aber die Invarianz von solchen Eigenschaften bezuglich der Ringautomorphismen gewahrleistet dies. Daher wird die Struktur der Automorphismen-klassengruppe fur gewisse Kategorien von assoziativen R-Algebren bestimmt, wobei R ein kommutativer Integritatsbereich mit Einselement ist. Die Idee fur die Strukturtheorie in diesen Kategorien $\mathbf{A}$ besteht im folgenden: Sei $\langle x\rangle$ die freie Algebra mit einem erzeugenden Element in $\mathbf{A}$. Fur jedes Objekt $A\in\mathbf{A}$ betrachtet man die Menge der Homomorphismen $[\langle x\rangle, A]$. Mit Hilfe einer Koalgebrastruktur von $\langle x\rangle$ in $\mathbf{A}$ erhalt man eine Algebrastruktur fur die Mengen $[\langle x\rangle, A]$. Dadurch bekommt man einen Funktor $\mathbf{A}\to\mathbf{A}$, der aquivalent ist mit dem identischen Funktor. Eine Reihe von interessanten Beispielen, Gegenbeispielen und Problemen beendet die Arbeit. *W. Vogel*

45 #3481 16A44

Clark, W. Edwin

A coefficient ring for finite non-commutative rings.

Proc. Amer. Math. Soc. **33** (1972), 25–28.

Following G. J. Janusz [Trans. Amer. Math. Soc. 122 (1966), 461–479; MR 35#1585] we let $\mathrm{GR}(p^r, r) = Z_{p^n}[x]/(f(x))$ where $f(x)$ is monic and irreducible modulo p, and call such a ring a Galois ring of characteristic p^n and rank r, which is determined up to isomorphism by p, n and r. The principal theorem of the paper under review is the following: A finite p-ring (a finite ring whose additive group is a p-group) contains a unique (up to isomorphism) subring such that $S/pS\cong R/\mathrm{rad}\,R$. Moreover, S is a direct sum of full matrix rings over Galois rings. This is a generalization of the previous results for finite commutative p-rings and for finite p-rings of characteristic p. The proof of the theorem is reduced to the case of a finite local p-ring. *H. Tominaga*

44 #6867 20.92

Brooks, Burrow P., Jr.; Clark, W. Edwin

On the categoricity of semigroup-theoretical properties.

Semigroup Forum **3** 1971/1972 *no.* 3 259–266

Die Autoren untersuchen die Frage, wann isomorphie-invariante Eigenschaften von Halbgruppen in kategorischer Sprache beschrieben werden konnen. Dazu wird (wie z.B. auch im Falle von Gruppen) die freie monogene Halbgruppe $\langle x\rangle$ kategoriell charakterisiert, und zwar als ein spezielles projektives Objekt. Durch Einfuhrung einer sog. Co-Halbgruppen-Struktur kann nun die Menge der Morphismen von $\langle x\rangle$ in eine Halbgruppe A selbst zu einer Halbgruppe gemacht werden. Diese Halbgruppe erweist sich stets als isomorph oder stets als anti-isomorph zu A. Hieraus folgt, dass die obige Beschreibung immer moglich ist, wobei aber einseitige Eigenschaften (z.B. die Existenz eines einseitigen Einselementes) nicht eindeutig festgelegt werden konnen. In diesem Zusammenhang wird noch gezeigt, dass die Automorphismenklassengruppe der Halbgruppenkategorie die Ordnung 2 hat. *Heinrich Seidel*

39 #260 16.50

Clark, W. Edwin

Murase's quasi-matrix rings and generalizations.

Sci. Papers College Gen. Ed. Univ. Tokyo **18** 1968 99–109

For a positive integer n, let $QM(n)$ be the semigroup with elements e_{ij}, where $1\le i\le n$, $i\le j$ are integers, together with the zero element 0. Multiplication is defined by $e_{hi}e_{jk}=e_{h,i-j+k}$ if $i\equiv j \pmod n$ and $e_{hi}e_{jk}=0$ otherwise. The contracted semigroup ring $K[QM(n)]$ over a division ring K is called an infinite quasi-matrix ring of degree n over K. It is shown that every quasi-matrix ring in the sense of I. Murase [same Papers 13 (1963), 131–158; MR 28#5086] is a homomorphic image of an infinite quasi-matrix ring and that every $K[QM(n)]$ is isomorphic to the ring of all $n\times n$ matrices (a_{ij}) over the ring of polynomials in x over K such that x divides a_{ij} whenever $i>j$. The following principal results are also proved. (1) $K[QM(n)]$ is prime, left and right Noetherian, and semisimple. (2) Every proper homomorphic image of $K[QM(n)]$ is Artinian and generalized uniserial. (3) Every ideal of $K[QM(n)]$ is principal. *J. Luh*

