Quandle Cocycle Invariants for Knot Table Used for Tangle Embeddings

Supplement to: Tangle Embeddings and Quandle Cocycle Invariants

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This file contains tables of quandle cocycle invariants used in the above mentioned paper *Tangle Embeddings and Quandle Cocycle Invariants*.

The tables are part of calculations performed by C. Smudde, and a more extensive database is available at http://shell.cas.usf.edu/quandle/.

The cocycles used for the dihedral quandles R_3, R_5 and R_7 are those described in Example 5.4 of the above mentioned paper:

$$\psi(x, y, z) = (1/p)(x - y)[\ (2z^p - y^p) - (2z - y)^p\ ] \pmod{p}.$$

For the other Alexander quandles $\mathbb{Z}_p[t]/(t^2 - t + 1)$, the cocycles $f(x, y, z) = (x - y)(y - z)^p$ were used as indicated in the tables. These cocycles take values in the same ring $\mathbb{Z}_p[t]/(t^2 - t + 1)$.

	R_3	R_5	R_7
3_1	$9 + 18u$	25	49
4_1	9	$25 + 50\,[u^3 + u^2]$	49
5_1	9	$25 + 50\,[u + u^4]$	49
5_2	9	25	$49 + 98\,[u^3 + u^5 + u^6]$
6_1	27	25	49
6_2	9	25	49
6_3	9	25	49
7_1	9	25	$49 + 98\,[u + u^4 + u^2]$
7_2	9	25	49
7_3	9	25	49
7_4	$9 + 18u$	$25 + 50\,[u^3 + u^2]$	49
7_5	9	25	49
7_6	9	25	49
7_7	$9 + 18u$	25	$49 + 98\,[u^4 + u^2 + u]$
8_1	9	25	49
8_2	9	25	49
8_3	9	25	49
8_4	9	25	49
8_5	$9 + 18u^2$	25	$49 + 98\,[u^4 + u^2 + u]$
8_6	9	25	49
8_7	9	25	49
8_8	9	125	49
8_9	9	125	49
8_{10}	27	25	49

Table 1: Quandle 3-cocycle invariants for knots up to 8_{10} for dihedral quandles

	R_3	R_5	R_7
8_{11}	27	25	49
8_{12}	9	25	49
8_{13}	9	25	49
8_{14}	9	25	49
8_{15}	$9 + 18u^2$	25	49
8_{16}	9	$25 + 50 \left[u^2 + u^3\right]$	$49 + 98 \left[u^6 + u^3 + u^5\right]$
8_{17}	9	25	49
8_{18}	$9 + 36 \left[u + u^2\right]$	$25 + 50 \left[u + u^4\right]$	49
8_{19}	$9 + 18u^2$	25	49
8_{20}	27	25	49
8_{21}	$9 + 18u^2$	$25 + 50 \left[u^4 + u\right]$	49
9_1	27	25	49
9_2	$9 + 18u^2$	$25 + 50 \left[u + u^4\right]$	49
9_3	9	25	49
9_4	$9 + 18u^2$	25	$49 + 98 \left[u + u^4 + u^2\right]$
9_5	9	25	49
9_6	27	25	49
9_7	9	25	49
9_8	9	25	49
9_9	9	25	49
9_{10}	$9 + 18u$	25	49
9_{11}	$9 + 18u^2$	25	49
9_{12}	9	$25 + 50 \left[u + u^4\right]$	$49 + 98 \left[u^2 + u + u^4\right]$
9_{13}	9	25	49
9_{14}	9	25	49
9_{15}	$9 + 18u^2$	25	49
9_{16}	$9 + 18u^2$	25	49
9_{17}	$9 + 18u$	25	49
9_{18}	9	25	49
9_{19}	9	25	49
9_{20}	9	25	49

Table 2: Quandle 3-cocycle invariants for knots for $8_{11} - 9_{20}$ for dihedral quandles