37 #1405 16.50

Clark, W. Edwin

A note on semiprimary PP-rings.

Osaka J. Math. **4** 1967 177–178

The author shows that, in a semiprimary ring, principal left ideals are projective if and only if left annihilators are direct summands and that these properties also hold with right replacing left. *J. P. Jans*

36 #6452 16.50 18.00

Clark, W. Edwin

Algebras of global dimension one with a finite ideal lattice.

Pacific J. Math. **23** 1967 463–471

Author's introduction: "Let A denote a finite-dimensional (associative) algebra over an algebraically closed field K. In this paper a specific representation is given for those algebras A which have global dimension one (or less) and have only a finite number of (two-sided) ideals. It is shown that every such algebra is isomorphic to a (contracted) semigroup algebra $K[S]$ over a subsemigroup S of the semigroup of all $n\times n$ matrix units $\{e_{ij}\}\cup\{0\}$ which (i) contains $e_{11},\cdots,e_{nn}$ and (ii) contains e_{ij} or e_{ji} whenever there are h and k such that e_{hi},e_{ik} and e_{hj},e_{jk} are in S. Conversely, if S satisfies (i) and (ii) then $K[S]$ has global dimension one or less and has a finite ideal lattice." *D. S. Rim*

36 #6447 16.32

Clark, W. Edwin

Generalized radical rings.

Canad. J. Math. **20** 1968 88–94

Let R be a ring and denote by $\circ$ the composition $a\circ b=a+b-ab$ for $a,b\in R$. It is known that $(R,\circ)$ is a semigroup and that R is a radical ring in the sense of Jacobson if and only if $(R,\circ)$ is a group. The author calls R a generalized radical ring if $(R,\circ)$ is the union of groups. A ring is said to be strongly regular if $a\in a^2R$ for all $a\in R$. An idempotent of a ring is called principal if it is an identity for the ring modulo its radical.

The author proves that every strongly regular ring is a generalized radical ring. It is also shown that a ring R possessing a principal idempotent is a generalized radical ring if and only it is a splitting extension of its radical by a strongly regular subring eRe for some idempotent $e\in R$. *K. G. Wolfson*

36 #5159 16.10

Clark, W. Edwin; Lewin, Jacques

On minimal ideals in the circle composition semigroup of a ring.

Publ. Math. Debrecen **14** 1967 99–104

The elements of a ring R form a semigroup $(R, \circ)$ under the circle composition defined by $a \circ b = a + b - ab$. The identity element of $(R, \circ)$ is the zero element 0 of R. There exists the minimum K of the ideals of $(R, \circ)$, and K is a completely simple ideal in $(R, \circ)$ if and only if R contains a principal idempotent e, i.e., e is modulo J the identity of R/J, where J is the Jacobson radical of R. It is well known that J is the maximum of all such ideals in the ring R in which every element a possesses an inverse in the semigroup $(R, \circ)$. If R contains a principal idempotent e, then $K = R \circ e \circ R$ is valid. The ideal of the ring R which is generated by $K - e$ is denoted by $I(R)$, where K is defined above; $I(R)$ is independent of the choice of e. This ideal is contained in J and is equal to the subring of R which is generated by the component $P_e = (1-e)R + R(1-e)$ in the direct sum (qua abelian groups) $R = eRe + P_e$. A linear variety of an additively written abelian group A consists of $a + M$, where $a \in A$ and M is a subgroup of A. The above subsemigroup $K = R \circ e \circ R$ of $(R, \circ)$ is a linear variety in the additive group $(R, +)$ of R if and only if $R = eRe + I(R)$ (direct qua abelian groups) is valid. In that case $K = e + P_e$ and $I(R) = P_e$. *E.-A. Behrens*

35 #5475 16.55 20.00

Clark, W. Edwin

Twisted matrix units semigroup algebras.

Duke Math. J. **34** 1967 417–423

The author defines a twisted semigroup algebra as follows. Let S be a semigroup with zero. Let F be a field and let $\varphi\colon S \times S \to F$ satisfy (i) $\varphi(s,t) = 0$ if and only if $st = 0$ and (ii) $\varphi(r, st)\varphi(s,t) = \varphi(rs,t)\varphi(r,s)$ whenever $rst \neq 0$. Let $F_\varphi[S]$ denote the vector space of all formal finite linear combinations $\sum \alpha_i s_i$ with $\alpha_i \in F$ and $s_i \in S$, $s_i \neq 0$. Define a product on $F_\varphi[S]$ by setting $s \cdot t = \varphi(s,t)st$, for s and t non-zero elements of S, and extending linearly to all of $F_\varphi[S]$. (ii) above insures the associativity of $F_\varphi[S]$. $F_\varphi[S]$ is termed a twisted semigroup algebra of S over F. If $\varphi(s,t) = 1$ [0] when $st \neq 0$ [$st = 0$], then $F_\varphi[S]$ is just the ordinary semigroup algebra of S over F.

Let $MU(n)$ denote the full semigroup of matrix units $\{e_{ij}\colon 1 \le i, j \le n\} \cup \{0\}$, where $e_{hi}e_{jk} = \delta_{ij}e_{hk}$. By a matrix units semigroup is meant a subsemigroup of $MU(n)$ which contains $e_{11}, \cdots, e_{nn}$.

The author proves the following structure theorem on finite-dimensional linear associative algebras, after establishing a number of interesting results on the characterization of a quasi-Baer, Artinian ring (an Artinian ring A with unity, such that the left annihilator of every ideal of A is generated, as a left ideal, by an idempotent). Theorem: Let A be a finite-dimensional algebra over an algebraically closed field F. Then, $A \cong F_\varphi[S]$ for some matrix units semigroup S if and only if A is quasi-Baer and has a finite ideal lattice. *L. M. Chawla*

34 #2621 16.48 16.46

Clark, W. Edwin

Baer rings which arise from certain transitive graphs.

Duke Math. J. **33** 1966 647–656

A ring A is called a Baer ring if the left [right] annihilator of each subset of A is a principal left [right] ideal generated by an idempotent. Let D be a division ring, and $M_n = M_n(D)$ be the ring of all $n \times n$ matrices with entries from D. Let $\{e_{ij}\}$ denote the usual n^2 matrix units of M_n. The set of all matrix units together with the zero matrix constitute the semigroup $MU(n)$. If S is any subsemigroup of $MU(n)$, the set $A(S, D)$ of all linear combinations over D of elements of S forms a ring. Now let S be any subsemigroup of $MU(n)$ containing e_{ii} for $i = 1, 2, 3, \cdots, n$. The main theorem of the paper then asserts that $A(S, D)$ is a Baer ring if and only if S also satisfies the following condition: $(*)$ if h, i, j, k are distinct integers such that e_{ki}, e_{ih}, e_{kj}, and $e_{jh} \in S$, then e_{ij} or $e_{ji} \in S$. This generalizes the known result that the complete ring of triangular matrices over a division ring is a Baer ring.

The title of the paper derives from the fact that there is a one-to-one correspondence between subsemigroups S of $MU(n)$ containing $e_{11}, e_{22}, \cdots, e_{nn}$ that satisfy $(*)$ and certain transitive directed graphs on n vertices. Actually no graph theory is involved in the arguments. *K. G. Wolfson*

32 #7659 20.92 16.44

Clark, W. Edwin

Weakly semi-simple finite-dimensional algebras.