	R_3	R_5	R_7
9_{21}	9	25	49
9_{22}	9	25	49
9_{23}	27	$25 + 50 \ [u + u^4]$	49
9_{24}	27	$25 + 50 \ [u^2 + u^3]$	49
9_{25}	9	25	49
9_{26}	9	25	49
9_{27}	9	25	343
9_{28}	$9 + 18u^2$	25	49
9_{29}	$9 + 18u$	25	49
9_{30}	9	25	49
9_{31}	9	$25 + 50 \ [u + u^4]$	49
9_{32}	9	25	49
9_{33}	9	25	49
9_{34}	$9 + 18u^2$	25	49
9_{35}	$27 + 54u$	25	49
9_{36}	9	25	49
9_{37}	$45 + 18 \ [u^2 + u]$	$25 + 50 \ [u^3 + u^2]$	49
9_{38}	$9 + 18u$	25	49
9_{39}	9	$25 + 50 \ [u^3 + u^2]$	49
9_{40}	$9 + 18u^2$	$25 + 150 \ [u^2 + u^3 + u + u^4]$	49
9_{41}	9	25	$637 + 294 \ [u^4 + u^2 +u + u^5 + u^3 + u^6]$
9_{42}	9	25	$49 + 98 \ [u^5 + u^6 + u^3]$
9_{43}	9	25	49
9_{44}	9	25	49
9_{45}	9	25	49
9_{46}	$45 + 18 \ [u + u^2]$	25	49
9_{47}	$27 + 54u^2$	25	49
9_{48}	$27 + 54u$	25	49
9_{49}	9	$25 + 150 \ [u + u^4 + u^3 + u^2]$	49

Table 3: Quandle 3-cocycle invariants for knots for the remaining 9 crossing knots for dihedral quandles

	$\mathbb{Z}_2[t]/(t^2-t+1)$	$\mathbb{Z}_3[t]/(t^2-t+1)$	$\mathbb{Z}_5[t]/(t^2-t+1)$	$\mathbb{Z}_7[t]/(t^2-t+1)$
$f =$	$(x-y)(y-z)^2$	$(x-y)(y-z)^3$	$(x-y)(y-z)^5$	$(x-y)(y-z)^7$
3_1	$16+48u^t$	$243+486u^{(2t+2)}$	$625+3750\,[u^{(t+3)}+$ $u^{(4t+2)}+u^{(3t+4)}+u^{(2t+1)}]$	117649
4_1	$16+48u^t$	81	625	2401
5_1	16	81	625	2401
5_2	16	81	625	2401
6_1	16	81	$243+486u^{(t+1)}$	2401
6_2	16	81	625	2401
6_3	16	81	625	2401
7_1	16	81	625	2401
7_2	$16+48u^t$	81	625	2401
7_3	$16+48u^t$	81	625	2401
7_4	16	$243+486u^{(t+1)}$	625	2401
7_5	16	81	625	2401
7_6	16	81	625	2401
7_7	16	729	625	2401
8_1	$16+48u^t$	81	625	2401
8_2	16	81	625	2401
8_3	16	81	$625+3750\,[u^{(t+3)}+$ $u^{(4t+2)}+u^{(3t+4)}+u^{(2t+1)}]$	2401
8_4	$16+48u^t$	81	625	2401
8_5	64	$243+486u^{(t+1)}$	$625+3750\,[u^{(t+3)}+$ $u^{(4t+2)}+u^{(3t+4)}+u^{(2t+1)}]$	117649
8_6	16	81	625	2401
8_7	16	81	625	2401
8_8	16	81	625	2401
8_9	16	81	625	2401
8_{10}	64	729	15625	117649

Table 4: Quandle 3-cocycle invariants for knots up to 8_{10} for some Alexander quandles

	$\mathbb{Z}_2[t]/(t^2-t+1)$	$\mathbb{Z}_3[t]/(t^2-t+1)$	$\mathbb{Z}_5[t]/(t^2-t+1)$	$\mathbb{Z}_7[t]/(t^2-t+1)$
$f=$	$(x-y)(y-z)^2$	$(x-y)(y-z)^3$	$(x-y)(y-z)^5$	$(x-y)(y-z)^7$
8_{11}	$16+48u^t$	729	$625+3750\,[u^{(3t+4)}+$ $+u^{(2t+1)}+u^{(4t+2)}+u^{(t+3)}]$	117649
8_{12}	16	81	15625	2401
8_{13}	$16+48u^t$	81	625	2401
8_{14}	16	81	625	2401
8_{15}	64	$243+486u^{(t+1)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
8_{16}	16	81	625	2401
8_{17}	16	81	625	2401
8_{18}	$64+192u^t$	$729+2916u^{(2t+2)}$ $+2916u^{(t+1)}$	$90625+75000\,[u^{(t+3)}+$ $u^{(4t+2)}+u^{(3t+4)}+u^{(2t+1)}]$	2401
8_{19}	16	$243+486u^{(t+1)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)})+u^{(t+3)}+u^{(4t+2)}]$	117649
8_{20}	64	729	15625	117649
8_{21}	64	$243+486u^{(t+1)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_1	$16+48u^t$	729	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_2	16	$243+486u^{(2t+2)}$	625	2401
9_3	16	81	625	2401
9_4	16	$243+486u^{(2t+2)}$	625	2401
9_5	16	81	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	2401
9_6	$16+48u^t$	729	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_7	16	729	625	2401
9_8	16	81	625	2401
9_9	16	81	625	2401
9_{10}	16	729	625	2401