Canad. J. Math. **18** 1966 433–442

Let B be a finite-dimensional algebra over a field F. If $\varphi: B \to F$ is an epimorphism (i.e., (B, φ) is a supplemented F-algebra in the sense of Cartan and Eilenberg [Homological algebra, Princeton Univ. Press, Princeton, N.J., 1956; MR 17, 1040]), then the multiplicative semigroup $S = \varphi^{-1}(1)$ is called a translate of the algebra $A = \operatorname{Ker}\varphi$. The author has shown [#7658 above] that S has a kernel K (= unique minimal ideal of S) which is a union of subgroups. Let $R(S)$ be the radical of the semigroup (i.e., union of all ideals I with $I^n \subseteq K$) and let $R(A)$ be the radical of the algebra A. It is shown here that for each $e = e^2 \in K$, $R(S) = R(A) + e$, whence A is nilpotent if and only if $R(S) = S$. The algebras of the title deal with the other extreme: a finite-dimensional algebra A is weakly semi-simple if it has a translate S with $R(S) = K$. For such an A, if f is a principal idempotent, then (i) fAf is semi-simple, and (ii) $A = fAf \oplus R(A)$ as a vector space. Conversely, a principal idempotent satisfying (i) or (ii) implies weak semi-simplicity. Algebras of class Q [Thrall, Canad. J. Math. 7 (1955), 382–390; MR 16, 992] are precisely the weakly semi-simple algebras A with $A^2 = A$ and $R(A)^3 = 0$. *F. W. Anderson*

32 #7658 20.92

Clark, W. Edwin

Affine semigroups over an arbitrary field.

Proc. Glasgow Math. Assoc. **7** 1965 80–92 (1965)

An affine semigroup S is a linear variety (i.e., a translate of a subspace of a vector space) endowed with an associative multiplication for which the mappings $x \to xa$ and $x \to ax$ $(x \in S)$ are affine mappings for all $a \in S$. The affine semigroups of Haskell Cohen and H. S. Collins [Trans. Amer. Math. Soc. 93 (1959), 97–113; MR 21#6400] are shown to be subsemigroups of such affine semigroups. Affine semigroups are characterized as follows: They are semigroups S isomorphic to semigroups $\varphi^{-1}(1)$, where φ is a Φ-epimorphism of an algebra A over Φ onto the field Φ and $\varphi^{-1}(1)$ is regarded as a subsemigroup of the multiplicative semigroup of A. If S is finite-dimensional, then it can be represented faithfully as a (multiplicative) semigroup of matrices over Φ (the vector space involved being that spanned by the matrices), and it can be decomposed, by an appropriate partition of the matrices, into a group, two vector spaces, and an algebra. The kernel K, which then always exists, of S may be characterized in terms of such a decomposition. Let $M(K)$ denote the linear variety generated by K. Then $M(K)^n = K$, if n is greater than the dimension of $M(K)$. If the characteristic of Φ is different from 2, if every element of K is idempotent, and if K is not a variety, then $M(K)$ contains a subsemigroup isomorphic to the semigroup [cf. Cohen and Collins, loc. cit.] $\Phi \oplus \Phi \oplus \Phi$, with multiplication $(x, y, z)(a, b, c) = (x, b, ab)$, where xb denotes multiplication in Φ. *G. B. Preston*

31 #1311 20.92

Clark, W. Edwin

Remarks on the kernel of a matrix semigroup. (English. Russian summary)

Czechoslovak Math. J. **15 (90)** 1965 305–310

If S is a matrix semigroup (matrices of finite order over a field), denote by $m(S)$ the set of all elements of S with minimal rank. If S contains a kernel K, then $K \subseteqq m(S)$.

An example is given which shows that $m(S)$ need not be equal to K. The same example gives a negative answer to a question which has been open for some time: If K is the kernel of a semigroup S, is a left ideal of K necessarily a left ideal of S?

Call a semigroup S pseudo-invertible if for any $a \in S$ there is a power a^n which is contained in a subgroup of S. It is proved that for a pseudo-invertible semigroup of matrices, $m(S) = K$ and K is completely simple. Conversely, for any matrix semigroup S for which K is completely simple, $K = m(S)$. *St. Schwarz*

1 369 267 05C35

Clark, W. Edwin (1-SFL); **Shekhtman, Boris** (1-SFL)

On the domination matrices of the $\mathcal{C}$-analogues of Kneser graphs. (English. English summary)

Proceedings of the Twenty-sixth Southeastern International Conference on Combinatorics, Graph Theory and Computing (Boca Raton, FL, 1995).

Congr. Numer. **107** (1995), 193–197.