Table 5: Quandle 3-cocycle invariants for knots for $8_{11}-9_{10}$ for some Alexander quandles

	$\mathbb{Z}_2[t]/(t^2-t+1)$	$\mathbb{Z}_3[t]/(t^2-t+1)$	$\mathbb{Z}_5[t]/(t^2-t+1)$	$\mathbb{Z}_7[t]/(t^2-t+1)$
$f=$	$(x-y)(y-z)^2$	$(x-y)(y-z)^3$	$(x-y)(y-z)^5$	$(x-y)(y-z)^7$
9_{11}	16	729	625	2401
9_{12}	$16+48u^t$	81	625	2401
9_{13}	$16+48u^t$	81	625	2401
9_{14}	$16+48u^t$	81	625	2401
9_{15}	16	729	625	2401
9_{16}	64	$243+486u^{(t+1)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_{17}	16	$243+486u^{(t+1)}$	625	2401
9_{18}	16	81	625	2401
9_{19}	16	81	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	2401
9_{20}	16	81	625	2401
9_{21}	$16+48u^t$	81	625	2401
9_{22}	64	81	625	2401
9_{23}	$16+48u^t$	729	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_{24}	64	729	15625	117649
9_{25}	64	81	625	2401
9_{26}	16	81	625	2401
9_{27}	16	81	625	2401
9_{28}	64	$243+486u^{(t+1)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_{29}	16	$243+486u^{(2t+2)}$	$625+3750\,[u^{(2t+1)}+$ $u^{(3t+4)}+u^{(t+3)}+u^{(4t+2)}]$	117649
9_{30}	64	81	625	2401

Table 6: Quandle 3-cocycle invariants for knots for $9_{11}-9_{30}$ for some Alexander quandles

$f =$	$\mathbb{Z}_2[t]/(t^2 - t + 1)$ $(x - y)(y - z)^2$	$\mathbb{Z}_3[t]/(t^2 - t + 1)$ $(x - y)(y - z)^3$	$\mathbb{Z}_5[t]/(t^2 - t + 1)$ $(x - y)(y - z)^5$	$\mathbb{Z}_7[t]/(t^2 - t + 1)$ $(x - y)(y - z)^7$
9_{31}	16	81	625	2401
9_{32}	16	81	625	2401
9_{33}	16	81	625	2401
9_{34}	16	$243 + 486u^{(2t+2)}$	625	2401
9_{35}	$16 + 48u^t$	729	625	2401
9_{36}	64	81	625	2401
9_{37}	$16 + 48u^t$	729	625	2401
9_{38}	16	$243 + 486u^{(2t+2)}$	$625 + 3750\,[u^{(2t+1)} + u^{(3t+4)} + u^{(t+3)} + u^{(4t+2)}]$	117649
9_{39}	64	81	625	2401
9_{40}	$64 + 192u^t$	$243 + 486u^{(t+1)}$	$625 + 3750\,[u^{(2t+1)} + u^{(3t+4)} + u^{(t+3)} + u^{(4t+2)}]$	117649
9_{41}	64	81	625	2401
9_{42}	64	81	625	2401
9_{43}	64	81	625	2401
9_{44}	64	81	625	2401
9_{45}	64	81	625	2401
9_{46}	16	729	625	2401
9_{47}	16	729	625	2401
9_{48}	16	729	625	2401
9_{49}	64	81	625	2401

Table 7: Quandle 3-cocycle invariants for knots for the remaining 9 crossing knots for some Alexander quandles